Top partners in little Higgs theories with *T* parity

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We consider a class of little Higgs theories with *T*-parity where *all* new particles responsible for canceling the standard model contributions to the one-loop quadratic divergences in the Higgs potential are odd under *T*-parity, including the heavy top partner which was previously taken to be *T*-even. The new construction significantly simplifies the spectrum in the top sector and completely changes the phenomenology of the top partner. At hadron colliders the signals of this class of *T*-invariant models appear to be even more similar to supersymmetry. We initiate a study on the collider phenomenology and discuss the challenge of distinguishing this class of models from supersymmetry at the Large Hadron Collider.

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I. INTRODUCTION

In the past few years a new approach to the electroweak symmetry breaking which is also weakly-coupled at the TeV scale, dubbed little Higgs theories, was proposed in Ref. [1]. In these theories the Higgs field arises as a pseudo-Nambu-Goldstone boson (PNGB). The low energy effective theory is described by a nonlinear sigma model with the peculiar property that the quadratically divergent contributions to the Higgs mass parameter are absent at one loop. This allows the cutoff of the effective theory to be raised to ~10 TeV without introducing fine-tunings in the Higgs potential. A cutoff at 10 TeV, on the other hand, is just high enough to suppress the noncalculable higher dimensional operators at the cutoff scale [2,3] in order to avoid the "little hierarchy problem."

There are several different types of little Higgs theories [4–14], as well as some examples of UV extensions above 10 TeV [15–18], and extensive phenomenological studies have been performed [19–35]. Despite many variations, the generic spectrum of little Higgs theories has new scalars, new fermions, and new gauge bosons at around 1 TeV to cancel the quadratically divergent contributions to the Higgs mass-squared which come from the standard model particles. The contributions of these new particles to the electroweak observables, which are calculable within the low-energy effective theory, turn out to be quite model-dependent. Generically tree-level corrections to the electroweak observables would require raising the mass of the new particles to be much higher than 1 TeV, thus reintroducing fine-tunings in the Higgs potential [32]. However, it was realized that these tree-level contributions can be eliminated in a systematic way in many little Higgs models by introducing a *T*-parity, under which standard model particles are even and the new heavy scalars and gauge bosons are odd [12–14]. The leading corrections to electroweak observables are then loop suppressed so that models with *T*-parity can be made consistent with the electroweak constraints without raising the mass of the new particles.

However, in the previous constructions of *T*-parity, the top sector is quite complicated. There are more than one new colored fermions at or below the TeV scale with both even and odd *T*-parities. In particular the heavy top partner which cancels the top-quark quadratic divergence is *T*-even. If this assignment is inevitable, then the new top partner can be a smoking gun in distinguishing little Higgs theories from other scenarios such as *R*-parity conserving supersymmetry (SUSY), and previous studies on the phenomenology in the top sector of little Higgs theories without *T*-parity, which rely on the single production of the heavy top partner [29,30,35], can still be applied to models with *T*-parity. In this paper we show that in fact there is a way to implement the *T*-parity in the top sector such that only one top partner is required to be ≲ TeV and it is odd under *T*-parity; the spectrum in the top sector is simplified. As a result, every standard model contribution to the one-loop quadratic divergence of the Higgs mass is canceled by that of a new particle with the same spin but opposite *T*-parity.

The introduction of *T*-parity has drastic effects on the phenomenology of little Higgs theories [13,31,33,36]. The *T*-odd particles have to be pair-produced in collider experiments and the lightest *T*-odd particle (LTP) is stable. If the LTP is neutral, it can be a good dark matter candidate and also gives rise to missing energy signals in colliders. Now with the *T*-odd top partner, the searches for the top partner based on single productions as well as decays into *Zt*, *Wb* and *ht* [29,30] are no longer valid. The *T*-odd top partners now also need to be pair-produced and their decays to LTP's give rise to missing energies plus jets, similar to the decays of top squarks in SUSY. One can see that *T*-parity in little Higgs theories plays a similar role as the *R*-parity in supersymmetric theories and the *KK*-parity in universal extra dimensions (UEDs) [37–39]. This similarity makes it a serious challenge to distinguish these scenarios at a hadron collider if such signals are discovered, as the spin information of the new particles is difficult to obtain.

In the following we concentrate on the top sector of the little Higgs theories. In the next section we discuss the implementation of T-parity with a T-odd heavy top partner for generic little Higgs theories. In Sec. III we discuss the impact of a T-odd top partner on the electroweak corrections. In Sec. IV we study the collider phenomenology of the T-odd top partner at the Large Hadron Collider (LHC). Then we conclude, which is followed by an appendix exhibiting how to implement our construction in two popular examples of little Higgs theories.

II. *T*-ODD HEAVY TOP PARTNERS

The central idea of the little Higgs mechanism is the "collective symmetry breaking:" no single interaction in the Lagrangian breaks enough global symmetries under which the Higgs particle is an exact Nambu-Goldstone boson; only in the presence of at least two interactions does the Higgs become a PNGB and acquire a mass term suppressed by the product of the two coupling constants in front of the two interactions involved. If one can make sure that, under naive dimensional analysis [40], each coupling comes with a loop factor, then the leading quadratic divergence in the Higgs mass can only arise at the two-loop order. In actual model-building the collective breaking is implemented in the scalar, the gauge, and the top sector separately, since each particle in the same representation of a global symmetry must have the same spin. This is why the standard model contribution in each sector is canceled by that of a new particle with the same spin. There are already many different types of little Higgs models in the literature and the way to implement T-parity in models based on symmetric spaces has been discussed in Refs. [12–14]. In this paper we focus on the top sector and discuss a new and simpler way to implement the T-parity and its phenomenological consequences.

