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Robustness of baryon-strangeness correlation and related ratios of susceptibilities

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Using quenched lattice QCD simulations we investigate the continuum limit of baryon-strangeness correlation and other related conserved charge-flavor correlations for temperatures $T_c < T \leq 2T_c$. By working with lattices having large temporal extents ($N_\tau = 12, 10, 8, 4$) we find that these quantities are almost independent of the lattice spacing, i.e., robust. We also find that these quantities have very mild dependence on the sea quark mass and acquire values which are very close to their respective ideal gas limits. Our results also confirm robustness of the Wroblewski parameter.

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I. INTRODUCTION

The recent results from the Relativistic Heavy Ion Collider (RHIC) [1] indicate the formation of a thermalized medium endowed with large collective flow and very low viscosity [2]. These findings suggest that quark gluon plasma (QGP) is a strongly interacting system for temperatures close to its transition temperature (T_c). Apart from the experimental indications, the most convincing evidence in favor of the existence of a strongly interacting QGP comes form the lattice QCD simulations [3,4]. These nonperturbative studies show that the thermodynamic quantities, like pressure and energy density, deviate form there respective ideal gas (of free quarks and gluons) values by about 20% even at temperature $T = 3T_c$. On the other hand, other lattice studies indicate the smallness of the viscous forces in QGP [5]. All these results point to the fact that close to T_c the nature of QGP is far from a gas of free quarks and gluons.

In order to uncover the nature of QGP in the vicinity of T_c and also to understand the underlying physics of these lattice results, many different suggestions have been made over the last decade. Descriptions in terms of various quasiparticles [6,7], resummed perturbation theories [8], effective models [9], etc., are few among many such attempts. Apart from all these, the newly proposed model of Shuryak and Zahed [10] has generated a considerable amount of interest in recent years. Motivated by the lattice results for the existence of charmonium in QGP [11], this model proposed a strongly interacting chromodynamic system of quasiparticles (with large thermal masses) of quarks, antiquarks, and gluons along with their numerous bound states. As different conserved charges, e.g., baryon number (B), electric charge (Q), third component of isospin (I) etc., are carried by different flavors (u, d, s) of quarks, in the conventional quasiparticle models, conserved charges come in strict proportion to number of u, d, s quarks. Thus conserved charges are strongly correlated with the flavors and the flavors have no correlations among themselves. On the other hand, in the model of [10], presence of bound states demands correlations among different flavors. Hence correlations between conserved charges and flavors depend on the mass spectrum of the bound states and the strong correlations among them are lost.

Based on the above arguments, in [12], it has been suggested that the quantity

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} \tag{1}$$

can be used to probe the degrees of freedom of QGP. Here $B = (U + D - S)/3$ is the net baryon number and U, D, S are the numbers of net (quarks minus antiquarks) up-quarks, down-quarks, and strange-quarks, respectively. The notation $\langle\cdot\rangle$ denotes the average taken over a suitable ensemble. It has been argued in [12] that for QGP where quarks are the effective degrees of freedom, i.e., where correlations among U, D, and S are absent, C_{BS} will have a value of 1 for all temperature $T > T_c$. On the other hand, for the model of [10] $C_{BS} = 0.62$ at $T = 1.5T_c$, while for a gas of hadron resonances $C_{BS} = 0.66$. Thus the knowledge of C_{BS} helps to identify the degrees of freedom in QGP.

By extending the idea of [12], recently in [13], many ratios like

$$C_{(KL)/L} = \frac{\langle KL\rangle - \langle K\rangle\langle L\rangle}{\langle L^2\rangle - \langle L\rangle^2} \equiv \frac{\chi_{KL}}{\chi_L}, \tag{2}$$

have been calculated using lattice QCD simulations with two flavors of dynamical light quarks and three flavors (two light and one heavy) of valance quarks. Here χ_L and χ_{KL} denote the susceptibilities corresponding to conserved charge L and correlation among conserved charges K and L respectively. The physical meaning of the ratios like $C_{(KL)/L}$ can be interpreted as follows—Create an excitation with quantum number L and then observe the value of a different quantum number K associated with this excitation. Thus these ratios identify the quantum numbers corresponding to different excitations and hence provide information about the degrees of freedom. The calculations of [13] found no evidence for the existence of bound states

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[10] even at temperatures very close to T_c. These finding are consistent with the results of [14], where the hypothesis of [10] has been tested by investigating the ratios of higher order baryon number susceptibilities obtained from lattice simulations.

As these lattice studies [13] involved simulations with dynamical quarks, they were done using small lattices having temporal lattice size $N_\tau = 4$. By comparing with the results from quenched simulations it has been shown [13] that $C_{(KL)/L}$ does not depend on N_τ for temperature $T = 2T_c$. It is clearly important to verify whether the same conclusion holds even close to T_c. Furthermore, it is known that in the case of quenched QCD with standard staggered quarks the diagonal quark number susceptibilities (QNS) have strong dependence on the lattice spacing even for the free theory [15,16]. On the other hand, the off-diagonal QNS are identically zero for an ideal gas and acquires a nonzero value only in the presence of interactions. So the lattice spacing dependence of the off-diagonal QNS is likely to be more complicated, as opposed to that for the diagonal QNS where these corrections are dominated by the lattice artifacts of the naive staggered action. Thus if these two QNS become comparable the ratios mentioned in Eq. (2) can have nontrivial dependence on the lattice spacing a and hence the continuum limit of these ratios can be different from that obtained using small lattices. Since the perturbative expressions for diagonal and off-diagonal QNS (for vanishingly small quark mass and chemical potential) are respectively [17]:

$$\frac{\chi_{ff}}{T^2} \simeq 1 + \mathcal{O}(g^2) \quad \text{and} \quad \frac{\chi_{ff'}}{T^2} \simeq -\frac{5}{144\pi^6} g^6 \ln g^{-1}, \quad (3)$$

it is reasonable to expect that the off-diagonal QNS may not be negligible at the vicinity of T_c where the coupling g is large. As the contributions of the bound states in the QNS become more and more important as one approaches T_c [18], on the lattice it is necessary to investigate the continuum limit of the these ratios of in order to verify the existence of bound states in a strongly coupled QGP. At present a continuum extrapolation of this kind can only be performed using quenched approximation due to the limitations of present day computational resources. A quenched result for these ratios will also provide an idea about the dependence of these ratios on the sea quark mass.

