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Lorentz violating inflation

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We explore the impact of Lorentz violation on the inflationary scenario. More precisely, we study the inflationary scenario in the scalar-vector-tensor theory where the Lorentz violating vector is constrained to be unit and timelike. It turns out that the Lorentz violating vector affects the dynamics of the chaotic inflationary model and divides the inflationary stage into two parts: the Lorentz violating stage and the standard slow roll stage. We show that the universe is expanding as a de Sitter space-time in the Lorentz violating stage although the inflaton field is rolling down the potential. More interestingly, we find the exact Lorentz violating inflationary solutions in the absence of the inflaton potential. In this case, the inflation is completely associated with the Lorentz violation.

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I. INTRODUCTION

Lorentz invariance has been considered as the most fundamental symmetry of physics. However, as far as we know, any symmetry is not realized exactly or can be spontaneously broken. Hence, it is important to investigate the possibility of Lorentz invariance violation. In fact, the observation of high energy cosmic rays reports super Greisen-Zatsepin-Kuzmin events although further confirmation is needed [1–3]. As argued in [4,5], this may suggest a Lorentz violation. Theoretically, any quantum theory of gravity requires drastic modification of the picture of space-time at the Planck scale [6]. In particular, the string theory may yield a Lorentz violation [7].

The impact of Lorentz violation on physics should be broad. Even in cosmology, many subjects should then be reconsidered. For example, the dark matter can be explained by the Lorentz violating gravity [8], it is shown that the Lorentz breaking is relevant to the dark energy problem [9,10], baryogenesis may be related to Lorentz violation [11,12], Lorentz violation affects the interpretation of cosmic rays [13,14], and we should study the impact of Lorentz violation on other phenomena such as nucleosynthesis [15] and primordial magnetic field [16]. Here, we shall concentrate on the role of Lorentz violation in the inflationary scenario.

Typically, Lorentz violation yields a preferred frame. In the case of the standard model of particles, there are strong constraints on the existence of a preferred frame (see the recent review [17]). In contrast, there is no reason to refuse any preferred frame in cosmology. Rather, there is a natural preferred frame which is defined by the cosmic microwave background radiation (CMB). Therefore, there is room to consider a gravitational theory which allows a preferred frame. The purpose of the present work is to clarify what occurs in the inflationary stage when we allow a preferred frame from the beginning. It turns out that there is a chance to detect the evidence of the Lorentz violation through the observation of the cosmic microwave background radiation and of the primordial gravitational waves.

When we talk about Lorentz violation, we have to specify the model somehow. Recently, various types of theories of gravity with Lorentz violation have been proposed. One is the ghost condensation model which has an unconventional kinetic term [18–20]. In the stable vacuum, the kinetic term has the expectation value. Hence, the Lorentz invariance is violated spontaneously. This violation mechanism is an interesting possibility. The inflationary scenario in the ghost condensation was also investigated in [21]. Another interesting one is a brane model of Lorentz violation [22–24]. From the string theoretical point of view, the braneworld picture seems to be natural. Hence, it is important to examine the Lorentz violation in the braneworld context. Of course, there are other interesting models. In particular, we should refer to the very interesting paper by Gasperini [25] where the relation between Lorentz violation and inflation is investigated using a frame dependent model of gravity.

In this paper, we will consider the spontaneous breaking of Lorentz symmetry due to a vector field [7,26–28]. When this vector field couples to gravity, we obtain a Lorentz violating theory of gravity, the so-called Einstein-Ather theory [29]. Interestingly, this theory has a wide parameter region where all of current experiments and observations can be explained [30–33]. When we consider the cosmology, parameters could depend on time. The time evolution of parameters can be regarded as a consequence of the

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dynamics of a scalar field. Thus, a natural generalization of the Einstein-Ather theory is the scalar-vector-tensor theory of gravity with a timelike unit vector field.