First let us review how the collective breaking in the top sector is achieved. To be specific, we illustrate it with $SU(3)$ global symmetries. A vectorlike pair of colored Weyl fermions, U and U^c, that are singlets under $SU(2)_W$ are introduced and couple to the Higgs scalar and the third generation quarks $q_3 = (d_3, u_3)$, $u_3^{c\prime}$ through interactions

$$\mathcal{L}_t = \lambda_1 f(d_3, u_3, U) V \begin{pmatrix} 0 \\ 0 \\ u_3^{c\prime} \end{pmatrix} + \lambda_2 f U^c U + \text{H.c.}, \quad (1)$$

where $V = V(\Sigma)$ is a function of the Goldstone bosons $\Sigma = \exp(2i\pi/f)$. Under $SU(2)_W$ the Goldstone bosons are decomposed into a Higgs doublet H, a triplet Φ and a singlet S,

$$\pi \sim \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi + S/\sqrt{6} & H \\ H^\dagger & -2S/\sqrt{6} \end{pmatrix}. \quad (2)$$

Each λ_i interaction individually preserves enough global symmetry to keep the Higgs massless; only when both λ_1

and λ_2 are present does the Higgs acquire a mass. Here we follow the convention in Ref. [29] to use letters u or U for weak eigenstates and t or T for mass eigenstates. When expanding $\mathcal{L}_t$ to the first order in the Higgs doublet H, we have

$$\mathcal{L}_t = \lambda_1 u_3^{c\prime}(fU + \sqrt{2}Hq_3) + \lambda_2 fU^c U + \cdots. \quad (3)$$

We see that a linear combination of $u_3^{c\prime}$ and U^c marries U to become massive at $\sim \lambda f$, whereas the orthogonal combination remains massless and has a Yukawa coupling to the Higgs doublet. To check the cancellation of one-loop quadratic divergence explicitly, one needs to go to the quadratic order in the Higgs doublet [29,35]. Writing in terms of mass eigenstates,

$$t = u_3, \qquad t^c = \frac{\lambda_2 u_3^{c\prime} - \lambda_1 U^c}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \quad (4)$$

$$T = U, \qquad T^c = \frac{\lambda_1 u_3^{c\prime} + \lambda_2 U^c}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \quad (5)$$

the Lagrangian now becomes

$$\mathcal{L}_t = m_T T^c T + \lambda_t h t^c t + \lambda_T h T^c t - \frac{\lambda_T'}{2m_T} h^2 T^c T + \cdots, \quad (6)$$

where

$$m_T = \sqrt{\lambda_1^2 + \lambda_2^2} f, \qquad \lambda_t = \frac{\sqrt{2}\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}},$$

$$\lambda_T = \frac{\sqrt{2}\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \qquad \lambda_T' = 2\lambda_1^2, \quad (7)$$

and h is the neutral component in the Higgs doublet $H = (h^+ h)^T$. There are three diagrams as shown in Fig. 1 contributing to the quadratic divergence of the Higgs mass and they involve λ_t, λ_T, and λ_T', respectively. The cancellation among the three contributions is guaranteed by the relation

$$\lambda_T' = \lambda_t^2 + \lambda_T^2. \quad (8)$$

The relation provides an important test of the little Higgs mechanism.

Previous implementations of T-parity in the top sector simply add to the lagrangian the images of the original interactions under T-parity to make it T-invariant. Because of the $hT^c t$ interaction involved in the diagram (b) of Fig. 1, the heavy top partner necessarily has the same even T-parity as the top quark. In these constructions, there are additional T-odd colored fermions introduced to avoid large Higgs mass generated at higher loops [12–14].

Here we show that a simple modification of Eq. (1) could make the implementation of T-parity easier, and the quadratically divergent contribution to the Higgs mass from the
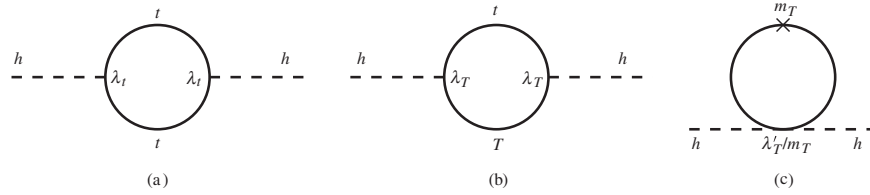
FIG. 1.   Cancellation of top quadratic divergences.

top quark loop can in fact be canceled by a T-odd top partner. We take the Lagrangian of the top sector to be

$$\mathcal{L}_t = \lambda f(d_3, u_3, T) V \begin{pmatrix} 0 \\ 0 \\ U_a^c \end{pmatrix} + \lambda f(d_3, u_3, -T) T[V]$$

$$\times \begin{pmatrix} 0 \\ 0 \\ U_b^c \end{pmatrix} + \text{H.c.}, \qquad (9)$$

where $T[V] \sim \Omega V^\dagger \Omega$, $\Omega = \text{diag}(1, 1, -1)$, is the T-image of V. The two terms are simply T-images of each other with U_a^c mapped to U_b^c and vice versa. Each term preserves enough global symmetry which protects the Higgs mass from being generated and the collective breaking is realized.

Expanding the Lagrangian (9) to quadratic order in the Higgs field we have

$$\mathcal{L}_t = \sqrt{2}\lambda f T T^c + 2\lambda h t t^c - \frac{\sqrt{2}\lambda}{f} h^2 T T^c + \cdots, \quad (10)$$

where

$$T^c = \frac{1}{\sqrt{2}}(U_a^c - U_b^c), \qquad t^c = \frac{1}{\sqrt{2}}(U_a^c + U_b^c) \qquad (11)$$

are odd and even under T-parity, respectively. In terms of the parameters in Eq. (6),

$$m_T = \sqrt{2}\lambda f, \qquad \lambda_t = 2\lambda, \qquad \lambda_T = 0, \qquad \lambda_T' = 4\lambda^2. \qquad (12)$$

One can see that the relation ensuring the cancellation of top quadratic divergences, Eq. (8), is satisfied, and the divergence is canceled by a vector-pair of T-odd fermions. Diagram (b) of Fig. 1 is now absent and the divergence is canceled between diagrams (a) and (c). In fact, this is similar to and inspired by how the top quadratic divergence is canceled in the "simple little Higgs" model [7].

In the above example, the one-loop quadratic divergence is canceled by introducing an $SU(2)_W$ singlet top partner to complete the left-handed top-bottom doublet into a triplet. One can easily see that it is also possible to cancel the quadratic divergence by completing the right-handed top quark into a triplet. In that case, the new T-odd partner will be an $SU(2)_W$ doublet instead. We will only consider the singlet case for simplicity.

We would like to emphasize that the structure of the couplings discussed here is quite general and model-independent. In fact, this construction can be applied to every little Higgs theory in the literature so far. For further demonstrations, we give explicit constructions of the new top sectors for the popular moose model and littlest Higgs model in the appendix.