The aim of this work is to carefully investigate the continuum limit of the ratios of the kind $C_{(KL)/L}$ for temperatures $T_c < T \leq 2T_c$ using quenched lattice QCD simulations. The plan of this paper is as follows. In Sec. II we will give the details of our simulations and present our results. In the Sec. III we will summarize and discuss our results.

II. SIMULATIONS AND RESULTS

The partition function of QCD for N_f flavors, each with chemical potential μ_f and mass m_f, at temperature T has the form

$$\mathcal{Z}(T, \{\mu_f\}, \{m_f\}) = \int \mathcal{D}\mathcal{U} e^{-S_G(\mathcal{U})} \prod_f \det M_f(T, \mu_f, m_f), \quad (4)$$

where S_G is the gauge part of the action and M is the Dirac operator. We have used standard Wilson action for S_G and staggered fermions to define M. The temperature T and the spatial volume V are expressed in terms of lattice spacing a by the relations $T = 1/(aN_\tau)$ and $V = (aN_s)^3$, N_s and N_τ being the number of lattice sites in the spatial and the Euclidean time directions, respectively. The flavor diagonal and the flavor off-diagonal quark number susceptibilities (QNS) are given by

$$\chi_{ff} = \left(\frac{T}{V}\right)\frac{\partial^2 \ln\mathcal{Z}}{\partial \mu_f^2} = \left(\frac{T}{V}\right)[\langle \mathbf{Tr}(M_f^{-1}M_f'' - M_f^{-1}M_f'M_f^{-1}M_f')\rangle + \langle\{\mathbf{Tr}(M_f^{-1}M_f')\}^2\rangle], \quad \text{and} \quad (5)$$

$$\chi_{ff'} = \left(\frac{T}{V}\right)\frac{\partial^2 \ln\mathcal{Z}}{\partial \mu_f \partial \mu_{f'}} = \left(\frac{T}{V}\right)\langle \mathbf{Tr}(M_f^{-1}M_f')\mathbf{Tr}(M_{f'}^{-1}M_{f'}')\rangle, \quad (6)$$

respectively. Here the single and double primes denote first and second derivatives with respect to the corresponding μ_f and the angular bracket denote averages over the gauge configurations.

In this paper we report results of these susceptibilities on lattices with $N_\tau = 4$, 8, 10, and 12 for the temperatures $1.1T_c \leq T \leq 2T_c$, chemical potential $\mu_f = 0$, and using quenched approximations. The details of our scale setting procedure are given in [3]. We have generated quenched gauge configurations by using the Cabbibo-Marinari pseudoheatbath algorithm with Kennedy-Pendleton updating of three $SU(2)$ subgroups on each sweep. Since for $m_q/T_c \leq 0.1$ QNS are almost independent of the bare valance quark mass (m_q) [15], we have used $m_q/T_c = 0.1$ for the light u and d-flavors. Motivated by the fact that for the full theory $m_s/T_c \sim 1$ we have used $m_q/T_c = 1$ for the heavier s-flavor. The fermion matrix inversions were done by using conjugate gradient method with the stopping criterion $|r_n|^2 < \epsilon|r_0|^2$, r_n being the residual after the n-th step and $\epsilon = 10^{-4}$ [19]. The traces have been estimated by the stochastic estimator $\mathbf{Tr}A = \sum_{i=1}^{N_v} R_i^\dagger A R_i/2N_v$, where R_i is a complex vector whose components have been drawn

TABLE I. The couplings (β), lattice sizes ($N_\tau \times N_s^3$), number of independent gauge configurations (N_{stat}), and number of vectors (N_v) that have been used for our simulations are given for each temperature. The gauge configurations were separated by 100 sweeps.

T/T_c	β	Lattice size	N_{stat}	N_v	
				$m_q/T_c = 0.1$	$m_q/T_c = 1$
	5.7000	4×10^3	44	250	100
		$\times 16^3$	50	250	100
		$\times 20^3$	30	250	100
1.1	6.1250	8×18^3	48	250	100
	6.2750	10×22^3	38	250	100
	6.4200	12×26^3	41	250	100
	5.7880	4×10^3	52	100	100
1.25	6.2100	8×18^3	49	200	100
	6.3600	10×22^3	46	200	100
	6.5050	12×26^3	45	200	100
	5.8941	4×10^3	51	100	100
1.5	6.3384	8×18^3	49	150	100
	6.5250	10×22^3	49	150	100
	6.6500	12×26^3	48	150	100
	6.0625	4×10^3	51	100	100
2.0	6.5500	8×18^3	50	100	100
	6.7500	10×22^3	46	100	100
	6.9000	12×26^3	49	100	100

independently from a Gaussian ensemble with unit variance. The square of a trace has been calculated by dividing N_v vectors into L nonoverlapping sets and then using the relation $(\mathbf{Tr}A)^2 = 2\sum_{i>j=1}^{L}(\mathbf{Tr}A)_i(\mathbf{Tr}A)_j/L(L-1)$. We have observed that as one approaches T_c from above these products, $\chi_{ff'}$, becomes more and more noisy for larger volumes and smaller quark masses. So in order to reduce the errors on $\chi_{ff'}$ number of vectors N_v have been increased (for the larger lattices and the smaller quark masses) with decreasing temperature. Details of all our simulations are provided in Table I.

