# Supersymmetry and *CP* violation in $|\Delta S| = 1$ $\tau$ decays

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We compute the supersymmetry (SUSY) effective Hamiltonian that describes the $|\Delta S| = 1$ semi-leptonic decays of tau leptons. We provide analytical expressions for supersymmetric contribution to $\tau \to u\bar{s}\nu_\tau$ transition in mass insertion approximation. We show that SUSY contributions may enhance the *CP* asymmetry of $\tau \to K\pi\nu_\tau$ decays by several orders of magnitude with respect to the standard model expectations. However, the resulting asymmetry is still well below the current experimental limits obtained by CLEO Collaborations. We emphasize that measuring *CP* rate asymmetry in this decay larger than $10^{-6}$ would be a clear evidence of physics beyond the supersymmetric extensions of the standard model.

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## I. INTRODUCTION

Strangeness-changing tau lepton decays play an important role in testing the dynamics of $|\Delta S| = 1$ weak interactions [1]. Determination of basic parameters of the standard model (SM) and tests of fundamental symmetries can be done using such tau decays. For instance, measurements of the spectral functions of tau decays into strange mesons have been used recently to obtain information on the mass of the strange quark and on the $V_{\rm us}$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element [2]. Furthermore, searches for *CP* violation effects in the double kinematical distributions of $\tau^\pm \to K_S\pi^\pm\nu_\tau$ decays have been performed recently by the CLEO Collaborations [3]. These exclusive decays can be used to provide further tests on the violation of the *CP* symmetry [3–6]. A "known" *CP* rate asymmetry of $O(10^{-3})$ has been pointed out to exist between $\tau^- \to K_{L,S}\pi^-\nu_\tau$ and their *CP* conjugate decays [5]. On the other hand, within the SM, the *CP* rate asymmetry turns out to be negligibly small (of order $10^{-12}$) in $\tau^\pm \to K^\pm\pi^0\nu_\tau$ decays [6], opening a large window to consider the effects of new physics contributions.

Supersymmetry (SUSY) is one of the most interesting candidates for physics beyond the SM. In SUSY models there are new sources of *CP* and flavor violation that may lead to significant impacts on the *CP* asymmetries of $\tau$ decays. In this paper we analyze SUSY contributions to the *CP* violating effects in $\tau \to K\pi\nu_\tau$ decays. We consider the effects due to the chargino and neutralino exchanges by using the mass insertion approximation (MIA) which is a very effective tool for studying SUSY contributions to flavor changing neutral current processes (FCNC) in a model-independent way. We take into account all the relevant operators involved in the effective Hamiltonian for $|\Delta S| = 1$ $\tau$ decays and provide an analytical expression for the corresponding Wilson coefficients.

The elementary process underlying $|\Delta S| = 1$ decays is $\tau^- \to s\bar{u}\nu_\tau$. The lowest order contribution to this decay in the SM is mediated by the exchange of a single $W^-$ boson, which is Cabibbo suppressed. We consider in this paper the higher order effects induced by supersymmetry in the effective Hamiltonian to describe this low-energy process. The observable effects induced in some of the dominant $|\Delta S| = 1$ exclusive processes are considered. The main focus of our paper is to provide an specific mechanism to generate *CP* violating couplings in the scalar form factor of $\tau \to K\pi\nu_\tau$ decays as studied by the CLEO Collaboration in [3].

This paper is organized as follows. In Sec. II we briefly review the SM contribution to the $|\Delta S| = 1$ $\tau$ decays. As pointed out before, the resulting *CP* rate asymmetry in $\tau^\pm \to K^\pm\pi^0\nu_\tau$ is negligible. Moreover, we show that this result remains intact in the case of minimal extension of the SM with right-handed neutrinos. In Sec. III we derive the SUSY effective Hamiltonian for the $\tau^- \to s\bar{u}\nu_\tau$ transitions. In Sec. IV we consider the effects of SUSY contributions on the branching ratios of two dominant exclusive $|\Delta S| = 1$ decays, namely $\tau^- \to K^-\nu_\tau$ and $\tau^- \to (K\pi)^-\nu_\tau$. Section V is devoted to analyzing the SUSY contributions to the *CP* asymmetry in $\tau \to K\pi\nu_\tau$ decay. We show that within SUSY models, one can generate the *CP* asymmetry, however it is below the experimental limits. Finally, we give our main conclusions in Sec. VI. We have also included an appendix to provide the complete expressions of the Wilson coefficients derived from SUSY.

## II. $|\Delta S| = 1$ $\tau$ DECAYS IN THE STANDARD MODEL

Strangeness-changing $|\Delta S| = 1$ decays of tau leptons are driven by the $\tau^- \to \bar{u}s\nu_\tau$ elementary process. In the SM they occur at the tree level, as shown in Fig. 1. The SM effective Hamiltonian underlying these decays is given by

 

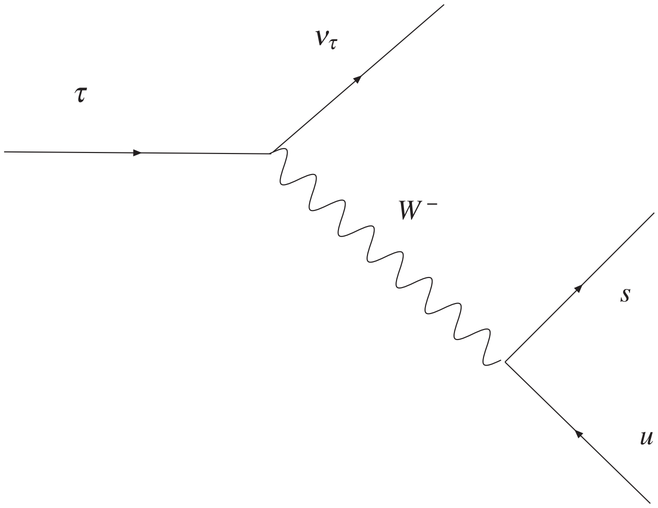


FIG. 1. SM tree-level contributions to $\tau^- \to \bar{u}s\nu_\tau$ transition.