III. ELECTROWEAK CORRECTIONS

So far we have concentrated on the top sector of the model. It will be useful to summarize the features and generic spectrum of little Higgs theories with T-parity before we go on to discuss the interactions of T-odd top partners, which are crucial to understand the electroweak corrections, as well as the collider phenomenology.

Generic little Higgs theories have at around 1 TeV new particles responsible for canceling the standard model quadratic divergences. They are a new set of heavy $SU(2) \times U(1)$ gauge bosons ($W_H^\pm$, W_H^0, B_H), new heavy scalars Φ, which could be triplet or singlet under $SU(2)_W$, and new fermions (T^c, T) that are partners of the top quark. It is shown, in Refs. [12–14] and the previous section, that these new particles can all be charged under the T-parity. In addition to the new particles involved in the cancellation of quadratic divergences, there is also a vectorlike T-odd doublet partner for every standard model fermionic doublet. (We will call them the "mirror fermions" though they are vectorlike and contain both chiralities.) These mirror fermions serve to cut off the contributions to the standard model four-fermion interactions from the Goldstone boson loop [14,33]. The four-lepton interactions are now strongly constrained by the LEP II data and the mirror leptons are required to be $\lesssim$ 2 TeV. The constraints on four-quark operators are weaker so the mirror quarks can be much heavier. Such heavy states may not be directly observed at the LHC, but they may induce higher dimensional operators of the light states, suppressed by their masses. So we include the effects of these mirror fermions in the discussions below.

As emphasized already, the motivation for T-parity is to eliminate leading order corrections to precision electroweak observables in little Higgs models. Without T-parity the scale of symmetry breaking f would generally be required to be higher than 3–4 TeV which reintroduces fine-tunings in the Higgs potential. With T-parity the electroweak corrections from T-odd particles are only loop-

induced. In Refs. [13,33] it is found that, in some region of the parameter space for the $SU(5)/SO(5)$ model, f only needs to be larger than 500 GeV in order to satisfy the precision electroweak measurements. The strongest constraints there come from corrections to the ρ parameter and $Z \to b\bar{b}$ vertex due to heavy gauge bosons as well as heavy top partners, but not from the triplet scalars since they are T-odd and not allowed to have a nonzero vacuum expectation value (VEV).

In the old way of implementing collective breaking in the top sector, the leading electroweak corrections from the heavy top partners originate from the λ_T term in the effective Lagrangian, Eq. (6), which gives a small mixing between t and T after the electroweak symmetry breaking when the Higgs gets a VEV: $\langle h \rangle = v/\sqrt{2}$. After diagonalizing the mass matrix, there is the physical top quark t_L and a heavy top partner T_H [21]

$$t_L = c_L t - s_L T, \qquad T_H = s_L t + c_L T; \qquad (13)$$

$$s_L = \frac{\lambda_T}{\sqrt{2}} \frac{v}{m_T} + \mathcal{O}(v^3/m_T^3), \quad c_L = 1 - \frac{1}{2}\left(\frac{\lambda_T}{\sqrt{2}} \frac{v}{m_T}\right)^2. \quad (14)$$

The important observation at this point is that, since (T, T^c) is a Dirac-pair of $SU(2)$ singlets, at leading order the heavy top partner T_H enters into precision electroweak observables only through the mixture with (t, t^c). Explicit calculations for the littlest Higgs theories with T-parity [13,33] showed that the corrections to the ρ parameter and $Z \to b\bar{b}$ vertex from the heavy top partners are, assuming $m_T \gg m_t$,

$$\delta\rho = \frac{3}{32\pi^2} \frac{e^2}{s_w^2 c_w^2} \frac{m_t^2}{m_Z^2} s_L^2 \left[\log\frac{m_T^2}{m_t^2} - 1 + \frac{1}{2}\left(\frac{\lambda_1}{\lambda_2}\right)^2\right], \quad (15)$$

$$\delta g_L^{b\bar{b}} = \frac{1}{8\pi} \frac{\alpha g}{c_w s_w^2} \frac{m_t^2}{m_W^2} s_L^2 \log\frac{m_T^2}{m_t^2}, \quad (16)$$

where $s_w^2 = 1 - c_w^2 = \sin^2\theta_w$ and θ_w is the Weinberg angle. Indeed, both equations above are proportional to the mixing parameter s_L.

In our new implementation of T-parity in the top sector, there is no mixing between t and T^c even after electroweak symmetry breaking since $\lambda_T = 0$ in Eq. (10), and hence $s_L = 0$. Moreover, this result holds to all orders in f and v as long as T-parity remains unbroken since (T, T^c) is odd whereas (t, t^c) is even. However, the T-odd top partners do contribute to $\delta\rho$ and $\delta g_L^{b\bar{b}}$ at higher orders, which generally involve the following four type of vertices, with the magnitude of their coefficients indicated up front,

$$\text{(I)} \quad \text{(I)} \quad \frac{v}{f}\{T^c\Phi^0 t, T^c\Phi^+ b\} \quad (17)$$

$$\text{(II)} \quad \text{(II)} \quad g\frac{v}{f}\{(T\bar{\sigma}_\mu t)W_H^{0\mu}, (T\bar{\sigma}_\mu b)W_H^{+\mu}\} \quad (18)$$

$$\text{(III)} \quad \text{(III)} \quad g\frac{v^2}{f^2}(T\bar{\sigma}_\mu T)W_L^{0\mu} \quad (19)$$

$$\text{(IV)} \quad \text{(IV)} \quad g'(T\bar{\sigma}_\mu T)B_L^\mu \quad (20)$$

Type (I) vertices arise from expanding the top sector Lagrangian (9) to order $1/f$, after the Higgs gets a VEV from electroweak symmetry breaking. Type (II) and (III) vertices can come from integrating out the mirror fermions as shown in Fig. 2(a) and 2(b), which give rises to the interactions

$$\text{(II)}' \quad \frac{1}{m_{\text{mirr}}}(TH^\dagger\bar{\sigma}_\mu\tau_a q_3)W_H^{a\mu} \quad (21)$$

$$\text{(III)}' \quad \frac{1}{m_{\text{mirr}}^2}(T\bar{\sigma}_\mu T)(H^\dagger\tau_a H)W_L^{a\mu} \quad (22)$$

where τ_a's are the $SU(2)$ generators, and m_{mirr} is the mass of the mirror fermions. After electroweak symmetry breaking, we obtain type (II) and (III) interactions. The mass of the top mirror fermion is parametrically of the same order as f though it can be much heavier in practice. There are also vertices similar to (II) with W_H^0 replaced by B_H and g replaced by g'. Type (IV) vertex is simply the hypercharge interaction of T. More detailed discussions on the interactions involving T-odd fermions can be found in the appendix.

