Electrically charged black hole with scalar hair

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An electrically charged black hole solution with scalar hair in four dimensions is presented. The self-interacting scalar field is real and it is minimally coupled to gravity and electromagnetism. The event horizon is a surface of negative constant curvature and the asymptotic region is locally an AdS spacetime. The asymptotic falloff of the fields is slower than the standard one. The scalar field is regular everywhere except at the origin and is supported by the presence of electric charge which is bounded from above by the AdS radius. In turn, the presence of the real scalar field smooths the electromagnetic potential everywhere. Regardless the value of the electric charge, the black hole is massless and has a fixed temperature. The entropy follows the usual area law. It is shown that there is a nonvanishing probability for the decay of the hairy black hole into a charged black hole without scalar field. Furthermore, it is found that an extremal black hole without scalar field is likely to undergo a spontaneous dressing up with a nontrivial scalar field, provided the electric charge is below a critical value.

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I. INTRODUCTION

It has been recently shown that general relativity with a minimally coupled self-interacting real scalar field, in four dimensions, admits an exact hairy black hole solution [1]. This result is somehow unexpected since it circumvents the so called no-hair conjecture, which originally stated that a black hole should be characterized only in terms of its mass, angular momentum, and electric charge [2–4] (for recent discussions see e.g., [5]). The thermodynamical analysis of the hairy black hole reveals the existence of a second order phase transition, so that below a critical temperature, a black hole in vacuum undergoes a spontaneous dressing up with a nontrivial scalar field. According to the analysis of Ref. [6] the solution is expected to be stable against linear perturbations, and progress towards the proof of nonlinear stability for solutions with the same asymptotic behavior has been performed in [7]. It can be seen also that below the critical temperature, these hairy black holes curiously admit only a finite number of quasinormal modes [8]. Furthermore, this black hole can be uplifted as a solution of 11-dimensional supergravity [9]. The black hole solution in [1] can be seen as the neutral case of the electrically charged hairy black hole found in [10].

Numerical hairy black hole solutions of this sort also have been found in Refs. [11–14], and another exact solution was found in [15]. In the case of conformally coupled scalar fields, exact black hole solutions were known to exist since the 70s [16]; however, the scalar field in this case diverges at the horizon. This last obstacle can be avoided considering a cosmological constant and a quartic self-interaction term for the scalar field [1,10,17]. The thermodynamics and the stability for the black hole with positive cosmological constant have been discussed in Refs. [18,19], respectively. Further aspects of this class of solutions have been studied in [20], and numerical black hole solutions for nonminimally coupled scalar fields have been found in Refs. [6,12,21]. In three dimensions, black holes dressed with conformally and minimally coupled scalar fields were found in [22,23], and some of their properties were analyzed in Refs. [24,25].

In this paper, an electrically charged black hole solution of gravity minimally coupled to a real self-interacting scalar field and electromagnetism in four dimensions is presented. The self-interacting potential induces a negative cosmological constant which allows the event horizon to be a surface of negative constant curvature which surrounds the singularity at the origin, and the asymptotic region is locally an AdS spacetime. The self-interacting potential considered here differs from the one considered in [1], but it has the same mass term.[1]

The effect of having asymptotically AdS black holes solutions whose horizons have a nontrivial topology is known to occur in vacuum [26,27], as well as in the presence of the electromagnetic field [28,29]. For the black hole solution presented here, the asymptotic falloff of the fields at the asymptotic region is slower than the standard one, as in [30], for a localized distribution of matter. The scalar field is regular everywhere except at the origin, and is supported by the presence of electric charge which is bounded from above by the AdS radius. In turn, the presence of the real scalar field smooths the electromagnetic potential everywhere. The thermodynamics is discussed in Sec. III, where it is found that regardless the value of the

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[1]The comparison can be seen precisely in the conformal frame. In this case the action is mapped to gravity with a negative cosmological constant and a conformally coupled scalar field without self-interaction, while in Ref. [1], a quartic self-interaction term with a fixed coupling constant was considered.

electric charge, the black hole is massless and it has a fixed temperature. The entropy follows the usual area law, as expected. In Sec. IV, it is shown that there is a nonvanishing probability for the decay of the hairy black hole into a black hole without scalar field. Possible decays including the extremal bare black hole are also analyzed, and it is found that an extremal black hole without scalar field is likely to undergo a spontaneous dressing up with a nontrivial scalar field, provided the electric charge is below certain critical value. Section V is devoted to some concluding remarks.