Recently, Lim has studied the inflationary scenario in the context of the Einstein-Ather theory [34]. In this paper, we will reconsider the inflationary scenario based on a Lorentz violating scalar-vector-tensor theory of gravity. In particular, the coupling between the inflaton and the Lorentz violating vector is incorporated in our model. Our primary concern is how the Lorentz violation can affect the inflationary scenario when we include this coupling. First, we show how the chaotic inflationary scenario is affected by the Lorentz violation. In the conventional theory of gravity, there is a power law inflation model which is an exact solution with the exponential potential. Hence, it is legitimate to seek for exact solutions also in the Lorentz violating scalar-vector-tensor theory of gravity. Indeed, we find three kinds of exact solutions in the absence of the inflaton potential. We also discuss the observability of Lorentz violation.

The organization of this paper is the following: in Sec. II, we introduce the scalar-vector-tensor theory where the Lorentz symmetry is spontaneously broken due to the unit-norm vector field. In Sec. III, we study the Lorentz violating chaotic inflation. In Sec. IV, we examine the model without the inflaton potential and find the exact inflationary solutions. In Sec. V, the cosmological tensor perturbations are discussed. The final section is devoted to the conclusion. In the appendix, we demonstrate the alignment of two preferred frames, namely, the cosmological and the vector frame.

II. LORENTZ VIOLATING SCALAR-VECTOR-TENSOR THEORY

In this section, we present our model with which we discuss the inflationary scenario.

We assume that the Lorentz symmetry exists, but that it is spontaneously broken by the presence of a vector field u^μ with expectation values

$$\langle 0|u^\mu u_\mu|0\rangle = -1. \tag{1}$$

The mechanism which gives this expectation value is discussed in Ref. [7]. Here, we have chosen the timelike expectation value for the reason explained in Ref. [35]. The Nambu-Goldstone modes in the Minkowski spacetime can be represented by

$$u^\mu = \frac{1}{\sqrt{1-\boldsymbol{\psi}^2}}(1, \boldsymbol{\psi}), \tag{2}$$

where $\boldsymbol{\psi}$ is a spatial vector field. Now, the action for the Nambu-Goldstone boson in curved space-time with the metric $g_{\mu\nu}$ becomes

$$S = \int d^4x\sqrt{-g}[-\beta_1\nabla^\mu u^\nu\nabla_\mu u_\nu - \beta_2\nabla^\mu u^\nu\nabla_\nu u_\mu - \beta_3(\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u^\nu\nabla_\mu u^\alpha\nabla_\nu u_\alpha + \lambda(u^\mu u_\mu + 1)], \tag{3}$$

where β_i are arbitrary parameters and λ is the Lagrange multiplier. Here we have taken into account the expectation value by just adopting it as a constraint

$$u^\mu u_\mu = -1. \tag{4}$$

Thanks to the constraint, this is the most general low energy action which has up to second order derivatives. Note that we take u^μ as the dimensionless vector. Hence, each β_i has the dimension of mass squared. In other words, $\sqrt{\beta_i}$ gives the mass scale of the symmetry breakdown.

It is straightforward to couple this Nambu-Goldstone modes to gravity by just adding the Einstein-Hilbert term. The resultant theory is called the Einstein-Ather or the vector-tensor theory. Here, we adopt the latter name. Remarkably, this vector-tensor theory is in agreement with current experiments as long as certain relations between β_i hold [30–33].

We will consider the inflationary scenario in this Lorentz violating gravity. Now, it is possible that the inflaton couples to the vector in the following way

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}R - \beta_1(\phi)\nabla^\mu u^\nu\nabla_\mu u_\nu - \beta_2(\phi)\nabla^\mu u^\nu\nabla_\nu u_\mu - \beta_3(\phi)(\nabla_\mu u^\mu)^2 - \beta_4(\phi)u^\mu u^\nu\nabla_\mu u^\alpha\nabla_\nu u_\alpha + \lambda(u^\mu u_\mu + 1) - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\Big], \tag{5}$$

where we have chosen the Einstein frame. The above action (5) has the general coordinate invariance, hence there exists an invariance under the local Lorentz transformations. The Lorentz violation appears spontaneously when we consider a particular solution. In fact, due to the constraint obtained by the variation with respect to λ, we have to choose a particular direction. Therefore, the Lorentz invariance must be violated. The extent of Lorentz violation is characterized by the magnitude of β_i. Since β_i today can be different from β_i in the very early universe, we do not have any constraint on β_i in the inflationary stage. Of course, ultimately, β_i has to approach the observationally allowed values today. Conservatively, we have the constraint on the present value $\beta_i < 10^{-7}$ in the Planck unit [17]. If we allow a relation between the different β_i, then we do not have any constraint on β_i.