Let us consider the ρ parameter first. At one-loop order (T, T^c) contribute to the self-energy of W_L^0 through type (III) vertex, as shown in Fig. 3(a), and thus enters into the ρ parameter

$$\delta\rho_T = \frac{3}{16\pi^2} \frac{e^2}{s_w^2 c_w^2 m_Z^2} \mathcal{O}\left(g^2\frac{v^4}{f^4}\right)m_T^2 \log\frac{m_{\text{mirr}}^2}{m_T^2}. \quad (23)$$

In the above we have replaced the cutoff dependence in the logarithm with m_{mirr}, since type (III) vertex arises from integrating out the mirror fermions. For the littlest Higgs model, this contribution is parametrically comparable to the one coming from the heavy gauge bosons [13,33]

$$\delta\rho_{W_H} = -\frac{9}{64\pi^2} \frac{g^2 e^2}{s_w^2 c_w^2 m_Z^2} \Delta m^2 \log\frac{\Lambda^2}{m_{W_H}^2}, \quad (24)$$
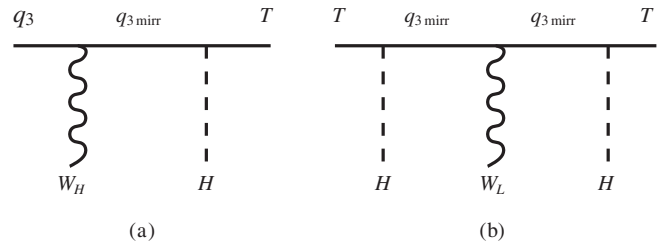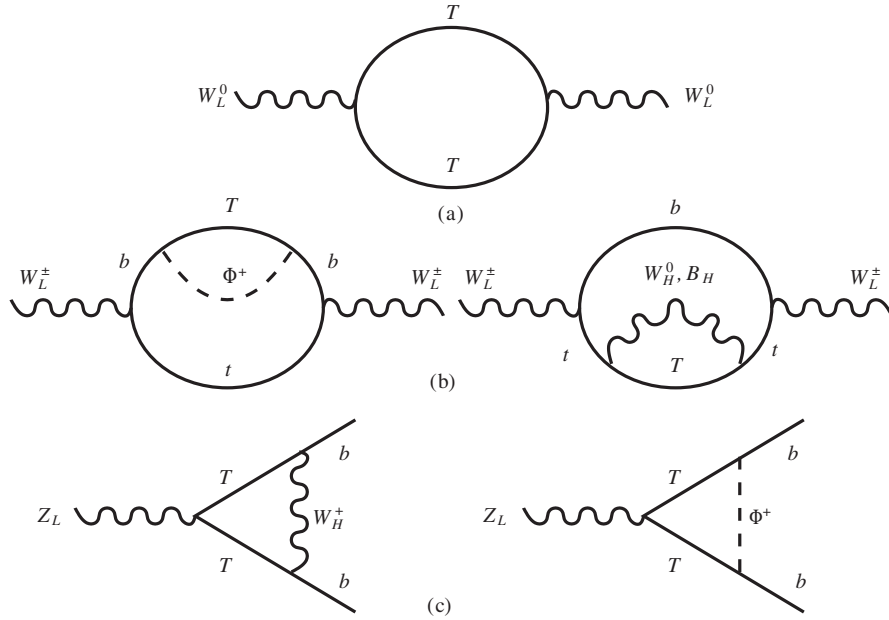

(a)   (b)

FIG. 2.   Generation of type (II)′ and (III)′ vertices by integrating out the mirror fermions.

FIG. 3. Examples of diagrams contributing to ρ parameters and $Zb\bar{b}$ vertex.

where

$$\Delta m^2 = m_{W_H^0}^2 - m_{W_H^\pm}^2 = \frac{1}{2}g^2 f^2 \sin^4\frac{v}{f}, \qquad (25)$$

and $\Lambda = 4\pi f \sim 10$ TeV is the cutoff of the nonlinear sigma model. A partial cancellation is possible between $\delta\rho_T$ and $\delta\rho_{W_H}$ as they come with opposite signs. Therefore, the constraint from the calculable contributions within the low-energy effective theory is expected to be even weaker than $f \gtrsim 500$ GeV quoted previously in the T-invariant littlest Higgs model with a T-even top partner [13,33]. Nevertheless, it is still desirable to keep a reasonable hierarchy between v and f so that the cutoff $\Lambda = 4\pi f$ is high enough to suppress the unknown contributions above the cutoff of the theory. There are also two-loop diagrams in $\delta\rho_T$ which involves type (I) and (II) vertices, two examples of which are shown in Fig. 3(b). They are not expected to give significant contributions.

For the case of $Zb\bar{b}$ vertex, there are now one-loop diagrams using the combinations of the interactions of type (I)–(IV). Two such diagrams are given in Fig. 3(c). These diagrams are suppressed by at least $(v/f)^2$ from the vertices. Such $(v/f)^2$ suppressions are parametrically of the same order as the s_L^2 factor in the case of a T-even top partner. However, because the exchanged particle is now a heavy particle such as Φ or W_H, instead of W_L as would be the case for a T-even top partner, there is an additional suppression factor of $(m_W/m_{\text{heavy}})^2$ comparing to the old expression in Eq. (16), which makes the correction to the $Zb\bar{b}$ vertex even smaller.

IV. LHC PHENOMENOLOGY

In this section, we outline the collider phenomenology of this class of little Higgs models at LHC. We will focus on general features of the signals and leave more detailed study for the future.

A. Decay modes

Characterization of the decay modes is crucial for identifying new physics signals. We focus our attention on the following states

$$T, \qquad A_H, \qquad W_H. \qquad (26)$$

They are the states which are responsible for canceling the one-loop quadratic divergence and are generically present in little Higgs models. There should also be scalars, whose nature is more model-dependent. These scalars cancel the quadratic divergence from the Higgs self-coupling and in general have small production cross sections at the LHC. Hence they are more difficult to discover. All other states can be pushed up to multi-TeV or higher so they may not be relevant at the LHC. Of course, if some of the extra states happen to be light, they can give additional signals beyond what is discussed here.