II. BLACK HOLE SOLUTION

Let us consider gravity minimally coupled to a real self-interacting scalar field and electromagnetism in four dimensions. The action is given by

$$I[g_{\mu\nu}, \phi, A_\mu] = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}\right], \tag{1}$$

where G is the Newton constant, and the self-interaction potential is chosen as

$$V(\phi) = -\frac{3}{8\pi G l^2}\cosh^4\left(\sqrt{\frac{4\pi G}{3}}\phi\right). \tag{2}$$

This potential has a global maximum at $\phi = 0$, giving rise to a negative cosmological constant, which can be written in terms of the AdS radius as $\Lambda = -3l^{-2}$. Its mass term, $m^2 = V''|_{\phi=0} = -2l^{-2}$, satisfies the Breitenlohner-Freedman bound, which ensures the perturbative stability of AdS spacetime [31].

The field equations are given by

$$G_{\mu\nu} = 8\pi G(T^{\phi}_{\mu\nu} + T^{\rm em}_{\mu\nu}), \tag{3}$$

$$\Box\phi = \frac{dV}{d\phi}, \tag{4}$$

$$\partial_\nu(\sqrt{-g}F^{\mu\nu}) = 0, \tag{5}$$

where the scalar and electromagnetic pieces of stress-energy tensor are

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - g_{\mu\nu}V(\phi), \tag{6}$$

$$T^{\mu\nu}_{\rm em} = -\frac{1}{4\pi}\left(F^{\mu}{}_{\alpha}F^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right), \tag{7}$$

respectively.

The field equations are solved by the following static metric:

$$ds^2 = -\left(1 + \frac{Gq^2}{r^2}\right)^{-1}\left(\frac{r^2}{l^2} - 1 + \frac{Gq^2}{l^2}\right)dt^2 + \left(1 + \frac{Gq^2}{r^2}\right)^{-2}\left(\frac{r^2}{l^2} - 1 + \frac{Gq^2}{l^2}\right)^{-1}dr^2 + r^2 d\sigma^2, \tag{8}$$

provided the scalar field and the electromagnetic potential are given by

$$\phi = \sqrt{\frac{3}{4\pi G}}\,\mathrm{Arctanh}\sqrt{\frac{Gq^2}{r^2 + Gq^2}}, \tag{9}$$

$$A = -\frac{q}{\sqrt{r^2 + Gq^2}}dt, \tag{10}$$

respectively. In (8), $d\sigma^2$ is the line element of the base manifold Σ, which has negative constant curvature (rescaled to -1), so that it is locally isometric to the hyperbolic manifold H^2. Thus, a smooth base manifold Σ can be obtained through a quotient of the form $\Sigma = H^2/\Gamma$, where Γ is a freely acting discrete subgroup of $O(2, 1)$. The metric (8) describes an asymptotically locally AdS spacetime, and if Σ is assumed to be compact without boundary, it has a single timelike Killing vector given by ∂_t.

The integration constant q, corresponds to the electric charge which is given by

$$Q = \frac{\sigma}{4\pi}q, \tag{11}$$

where σ denotes the area of Σ, and as it is shown below, the mass of this solution vanishes for any value of q.

The curvature and the scalar field are singular at the origin $r = 0$, but the electromagnetic potential is regular everywhere.

The metric (8) describes a black hole solution with topology $\mathbb{R}^2 \times \Sigma$, with an event horizon located at

$$r_+ = \sqrt{l^2 - Gq^2}, \tag{12}$$

provided the electric charge is bounded from above by

$$q^2 < \frac{l^2}{G}. \tag{13}$$

For $q^2 = l^2/G$, the spacetime has a nut on the null curve $r = 0$, which coincides with the singularity.