In our setup, the preferred frame is selected by the constrained vector field u^μ which violates Lorentz symmetry. In cosmology, there also exists a natural preferred frame, the so-called CMB rest frame. As is shown in the appendix, these two frames are the same in practice. This degeneracy has not been well appreciated so far. Indeed, the cosmology has been mostly discussed in the context of the Lorentz invariant Einstein gravity. Once this degeneracy is noticed, however, the Lorentz violating scalar-vector-tensor theory of gravity (5) turns out to be a possible framework to describe the inflationary universe.

If $\beta_i = 0$, the action (5) is reduced to the conventional one. In that case, we have the chaotic inflation for a generic potential V. For the exponential potential, we have the exact power law inflation. Once we switched on β_i, the Lorentz violating vector affects the inflaton dynamics. Hence, our first concern is how the Lorentz violation modifies the picture of the chaotic inflationary scenario. Our second aim is to find the exact inflationary scenario with the Lorentz violation. Interestingly, we find exact solutions in the absence of the inflaton potential.

III. LORENTZ VIOLATING CHAOTIC INFLATION

Now, let us consider the chaotic inflationary scenario and clarify to what extent the Lorentz violating vector affects the inflationary scenario.

In principle, the preferred frame determined by the vector u^μ can be different from the CMB rest frame. That would imply an anisotropic universe. However, alignment of these frames had been achieved during the cosmic expansion as is explained in the appendix. Hence, we can start with the homogeneous and isotropic space-time. Usually, the metric ansatz is imposed after taking the variation. With a homogeneous ansatz, however, we can perform the reduction at the action level provided that the lapse function is kept as a variable. Hence, we parametrize the metric as

$$ds^2 = -\mathcal{N}^2(t)dt^2 + e^{2\alpha(t)}\delta_{ij}dx^i dx^j, \tag{6}$$

where we have included the lapse function $\mathcal{N}$. After the variation, one can choose the lapse function freely because it is nothing but a choice of the time coordinate. Note that the shift function is not necessary because the homogeneous ansatz kills the freedom of the spatial coordinate transformation. In other words, we have no momentum constraint in the homogeneous system. The scale of the universe is determined by α. Since the spatial isotropy does not allow spatial components of u^μ, we have to take

$$u^\mu = \left(\frac{1}{\mathcal{N}}, 0, 0, 0\right), \tag{7}$$

where the normalization is determined by the constraint (4). Given these, we can calculate necessary quantities, for example, $\nabla_i u^j = \dot{\alpha}/\mathcal{N}\delta_i^j$ and other components vanish. Now, substituting these quantities into the action (5), we obtain

$$S = \int dt \frac{1}{\mathcal{N}} e^{3\alpha}\left[-\frac{3}{8\pi G}(1 + 8\pi G\beta)\dot{\alpha}^2 + \frac{1}{2}\dot{\phi}^2 - \mathcal{N}^2 V(\phi)\right], \tag{8}$$

where $\beta(\phi) = \beta_1 + 3\beta_2 + \beta_3$. Note that β_4 does not contribute to the background dynamics. One might think that the reduced action (8) looks like a nonminimally coupled scalar field [36,37]. However, the structure of the equations of motion is quite different. In particular, the feature of perturbations is completely different.