More specifically, we will narrow our attention further down to the decay chain of T since it has the largest production cross sections. If W_H is involved in the decay chain, its decay will be important as well.

We summarize various decay channels of T as follows:
(1) T can decay through $T \to tA_H$, $T \to tW_H^0$, and $T \to bW_H^+$.
(2) Since T is a singlet, it does not have direct renormalizable interactions with W_H, but there can be higher

dimensional operators such as $\bar{T}H^\dagger \slashed{W}_H q_3$. After plugging in the Higgs VEV, the coupling $\bar{T}\slashed{W}_H q_3$ is suppressed at least by v/f. This is the type (II) vertex discussed in the previous section. As a result, the width of $T \to tW_H^0$, and $T \to bW_H^+$ are suppressed (at least) by v^2/f^2. Therefore, their branching ratio will be suppressed with respect to the dominant mode $T \to tA_H$ by about

$$\left(\frac{g}{g'}\right)^2 \frac{v^2}{f^2} \sim 10\%\left(\frac{1 \text{ TeV}}{f}\right)^2 \qquad (27)$$

(3) $T \to bW_H^+$ with $W_H^+ \to W^+ A_H$ will produce final states identical to $T \to tA_H$. Therefore, they are less distinct.

(4) $T \to tW_H^0$, with $W_H^0 \to hA_H$ could be more interesting. There will be more b's in the final state. In addition, since W_H^0 decay predominantly to hA_H, this decay channel could be different from the typical second neutralino decay of SUSY; $\tilde{N}_2$ could have sizable branching fractions to both $h\tilde{N}_1$ and $Z\tilde{N}_1$ (in contrast, there is no $W_H^0 \to ZA_H$ decay), although the latter is somewhat suppressed when $\tilde{N}_1$ is dominantly $\tilde{B}$.

B. Production

We now turn to the discussion of the production of new heavy states. Analogous to supersymmetry with exact R-parity, all new T-odd particles in little Higgs models will have to be pair produced. This makes their searches at colliders more challenging. On the other hand, because of the T-parity, there is no strong constraint from the precision electroweak data and the new particles could be much lighter than those in models without T-parity, which makes them more accessible at the colliders.

Pair productions and decays of the heavy gauge bosons will produce jets, leptons, and missing energies in the final states, which is similar to decays of neutralino and chargino productions in supersymmetry. However, it would be difficult to separate them from the standard model backgrounds, such as WZ and WW.

In addition, the top partner is a Dirac fermion charged under color. Therefore it should have decent production cross sections.

(1) As shown in Fig. 4, the production cross section is sizable. Assuming $M_T = 1$ TeV, and $L = 100$ fb^{-1}, thousands of $T\bar{T}$ pairs could be produced at the LHC.

(2) If the dominant decay channel is $T \to tA_H$, it will be challenging to suppress the top background. A large missing energy cut will be necessary and a detailed study is needed to tell how well the signals can be separated from the backgrounds.

(3) There could be additional decay channels with sizable branching fractions. In particular, the decay mode $T \to tW_H^0$ could give us additional signatures.
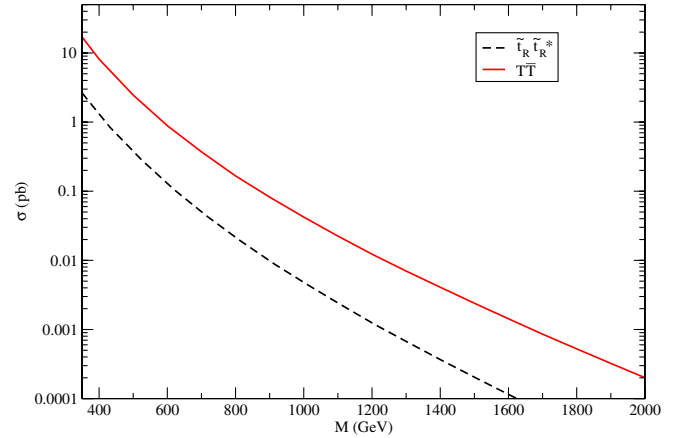

FIG. 4 (color online). Production cross sections of T at the LHC as a function of its mass (solid line). For comparison, we also plot the production cross section of $\tilde{t}_R$ in SUSY.

C. Comparison with SUSY

The heavy top partner T cancels the one-loop quadratic divergence in the Higgs potential coming from the top quark. In this sense, its role is analogous to the top squark in supersymmetry. More generally, it is a common feature of models with a natural mechanism to stabilize the electroweak scale to have a state with toplike quantum numbers. Such a state could be copiously produced at the LHC. Its generic signature could be a set of energetic b-rich events above the $t\bar{t}$ background.

The most important task after discovering such a state is obviously measuring its properties. It is likely to be a nontrivial task as we will demonstrate with our current example.

We will compare the signatures of T with those of $\tilde{t}_R$ in SUSY. As we commented previously, if the spectrum and the couplings of the model are such that there are several decay channels of T, there may be some distinctive features by examining the decay branching ratios of W_H^0 as compared with those of $\tilde{N}_2$ in supersymmetric theories. However, such a comparison relies on assumptions about the details of the spectrum and is not generic. Therefore, we will focus our attention on comparing the case in which T predominantly decays to tA_H with the case in which $\tilde{t}_R \to t\tilde{B}$.