In the following sections we present our results. The notations we use are the same as in [13]. Since we use equal masses for the two light u and d flavors, the flavor diagonal susceptibilities in this context are $\chi_{uu} = \chi_{dd} \equiv \chi_u$ and the flavor off-diagonal susceptibilities are $\chi_{du} = \chi_{ud}$. For the heavy flavor s the flavor diagonal susceptibility is denoted as $\chi_{ss} \equiv \chi_s$ and the flavor off-diagonal susceptibilities are $\chi_{ds} = \chi_{us}$. Expressions for all the susceptibilities used here have been derived in the appendix of [13].

A. Susceptibilities

In order to understand the cutoff dependence of $C_{(KL)/L}$ let us start by examining the same for the diagonal and off-diagonal QNS. We have found that the for all the temperatures the diagonal QNS (χ_u and χ_s) depend linearly on $a^2 \propto 1/N_\tau^2$, i.e., the finite lattice spacing corrections to the diagonal QNS have the form $\chi_{ff}(a, m_f, T) = \chi_{ff}(0, m_f, T) + b(m_f, T)a^2 + \cdots$. As an illustration of this we have shown our data for $1.1T_c$ and $1.25T_c$ in Fig. 1. Similar variations were found for the other temperatures also. We have made continuum extrapolations of the diagonal QNS by making linear fits in $1/N_\tau^2$. Our continuum extrapolated results match, within errors, with the available data of [15] at $1.5T_c$ and $2T_c$.

In Fig. 2 we present some of our typical results for the off-diagonal QNS. Note that here the scales are ~100 magnified as compared to Fig. 1. The sign of our off-diagonal QNS is consistent with the perturbative predictions of [17], as well as with the lattice results of [19,20]. The order of magnitude of our off-diagonal QNS matches with the results of [19] which uses the same unimproved staggered fermion action as in the present case. As can be seen from Fig. 2, within our errors, we have not found any perceptible dependence $\chi_{ff'}$ on the lattice spacing a. Hence to good approximation $\chi_{ff'}(a, m_f, m_{f'}, T) \approx \chi_{ff'}(0, m_f, m_{f'}, T)$. Also for the other temperatures, which are not shown in Fig. 2, similar variations were found.

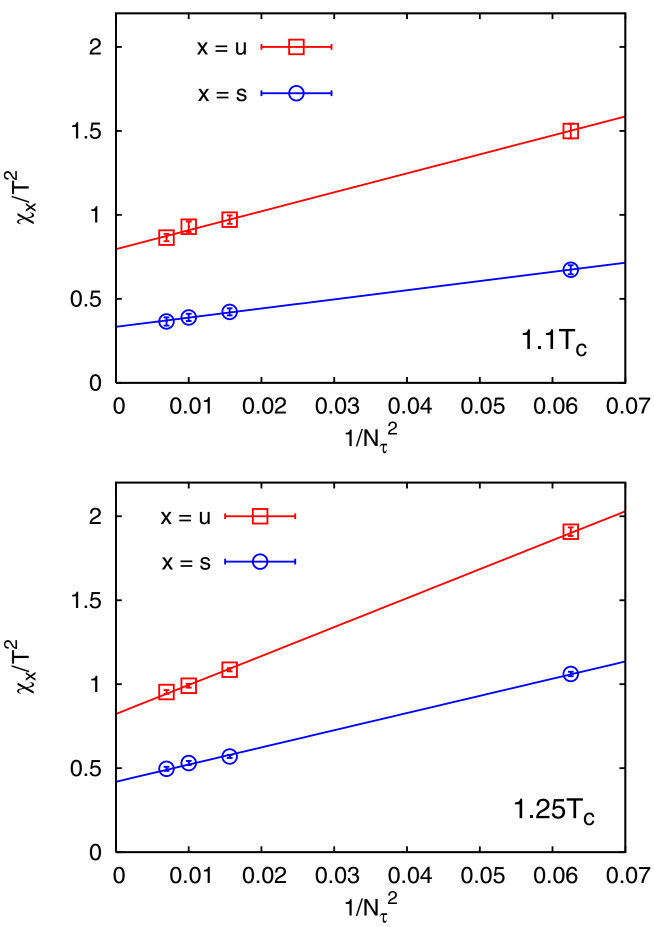

FIG. 1 (color online). We show the N_τ ($\propto 1/a$) dependence of χ_u/T^2 (squares) and χ_s/T^2 (circles) for $1.1T_c$ (top panel) and for $1.25T_c$ (bottom panel). The continuum extrapolations (linear fits in $1/N_\tau^2$) are shown by the lines.