$$\mathcal{H}_{\rm SM} = \frac{G_F}{\sqrt{2}} V_{\rm us}(\bar{\nu}_\tau \gamma^\mu L\tau)(\bar{s}\gamma_\mu Lu), \quad (1)$$

where $V_{\rm us}$ is the $u\bar{s}$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element and $L, R = 1 \mp \gamma_5$. The amplitudes for the dominant exclusive processes derived from this Hamiltonian are:

$$\mathcal{A}^{\rm SM}(\tau^-(p) \to K^-(q)\nu_\tau(p')) = i\frac{G_F}{\sqrt{2}} V_{\rm us} f_K m_\tau r, \quad (2)$$

$$\mathcal{A}^{\rm SM}(\tau^-(p) \to K(q)\pi(q')\nu_\tau(p')) = i\frac{G_F}{\sqrt{2}} V_{\rm us} C_K[f_V(t)Q^\mu \bar{\nu}(p')\gamma_\mu L\tau(p) + m_\tau f_S(t) r], \quad (3)$$

where letters within parenthesis denote the momenta of the particles, $f_K$ is the $K^-$ decay constant, $t = (q + q')^2$ is the square of the momentum transfer, $C_K = 1(1/\sqrt{2})$ for $K^0\pi^-(K^-\pi^0)$ state, $r = \bar{\nu}(p')R\tau(p)$, $\Delta^2 = m_K^2 - m_\pi^2$, and

$$Q_\mu = (q - q')_\mu - \frac{\Delta^2}{t}(q + q')_\mu. \quad (4)$$

In the SM, the two-body decay of Eq. (2) is a clean prediction if one uses $f_K \simeq 159.8$ MeV from $K^- \to \mu^-\nu_\mu$ decay [7]. Since new physics can affect $\tau \to K\nu$ and $K \to \mu\nu$ decays in a nonuniversal way, these decays can be used to obtain interesting bounds on new physics couplings.

On the other hand, the three-body decay of Eq. (3) can exhibit eventually the effects of $CP$ violation [3–6]. However, the decay rate of this process is given by [8]

$$\Gamma(\tau \to K\pi\nu_\tau) = \frac{G_F^2 m_\tau^5}{768\pi^3}|V_{\rm us}|^2 I_{\rm SM}, \quad (5)$$

where

$$I_{\rm SM} = \frac{1}{m_\tau^6}\int_{(m_K+m_\pi)^2}^{m_\tau^2} \frac{dt}{t^3}(m_\tau^2 - t)^2\bigg[|f_V|^2\Big(1 + \frac{2t}{m_\tau^2}\Big) \times [\lambda(t, m_K^2, m_\pi^2)]^{3/2} + 3|f_S|^2\Delta^4[\lambda(t, m_K^2, m_\pi^2)]^{1/2}\bigg]. \quad (6)$$

The function $\lambda(x, y, z)$ is given by $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. It is clear from the expression of $\Gamma(\tau \to K\pi\nu_\tau)$ that within the SM, the direct $CP$ asymmetry identically vanishes. As is well known, a necessary condition to generate a $CP$ rate asymmetry is that at least two terms of the amplitude for a given physical process have different weak and strong phases. However, in the SM the relative weak phase between the scalar $f_S$ and the vector $f_V$ form factors of $\tau \to K\pi\nu$ is zero. Furthermore, since the form factors $f_{V,S}(t)$ in Eq. (3) belong, respectively, to the (orthogonal) $l = 1$ and $l = 0$ angular momentum configurations of the $K\pi$ system, the $f_S f_V$ term in the squared amplitude vanish upon the integration over the variable $u = (p - p')^2$, therefore the $CP$ violating terms vanish in the integrated rate $\Gamma$ and in the hadronic spectrum $d\Gamma/dt$. Thus, within the SM $CP$ violating effects in this three-body channel can manifest only in the double differential decay distribution where the interference of $f_V$ and $f_S$ is present (see Ref. [3]).

A different mechanism to generate a $CP$ rate asymmetry in the SM for $\tau^\pm \to K^\pm\pi^0\nu_\tau$ was considered in [6]. In this case the two amplitudes with different weak and strong phases contribute to the same $l = 1$ angular momentum configuration. This asymmetry turns out to be negligibly small since it is suppressed by the CKM factor $V_{\rm td} \simeq 10^{-3}$ and also by a higher order suppression factor $g^2/4\pi M_W^2 \simeq 10^{-8}$. Thus, the resulting $CP$ rate asymmetry is expected to be negligible, as confirmed in Ref. [6]. Therefore, this decay can be suitable to search for the effects of $CP$ violation induced by new physics.

The minimal extension of the SM with right-handed neutrinos $\nu$SM, where nonvanishing neutrino masses can be obtained, allows for a new source of the $CP$ violation through the $U_{\rm MNS}$ mixing matrix. In this scenario, the amplitude of the decay $\tau^- \to K^-\pi^0\nu_\tau$ will be given by

$$\mathcal{A}^{\nu\mathcal{SM}}(\tau^- \to K^-\pi^0\nu_\tau) = (U_{\rm MNS}^*)_{33}|\mathcal{A}^{\mathcal{SM}}|e^{i\delta_{\rm SM}}. \quad (7)$$

It is remarkable that, although the amplitude $\mathcal{A}^{\nu\mathcal{SM}}$ has a weak $CP$ violating phase, the $CP$ asymmetry still vanishes. Therefore, any measurement of a nonvanishing $CP$ asymmetry will be a hint for a new physics beyond the SM. In the rest of the paper we will focus on the NP contributions to $CP$ violation in the three-body decay induced by SUSY.

## III. SUSY CONTRIBUTIONS TO $|\Delta S| = 1$ $\tau$ DECAYS

The effective Hamiltonian $H_{\rm eff}$ derived from SUSY can be expressed as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{us}} \sum_i C_i(\mu) Q_i(\mu), \tag{8}$$

where $C_i$ are the Wilson coefficients and $Q_i$ are the relevant local operators at low-energy scale $\mu \simeq m_\tau$. The operators are given by

$$Q_1 = (\bar{\nu}\gamma^\mu L\tau)(\bar{s}\gamma_\mu Lu), \tag{9}$$

$$Q_2 = (\bar{\nu}\gamma_\mu L\tau)(\bar{s}\gamma_\mu Ru), \tag{10}$$

$$Q_3 = (\bar{\nu}R\tau)(\bar{s}Lu), \tag{11}$$

$$Q_4 = (\bar{\nu}R\tau)(\bar{s}Ru), \tag{12}$$

$$Q_5 = (\bar{\nu}\sigma_{\mu\nu}R\tau)(\bar{s}\sigma^{\mu\nu}Ru), \tag{13}$$

where $L$, $R$ are as defined in the previous section and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The Wilson coefficients $C_i$, at the electroweak scale, can be expressed as $C_i = C_i^{\text{SM}} + C_i^{\text{SUSY}}$, where $C_i^{\text{SM}}$ are given by

$$C_1^{\text{SM}} = 1, \qquad C_{2,3,4,5}^{\text{SM}} = 0. \tag{14}$$

SUSY contributions to the Hamiltonian of $\tau^- \to \bar{u}s\nu_\tau$ transitions can be generated through two topological box diagrams as shown in Figs. 2 and 3. Other SUSY contributions (vertex corrections) are suppressed either due to small Yukawa couplings of light quarks or because they have the same structure as the SM in the hadronic vertex. In our computations of Wilson coefficients, we will work in the mass insertion approximation (MIA), where gluino and neutralino are flavor diagonal. Denoting by $(\Delta_{AB}^f)_{ab}$ the *off*-diagonal terms in the sfermion mass matrices where $A$, $B$ indicate chirality, $A, B = (L, R)$, the $A - B$ sfermion propagator can be expanded as