Since fermions have very different cross sections comparing to bosons, they might be distinguishable from bosons based on their production rate.[1] However, in order to make a meaningful comparison, one must get a handle on the mass of new particles. One possible observable is the average effective mass. We define the effective mass by

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[1] We assume it is possible to separate a significant portion of the signal from the $t\bar{t}$ background. It is obviously a crucial assumption which deserves further study. Although it will not change the qualitative feature outlined here, inclusion of the effect of the background will certainly make such a comparison even more difficult than what is illustrated in this study.
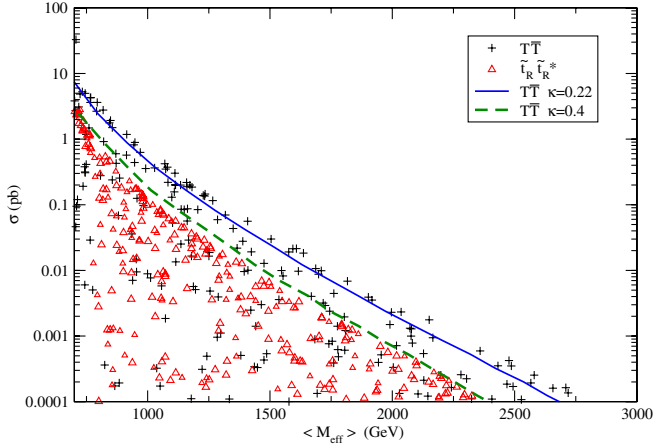
FIG. 5 (color online). Comparison between T and $\tilde{t}_R$. $\kappa = m_{A_H}/m_T$ or $\kappa = m_{\text{LSP}}/m_{\tilde{t}_R}$.

$$M_{\text{eff}} = \not{E}_T + \sum_i |P_T^i| (\text{Leading jets, \# of jets} \geq 2,$$

$$\not{E}_T \geq 200 \text{ GeV}). \tag{28}$$

We use the average of the effective mass as an example of a potential observable related to relevant mass scales in the problem.

We plot the cross sections versus the average effective mass in Fig. 5. We see that for a significant portion of parameter space when there is a large separation between m_T and m_{A_H}, $\kappa = m_{A_H}/m_T < 0.4$, these two cases are distinguishable. The ratio κ is model-dependent. For the littlest Higgs model presented in the appendix, $\kappa = \sqrt{2/5}(g'/\lambda_t) \approx 0.22$, so it can be distinguished.

For larger κ, the decay products are softer. There can be a SUSY model giving the same values for the two observables, the total cross sections and the effective mass, so they are not distinguishable in this strategy. Such an effect persists even as we use other generic P_T-like observables, which is anticipated since most of those observables only measure mass difference. This is a general lesson for this kind of comparison based on the total cross section in combination with this class of observables proportional to mass difference between the heavy toplike state and the lightest stable neutral state. Therefore, it is difficult to use them to completely break the degeneracy.

On the other hand, it is a degeneracy between two specific models with very distinct mass spectra: the SUSY model has a much lighter spectrum comparing to the little Higgs model. Therefore, some kinematic distributions, such as forward jets, or the opening angle of the b-lepton pair might help to distinguish these two scenarios. This requires more detailed studies in the future. Of course, spin-correlations will also be very useful in principle to distinguish these two scenarios [41–43], which also deserves further studies.

V. CONCLUSION

In this paper we have shown that there is a class of little Higgs models in which every new particle responsible for canceling the one-loop quadratic divergence, including the heavy top partner, is odd under T-parity. Such a construction largely simplifies the implementation of T-parity in the top sector. We discussed the generic form of the top sector Lagrangian along with a comparison with earlier constructions. A couple of explicit examples of little Higgs models with T-odd top partners are demonstrated in the appendix.

The odd T-parity of the top partner completely changes its collider phenomenology. The main phenomenological features of such a T-odd top partner is outlined in this paper. Because of the conserved T-parity, such states have to be pair produced. On the other hand, for the same reason, the top partner could be much lighter without violating the constraints from the electroweak precision tests, and therefore could have a decent production rate at the LHC.

Generically, the collider signature for such a state is similar to other TeV scale new physics models with a Z_2 parity, such as low energy supersymmetry. Such a discrete symmetry is also independently motivated by the dark matter in the universe, which could be composed of the lightest particle charged under this symmetry. Therefore, this kind of model provides an interesting playground for comparison with supersymmetry. We discuss the challenge of distinguishing these models from supersymmetry at hadron colliders. The parameter spaces of these two classes of models which produce similar collider signature generically have quite different overall mass spectra. This difference points to a possible direction of distinguishing them by using more detailed kinematic distributions. Further detailed studies are certainly required to check the viability of this strategy at the LHC.

APPENDIX: EXPLICIT MODELS

In Sec. II we demonstrated using a simple toy model the general idea of constructing a top Yukawa sector in which the top partner is T-odd. In this appendix we show how to apply the general idea to the minimal moose and the $SU(5)/SO(5)$ littlest Higgs models with T-parity. We will focus on implementing T-parity in the fermionic

sector. For T-parity invariant gauge and scalar sectors we refer the readers to Refs. [12–14].

First we start with the minimal moose model in Ref. [4]. The model consists of two sites with four copies of $SU(3)$ global symmetry on each site,[2] broken down to the diagonal $SU(3)$'s by four link fields $\Sigma_i \rightarrow L_i \Sigma_i R_i^\dagger$, $i = 1, 2, 3, 4$. Within each site an $SU(2) \times U(1)$ subgroup is weakly gauged with equal strength, the diagonal of which remains unbroken and is identified with the standard model $SU(2)_W \times U(1)_Y$. In the fermionic sector we introduce two copies of electroweak doublet fermions, charged under the L- and R-site, respectively,

$$Q_a = \begin{pmatrix} d_a \\ u_a \\ 0 \end{pmatrix} \rightarrow L_i Q_a, \qquad Q_b = \begin{pmatrix} d_b \\ u_b \\ 0 \end{pmatrix} \rightarrow R_i Q_b.$$

(A1)

Under T-parity they transform as $Q_a \leftrightarrow Q_b$. Both Q_a and Q_b carry additional $U(1)$ gauge charges on top of the ones corresponding to the T_8 generators of $SU(3)$'s in order to have the correct hypercharges. The T-even linear combination $(Q_a + Q_b)/\sqrt{2}$ will be taken to be the standard model fermions. To make the other combination massive, we introduce a conjugate doublet transforming nonlinearly by way of CCWZ [44,45]

$$Q_c^c = \begin{pmatrix} d_c^c \\ u_c^c \\ 0 \end{pmatrix} \rightarrow U_i Q_c^c,$$