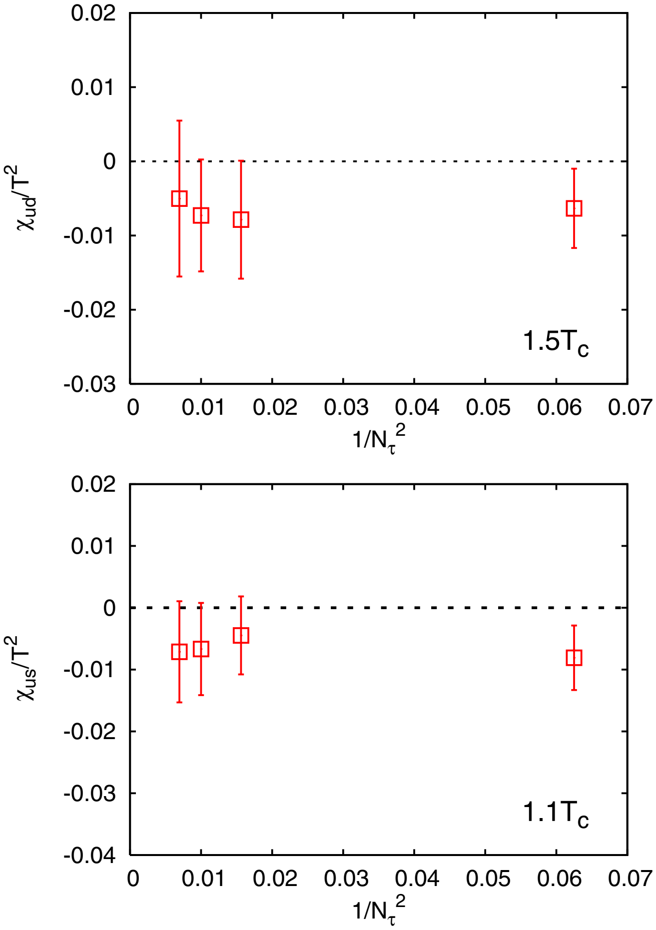

FIG. 2 (color online). N_τ dependence of the off-diagonal QNS χ_{ud}/T^2 at $1.5T_c$ (top panel) and χ_{us}/T^2 at $1.1T_c$ (bottom panel) have been shown.

Results of our continuum extrapolations of the diagonal and off-diagonal QNS are listed in Table II.

For the sake of completeness we also present our continuum extrapolated results for the two very important quantities, the baryon number susceptibility (χ_B) and the electric charge susceptibility (χ_Q). These quantities are related to the event-by-event fluctuations of baryon number and electric charge [21] which have already been measured at RHIC [22]. The definitions that we use for χ_B and χ_Q are [13]

$$\chi_B = \tfrac{1}{9}(2\chi_u + \chi_s + 2\chi_{ud} + 4\chi_{us}), \quad \text{and} \\ \chi_Q = \tfrac{1}{9}(5\chi_u + \chi_s - 4\chi_{ud} - 2\chi_{us}). \tag{7}$$

In Fig. 3 we show the continuum results for χ_B/T^2 and χ_Q/T^2. Continuum extrapolations have been performed by making linear fits in $a^2 \propto 1/N_\tau^2$. Continuum limit of these quantities were also obtained in [15] for $T \geq 1.5T_c$, though using different definitions for these quantities. Nevertheless, given the compatibility of our diagonal QNS with that of [15] and the smallness of the off-diagonal QNS for $T \geq 1.5T_c$ our continuum results for χ_B and χ_Q are compatible with that of [15], for any chosen definitions for these quantities.

B. Ratios

Wroblewski parameter (λ_s) [23] is a quantity of extreme interest due to its relation to the enhancement of strangeness production in QGP [24]. The rate of production of quark pairs in a equilibrated plasma is related to the imaginary part of the complex QNS by fluctuation-dissipation theorem. If one assumes that the plasma is in chemical (and thermal) equilibrium and the typical energy scales for the production of u, d, and s quarks are well separated from the inverse of the characteristic time scale of the QCD plasma, then using Kramers-Kroing relation one can relate λ_s to the ratio of QNS [25]:

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} = \frac{\chi_s}{\chi_u}. \tag{8}$$

(In the above equation $\langle f\bar{f}\rangle$ should be interpreted as quark number density and not as quark antiquark condensates.) We have found that λ_s, which is a ratio of two diagonal QNS, remains constant (within ~5%) with varying lattice spacings for all temperatures in $1 < T/T_c \leq 2$. We have illustrated this in the top panel of Fig. 4 by plotting λ_s with $1/N_\tau^2$ for the temperature $1.1T_c$. These results are somewhat surprising since the order a^2 corrections are not negligible for the individual diagonal QNS. But for the ratio of the diagonal QNS for two different bare valance quark masses these order a^2 corrections happen to be

TABLE II. Parameters for the continuum extrapolations of the diagonal (χ_u, χ_s) and off-diagonal (χ_{ud}, χ_{us}) QNS. For the diagonal QNS continuum, extrapolations are made by fitting $a + b/N_\tau^2$ to our data for the three largest lattice sizes. For the off-diagonal QNS continuum, extrapolations are made by fitting our data to a constant c. Numbers in the bracket denote the errors on the fitting parameters and $\chi^2_{\rm d.o.f.}$ refers to the value of the chi-square per degrees of freedom for that particular fit.