$$\langle \tilde{f}_A^a \tilde{f}_B^{b*} \rangle = i(k^2 I - \tilde{m}^2 I - \Delta_{AB}^f))_{ab}^{-1}$$
$$\simeq \frac{i\delta_{ab}}{k^2 - \tilde{m}^2} + \frac{i(\Delta_{AB}^f)_{ab}}{(k^2 - \tilde{m}^2)^2} + O(\Delta^2), \tag{15}$$

where $\tilde{f}$ denotes any scalar fermion, $a$, $b = (1, 2, 3)$ are flavor indices, $I$ is the unit matrix, and $\tilde{m}$ is the average sfermion mass. It is convenient to define a dimensionless quantity $(\delta_{AB}^f)_{ab} \equiv (\Delta_{AB}^f)_{ab}/\tilde{m}^2$. As long as $(\Delta_{AB}^f)_{ab}$ is smaller than $\tilde{m}^2$ we can consider only the first order term in $(\delta_{AB}^f)_{ab}$ of the sfermion propagator expansion. In our analysis we will keep only terms proportional to the third generation Yukawa couplings and terms of order $\lambda$ where $\lambda = V_{\text{us}}$.

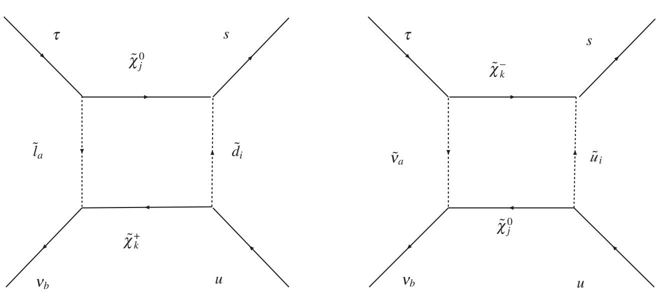

FIG. 2. SUSY box contributions to $\tau^- \to \bar{u}s\nu_\tau$ transition.

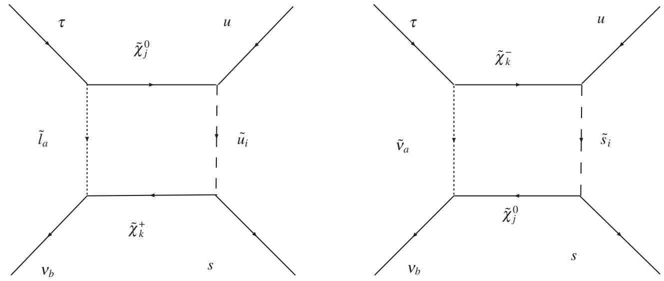

FIG. 3. Crossed diagrams of Fig. 2.

The complete expressions for the Wilson coefficients $C_i$ at $m_W$ scale induced by SUSY computed from Figs. 2 and 3 can be found in the appendix. As can be seen from this appendix, the $C_i$ are given in terms of several mass insertions that represent the flavor transitions between different generations of quarks or leptons. In general, these mass insertions are complex and of order one. However, the experimental limits of several flavor changing neutral currents impose severe constraints on most of these mass insertions. In the following, we summarize all the important constraints on the relevant mass insertions for our process.

(1) From the experimental measurements of BR$(\mu \to e\gamma) < 1.2 \times 10^{-11}$, the following bounds on $|(\delta_{12}^l)_{AB}|$ and $|(\delta_{12}^\nu)_{AB}|$ are obtained [9]: For $M_1 \sim M_2 = 100$ GeV, $\mu = \tilde{m}_l = 200$ GeV, and $\tan\beta \simeq 10$,

$$|(\delta_{12}^l)_{LL}| \lesssim 10^{-3}, \qquad |(\delta_{12}^l)_{LR}| \lesssim 10^{-6}, \tag{16}$$

$$|(\delta_{12}^\nu)_{LL}| \lesssim 6 \times 10^{-4}, \qquad |(\delta_{13}^\nu)_{LL}| \lesssim 4 \times 10^{-4},$$
$$|(\delta_{23}^\nu)_{LL}| \lesssim 7 \times 10^{-4}. \tag{17}$$

(2) From BR$(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$, one gets the following constraint on $|(\delta_{23}^l)_{LR})|$ [10,11]:

$$|(\delta_{23}^l)_{LR}| \lesssim 10^{-2}, \tag{18}$$

and from BR$(\tau \to e\gamma) < 3.1 \times 10^{-7}$, one finds [10,11]:

$$|(\delta_{13}^l)_{LR}| \lesssim 10^{-1}. \tag{19}$$

(3) The mass insertions $(\delta_{12}^d)_{AB}$ are constrained by the $\Delta M_K$ and $\epsilon_K$ as follows [10]:

$$|(\delta_{12}^d)_{LL}| \lesssim 4 \times 10^{-2}, \qquad |(\delta_{12}^d)_{LR}| \lesssim 4 \times 10^{-3}, \tag{20}$$

$$\sqrt{|\mathrm{Im}[(\delta^d_{12})_{LL}]^2|} \lesssim 3\times 10^{-3}, \qquad \sqrt{|\mathrm{Im}[(\delta^d_{12})_{LR}]^2|} \lesssim 3\times 10^{-4}. \tag{21}$$

(4) The mass insertion $(\delta^u_{12})_{AB}$ are constrained by the $\Delta M_D$ as follows [12]:

$$|(\delta^u_{12})_{LL}| \lesssim 1.7\times 10^{-2}, \qquad |(\delta^u_{12})_{LR}| \lesssim 2.4\times 10^{-2}. \tag{22}$$

Here, three comments are in order. (i) Because of the Hermiticity of the $LL$ sector in the sfermion mass matrix, $(\delta^f_{AB})_{LL} = (\delta^f_{AB})^\dagger_{LL} = (\delta^f_{BA})^*_{LL}$, where $A$, $B = 1, 1, 2, 3$. (ii) The above constraints imposed on the mass insertions $(\delta^{q,l}_{AB})_{LL,LR}$ are derived from supersymmetric contributions through exchange of gluino or neutralino which preserves chirality, therefore the same constraints are also imposed on the mass insertions $(\delta^{q,l}_{AB})_{RR,RL}$. (iii) The mass insertions $(\delta^f_{AB})_{LR(RL)}$ are not, in general, related to the mass insertions $(\delta^f_{BA})_{LR(RL)}$. Taking the above constraints into account, one finds that the dominant contribution to the $\tau^- \to u\bar{s}\nu_\tau$ is given in terms of $(\delta^\nu_{32})_{LR}$, $(\delta^\nu_{32})_{RL}$, $(\delta^d_{21})_{RL}$, and $(\delta^u_{21})_{LR}$. Notice that the effective Hamiltonian (Eq. (8)) derived in this section can induce supersymmetric effects in all the $|\Delta S| = 1$ exclusive $\tau$ lepton decay. In the following section we consider two examples.