(A2)

where the matrix U_i nonlinearly realizes the $SU(3)$ global symmetry through $\xi_i \rightarrow L_i \xi_i U_i^\dagger = U_i \xi_i R_i^\dagger$, $\xi_i^2 = \Sigma_i$ [12–14]. Moreover, $\xi_i \rightarrow \Omega \xi_i^\dagger \Omega$ under T-parity, whereas the conjugate doublet transforms as $Q_c^c \rightarrow -\Omega Q_c^c$, where $\Omega = \mathrm{diag}(1, 1, -1)$. We can write down the following T-invariant Yukawa-type interactions

$$\kappa f(\bar{Q}_a \xi_2 Q_c^c - \bar{Q}_b \Omega \xi_2^\dagger Q_c^c),$$

(A3)

where the coefficient κ in general is different for different generations. Equation (A3) gives a mass to the linear combination $Q_a - \Omega Q_b$, that is the T-odd Dirac-pair $q_\mathrm{mirr} = (q_c^c, (q_a - q_b)/\sqrt{2})$. This is the mirror fermion we refer to in Sec. III. The construction described here avoids large contributions to the four-fermion operators [14], which would arise from the CCWZ kinetic terms if the standard model fermions are also realized nonlinearly as in (A2). With the present construction, the standard model fermions now have standard kinetic terms without involving any Goldstone bosons, hence giving no large contributions to the four-fermion operators. Although strong

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[2]This model does not have a custodial $SU(2)$ symmetry and contains a large contribution to the $\Delta \rho$ parameter in the nonlinear sigma model Lagrangian [26], which can easily be cured by replacing $SU(3)$ with a group which contains a custodial $SU(2)$ such as $SO(5)$ [8].

TABLE I. The $U(1)$ charge assignments for Q_a, Q_b, U_a^c, U_b^c fermions. Q_c^c is not listed because it transforms nonlinearly. The physical $U(1)_Y$ hypercharge is the sum of the $U(1)$ charges.

	q_a	U_a	U_a^c	q_b	U_b	U_b^c
$U(1)_a$	$\frac{1}{12}$	$\frac{7}{12}$	$-\frac{7}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{12}$
$U(1)_b$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{12}$	$\frac{1}{12}$	$\frac{7}{12}$	$-\frac{7}{12}$

constraints on four-fermion operators exist only for the light generations and the lepton sector, we use the same construction for the top sector for consistency.

Next we consider the top sector, for which we introduce complete $SU(3)$ triplets of quarks,

$$Q_{3a} = \begin{pmatrix} b_a \\ t_a \\ U_a \end{pmatrix}, \qquad Q_{3b} = \begin{pmatrix} b_b \\ t_b \\ U_b \end{pmatrix}, \qquad Q_{3c}^c = \begin{pmatrix} b_c^c \\ t_c^c \\ U_c^c \end{pmatrix}$$

(A4)

Under T-parity, $Q_{3a} \leftrightarrow Q_{3b}$ and $Q_{3c}^c \rightarrow -\Omega Q_{3c}^c$. In particular, $U_a \leftrightarrow U_b$ and $U_c^c \rightarrow U_c^c$.

The mirror triplet Q_{3c}^c marries a linear combination $Q_{3a} - \Omega Q_{3b}$ and become massive through the interaction in Eq. (A3). Note that there is a massive T-even state consisting of U_c^c and $U_a + U_b$. However, it can be made very heavy without affecting the cancellation of the quadratic divergence from the top loop. We assume that it is much heavier than the T-odd top partner discussed below. To complete the spectrum we add two $SU(2)$ singlets U_a^c, U_b^c, with the T-parity transformation rule $U_a^c \leftrightarrow U_b^c$, as discussed in Sec. II. There is some freedom in the $U(1)$ charge assignments. One convenient choice was given in Ref. [12] which we list here in Table I. The top Yukawa coupling can be written as

$$\mathcal{L}_t = \lambda f Q_{3a} \Sigma_1^\dagger \begin{pmatrix} 0 \\ 0 \\ U_b^c \end{pmatrix} + \lambda f Q_{3b} \Omega \Sigma_1 \Omega \begin{pmatrix} 0 \\ 0 \\ U_a^c \end{pmatrix} + \mathrm{H.c.},$$

(A5)

which in fact introduces mixing between the light T-even and the heavy T-even fermions. This mixing can be minimized by taking the mass of the heavy T-even fermions to be around the cutoff scale ~ 10 TeV. In other words, the heavy T-even fermions can be decoupled from the spectrum below the cutoff without affecting the cancellation of top quadratic divergences. Alternatively, if we add additional interactions in the top sector as follows

$$\mathcal{L}_t' = \lambda' f Q_{3b} \Omega \Sigma_2 \Sigma_1^\dagger \begin{pmatrix} 0 \\ 0 \\ U_b^c \end{pmatrix} + \lambda' f Q_{3a} \Sigma_2^\dagger \Sigma_1 \Omega \begin{pmatrix} 0 \\ 0 \\ U_a^c \end{pmatrix}$$

$$+ \mathrm{H.c.},$$

(A6)

then the mixing is suppressed when λ' is close to λ. In fact, the mixing is completely eliminated in the limit $\lambda' = \lambda$,

since when $\Sigma_2 = \langle \Sigma_2 \rangle = \mathbb{1}$

$$\mathcal{L}_t + \mathcal{L}'_t = \lambda f (Q_{3a} + \Omega Q_{3b}) \Sigma_1^\dagger \begin{pmatrix} 0 \\ 0 \\ U_b^c \end{pmatrix}$$

$$+ \lambda f (Q_{3a} + \Omega Q_{3b}) \Sigma_1 \Omega \begin{pmatrix} 0 \\ 0 \\ U_a^c \end{pmatrix} + \text{H.c.} \quad \text{(A7)}$$

which is reminiscent of (9). The Σ_2 field is inserted to ensure gauge invariance. When expanding $\mathcal{L}_t + \mathcal{L}'_t$ to quadratic order in the Higgs field which resides in Σ's, we obtain Eq. (10) with the following T-even top quarks and T-odd top partners:

$$(t, t^c) = ((t_a + t_b)/\sqrt{2}, (U_a^c + U_b^c)/\sqrt{2}),$$
$$(T, T^c) = ((U_a - U_b)/\sqrt{2}, (U_a^c - U_b^c)/\sqrt{2}). \quad \text{(A8)}$$

The mass of the T-odd heavy partner plays an important role in collider phenomenology. In this model there are two light Higgs doublets, so the Yukawa coupling λ of the top sector depends on the ratio of the Higgs VEVs. In addition, m_{W_H} and m_{A_H} receives contributions from VEVs of all link fields Σ_i while m_T does not. Therefore, the ratio between the masses of the top partner T and the heavy gauge bosons W_H, A_H is not completely fixed, but depends on model parameters.