Let us deduce the equations of motion. Taking the variation with respect to α and ϕ yields the evolution equations for them. As the action (8) has the time reparametrization invariance, we also have the Hamiltonian constraint which can be obtained by taking the variation with respect to the lapse function $\mathcal{N}$. We set $\mathcal{N} = 1$ after taking the variation to choose the cosmic time. Then, we define the dimensionless derivative Q' by

$$\dot{Q} = \frac{dQ}{d\alpha}\frac{d\alpha}{dt} \equiv Q'\frac{d\alpha}{dt}. \tag{9}$$

Thus, the equations of motion are finally deduced as

$$\left(1 + \frac{1}{8\pi G\beta}\right)H^2 = \frac{1}{3}\left[\frac{1}{2}\frac{H^2\phi'^2}{\beta} + \frac{V}{\beta}\right] \tag{10}$$

$$\left(1 + \frac{1}{8\pi G\beta}\right)\frac{H'}{H} + \frac{1}{2}\frac{\phi'^2}{\beta} + \frac{\beta'}{\beta} = 0 \tag{11}$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} + 3\beta_{,\phi} = 0, \tag{12}$$

where $\beta_{,\phi}$ denotes the derivative with respect to ϕ. We have taken $H = \dot{\alpha}$ as an independent variable. As is usual with gravity, these three equations are not independent. Usually, the second one is regarded as a redundant equation.

The above equation changes its property at the critical value ϕ_c defined by

$$8\pi G\beta(\phi_c) = 1. \tag{13}$$

When we consider the inflationary scenario, we usually require to have enough e-folding number, say $N = 70$. Let ϕ_i be the corresponding initial value of the scalar field. If $\phi_c > \phi_i$, the effect of Lorentz violation on the inflationary scenario would be negligible. However, if $\phi_c < \phi_i$, the standard scenario should be modified (see Fig. 1). It depends on the models. To make the discussion more specific, we choose the model

$$\beta = \xi\phi^2, \qquad V = \tfrac{1}{2}m^2\phi^2, \tag{14}$$

where ξ and m are parameters. For this model, we have

$$\phi_c = \frac{M_{\rm pl}}{\sqrt{8\pi\xi}}. \tag{15}$$

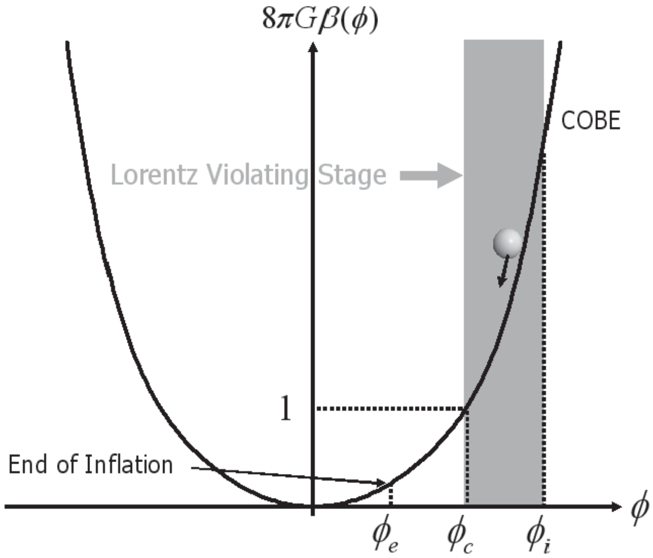

FIG. 1. There is a critical point where the coupling between the Lorentz violating vector and the inflaton becomes ineffective.

As $\phi_i \sim 3M_{\rm pl}$ approximately in the standard case, the condition $\phi_i > \phi_c$ implies the criterion $\xi > 1/(72\pi) \sim 1/226$ for the Lorentz violation to be relevant to the inflation. For other models, a similar criterion can be easily obtained.

Now, we suppose that the Lorentz violation is relevant and analyze the two regimes separately.

A. Lorentz violating stage

For a sufficiently larger value of ϕ, both the coupling function β and the potential function V are important in the model (14). During this period, the effect of Lorentz violation on the inflaton dynamics must be large. In the Lorentz violating regime, $8\pi G\beta \gg 1$, we have

$$H^2 = \frac{1}{3\beta}\left[\frac{1}{2}H^2\phi'^2 + V\right] \qquad (16)$$

$$\frac{H'}{H} + \frac{1}{2\beta}\phi'^2 + \frac{\beta'}{\beta} = 0 \qquad (17)$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} + 3\beta_{,\phi} = 0. \qquad (18)$$