## IV. CONSTRAINTS FROM TWO-BODY $\tau$ DECAYS

In this section we analyze the possible constraints that may be imposed on the SUSY contributions to $\tau \to \bar{s}u\nu_\tau$ from the exclusive $|\Delta S| = 1$ decay: $\tau^-(p) \to K^-(q)\nu_\tau(p')$. The decay amplitude considering effects of SUSY contributions reads:

$$\mathcal{A}(\tau^- \to K^-\nu_\tau) = \mathcal{A}_{\mathrm{SM}} + \mathcal{A}_{\mathrm{SUSY}}, \tag{23}$$

which can be written explicitly as:

$$\mathcal{A}(\tau^- \to K^-\nu_\tau) = i\frac{G_F}{\sqrt{2}} V_{\mathrm{us}} f_K m_\tau (\bar{\nu}(p')R\tau(p)) \times \{1 + \delta_{\mathrm{SUSY}}(\tau)\}. \tag{24}$$

The decay $K^- \to \mu^-\nu$, which fixes $f_K$ in the absence of new physics, would also be modified by the effects of new physics:

$$\mathcal{A}(K^- \to \mu^-\nu) = i\frac{G_F}{\sqrt{2}} V_{\mathrm{us}} f_K m_\mu (\bar{\nu}(p')R\mu(p)) \times \{1 + \delta_{\mathrm{SUSY}}(K)\}. \tag{25}$$

In order to estimate the size of SUSY contributions in such decays, one defines the ratio:

$$R_{\tau/K} = \frac{\Gamma(\tau \to K\nu\tau(\gamma))}{\Gamma(K \to \mu\nu(\gamma))} = \frac{m_\tau^3}{2m_K m_\mu^2}\cdot\frac{(1-\frac{m_K^2}{m_\tau^2})^2}{(1-\frac{m_\mu^2}{m_K^2})^2}.(1+\delta R_{\tau/K}) \times [1 + 2\,\mathrm{Re}(\delta_{\mathrm{SUSY}}(\tau) - \delta_{\mathrm{SUSY}}(K))].$$

In the above equation $(\gamma)$ means that complete SM radiative corrections of $O(\alpha)$ have been included. The long-distance radiative corrections, which do not cancel in the above ratio, were computed in [13] and read $\delta R_{\tau/K} = (0.90^{+0.17}_{-0.26})\%$. Using the experimental rates for the involved decays: $\Gamma(\tau \to K\nu) = (2.36 \pm 0.08)\times 10^{10} s^{-1}$, $\Gamma(K \to \mu\nu) = (0.5118 \pm 0.0018)\times 10^8 s^{-1}$ [7] into Eq. (21) we get:

$$\mathrm{Re}[\delta_{\mathrm{SUSY}}(K) - \delta_{\mathrm{SUSY}}(\tau)] = 0.02 \pm 0.03. \tag{26}$$

This equation is actually a model-independent result. Using the expression for the Wilson coefficient induced by SUSY, it is easy to see that the dominant contribution to $\mathrm{Re}(\delta_{\mathrm{SUSY}}(\tau) - \delta_{\mathrm{SUSY}}(K))$ comes from $C_1^{\mathrm{SUSY}}$. In terms of the SUSY parameters, it can be translated as follows

$$\mathrm{Re}[\delta_{\mathrm{SUSY}}(K) - \delta_{\mathrm{SUSY}}(\tau)] \simeq \mathrm{Re}[C_1^{\mathrm{SUSY}}(K) - C_1^{\mathrm{SUSY}}(\tau)], \tag{27}$$

where $C_1^{\mathrm{SUSY}}(K,\tau)$ are, respectively, the Wilson coefficients corresponding to the operators responsible for $K \to \mu\nu_\mu$ and $\tau \to K\pi\nu_\tau$. Using as input parameters $M_1 = 100$, $M_2 = 200$ GeV and $\mu = M_{\tilde{q}} = 400$ GeV and $\tan\beta \simeq 20$, one gets

$$\mathrm{Re}[C_1^{\mathrm{SUSY}}(K) - C_1^{\mathrm{SUSY}}(\tau)] \simeq -0.03(\delta^u_{21})_{LL} - 0.006(\delta^l_{23})_{LL}, \tag{28}$$

where we keep the dominant contributions only. Using the bounds on the $\delta$'s given in the previous section, the term which is mostly unconstraint is $(\delta^l_{23})_{LL}$. It is clear that still we are far from the experimental limit given in (26) but it is important to notice that experimental data should strongly improve in a close future [14].

## V. SUSY CONTRIBUTION TO *CP* ASYMMETRY IN $\tau \to K\pi\nu_\tau$

Having analyzed the constraints from two-body $\tau$ decays imposed on the supersymmetric contributions to $\tau \to \bar{s}u\nu_\tau$ transition, now we can study the supersymmetric effects on the $CP$ violation in the three-body decay $\tau \to K\pi\nu_\tau$. We will show that although supersymmetry enhances the asymmetry of this process by many more orders of magnitude than the SM expectation, the resulting $CP$ asymmetries are still smaller than the current experimental reaches.

Given the spin-parity properties of the $K\pi$ system, we can write the total amplitude (SM and SUSY) of the $\tau(p) \to K(q)\pi(q')\nu_\tau(p')$ decay as

$$\mathcal{A}_T(\tau \to K\pi\nu) = \frac{G_F V_{us}}{\sqrt{2}}[(1 + C_1)\langle K\pi|\bar{s}\gamma_\mu u|0\rangle\bar{\nu}(p') \times \gamma^\mu L\tau(p) + (C_3 + C_4) \times \langle K\pi|\bar{s}u|0\rangle\bar{\nu}(p')R\tau(p) + C_5\langle K\pi|\bar{s}\sigma_{\mu\nu}u|0\rangle\bar{\nu}(p')\sigma^{\mu\nu}R\tau(p)], \quad (29)$$

where $C_i$ stand for $C_i^{\rm SUSY}$, since the $C_i^{\rm SM}$ are explicitly included. It is now clear that the resulting $CP$ asymmetry depends on the relative ratio among the SUSY Wilson coefficients. For example, in the case that $C_1$ is giving the dominant SUSY contribution and $C_{3,4,5}$ effects can be negligible, then the $CP$ asymmetry of $\tau \to K\pi\nu_\tau$ will vanish identically as in the SM. We consider the following two interesting scenarios:

(i) The case of $C_3$ or $C_4$ gives relevant contributions while $C_5$ is negligible. In this case, SUSY induces a relative weak phase between the vector and scalar form factors describing this process.

(ii) The case of $C_5$ gives relevant contributions while $C_{3,4}$ are negligible. In this case, SUSY induces a relative weak phase between the vector and tensor form factors.