At this stage it is worth pointing out that to the first order in the physical Higgs field, Eq. (A5) contains the vertex $T^c H q_{3\text{mirr}}$, which was used in Fig. 2(a) and 2(b) to obtain the interactions (II)$'$ and (III)$'$ in Eqs. (21) and (22) after the mirror fermions are integrated out. On the other hand, the kinetic terms for the fermions,

$$\bar{Q}_a \bar{\sigma}_\mu D_a^\mu Q_a + \bar{Q}_b \bar{\sigma}_\mu D_b^\mu Q_b, \quad \text{(A9)}$$

when expressed in terms of the mass eigenstates give rise to the vertices $g(q_3 \bar{\sigma}_\mu q_{3\text{mirr}}) W_H^\mu$ and $g'(T \bar{\sigma}_\mu T) B_L^\mu$. The former was used again in Fig. 2(a), whereas the latter is the type (IV) interaction in Eq. (20).

Next we turn to the littlest Higgs model which is based on the coset space $SU(5)/SO(5)$. The nonlinear sigma model field has a vacuum expectation value

$$\langle \Sigma \rangle \equiv \Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}, \quad \text{(A10)}$$

where Σ is a (5×5) symmetric tensor under $SU(5)$: $\Sigma \rightarrow V \Sigma V^T$, $V \in SU(5)$. Again we introduce two copies of fermionic doublets, embedded in incomplete $SU(5)$ multiplets, as well as the conjugate fermions which transform linearly only under the unbroken $SO(5)$:

$$Q_a = \begin{pmatrix} q_a \\ 0 \\ 0 \end{pmatrix} \rightarrow V Q_a, \qquad Q_b = \begin{pmatrix} 0 \\ 0 \\ q_b \end{pmatrix} \rightarrow V^* Q_b,$$

$$Q_c^c = \begin{pmatrix} q_c^c \\ \chi^c \\ \tilde{q}_c^c \end{pmatrix} \rightarrow U Q_c^c, \quad \text{(A11)}$$

where U again realizes the $SU(5)$ symmetry nonlinearly through $\Sigma = \xi^2 \Sigma_0$, $\xi \rightarrow U \xi \Sigma_0 V^T \Sigma_0 = V \xi U^\dagger$ [12–14]. The action of T-parity on the fermions takes $Q_a \leftrightarrow \Sigma_0 Q_b$ and $Q_c^c \rightarrow -\Omega_5 Q_c^c$, with $\Omega_5 = \text{diag}(1, 1, -1, 1, 1)$. The T-odd combination of doublets is $(q_a - q_b)/\sqrt{2}$ and gets a mass through

$$\kappa f (Q_a \xi Q_c^c - Q_b \Sigma_0 \Omega_5 \xi^\dagger Q_c^c). \quad \text{(A12)}$$

The above equation is T-invariant since $\xi \rightarrow \Omega_5 \xi^\dagger \Omega_5$. The T-odd massive mirror fermions are $q_{\text{mirr}} = (q_c^c, (q_a - q_b)/\sqrt{2})$, whereas the remaining fermions χ^c and $\tilde{q}_c^c$ can be given Dirac masses by introducing additional particles [12–14].

In the top sector we introduce additional singlets as follows

$$Q_{3a} = \begin{pmatrix} q_{3a} \\ U_a \\ 0 \end{pmatrix}, \qquad Q_{3b} = \begin{pmatrix} 0 \\ U_b \\ q_{3b} \end{pmatrix}, \qquad Q_{3c}^c = \begin{pmatrix} q_{3c}^c \\ \chi_3^c \\ \tilde{q}_{3c}^c \end{pmatrix}. \quad \text{(A13)}$$

As in the case with the minimal moose model, Eq. (A12) gives a Dirac mass to the T-even singlets (χ_3^c, $U_a + U_b$). The massless combinations are $q_{3a} + q_{3b}$ and $U_a - U_b$, and have even and odd T-parity, respectively. The top Yukawa coupling can be written as, introducing additional singlets $U_a^c \leftrightarrow U_b^c$ under T-parity,

$$\mathcal{L}_t = \tfrac{1}{2} \lambda f \epsilon_{ijk} \epsilon_{xy} (Q_{ai} \Sigma_{jx} \Sigma_{ky} U_a^c + (\Sigma_0 Q_b)_i \tilde{\Sigma}_{jx} \tilde{\Sigma}_{ky} U_b^c)$$
$$+ \text{H.c.}, \quad \text{(A14)}$$

where $\tilde{\Sigma} = \Sigma_0 \Omega_5 \Sigma^\dagger \Omega_5 \Sigma_0$ is the image of the Σ field under T-parity. Moreover, i, j, $k = 1, 2, 3$ and $x, y = 4, 5$. Again we can introduce additional interactions to remove the mixing between the light and heavy T-even fermions

$$\mathcal{L}'_t = \tfrac{1}{2} \lambda' f \epsilon_{lmn} \epsilon_{rs} [(\Omega_5 Q_b)_l \Sigma'_{mr} \Sigma'_{ns} U_a^c$$
$$+ (\Omega_5 \Sigma_0 Q_a)_l \tilde{\Sigma}'_{mr} \tilde{\Sigma}'_{ns} U_b^c] + \text{H.c.}, \quad \text{(A15)}$$

where $\Sigma' = \Omega_5 \Sigma^\dagger \Omega_5$ and $\tilde{\Sigma}' = \Sigma_0 \Sigma \Sigma_0$ is the T-image of Σ'. Moreover, l, m, $n = 3, 4, 5$ and $r, s = 1, 2$ in the above. After adding $\mathcal{L}'_t$ to $\mathcal{L}_t$ and taking $\lambda' = \lambda$, the top quarks and T-odd top partners have the same expression as in Eq. (A8) and the interactions of the T-odd fermions follow those discussed in the minimal moose model. Contrary to the previous case, the relation between the masses of the

T-odd gauge bosons and top partner in this simplest model is fixed, and their masses are given by

$$M_{W_H} = gf, \qquad M_{A_H} = \frac{1}{\sqrt{5}} g'f, \qquad M_T = \frac{1}{\sqrt{2}} \lambda_t f.$$

$$(A16)$$

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