To have the inflation, we impose the condition

$$H^2\phi'^2 \ll V \qquad (19)$$

as the slow roll condition. Consequently, Eq. (16) is reduced to

$$H^2 = \frac{1}{3\beta}V. \qquad (20)$$

Using Eq. (20), the slow roll condition (19) can be written as

$$\phi'^2 \ll \beta. \qquad (21)$$

Now, we also impose the condition $H'/H \ll 1$ as the quasi-de Sitter condition. Then, Eq. (17) gives us the condition

$$\beta' \ll \beta. \qquad (22)$$

We also require the standard condition

$$\phi'' \ll \phi'. \qquad (23)$$

Thus, we have the slow roll Eqs. (20) and

$$\phi' + \frac{V_{,\phi}}{3H^2} + \beta_{,\phi} = 0. \qquad (24)$$

For our example (14), we can easily solve Eqs. (20) and (24) as

$$\phi(\alpha) = \phi_i e^{-4\xi\alpha}. \qquad (25)$$

For this solution to satisfy slow roll conditions (21)–(23), we need $\xi < 1/16$. Thus, we have the range $1/226 < \xi < 1/16$ of the parameter for which the Lorentz violating inflation is relevant. Note that, in our model (14), the Hubble parameter (20) becomes constant to the lowest order in the slow roll approximation as

$$H^2 = \frac{m^2}{6\xi}, \qquad (26)$$

even though the inflaton is rolling down the potential. This is a consequence of Lorentz violation. When the higher order corrections in the slow roll approximation are taken into account, the value of H receives the small corrections, however, the Hubble constant remains constant. In fact, perturbative calculations give a full order result

$$H^2 = \frac{\frac{m^2}{6\xi}}{1 - \frac{8\xi}{3}}.$$

B. Standard slow roll stage

After the inflaton crosses the critical value ϕ_c, the dynamics is governed entirely by the potential V. In the standard slow roll regime $8\pi G\beta \ll 1$, we have

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}H^2\phi'^2 + V\right] \qquad (27)$$

$$\frac{H'}{H} + 4\pi G\phi'^2 = 0 \qquad (28)$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} = 0 \qquad (29)$$

The following arguments are standard. The usual slow roll conditions give the slow roll equations

$$H^2 = \frac{8\pi G}{3}V \qquad (30)$$

$$\phi' + \frac{V_{,\phi}}{3H^2} = 0. \tag{31}$$

In the simplest case $V = \frac{1}{2}m^2\phi^2$, the evolution of the inflaton can be solved as

$$\phi^2(\alpha) = \phi_c^2 - \frac{\alpha}{2\pi G}. \tag{32}$$

The scale factor $a(t) = e^\alpha$ can be also obtained as

$$a(t) = \exp[2\pi G(\phi_c^2 - \phi^2(t))]. \tag{33}$$

The standard inflation stage ends and the reheating commences when the slow roll conditions are violated.

C. *e*-folding number

Now it is easy to calculate the e-folding number. Let ϕ_i be the value of the scalar field corresponding to the e-folding number $N = 70$. The total e-folding number reads

$$N = \frac{1}{4\xi}\log\frac{\phi_i}{\phi_c} + 2\pi G(\phi_c^2 - \phi_e^2), \tag{34}$$

where $\phi_e \sim 0.3M_{\rm pl}$ is the value of scalar field at the end of inflation. Note that the first term arises from the Lorentz violating stage. As an example, let us take the value $\xi = 10^{-2}$. Then, $\phi_c \sim 2M_{\rm pl}$. The contribution from the end of inflation is negligible. Therefore, we get $\phi_i \sim 12M_{\rm pl}$.

In this simple example, the coupling to the Lorentz violating sector disappears after reheating. Hence, the subsequent homogeneous dynamics of the universe is the same as that of Lorentz invariant theory of gravity. However, it is possible to add some constants to β_i, which are consistent with current experiments. In that case, the effect of the Lorentz violation is still relevant to the subsequent history.

So far, we have considered a special model where the coupling β has the same power as the potential V. It is straightforward to extend our consideration to more general cases.

IV. LORENTZ VIOLATING INFLATION WITHOUT POTENTIAL

In this section, we will investigate a purely Lorentz violating inflationary model.