Let us start by analyzing the $CP$ asymmetry in the first scenario. Using the definition of the hadronic matrix element introduced in Eq. (3):

$$\langle K\pi|\bar{s}\gamma_\mu u|0\rangle = C_K\{f_V(t)Q_\mu + f_S(t)(q + q')_\mu\}, \quad (30)$$

we can obtain the hadronic matrix element of the scalar current by taking the divergence in the usual form:

$$\langle K\pi|\bar{s}u|0\rangle = \frac{C_K t}{m_s - m_u} f_S(t), \quad (31)$$

where $m_{s,u}$ denote $s$, $u$ current quark masses. Thus, we finally get the amplitude:

$$\mathcal{A}_T(\tau \to K\pi\nu) = \frac{C_K G_F V_{us}}{\sqrt{2}}(1 + C_1)\Big\{f_V Q^\mu \bar{u}(p')\gamma_\mu L u(p) + \left[m_\tau + \left(\frac{C_3 + C_4}{1 + C_1}\right)\frac{t}{m_s - m_u}\right] \times f_S \bar{u}(p')R u(p)\Big\}.$$

It is remarkable that in this case, SUSY effects just modify the normalization of the SM amplitude and the relative size of the vector and scalar contributions.

When we compare this expression with the decay amplitude given in Eq. (2) of Ref. [3]:

$$\mathcal{A}(\tau^- \to K\pi\nu_\tau) \sim \bar{u}(p')\gamma_\mu L u(p) f_V Q^\mu + \Lambda\bar{u}(p')R u(p) f_S M, \quad (32)$$

where $M = 1$ GeV is a normalization mass scale, we obtain the relation

$$\Lambda M = m_\tau + \left(\frac{C_3 + C_4}{1 + C_1}\right)\frac{t}{m_s - m_u}. \quad (33)$$

The first term in the right-hand side (r.h.s.) of this equation is the usual contribution of the SM, which is real, and the second term arises from the SUSY contributions and contains a $CP$ violating phase, hence it can generate a nonvanishing $CP$ asymmetry.

The square of the matrix element becomes:

$$\sum_{\rm pols}|\mathcal{A}|^2 \sim |f_V|^2(2p.Qp'.Q - p.p'Q^2) + |\Lambda|^2|f_S(t)|^2 M^2 p.p' + 2\,{\rm Re}\Lambda \cdot {\rm Re}(f_S f_V^*)M m_\tau p'.Q - 2\,{\rm Im}\Lambda \cdot {\rm Im}(f_S f_V^*)M m_\tau p'.Q. \quad (34)$$

The last term in the previous equation is odd under a $CP$ transformation but we should notice that the last two terms disappear once we integrate on the kinematical variable $u$ of the process in consideration. This means that it is not possible to generate a $CP$ asymmetry in total decay rates corresponding to this process. So the only way to generate a $CP$ asymmetry is to look for the double differential distribution $(d^2\Gamma/dudt)$ or a variant of it as CLEO Collaboration did in Ref. [3]. The CLEO Collaboration has recently studied the ratio of $CP$-odd to $CP$-even terms of this squared amplitude for $\tau^\pm \to K_S\pi^\pm\nu_\tau$ decays and has obtained the following bound: $-0.172 < {\rm Im}(\Lambda) < 0.067$ at 90% C.L. Using Eq. (33), we can translate this bound into:

$$-0.010 \leq {\rm Im}\left(\frac{C_3 + C_4}{1 + C_1}\right) \leq 0.004, \quad (35)$$

where we have used $m_s - m_u = 100$ MeV, and the average value $\langle t\rangle \approx (1332.8\ {\rm MeV})^2$. Now, as input parameters $M_1 = 100$ and $M_2 = 200$ GeV and $\mu = M_{\tilde{q}} = 400$ GeV and $\tan\beta = 20$, one gets

$${\rm Im}\left(\frac{C_3 + C_4}{1 + C_1}\right) \simeq 1.3 \times 10^{-5}\,{\rm Im}(\delta_{21}^d)_{RL}. \quad (36)$$

Still the experimental bound is too loose to give us information on $(\delta_{21}^d)_{RL}$. Notice however that forthcoming measurements of the $CP$ asymmetry in the $K_S\pi$ channel will be significantly improved at $B$ factories [14] since their data sample is larger by 2 orders of magnitude than the one used by CLEO [3] in their analysis.

Now, we turn to take into consideration the $O_5$ operator which is naturally induced by SUSY corrections to Wilson coefficients. This operator could interfere with the $O_1$

operator which contains SM contributions and the strong phases. So in principle, using this interference between $O_5$ and $O_1$, it should be possible to generate a $CP$ asymmetry directly in the total decay rate which is completely forbidden in SM. Let us keep the dominant contribution to $\tau^- \to (K\pi)^- \nu_\tau$ and the $O_5$ contribution:

$$\mathcal{A}_T(\tau \to K\pi\nu) = \frac{G_F V_{\rm us}}{\sqrt{2}}(1+C_1)\bigg\{f_V(t)Q_\mu \bar{u}(p')\gamma^\mu L u(p) + \frac{C_5}{1+C_1}\langle K\pi|\bar{s}\sigma_{\mu\nu}u|0\rangle \bar{u}(p')\sigma^{\mu\nu} R u(p)\bigg\}. \tag{37}$$

In this expression, we have neglected the $f_S$ effect since its contribution to total decay rate is numerically small (around 3% at most, see [8]) and conserves $CP$. The most general form of the antisymmetric matrix element of the hadronic tensor current is given by

$$\langle K\pi|\bar{s}\sigma_{\mu\nu}u|0\rangle = \frac{ia}{m_K}[(p_\pi)^\mu (p_K)^\nu - (p_\pi)^\nu (p_K)^\mu], \tag{38}$$

where $a$ is a dimensionless quantity which fixes the scale of the hadronic matrix element. It is important to remember that $f_V(t)$ contains the strong phases as it can be parametrized [8]:

$$f_V(t) = \frac{f_V(0) m_{K^*}^2}{m_{K^*}^2 - t - i m_{K^*}\Gamma_{K^*}}. \tag{39}$$