T/T_c	χ_u/T^2			χ_s/T^2			χ_{ud}/T^2		χ_{us}/T^2	
	a	b	$\chi^2_{\rm d.o.f.}$	a	b	$\chi^2_{\rm d.o.f.}$	$c \times 10^3$	$\chi^2_{\rm d.o.f.}$	$c \times 10^3$	$\chi^2_{\rm d.o.f.}$
1.1	0.79(1)	11.3(5)	0.3	0.33(1)	5.4(1)	0.1	−4(4)	0.5	−6(4)	0.1
1.25	0.84(1)	15(1)	0.5	0.45(1)	10(1)	0.8	−0.2(1.0)	0.1	−0.7(1.0)	0.6
1.5	0.83(1)	17.3(3)	0.5	0.55(1)	12.5(5)	0.9	−7(5)	0.1	2(2)	0.6
2.0	0.86(2)	19.7(2)	0.7	0.70(2)	17(2)	0.8	2(3)	0.5	−0.1(1.0)	0.8

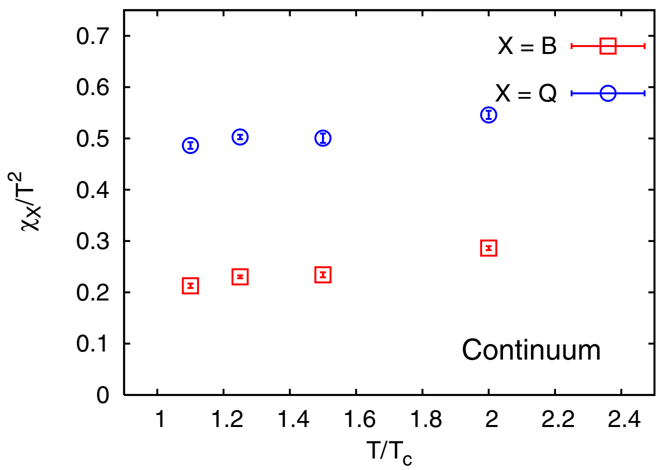

FIG. 3 (color online). The continuum results for χ_B/T^2 (squares) and χ_Q/T^2 (circles) have been shown.

negligible and thus seems to be quark mass independent. This indicates that the finite lattice spacing corrections to the diagonal QNS is constrained to have the form $\chi_{ff}(a, m_f, T) = \chi_{ff}(0, m_f, T)[1 + b(T)a^2 + \cdots]$, as opposed to the more general form $\chi_{ff}(a, m_f, T) = \chi_{ff}(0, m_f, T) + b(m_f, T)a^2 + \cdots$.

Our continuum results for the Wroblewski parameter have been shown in the bottom panel of Fig. 4. In view of the constancy of λ_s we have made the continuum extrapolations by making a constant fit to $a^2 \propto 1/N_\tau^2$. Our continuum limit for λ_s is consistent with the previously reported [15] continuum values for $T \geq 1.5T_c$. Our continuum results for λ_s are very close to the results of [13] for the whole temperature range of $T_c < T \leq 2T_c$. Closeness of our quenched results with the results from the dynamical simulations of [13] suggest that the Wroblewski parameter has practically no dependence on the mass of the sea quarks. These observations along with the fact that λ_s has very mild dependence on the valance quark mass [26] shows that the present day lattice QCD results for the Wroblewski parameter are very reliable. The

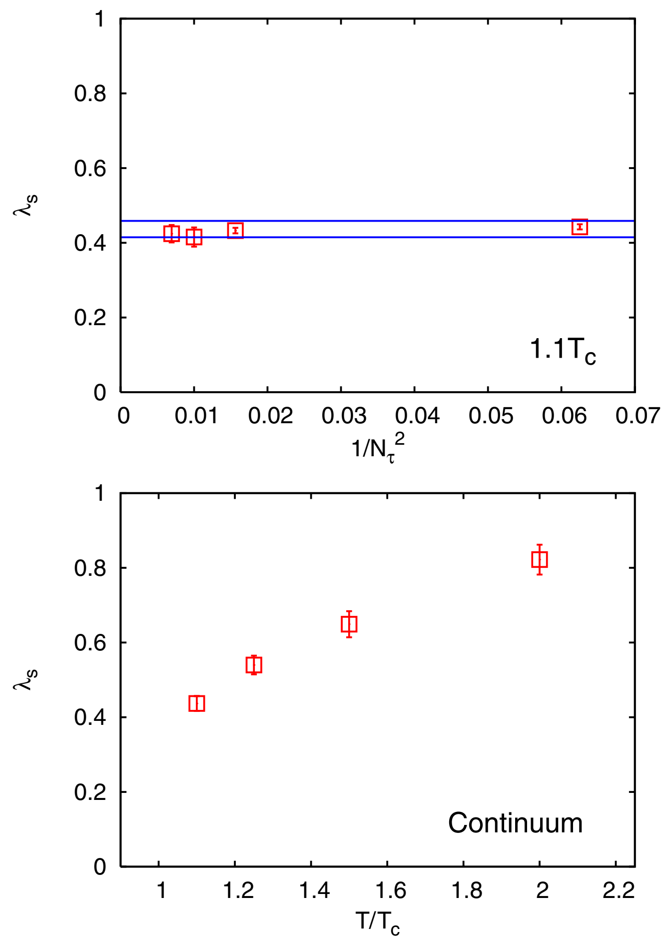

FIG. 4 (color online). In the top panel robustness of the Wroblewski parameter (λ_s) with changing lattice spacings has been shown for $1.1T_c$. The lines indicate the 5% error band of a constant fit to this data. In the bottom panel we show our continuum results for λ_s (see text for details).