A. Exact solutions

It is interesting to observe that we have inflation even in the case $V = 0$. In this case, Eqs. (10)–(12) read

$$1 = \frac{\phi'^2}{6\bar{\beta}} \tag{35}$$

$$\frac{H'}{H} + \frac{1}{2\bar{\beta}}\phi'^2 + \frac{\bar{\beta}'}{\bar{\beta}} = 0 \tag{36}$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + 3\bar{\beta}_{,\phi} = 0, \tag{37}$$

where we have defined the variable $\bar{\beta} = \beta + 1/(8\pi G)$. Substituting Eq. (35) into Eq. (36), we have

$$(\bar{\beta}H)' + 3\bar{\beta}H = 0 \tag{38}$$

It yields $\bar{\beta}H \propto e^{-3\alpha}$.

The condition for the accelerating universe $\ddot{a} > 0$ is now

$$\frac{H'}{H} > -1. \tag{39}$$

Using Eqs. (35) and (36), we can reduce the condition (39) to

$$(\log\bar{\beta})' < -2. \tag{40}$$

As the scalar is rolling down, Eq. (35) can be solved as $\phi' = d\phi/d\alpha = -\sqrt{6\bar{\beta}}$. Thus, finally, we obtain the condition for $\bar{\beta}$ as

$$\sqrt{\frac{6}{\bar{\beta}}}\frac{d\bar{\beta}}{d\phi} > 2. \tag{41}$$

Let us consider an exactly solvable model, $\bar{\beta} = \xi\phi^2$. In this simplest case, the condition (41) yields $\xi > 1/6$. We can solve Eq. (35) as

$$\phi \propto e^{-\sqrt{6\xi}\alpha}. \tag{42}$$

Therefore, we have

$$H \propto e^{-\alpha/p}, \qquad p = \frac{1}{3 - 2\sqrt{6\xi}}. \tag{43}$$

There are three cases to be considered, i.e., (i) $1/6 < \xi < 3/8$, (ii) $\xi = 3/8$, (iii) $3/8 < \xi$.

(i) $1/6 < \xi < 3/8$

In this case, $p > 0$. Hence, it is easy to solve Eq. (43)

$$\dot{\alpha} \sim e^{-\alpha/p} \tag{44}$$

as

$$a(t) \sim t^p, \qquad p > 1. \tag{45}$$

This is a power law inflation.

(ii) $\xi = 3/8$

In this case, $1/p = 0$. This is nothing but the de Sitter solution

$$a(t) \sim e^{Ht}. \tag{46}$$

The Hubble constant should be determined by the initial condition. Although the space-time itself is de Sitter, the scalar field shows a nontrivial time evolution (42). Therefore, it is interesting to calculate the curvature perturbations in this model.

(iii) $3/8 < \xi$

In this case, $p \equiv -|p| < 0$. Hence, the solution becomes

$$a(t) \sim (-t)^{-|p|}, \qquad t < 0. \tag{47}$$

Thus, this solution represents a superinflationary universe. As the weak energy condition is effectively violated for this case, we need to check the stability of the solution. Moreover, this kind of universe encounters a singularity in the future (at $t = 0$). It is possible to resolve this singularity by adding the term appeared in the string 1-loop corrections [38,39]. It is also important to study if the behavior of perturbations is also similar or not [40,41]. We leave these issues for the future work.

B. Inflationary scenario

In the absence of the inflaton potential, we have obtained exact solutions, i.e., the power law inflation, the de Sitter inflation, and the superinflation. If we slightly modify $\bar{\beta}$, the inflation will end when the condition (41) is violated. Note that when the scalar varies from ϕ_i to ϕ_e, the e-folding number of the universe can be calculated as

$$N = \frac{1}{\sqrt{6\xi}} \log \frac{\phi_i}{\phi_e}. \tag{48}$$

We expect that the reheating would occur during the oscillation phase. It should be stressed that the above inflations are completely associated with the Lorentz violation.