The tensor form factor which is given by

$$f_T = \frac{aC_5}{1+C_1} \tag{40}$$

has no strong phases but of course could have a weak phase (arises either from $C_1$ or $C_5$) and it can be at most a slightly varying function of $t$. So, one can compute now the $CP$ asymmetry in the total decay rate:

$$a_{CP} = \frac{\Gamma(\tau^- \to K^-\pi^0\nu_\tau) - \Gamma(\tau^+ \to K^+\pi^0\nu_\tau)}{\Gamma(\tau^- \to K^-\pi^0\nu_\tau) + \Gamma(\tau^+ \to K^+\pi^0\nu_\tau)} \tag{41}$$

$$= \frac{a}{\Gamma_{\rm SM}} {\rm Im}C_5 \frac{-G_F^2|V_{\rm us}|^2}{128\pi^3 m_\tau^2 m_K} \int_{(m_K+m_\pi)^2}^{m_\tau^2} dt \frac{(m_\tau^2 - t)}{t^2} \times {\rm Im}(f_V(t))\lambda^{1/2}\{m_\pi^2(t+\Delta^2)^2 + m_\tau^2\lambda + [(t-m_\pi^2)^2 - m_K^2(t+m_\pi^2)](t-\Delta^2)\}, \tag{42}$$

where $\lambda$, $\Delta$, and $\Gamma_{\rm SM}$ are given in Sec. II. Integrating numerically on $t$, one gets

$$a_{CP} \simeq \frac{a}{2}{\rm Im}C_5 \tag{43}$$

$$\simeq 1.4\times 10^{-7} a\,{\rm Im}(\delta_{21}^u)_{LR}, \tag{44}$$

where to get the last equation, we use the same SUSY parameters as before. Again within SUSY extensions of the SM, this $CP$ asymmetry is small (even if it is practically 5 orders of magnitude bigger than the one expected in SM [6]). Clearly the observation of a $CP$ asymmetry in this channel at a range bigger than $10^{-6}$ will be not only a clear evidence of physics beyond the standard model but also an evidence we need physics beyond supersymmetric extensions of the standard model.

## VI. CONCLUSION

In summary, in this paper we have computed the effective Hamiltonian derived from SUSY for $|\Delta S| = 1$ tau lepton decays using the mass insertion approximation. Although experimental data for such decays are not precise enough at the present to give constraints on the fundamental parameters of SUSY, we have shown how physics beyond the standard model as supersymmetric extensions of the SM could induce $CP$ violating asymmetry in the double differential distribution as the CLEO Collaboration did in Ref. [3] and could also induce $CP$ asymmetry in total decay rate due to interference between $O_5$ and $O_1$ operators. We have argued that any $CP$ asymmetry in the channel under consideration bigger than $10^{-6}$ will be a clear evidence of not only physics beyond the standard model but also an evidence of physics beyond SUSY extensions of the SM. We also provided a model-independent constraint on the new physics contribution to $\tau \to K\nu$. In particular, it is interesting to observe that SUSY can provide a specific mechanism to generate a $CP$ violating term in the probability distribution of $\tau \to K\pi\nu_\tau$ decays. Forthcoming and more precise data for observables of the exclusive processes considered in this paper will either provide better constraints on SUSY parameters or give a mechanism to explain discrepancies with the SM if they are observed.

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## APPENDIX: SUSY CONTRIBUTIONS TO WILSON COEFFICIENTS OF $\tau^- \to s\bar{u}\nu_\tau$

Here we provide the complete expressions for the supersymmetric contributions, at leading order in MIA, for the Wilson coefficients of $\tau^- \to s\bar{u}\nu_\tau$ transition, $C_i(M_W)$, $i = 1, \ldots, 5$. As mentioned in Sec. III, the dominant SUSY contributions are given by chargino-neutralino box diagram exchanges, as illustrated in Fig. 2.

The effective Hamiltonian $H_{\rm eff}$ derived from SUSY can be expressed as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{\rm us} \sum_i C_i(\mu) Q_i(\mu), \tag{A1}$$

$$= \sum_i \tilde{C}_i(\mu) Q_i(\mu), \tag{A2}$$

where $C_i$ are the dimensionless Wilson coefficients and $Q_i$ are the relevant local operators at the low-energy scale $\mu \simeq m_\tau$. The operators are given by

$$Q_1 = (\bar{\nu}\gamma^\mu L\tau)(\bar{s}\gamma_\mu Lu), \tag{A3}$$

$$Q_2 = (\bar{\nu}\gamma_\mu L\tau)(\bar{s}\gamma_\mu Ru), \tag{A4}$$

$$Q_3 = (\bar{\nu}R\tau)(\bar{s}Lu), \tag{A5}$$

$$Q_4 = (\bar{\nu}R\tau)(\bar{s}Ru), \tag{A6}$$

$$Q_5 = (\bar{\nu}\sigma_{\mu\nu}R\tau)(\bar{s}\sigma^{\mu\nu}Ru). \tag{A7}$$

In terms of the vertex, one can write the complete vertex as a product of the vertex coming from the leptonic sector and of the vertex coming from the hadronic sector. In this respect we can also write the Wilson coefficients as

$$\tilde{C}_i = C^l_{i(\tau-\chi^-)}(C^q_{i(\tau-\chi^- -s)} + C^q_{i((\tau-\chi^- -u)}) + C^l_{i(\tau-\chi^0)}(C^q_{i(\tau-\chi^0-s)} + C^q_{i((\tau-\chi^0-u)}),$$

where the $C^l_i$ is due to the leptonic vertex and $C^q_i$ is from the quark sectors. If we expand $C^{l,q}_i$ in terms of the mass insertions, one finds that the leading contributions are given by

$$\begin{aligned}\tilde{C}_1 = {}& C^{l(0)}_{1(\tau-\chi^-)}(C^{q(0)}_{1(\tau-\chi^- -s)}\tilde{I}(x_i,x_j) + C^{q(0)}_{1((\tau-\chi^- -u)}I(x_i,x_j)) + C^{l(0)}_{1(\tau-\chi^0)}(C^{q(0)}_{1(\tau-\chi^0-s)}I(x_i,x_j) + C^{q(0)}_{1(\tau-\chi^0-u)}\tilde{I}(x_i,x_j))\\ &+ C^{l(0)}_{1(\tau-\chi^-)}(C^{q(1)}_{1(\tau-\chi^- -s)}\tilde{I}_n(x_i,x_j) + C^{q(1)}_{1(\tau-\chi^- -u)}I_n(x_i,x_j)) + C^{l(0)}_{1(\tau-\chi^0)}(C^{q(1)}_{1(\tau-\chi^0-s)}I_n(x_i,x_j) + C^{q(1)}_{1(\tau-\chi^0-u)}\tilde{I}_n(x_i,x_j)\\ &+ C^{l(1)}_{1(\tau-\chi^-)}(C^{q(0)}_{1(\tau-\chi^- -s)}\tilde{I}_n(x_i,x_j) + C^{q(0)}_{1(\tau-\chi^- -u)}I_n(x_i,x_j)) + C^{l(1)}_{1(\tau-\chi^0)}(C^{q(0)}_{1(\tau-\chi^0-s)}I_n(x_i,x_j) + C^{q(0)}_{1(\tau-\chi^0-u)}\tilde{I}_n(x_i,x_j))\\ &+ O(\delta^2).\end{aligned} \tag{A8}$$