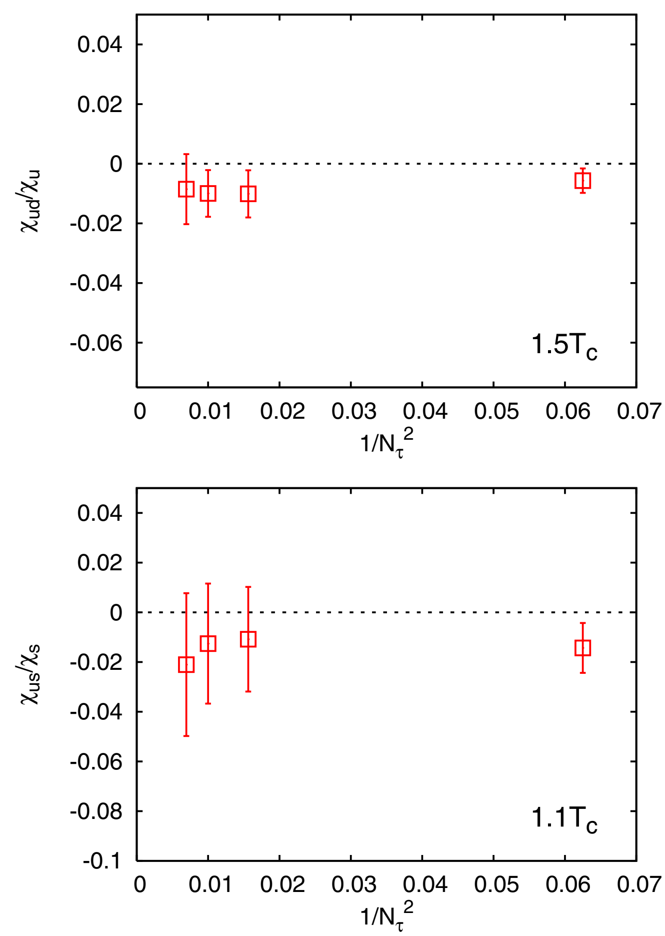

FIG. 5 (color online). The top panel shows the N_τ dependence of χ_{ud}/χ_u at $1.5T_c$. The bottom panel shows the same for χ_{us}/χ_s at $1.1T_c$.

robustness of the Wroblewski parameter is very encouraging specially since in the vicinity of T_c the lattice results for this quantity almost coincides with the value ($\lambda_s \approx 0.43$) extracted by fitting the experimental data of RHIC with a hadron gas fireball model [27].

After examining the ratio of the diagonal QNS, let us focus our attention on the ratios of off-diagonal to diagonal QNS. Given our results for the diagonal and off-diagonal QNS, it is clear that these will have the form $\chi_{ff'}(a, m_f, m_{f'}, T)/\chi_{ff}(a, m_f, T) \approx [\chi_{ff'}(0, m_f, m_{f'}, T)/\chi_{ff}(0, m_f, T)][1 - b(T)a^2]$. Since $b(T)$ is positive, i.e., χ_{ff} decreases with decreasing lattice spacing, this ratio is expected to decrease (as $\chi_{ff'}$ is negative) and move away from zero. However, due to the smallness of these ratios itself, within our numerical accuracies, we have been unable to identify any such effect. This has been exemplified in Fig. 5 where χ_{ud}/χ_u at $1.5T_c$ (top panel) and χ_{us}/χ_s at $1.1T_c$ (bottom panel) have been shown.

Following the main theme of this paper we now present the lattice spacing dependence of ratios the like $C_{(KL)/L}$. Two such ratios that can directly probe the degrees of freedom in a QGP are [12,13]

$$C_{BS} \equiv -3C_{(BS)/S} = -3\frac{\chi_{BS}}{\chi_S} = \frac{\chi_s + 2\chi_{us}}{\chi_s} = 1 + \frac{2\chi_{us}}{\chi_s}, \quad \text{and} \tag{9a}$$

$$C_{QS} \equiv 3C_{(QS)/S} = 3\frac{\chi_{QS}}{\chi_S} = \frac{\chi_s - \chi_{us}}{\chi_s} = 1 - \frac{\chi_{us}}{\chi_s}. \tag{9b}$$

These quantities probe the linkages of the strangeness carrying excitations to baryon number (C_{BS}) and electric