This black hole has the same causal structure as the Schwarzschild-AdS black hole, where at each point of the Penrose diagram the sphere is replaced by Σ. The horizon radius satisfies the bound $r_+ \leq l$, which is saturated for $q = 0$. Note that the scalar field cannot be switched off keeping the electric charge fixed. This means that the scalar field can be switched off only if the electric charge vanishes, and then the metric reduces to

$$d\bar{s}^2 = -\left(\frac{r^2}{l^2} - 1\right)dt^2 + \left(\frac{r^2}{l^2} - 1\right)^{-1}dr^2 + r^2 d\sigma^2, \tag{14}$$

which corresponds to a negative constant curvature spacetime.[2]

The electromagnetic potential at the origin has a fixed value that only depends on the sign of the electric charge and the Newton constant

$$A_t|_{r=0} = -\frac{\mathrm{sgn}(q)}{\sqrt{G}},$$

and at the horizon is given by $A_t|_{r=r_+} = -q/l$. It is remarkable that the presence of the real scalar field produces a backreaction on the metric that regularizes the Maxwell field everywhere. The field strength two-form reads

$$F = q\frac{r}{(r^2 + Gq^2)^{3/2}} dr \wedge dt,$$

which at the origin grows linearly as $\mathrm{sgn}(q)q^{-2}G^{-3/2}r$, has an extremum at $r = |q|\sqrt{G/2}$ given by $2(3\sqrt{3}Gq)^{-1}$, and asymptotically decays as $q/r^2 + G\,\mathcal{O}(r^{-4})$. Note that in an orthonormal frame the electric field, however, has the usual form, $F^{01} = q/r^2$.

The scalar field at the horizon is given by

$$\phi|_{r=r_+} = \sqrt{\frac{3}{4\pi G}}\,\mathrm{Arctanh}\left(\sqrt{G}\frac{|q|}{l}\right),$$

and asymptotically behaves as

$$\phi = \sqrt{\frac{3}{4\pi}}\frac{|q|}{r} + \mathcal{O}(r^{-3}). \tag{15}$$

As discussed in [23], and further developed in [34–36], the presence of scalar fields with a slow falloff as in Eq. (15) has generically two effects: It gives rise to a strong backreaction that relaxes the standard asymptotic form of the geometry, and it generates additional contributions to the charges that depend explicitly on the scalar fields at infinity which are not already present in the gravitational part. These effects have also been discussed recently following the covariant phase space method [37]. The asymptotic form of the metric (8) reads

$$g_{tt} = -\left(\frac{r^2}{l^2} - 1\right) + \mathcal{O}(r^{-2}),$$

$$g_{rr} = \frac{l^2}{r^2} + \left(1 - 3G\frac{q^2}{l^2}\right)\frac{l^4}{r^4} + \mathcal{O}(r^{-6}),$$

which manifestly deviates from the standard behavior [30].

The mass of the black hole under consideration can be computed explicitly from a surface integral as in [36]. For a scalar field satisfying $m^2 = V''|_{\phi=0} = -2l^{-2}$, the mass is given by

$$M = Q_G(\partial_t) + Q_\phi(\partial_t), \tag{16}$$

where $Q_G(\partial_t)$ stands for the standard formula [30,38], and the contribution from the scalar field is

$$Q_\phi(\partial_t) = \frac{1}{6}\int d^2\sigma r^3\left[\left(\frac{r}{l}\partial_r\phi\right)^2 - m^2\phi^2 - \frac{1}{3}V'''|_{\phi=0}\phi^3\right]. \tag{17}$$

For the potential (2), the coefficient $V'''|_{\phi=0}$ of the cubic term in (17) vanishes, and as expected, evaluating (16) for the black hole, the divergences coming from Q_G and Q_ϕ are cancelled. Thus the mass, which is given by the remaining finite term, is found to vanish. This also can be seen from the asymptotic form of the fields, since the terms coming from the metric that contribute to mass, which are the ones of order r^{-1} in g_{tt}, and order r^{-5} in g_{rr}, are absent. Moreover, the contribution to the mass coming from the scalar field requires that both leading orders in the scalar field, i.e., the orders r^{-1} and r^{-2}, must be simultaneously present. Therefore, the contribution to the mass coming from the scalar field also vanishes since the term of order r^{-2} does not appear in the asymptotic form of ϕ given by Eq. (15). These results also could be discussed following different covariant approaches as in [39,40].