V. EVOLUTION OF TENSOR PERTURBATIONS

Needless to say, the evolution of cosmological perturbations needs to be studied. Because of the Lorentz violation, the velocity of the gravitational waves are different from the velocity of the light. This and the nontrivial coupling functions β_i would cause interesting consequence on the spectrum of tensor, vector and scalar perturbations. In particular, the vector perturbations are intriguing since there are no vector perturbations in the Lorentz invariant inflationary scenario. However, as the calculation is very complicated, we leave the complete analysis for future publication. Instead, here, we discuss the simplest case, namely, tensor perturbations. Even in this case, we can make some interesting predictions.

The tensor part of perturbations can be described by

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}(t, x^i))dx^i dx^j, \tag{49}$$

where the perturbations satisfy $h^i_i = h^{;j}_{ij} = 0$. The quadratic part of the action is given by

$$S = \int d^4x \frac{a^3}{16\pi G}\left[\frac{1}{4}\gamma \dot{h}_{ij}\dot{h}^{\prime ij} - \frac{1}{4a^2} h_{ij,k}h^{ij,k}\right] \tag{50}$$

where we have defined

$$\gamma = 1 - 16\pi G(\beta_1 + \beta_3). \tag{51}$$

Apparently, the velocity of the gravitational waves is not 1. To have a real velocity, we have to impose $\gamma > 0$. Hence, we assume β_1 and β_3 are constant, but β_2 is time dependent and so is β.

In the case of chaotic inflation model, the Hubble parameter is constant (26) during the Lorentz violating stage. Therefore, the spectrum is extremely flat although the inflaton is rolling down the potential.

VI. CONCLUSION

We have examined the impact of a model of Lorentz violation on the inflationary scenario. As a specific model, we have considered the spontaneous violation of the Lorentz symmetry due to a vector field. More specifically, we have investigated scalar-vector-tensor theory of gravity where the vector is constrained to be unit and timelike.

First, we have examined the chaotic inflationary scenario and found that the Lorentz violation modifies the dynamics of the inflaton for a certain parameter region in our model. We have shown that the inflationary stage breaks into two parts; the Lorentz violating stage and the standard slow roll stage. We found that the universe is expanding as a de Sitter space-time in the Lorentz violating stage although the inflaton field is rolling down the potential. Moreover, we have calculated the e-folding number by taking into account the above modification and shown that we can get enough e-folding.

In this paper, we have considered the simplest case $\beta \sim V \sim \phi^2$. In other cases, for instance, $\beta \sim \phi^4$ and $V \sim \phi^2$, the Hubble parameter H increases during the Lorentz violating stage. In the standard slow roll stage, the Hubble parameter H decreases. Therefore, we can easily generate the spectrum with the initial (steep) blue spectrum and the later (slightly) red spectrum. This may explain the deficiency of the CMB power spectrum at large scales observed by WMAP [42].

We have also shown that the inflation can be realized without the inflaton potential. Depending on the value of the parameter ξ, we have obtained exact solutions, i.e. the power law inflation, de Sitter inflation, and the superinflation. Interestingly, even in the exact de Sitter case, the dynamics of the scalar field turns out to be nontrivial. As the weak energy condition is effectively violated for the superinflationary solution, we definitely need to check the stability of the solution. In all cases, the inflation ends when the coupling function $\bar{\beta}$ is slightly modified from exactly solvable cases. These exactly solvable models are important to understand the evolution of cosmological perturbations in the Lorentz violating theory of gravity.

It would be interesting to study the evolution of fluctuation completely. We briefly discussed the case of tensor perturbations. If the vector modes of perturbations can survive till the last scattering surface, they leave the remnant of the Lorentz violation on the CMB polarization spectrum. It is also intriguing to seek for a relation to the large scale anomaly discovered in CMB by WMAP [43–46]. The calculation of the curvature perturbation is much more complicated. However, it must reveal more interesting phenomena due to Lorentz violating inflation. The tensor-scalar ratio of the power spectrum would be also interesting. These are now under investigation [47].