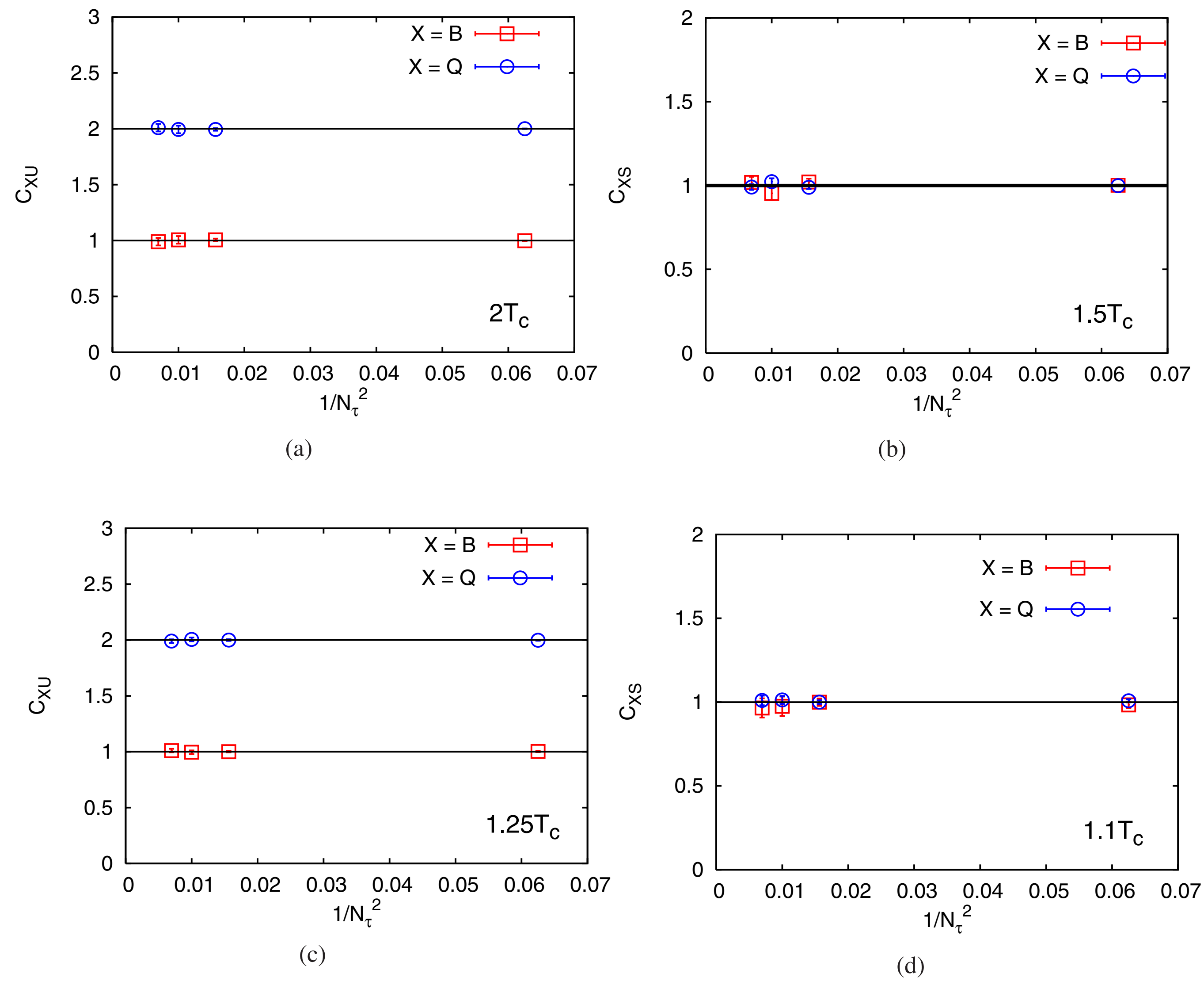

FIG. 6 (color online). Lattice spacing dependence of C_{XU} and C_{XS} are shown for temperatures $2T_c$ [panel (a)], $1.5T_c$ [panel (b)], $1.25T_c$ [panel (c)], and $1.1T_c$ [panel (d)] by plotting these quantities as a function of $1/N_\tau^2$ ($\propto a^2$), for $N_\tau = 4, 8, 10, 12$. The lines indicate the ideal gas values for these ratios.

charge (C_{QS}) and hence give an idea about the average baryon number and the average electric charge of all the excitations carrying the s flavors. These ratios are normalized such that for a pure quark gas, i.e., where unit strangeness is carried by excitations having $B = -1/3$ and $Q = 1/3$, $C_{BS} = C_{QS} = 1$. A value of C_{BS} and C_{QS} significantly different from 1 will indicate that the QGP phase may contain some other degrees of freedom apart form the quasiquarks.

Similar ratios can also be formed for the light quark sector [13], e.g., for the u flavor the ratios

$$C_{BU} \equiv 3C_{(BU)/U} = 3\frac{\chi_{BU}}{\chi_U} = \frac{\chi_u + \chi_{ud} + \chi_{us}}{\chi_u} = 1 + \frac{\chi_{ud}}{\chi_u} + \frac{\chi_{us}}{\chi_u}, \quad \text{and} \tag{10a}$$

$$C_{QU} \equiv 3C_{(QU)/U} = \frac{3\chi_{QU}}{\chi_U} = \frac{2\chi_u - \chi_{ud} - \chi_{us}}{\chi_u} = 2 - \frac{\chi_{ud}}{\chi_u} - \frac{\chi_{us}}{\chi_u} \tag{10b}$$

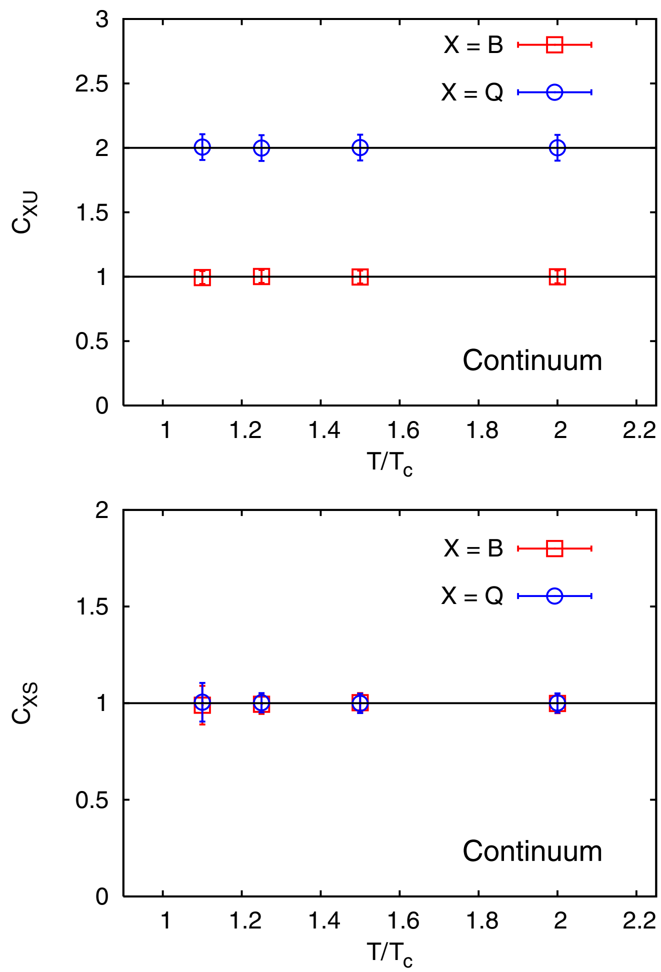

FIG. 7 (color online). Continuum results for C_{XU} (top panel) and C_{XS} (bottom panel). The lines indicate the ideal gas values for these quantities. See text for details.