III. THERMODYNAMICS

The thermodynamics for the electrically charged black hole with scalar hair is discussed using the Euclidean approach. For this purpose it is useful to consider a minisuperspace of static Euclidean metrics given by

$$ds^2 = N(r)^2 f(r)^2 dt^2 + f(r)^{-2}dr^2 + r^2 d\sigma^2, \tag{18}$$

where the Euclidean time has period β, and the radius ranges as $r \geq r_+$. The scalar and electromagnetic fields are assumed to be of the form $\phi = \phi(r)$, and $A = A_t(r)dt$, respectively. The temperature T corresponds to the inverse of β, and it is fixed requiring that the allowed class of the geometries (18) should contain no conical singularities at the horizon. This condition implies that

$$\beta(N(r)(f^2(r))')|_{r=r_+} = 4\pi, \tag{19}$$

which for the black hole solution (8), yields the following temperature:

$$T = \beta^{-1} = \frac{1}{2\pi l}. \tag{20}$$

Note that the temperature is determined only by the AdS radius and it is independent of the horizon size.

The Euclidean path integral in the saddle point approximation around the Euclidean solution is identified with the partition function of a thermodynamical ensemble [41]. Here we consider the Euclidean continuation of the action

[2] Spacetimes of the form (14) admit Killing spinors provided Σ is a noncompact surface [32]. In this case, the metric describes the supersymmetric ground state of a warped black string. Its stability under gravitational perturbations has been explicitly proved in [33].

(1) in Hamiltonian form. Following the analysis performed in Ref. [1], and including electric charge, one obtains that the reduced Hamiltonian action is

$$I = -\frac{\beta\sigma}{4\pi}\int_{r_+}^{\infty}(N(r)\mathcal{H}(r) + A_t p')dr + B, \tag{21}$$

where B is a surface term, and σ is the area of the base manifold Σ. The reduced Hamiltonian is given by

$$\mathcal{H} = \frac{r^2}{2G}\left[\left(\frac{(f^2)'}{r} + \frac{1}{r^2}(1+f^2)\right) + 4\pi G(f^2(\phi')^2 + 2V(\phi)) + \frac{Gp^2}{r^4}\right],$$

and p is defined in terms of electric field as

$$p = \frac{r^2}{N}A_t'.$$

The Euclidean black hole solution is static and satisfies the constraints $\mathcal{H} = 0$, $p' = 0$. Therefore, the action (21) evaluated on the classical solution is just given by the boundary term B. The boundary term is fixed by requiring the action (21) to attain an extremum for the minisuperspace considered here [42].

In what follows, we work in the grand canonical ensemble, so that we consider variations of the action keeping fixed the temperature and the "voltage", i.e., β and $\Phi = A_t(\infty) - A_t(r_+)$ are constants.

The variation of the required boundary term is

$$\delta B \equiv \delta B_G + \delta B_\phi + \delta B_{\rm em}, \tag{22}$$

where

$$\delta B_G = \frac{\beta\sigma}{8\pi G}[Nr\delta f^2]_{r_+}^{\infty}, \tag{23}$$

and the contribution from the matter sector is given by

$$\delta B_\phi = \beta\sigma N r^2 f^2 \phi' \delta\phi|_{r_+}^{\infty}. \tag{24}$$

$$\delta B_{\rm em} = \frac{\beta\sigma}{4\pi}\Phi\delta p. \tag{25}$$

The variation of the fields at infinity for the black hole solution (8)–(10) reads

$$\delta f^2|_\infty = \frac{3G}{l^2}\delta q^2 + O(r^{-2}), \tag{26}$$

$$\delta\phi|_\infty = \sqrt{\frac{3}{\pi}}\frac{1}{4|q|r}\delta q^2 + O(r^{-3}), \tag{27}$$

$$\delta p|_\infty = \delta q \tag{28}$$

and thus, one obtains

$$\delta B_G|_\infty = \frac{3\beta\sigma}{8\pi l^2} r\delta q^2 + O(r^{-1}). \tag{29}$$