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APPENDIX: ALIGNMENT OF PREFERRED FRAMES

Here, we would like to show that the alignment of two frames, the CMB rest frame and the frame determined by u^μ, will occur during the cosmological evolution. For simplicity, we ignore the scalar field, instead we add a cosmological constant term to the action. The action is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R - 2\Lambda) - \beta_1\nabla^\mu u^\nu\nabla_\mu u_\nu - \beta_2\nabla^\mu u^\nu\nabla_\nu u_\mu - \beta_3(\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u^\nu\nabla_\mu u^\alpha\nabla_\nu u_\alpha + \lambda(u^\mu u_\mu + 1)\right]. \tag{A1}$$

We consider the Bianchi Type I metric as an ansatz:

$$ds^2 = -\mathcal{N}^2(t)dt^2 + e^{2\alpha(t)}[e^{-4\sigma_+(t)}dx^2 + e^{2\sigma_+(t)}\{e^{2\sqrt{3}\sigma_-(t)}dy^2 + e^{-2\sqrt{3}\sigma_-(t)}dz^2\}] \tag{A2}$$

and now the vector field can be tilted as

$$u^\mu = \left(\frac{1}{\mathcal{N}(t)}\cosh\theta(t), e^{-\alpha(t)+2\sigma_+(t)}\sinh\theta(t), 0, 0\right). \tag{A3}$$

Since the anisotropic space-time is parametrized generically, the above ansatz for the vector field is sufficiently general. Thus, in general, the cosmic frame is different from the preferred frame determined by u^μ. Substituting the metric and the vector field into the action (A1), we obtain

$$S = \int dt\frac{1}{\mathcal{N}}e^{3\alpha}[-A\dot{\alpha}^2 - \lambda\mathcal{N}^2 + B(\dot{\sigma}_+^2 + \dot{\sigma}_-^2) + D\dot{\theta}^2 - E\theta\dot{\theta}\,\dot{\alpha} - F\theta^2\dot{\alpha}^2], \tag{A4}$$

where

$$A = \frac{3}{8\pi G}\{1 + 8\pi G(\beta_1 + 3\beta_2 + \beta_3)\}, \tag{A5}$$

$$B = \frac{3}{8\pi G}\{1 - 16\pi G(\beta_1 + \beta_3)\}, \tag{A6}$$

$$D = \beta_1 - \beta_4, \tag{A7}$$

$$E = 2(3\beta_2 + \beta_3 + \beta_4), \tag{A8}$$

$$F = 2\beta_1 + 9\beta_2 + 3\beta_3 + \beta_4, \tag{A9}$$

$$\lambda = \frac{\Lambda}{8\pi G}. \tag{A10}$$

By taking the variation of (A4), we obtain

$$A\dot{\alpha}^2 - \lambda = 0, \tag{A11}$$

$$\frac{d}{dt}(e^{3\alpha}\dot{\sigma}_\pm) = 0, \tag{A12}$$

$$A\frac{d}{dt}(e^{3\alpha}\dot{\alpha}) - 3\lambda e^{3\alpha} = 0, \tag{A13}$$

$$E\theta\frac{d}{dt}(e^{3\alpha}\dot{\alpha}) - 2D\frac{d}{dt}(e^{3\alpha}\dot{\theta}) - 2Fe^{3\alpha}\theta\dot{\alpha}^2 = 0, \tag{A14}$$

where we kept up to the first order with respect to $\sigma_\pm$ and θ. From Eq. (A12), it turns out that the anisotropy decays as the universe expands. Now, we can deduce the master equation for the tilt θ as

$$\ddot{\theta} + 3\dot{\alpha}\,\dot{\theta} + \left(\frac{F}{D}\dot{\alpha}^2 - \frac{3E}{2AD}\lambda\right)\theta = 0. \tag{A15}$$

Using Eq. (A11) and the definitions (8) and (9), we have

$$\ddot{\theta} + 3\dot{\alpha}\,\dot{\theta} + \frac{2\lambda}{A}\theta = 0. \tag{A16}$$

For the effective gravitational coupling to be positive, we need $A > 0$. Thus, (A16) tells us that the tilt θ will vanish during the cosmic expansion.

Namely, the CMB rest frame and the preferred frame determined by u^μ are the same in practice. What we did in this paper was reveal what this degeneracy means in inflationary cosmology.

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