With

$$C^{l(0)}_{1(\tau-\chi^0)} = \frac{-g^2}{\sqrt{2}}(N^*_{i2} + \tan\theta_w N^*_{i1})U^*_{j1}(U^*_{\rm MNS})_{33} - y^2_\tau N^*_{i3}U^*_{j2}(U^*_{\rm MNS})_{33}, \tag{A9}$$

$$C^{q(0)}_{1(\tau-\chi^0-u)} = \left(\frac{1}{8}\right)\frac{g^2}{\sqrt{2}}\left(N^*_{i2} + \frac{1}{3}\tan\theta_w N^*_{i1}\right)(V^*_{\rm CKM})_{12}V_{j1}, \tag{A10}$$

$$C^{q(0)}_{1(\tau-\chi^0-s)} = \left(\frac{1}{16}\right)\frac{-g^2}{\sqrt{2}}\left(N_{i2} - \frac{1}{3}\tan\theta_w N_{i1}\right)U^*_{j1}(V^*_{CKM})_{12}, \tag{A11}$$

$$\begin{aligned}C^{l(1)}_{1(\tau-\chi^0)} = {}& -\frac{g^2}{\sqrt{2}}(N^*_{i2} + \tan\theta_w N^*_{i1})U^*_{j1}(U^*_{\rm MNS})_{a3}(\delta^l_{LL})_{a3}\\ &+ \frac{g}{\sqrt{2}}(N^*_{i2} + \tan\theta_w N^*_{i1})U^*_{j2}(U^*_{\rm MNS})_{33}(h_e)_{33}(\delta_{RL})_{33}\\ &+ g(h_e)_{33}N^*_{i3}U^*_{j1}(\delta^l_{LR})_{a3}(U^*_{\rm MNS})_{a3}\\ &- (h_e)^2_{33}N^*_{i3}U^*_{j2}(\delta^l_{RR})_{33}(U^*_{\rm MNS})_{33},\end{aligned} \tag{A12}$$

and

$$\begin{aligned}C^{q(1)}_{1(\tau-\chi^0-s)} = {}& \left(\frac{1}{16}\right)\left(\frac{-g^2}{\sqrt{2}}\left(N_{i2} - \frac{1}{3}\tan\theta_w N_{i1}\right)U^*_{j1}(V^*_{\rm CKM})_{1a}\right.\\ &\times(\delta^d_{LL})_{2a} + \frac{g}{\sqrt{2}}\left(N_{i2} - \frac{1}{3}\tan\theta_w N_{i1}\right)\\ &\left.\times U^*_{j2}(V^*_{\rm CKM})_{13}(h_d)_{33}(\delta^d_{LR})_{23}\right),\end{aligned} \tag{A13}$$

$$\begin{aligned}C^{q(1)}_{1(\tau-\chi^0-u)} = {}& \left(\frac{1}{8}\right)\left(\frac{g^2}{\sqrt{2}}\left(N^*_{i2} + \frac{1}{3}\tan\theta_W N^*_{i1}\right)\right.\\ &\times(V^*_{\rm CKM})_{a2}(\delta^u_{LL})_{a1}V_{j1}\\ &- \frac{g}{\sqrt{2}}\left(N^*_{i2} + \frac{1}{3}\tan\theta_W N^*_{i1}\right)\\ &\left.\times(h_u)_{33}V_{j2}(V^*_{\rm CKM})_{32}(\delta^u_{RL})_{31}\right).\end{aligned} \tag{A14}$$

$$C^{l(0)}_{1(\tau-\chi^-)} = \frac{g^2}{\sqrt{2}}V^*_{j1}(U^*_{\rm MNS})_{33}(N_{i2} - \tan\theta_w N_{i1}) - (h_\nu)^2_{33}V^*_{j2}(U^*_{\rm MNS})_{33}N_{i4}, \tag{A15}$$

$$C^{q(0)}_{1(\tau-\chi^- -u)} = \left(\frac{1}{16}\right)\left(\frac{g^2}{\sqrt{2}}V_{j1}\left(N^*_{i2} + \frac{1}{3}\tan\theta_w N^*_{i1}\right)(V^*_{\rm CKM})_{12}\right), \tag{A16}$$

$$C^{q(0)}_{1((\tau-\chi^- -s)}=\Big(\frac{1}{8}\Big)\Big(-\frac{g^2}{\sqrt{2}}\Big(N_{i2}-\frac{1}{3}\tan\theta_w N_{i1}\Big)(V^*_{\rm CKM})_{12}U^*_{j1}\Big), \tag{A17}$$

$$\begin{aligned}C^{l(1)}_{1(\tau-\chi^-)}=&\frac{g^2}{\sqrt{2}}V^*_{j1}(U^*_{\rm MNS})_{3a}(N_{i2}-\tan\theta_w N_{i1})(\delta^\nu_{LL})_{3a}\\&+gV^*_{j1}(h_\nu)_{33}N_{i4}(\delta^\nu_{RL})_{3a}(U^*_{\rm MNS})_{3a}\\&-\frac{g}{\sqrt{2}}(N_{i2}-\tan\theta_w N_{i1})V^*_{j2}(h_\nu)_{aa}(U^*_{\rm MNS})_{3a}(\delta^\nu_{LR})_{3a}\\&-(h_\nu)_{33}N_{i4}V^*_{j2}(h_\nu)_{aa}(\delta^\nu_{RR})_{3a}(U^*_{\rm MNS})_{3a}.\end{aligned} \tag{A18}$$

In the case of decoupling of the sneutrino-right, the last terms are strongly suppressed,

$$\begin{aligned}C^{q(1)}_{1(\tau-\chi^- -u)}=&\Big(\frac{1}{16}\Big)\Big(\frac{g^2}{\sqrt{2}}V_{j1}\Big(N^*_{i2}+\frac{1}{3}\tan\theta_w N^*_{i1}\Big)\\&\times(V^*_{\rm CKM})_{a2}(\delta^u_{LL})_{a1}\\&-\frac{g}{\sqrt{2}}(h_u)_{33}V_{j2}(V^*_{\rm CKM})_{32}(\delta^u_{RL})_{31}\\&\times\Big(N^*_{i2}+\frac{1}{3}\tan\theta_w N^*_{i1}\Big)\Big),\end{aligned} \tag{A19}$$

$$\begin{aligned}C^{q(1)}_{1(\tau-\chi^- -s)}=&\Big(\frac{1}{8}\Big)\Big(-\frac{g^2}{\sqrt{2}}\Big(N_{i2}-\frac{1}{3}\tan\theta_w N_{i1}\Big)\\&\times(V^*_{\rm CKM})_{1a}U^*_{j1}(\delta^d_{LL})_{2a}\\&+\frac{g}{\sqrt{2}}\Big(N_{i2}-\frac{1}{3}\tan\theta_w N_{i1}\Big)\\&\times(V^*_{\rm CKM})_{13}U^*_{j2}(h_d)_{33}(\delta^d_{LR})_{23}\Big).\end{aligned} \tag{A20}$$