The variation of the purely gravitational contribution to the boundary term $\delta B_G|_\infty$ has a linearly divergent term and is devoid of a finite piece. As discussed above, this reflects the fact that the scalar field produces a slow decay for the metric as compared with that of pure gravity with a standard localized distribution of matter [30]. This divergence is cancelled by the contribution coming from the scalar field

$$\delta B_\phi|_\infty = -\frac{3\beta\sigma}{8\pi l^2} r\delta q^2 + O(r^{-1}). \tag{30}$$

Choosing $A|_\infty$ to vanish, the total boundary term at infinity then vanishes

$$B|_\infty = 0. \tag{31}$$

The variation of the boundary term at the horizon, is obtained using

$$\delta f^2|_{r_+} = -(f^2)'|_{r_+}\delta r_+,$$

and Eqs. (19) and (23)–(25). Since, $\delta B_\phi|_{r_+}$ vanishes, the variation of the total boundary term is

$$\delta B|_{r_+} = \frac{\sigma}{4\pi}\left(-\frac{1}{4G}N(r_+)\beta(f^2)'|_{r_+}\delta(r_+^2) + \beta\Phi\delta p\right) = -\frac{\sigma}{4G}\delta(r_+^2) - \frac{\beta\sigma}{4\pi}\Phi\delta q, \tag{32}$$

where the last term in (32) is the contribution from the electric field. Hence, the boundary term at the horizon can be integrated as

$$B|_{r_+} = -\frac{\sigma}{4G}r_+^2 - \frac{\beta\sigma}{4\pi}\Phi q. \tag{33}$$

The value of the Euclidean action on shell is then given by the boundary terms in Eqs. (31) and (33), which reads

$$I = \frac{\sigma}{4G}r_+^2 + \frac{\beta\sigma}{4\pi}\Phi q, \tag{34}$$

up to an arbitrary additive constant. The Euclidean action is related to the free energy (in units where $\hbar = k_B = 1$) as $I = -\beta F$, which in the grand canonical ensemble is given by

$$I = S - \beta M + \beta\Phi Q. \tag{35}$$

Here M, Q, and S stand for the mass, electric charge and entropy, respectively. Thus, once the free energy is identified with the Euclidean action, these quantities must satisfy the first law of thermodynamics. Expressions (34) and (35), allow one to obtain the mass, the electric charge, and the entropy from the standard thermodynamical relations

$$M = \left(\beta^{-1}\Phi\frac{\partial}{\partial\Phi} - \frac{\partial}{\partial\beta}\right)I = 0,$$

$$Q = \beta^{-1}\frac{\partial I}{\partial\Phi} = \frac{\sigma}{4\pi}q, \qquad S = \left(1 - \beta\frac{\partial}{\partial\beta}\right)I = \frac{\sigma}{4G}r_+^2.$$

Thus, as expected, the entropy follows the area law since the horizon area is given by σr_+^2, and the mass is shown to vanish from an independent method.

IV. THERMAL DECAY

For a fixed temperature and electromagnetic potential, the action principle (1) also admits an electrically charged solution for the same boundary conditions but without hair, i.e. with $\phi \equiv 0$. This solution [29] is described by

$$ds^2 = -\left[\frac{\rho^2}{l^2} - 1 - \frac{2G\mu_0}{\rho} + \frac{Gq_0^2}{\rho^2}\right]dt^2 + \left[\frac{\rho^2}{l^2} - 1 - \frac{2G\mu_0}{\rho} + \frac{Gq_0^2}{\rho^2}\right]^{-1} d\rho^2 + \rho^2 d\sigma^2, \quad (36)$$

with

$$A = -\frac{q_0}{\rho}dt, \quad (37)$$

for which the mass and the electric charge by $M_0 = \sigma\mu_0/4\pi$, and $Q_0 = \sigma q_0/4\pi$.

Since the hairy black hole (8) has a fixed temperature given by (20), the matching of this temperature with the one for the black hole in Eq. (36) reads[3]

$$\beta_0 = \frac{4\pi\rho_+}{3\frac{\rho_+^2}{l^2} - \frac{q_0^2 G}{\rho_+^2} - 1} = \beta = 2\pi l,$$

where β_0 stands for the Euclidean period of the black hole without scalar hair. This condition is fulfilled when the mass and the electric charge of the black hole without scalar hair relate with the horizon radius ρ_+ in the following way:

$$G\mu_0 l = \rho_+^2\left(\frac{2\rho_+}{l} - \frac{l}{\rho_+} - 1\right), \qquad Gq_0^2 = \frac{\rho_+^3}{l}\left(\frac{3\rho_+}{l} - \frac{l}{\rho_+} - 2\right). \quad (38)$$

Analogously, matching the voltages amounts to match the electromagnetic potentials (10) and (37) at the horizon, which leads to

$$\frac{q}{l} = \frac{q_0}{\rho_+}. \quad (39)$$

Note that since $q_0^2 \geq 0$, both black holes can have the same temperature[4] provided $\rho_+ \geq l$. This raises the question of whether one black hole can decay into the other. Since the partition function is given by $Z = \exp(I)$, this can be examined evaluating the difference between the corresponding Euclidean actions.

The Euclidean action evaluated on the hairy black hole (8)–(10) reads

$$I_\phi = \frac{\sigma l^2}{4G}\left[1 + G\frac{q^2}{l^2}\right], \quad (40)$$

which by virtue of the matching conditions (38) and (39), can be expressed as

$$I_\phi = \frac{\sigma}{2G}\rho_+^2\left[\frac{3}{2} - \frac{l}{\rho_+}\right].$$

For the black hole without scalar hair the Euclidean action is given by

$$I_0 = \frac{\sigma}{2G}\rho_+^2\left[\frac{\rho_+}{l} - \frac{1}{2}\right].$$

Therefore, the difference between both Euclidean actions is

$$\Delta I = I_0 - I_\phi = \frac{\sigma l}{2G}\rho_+\left[\frac{\rho_+}{l} - 1\right]^2, \quad (41)$$

which is always positive. This means that there is a nonvanishing probability for the decay of the hairy black hole into the black hole without scalar field, so that the black hole without scalar hair is thermodynamically favored.

For a fixed temperature and voltage, by virtue of (38), the difference between the black hole masses under the decay is always positive for the allowed range, $\rho_+ > l$. Analogously, the difference between the absolute values of the electric charges is

$$\Delta|q| = |q_0| - |q| = |q_0|\left(1 - \frac{l}{\rho_+}\right) > 0.$$

Similarly, since the entropy for the black hole without scalar field is $S_0 = \sigma\rho_+^2(4G)^{-1}$, the entropies for the allowed range are found to obey

$$\Delta S = S_0 - S_\phi = \frac{\sigma l^2}{2}\frac{\mu_0}{\rho_+} > 0.$$

In sum, as ΔI in Eq. (41) is positive, there is a nonvanishing probability for the decay of the black hole dressed with the scalar field into the bare black hole. Since the process takes place for black holes in a vacuum with $\rho_+ > l$, which has positive mass, in the decay process, the scalar black hole absorbs energy and electric charge from the thermal bath, increasing its horizon radius an consequently its entropy. This suggests that in this process the scalar field is at least partially absorbed by the black hole.

[3]Note that, since the base manifold is locally hyperbolic, at fixed temperature and voltage, there is only one black hole without scalar hair. For spherical symmetry there are two possible black holes without scalar hair satisfying these requirements.

[4]This bound is saturated for $\mu_0 = q_0 = 0$, but in this case, the matching of the voltages implies that $q = 0$ and $r_+ = l$. This means that when the bound is saturated, matter fields are switched off and both metrics coincide.

A. Transitions involving the extremal charged black hole without hair

The black hole without scalar hair described by Eqs. (36) and (37) admits an extremal case, for which the mass and the electric charge are fine tuned such that its temperature vanishes, and thus the Euclidean time period β_e is arbitrary. This opens two additional possible decay channels to be explored.