The contribution to $C_2$ is found to vanish identically, i.e.,

$$\tilde{C}_2=0, \tag{A21}$$

$$\begin{aligned}\tilde{C}_3=&C^{l(0)}_{3(\tau-\chi^-)}C^{q(1)}_{3(\tau-\chi^- -s)}I_n(x_i,x_j)\\&+C^{l(0)}_{3(\tau-\chi^0)}C^{q(1)}_{3(\tau-\chi^0-s)}I_n(x_i,x_j)+O(\delta^2),\end{aligned} \tag{A22}$$

where $I_n(x_i,x_j)$ is defined below and $x_i=m^2_{\chi^\pm_i}/\tilde{m}^2$ and $x_j=m^2_{\chi^0_j}/\tilde{m}^2$.

$$C^{l(0)}_{3(\tau-\chi^0)}=g(h_e)_{33}N_{i3}U^*_{j1}(U^*_{\rm MNS})_{33} \tag{A23}$$

$$-g\sqrt{2}\tan\theta_w N_{i1}U^*_{j2}(h_e)_{33}(U^*_{\rm MNS})_{33}, \tag{A24}$$

$$C^{l(0)}_{3(\tau-\chi^-)}=-(h_e)_{33}\frac{g}{\sqrt{2}}(N_{i2}-\tan\theta_w N_{i1})U_{j2}(U^*_{\rm MNS})_{33}, \tag{A25}$$

$$\begin{aligned}C^{q(1)}_{3(\tau-\chi^- -s)}=&\Big(-\frac{1}{8}\Big)\Big(\frac{g^2}{\sqrt{2}}\frac{2}{3}\tan\theta_w N^*_{i1}U^*_{j1}(V^*_{\rm CKM})_{1a}(\delta^d_{RL})_{2a}\\&-\frac{g}{\sqrt{2}}\frac{2}{3}\tan\theta_w N^*_{i1}U^*_{j2}(h_d)_{33}(V^*_{\rm CKM})_{13}(\delta^d_{RR})_{23}\Big),\end{aligned} \tag{A26}$$

$$\begin{aligned}C^{q(1)}_{3(\tau-\chi^0-s)}=&\Big(-\frac{1}{8}\Big)\Big(\frac{2}{3}\frac{g^2}{\sqrt{2}}\tan\theta_w U^*_{j1}N^*_{i1}(V^*_{\rm CKM})_{1a}(\delta^d_{RL})_{2a}\\&-\frac{2}{3}\frac{g}{\sqrt{2}}\tan\theta_w U^*_{j2}N^*_{i1}(V^*_{\rm CKM})_{13}(\delta^d_{RR})_{23}(h_d)_{33}\Big).\end{aligned} \tag{A27}$$

$$\begin{aligned}\tilde{C}_4=&C^{l(0)}_{4(\tau-\chi^-)}C^{q(1)}_{4(\tau-\chi^- -u)}\tilde{I}_n(x_i,x_j)\\&+C^{l(0)}_{4(\tau-\chi^0)}C^{q(1)}_{4((\tau-\chi^0-u)}\tilde{I}_n(x_i,x_j)+O(\delta^2),\end{aligned} \tag{A28}$$

where $\tilde{I}_n(x_i,x_j)$ is given below:

$$\begin{aligned}C^{l(0)}_{4(\tau-\chi^-)}=&-(h_e)_{33}\frac{g}{\sqrt{2}}(N_{i2}-\tan\theta_w N_{i1})U_{j2}(U^*_{\rm MNS})_{33}\\=&C^{l(0)}_{3(\tau-\chi^-)},\end{aligned} \tag{A29}$$

$$\begin{aligned}C^{l(0)}_{4(\tau-\chi^0)}=&g(h_e)_{33}N_{i3}U^*_{j1}(U^*_{\rm MNS})_{33}\\&-g\sqrt{2}\tan\theta_w N_{i1}U^*_{j2}(h_e)_{33}(U^*_{\rm MNS})_{33},\end{aligned} \tag{A30}$$

$$\begin{aligned}C^{q(1)}_{4(\tau-\chi^0-u)}=&\Big(-\frac{1}{8}\Big)\Big(-\frac{4}{3}\frac{g^2}{\sqrt{2}}\tan\theta_w N_{i1}V_{j1}(V^*_{\rm CKM})_{a2}(\delta^u_{LR})_{a1}\\&+\frac{4}{3}\frac{g}{\sqrt{2}}\tan\theta_w N_{i1}V_{j2}(h_u)_{33}(V^*_{\rm CKM})_{32}(\delta^u_{RR})_{31}\Big),\end{aligned} \tag{A31}$$

$$\begin{aligned}C^{q(1)}_{4(\tau-\chi^- -u)}=&\Big(-\frac{1}{8}\Big)\Big(-\frac{4}{3}\frac{g^2}{\sqrt{2}}\tan\theta_w V_{j1}N_{i1}(V^*_{\rm CKM})_{a2}(\delta^u_{LR})_{a1}\\&+\frac{4}{3}\frac{g}{\sqrt{2}}\tan\theta_w V_{j2}N_{i1}(h_u)_{33}(V^*_{\rm CKM})_{32}(\delta^u_{RR})_{31}\Big).\end{aligned} \tag{A32}$$

$$\tilde{C}_{5(\tau-\chi^0-u)}=-\tfrac{1}{4}\tilde{C}_{4(\tau-\chi^0-u)}, \tag{A33}$$

$$\tilde{C}_{5(\tau-\chi^- -u)}=\tfrac{1}{4}\tilde{C}_{4(\tau-\chi^- -u)}. \tag{A34}$$

The loop integrals $I_n(x_i,x_j)$ and $\tilde{I}_n(x_i,x_j)$ are defined as follows:

$$
\begin{aligned}
I(x_i, x_j) &= \frac{1}{16\pi^2 \tilde{m}^2}\left(\frac{1}{x_i - x_j}\right)\left(\frac{x_i^2 - x_i - x_i^2 \log x_i}{(1 - x_i)^2} - (x_i \leftrightarrow x_j)\right),\\
\tilde{I}(x_i, x_j) &= \frac{\sqrt{x_i x_j}}{16\pi^2 \tilde{m}^2}\left(\frac{1}{x_i - x_j}\right)\left(\frac{x_i^2 - x_i - x_i \log x_i}{(1 - x_i)^2} - (x_i \leftrightarrow x_j)\right),\\
I_n(x_i, x_j) &= \frac{1}{32\pi^2 \tilde{m}^2}\left(\frac{1}{x_i - x_j}\right)\left(\frac{2x_i^2 - 2x_i - 2x_i \log x_i}{(x_i - 1)^2} - \frac{x_i^3 - 4x_i^2 + 3x_i + 2x_i \log x_i}{(x_i - 1)^3} - (x_i \leftrightarrow x_j)\right),\\
\tilde{I}_n(x_i, x_j) &= \frac{-\sqrt{x_i x_j}}{32\pi^2 \tilde{m}^2}\left(\frac{1}{x_i - x_j}\right)\left(\frac{x_i^3 - 4x_i^2 + 3x_i + 2x_i \log x_i}{(x_i - 1)^3} - (x_i \leftrightarrow x_j)\right).
\end{aligned} \tag{A35}
$$


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