1. Stability of the nonextremal charged black hole without scalar hair

Let us begin analyzing the transition between the extreme and nonextreme charged black holes without scalar hair. Following the same procedure as in Sec. III, but taking into account that the Euclidean period is arbitrary, it is found that the entropy vanishes, as in Ref. [43] (see also [44,45]), and the Euclidean action is given by

$$I_e = \beta_e \frac{\sigma}{4\pi G l^2}\rho_e^3, \tag{42}$$

where ρ_e is the horizon radius of the extremal black hole.

For the nonextremal black hole without scalar field the Euclidean action can be expressed q_0 and ρ_+ as

$$I_0 = \frac{\sigma\rho_+^2}{4G}\left[\frac{1+\frac{\rho_+^2}{l^2}+\frac{q_0^2 G}{\rho_+^2}}{3\frac{\rho_+^2}{l^2}-\frac{q_0^2 G}{\rho_+^2}-1}\right].$$

For the decay process, the Euclidean time period of the extremal black hole is then fixed as $\beta_e = \beta_0$, and matching the voltages leads to

$$\frac{q_0}{\rho_+} = \frac{q_e}{\rho_e}.$$

This allows one to express the Euclidean action for the extremal black hole (42) in terms of the horizon radius and the electric charge of the nonextremal black hole as

$$I_e = \frac{\sigma l}{G}\rho_+\left[\frac{(\frac{1}{3}[1+\frac{Gq_0^2}{\rho_+^2}])^{3/2}}{3\frac{\rho_+^2}{l^2}-\frac{q_0^2 G}{\rho_+^2}-1}\right].$$

Thus, the difference between both Euclidean actions is

$$\Delta I = I_0 - I_e = \frac{\sigma}{4G}\frac{\rho_+^2+\frac{\rho_+^4}{l^2}+q_0^2 G - 4l\rho_+(\frac{1}{3}[1+\frac{Gq_0^2}{\rho_+^2}])^{3/2}}{3\frac{\rho_+^2}{l^2}-\frac{q_0^2 G}{\rho_+^2}-1},$$

which can be shown to be positive for the allowed range of the parameters. This ensures the stability of the nonextreme black hole, and one then concludes that there is a nonvanishing probability for the decay of the extremal into the nonextremal solution. This result is qualitatively similar with what was found in Ref. [46].

2. Spontaneous scalar field dressing up of the extremal black hole

Consider now the transition between the hairy and the extreme case. The Euclidean action for the hairy black hole I_ϕ is given by Eq. (40), and the one for the extremal solution without scalar field I_e is given by (42). In this case, according to Eq. (20), the Euclidean period of the extremal solution must be fixed as $\beta_e = \beta_\phi = 2\pi l$, and the matching voltages reads

$$\frac{q}{r_+} = \frac{q_e}{\rho_e}. \tag{43}$$

Since the Euclidean action I_ϕ in (40) depends only on the electric charge it is convenient to make use of (12) and (43) in order to express the Euclidean action I_e in (42) as

$$I_e = \frac{\sigma l^2}{2G}\left[\frac{1}{3}\left(\frac{l^2}{l^2 - Gq^2}\right)\right]^{3/2}.$$

Therefore, the difference between the Euclidean action of the hairy black hole and the one for the extremal case without scalar hair is given by

$$\Delta I = I_\phi - I_e = \frac{\sigma l^2}{2G}\left(\frac{1}{2}+\frac{Gq^2}{2l^2}-\left[\frac{1}{3}\left(\frac{l^2}{l^2-Gq^2}\right)\right]^{3/2}\right), \tag{44}$$

which changes of sign for a critical value of the electric charge q_c satisfying

$$\frac{Gq_c^2}{l^2} \approx 0.615\,713.$$

Remarkably, for $q^2 < q_c^2$, the difference of Euclidean actions (44) is positive, and hence the extremal bare black hole is likely to undergo a spontaneous dressing up with a nontrivial scalar field. For $q^2 > q_c^2$, it turns out that $\Delta I < 0$, and then there is a nonvanishing probability for the decay of hairy black hole decay into the extremal bare solution. For the critical point, $q^2 = q_c^2$, both solutions can coexist.

V. CONCLUDING REMARKS

It was shown that gravity minimally coupled to a real self-interacting scalar field and electromagnetism in four dimensions admits a charged hairy black hole solution. The event horizon is a surface of negative constant curvature and the asymptotic region has negative constant curvature. The self-interacting potential (2) is negative and unbounded from below, possessing a global maximum at $\phi = 0$, and it has a mass term satisfying the Breitenlohner-Freedman bound that guarantees the perturbative stability of global AdS spacetime [31]. For the topology considered here, it was shown that the stability of the locally AdS spacetime (14) under scalar perturbations, holds provided the mass satisfies the same Breitenlohner-Freedman bound [47]. The asymptotic falloff of the fields is slower than the standard one, and for the scalar field, only the branch with the slower falloff is switched on. In spite of this, the mass is still well defined and it is shown to vanish. The scalar field is regular everywhere except at the origin and is supported by the presence of electric charge which is bounded from above by the AdS radius. An upper bound for the electric charge also exists for the massless bare black hole with the same topology.

The presence of the real scalar field smooths the electromagnetic potential everywhere. The hairy black hole has a fixed temperature $T = (2\pi l)^{-1}$ regardless of the value of the electric charge, and the entropy follows the usual area law. It is also shown that there is a nonvanishing probability for the decay of the hairy black hole into a black hole without scalar field, where in the decay process, the hairy black hole absorbs energy and electric charge from the thermal bath, increasing its horizon radius and consequently its entropy. A similar behavior has been previously observed for black holes with scalar hair in Refs. [14,24]. It is worth pointing out that, although the boundary conditions considered here coincide with the ones for the hairy black hole found in [1], there is no phase transition where the bare nonextremal black hole spontaneously dresses up with a scalar field. Furthermore, since the bare black hole admits an extremal case, for which the Euclidean time period is arbitrary, two additional decay channels open up. Noteworthy, it is found that an extremal black hole without scalar field is likely to undergo a spontaneous dressing up with a nontrivial scalar field, provided the electric charge satisfies $q^2 < q_c^2$; while for $q^2 > q_c^2$ there is a nonvanishing probability for the decay of hairy black hole decay into the extremal bare solution.

In summary, for a fixed voltage and temperature $\beta = 2\pi l$, one obtains that if the electric charge of the hairy black hole satisfies $q^2 < q_c^2$, then the Euclidean actions satisfy:

$$I_0 > I_\phi > I_e,$$

which means that the nonextremal bare black hole is stable. Consequently, the hairy black hole is likely to decay into a nonextremal one, and moreover, the extremal black hole is able to decay into the hairy or into the nonextremal black hole, with different branching ratios. For $q^2 > q_c^2$, the Euclidean actions fulfill:

$$I_0 > I_e > I_\phi,$$

which means that for this range, the scalar black hole can decay into the extreme or into the nonextreme black hole with different probabilities. In the case $q^2 = q_c^2$, both the hairy and the extremal black hole can coexist, but they can decay into the nonextremal solution without a scalar field.

It would be interesting to explore the possible decays in different ensembles, as well as the regions where there is overlap between thermal and mechanical stability. It is worth pointing out that the analysis performed in Ref. [6], suggests that the hairy black hole solution found here should be stable against linear perturbations. The nonlinear stability for this solution also could be studied following the approach of Ref. [7].

As a final remark, it is worth pointing out that in the conformal frame, the potential (2) is mapped to a negative cosmological constant, and the scalar field becomes conformally coupled without self-interaction. Following [10], it can be seen also that in the conformal frame, the black hole solves the vacuum field equations since the stress-energy tensor for the scalar field cancels with the one for the electromagnetic field. The effect of having nontrivial matter fields with a vanishing total energy momentum tensor also have been discussed for flat spacetime in [48,49] and for three-dimensional gravity with negative cosmological constant in Refs. [23,24,50]. This effect also has been discussed for different setups in Refs. [51].

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