quantifies the average baryon number (C_{BU}) and and the average electric charge (C_{QU}) of all the excitations carrying u quarks. For a medium of pure quarks, i.e., where the u flavors are carried by excitations with baryon number $1/3$ and electric charge $2/3$, $C_{BU} = 1$, and $C_{QU} = 2$. Similar ratios can also be formed for the d quarks [13]. As can be seen from Eqs. (9) and (10) the lattice spacing dependence of C_{BS} etc. are governed by the cutoff dependence of the ratios $\chi_{ff'}/\chi_{ff}$. Since we have already emphasized that, within our numerical accuracies, the ratios $\chi_{ff'}/\chi_{ff}$ are almost independent of lattice spacings, it is expected that the same will also happen for the ratios $C_{(KL)/L}$. In accordance to this expectation we have found that for temperatures $1.1T_c \le T \le 2T_c$ these ratios are independent of lattice spacings within ~5% errors, see Fig. 6. Note that these ratios are not only independent of the lattice spacings but also acquire values which are very close to their respective ideal gas limits.

In Fig. 7 we present our continuum results for C_{XS} (bottom panel) and C_{XU} (top panel), where $X = B$, Q. Since these ratios remain almost constant with changing $1/N_\tau^2$ (see Fig. 6) we have made continuum extrapolations by making constant fits of our data to $1/N_\tau^2$. For the whole temperature range of interests ($T_c < T \le 2T_c$) these ratios have values which are compatible with that for a gas of pure quarks. This is exactly what has been found in [13] using partially quenched simulations with smaller lattices. For the d quarks also we have found similar results.

III. SUMMARY AND DISCUSSION

In this paper we have made a careful investigation of the continuum limit of different ratios of off-diagonal to diagonal susceptibilities in quenched QCD using lattices with large temporal extents ($N_\tau = 12$, 10, 8, and 4), for a very interesting range of temperature ($T_c < T \le 2T_c$) and for vanishing chemical potential. We have found that for this whole range of temperature the lattice results for the ratios like C_{BS}, C_{QS}, etc. are robust, i.e., they are almost independent (within ~5%) of the lattice spacing. We have also arrived at the same conclusion for the Wroblewski parameter which is of interest to the experiments in RHIC and Large Hadron Collider (LHC).

At this point, it is good to have some idea about how unquenching might change our results. It has been found [28] that in the temperature range $T \ge 1.25T_c$ there is only 5–10% change in the QNS in going from quenched to $N_f = 2$ dynamical QCD. On the other hand, since the order of the phase transition depends strongly on the number of dynamical flavors the change in QNS is likely to be much larger in the vicinity of the transition temperature for the quenched theory which has a first order phase transition. Though this may be true for the individual QNS, their ratios may have very mild dependence on the sea quarks content of the theory. Given the good compatibility of our results of C_{BS}, C_{QS}, etc. with the results of [13] it is

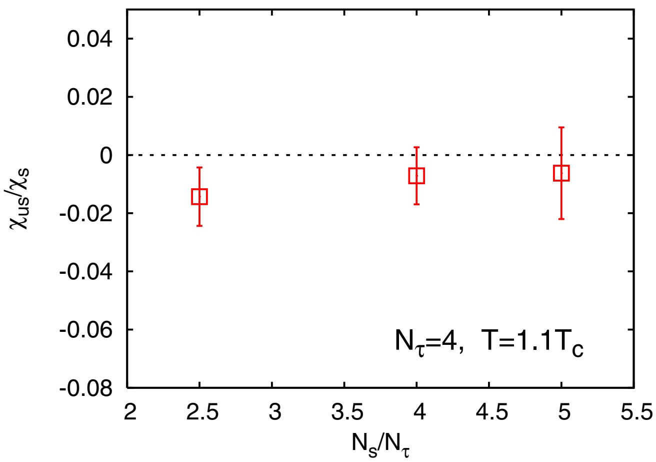

FIG. 8 (color online). Dependence of the ratio χ_{us}/χ_s on the aspect ratio has been shown for $N_\tau = 4$ at temperature $1.1T_c$.

clear that indeed these ratios have very mild dependence on the sea quark content of the theory. It is also known that [15] for bare valance quark mass of $m_q/T_c \leq 0.1$ the dependence of the QNS on the valance quarks mass is very small. Hence our results show that the ratios like $C_{(KL)/L}$ are robust not only in the sense that they do not depend on the lattice spacings but also they have very mild dependence on the quark masses.

All the results presented in this paper are for spatial lattice sizes $N_s = 2N_\tau + 2$, i.e., for aspect ratios $N_s/N_\tau =$ 2.5–2.17. In view of the fact that quenched QCD has a first order phase transition it is important to have some idea about the volume dependence of our results, specially in the vicinity of the transition temperature T_c. To check this dependence we have performed simulations using lattices having aspect ratios $N_s/N_\tau = 2.5$–5, for our smallest temporal lattice $N_\tau = 4$ and at temperature $1.1T_c$. In these simulations we have not found any significant volume dependence of any quantity which have been presented in this paper. As an illustration, in Fig. 8, we have shown the dependence of χ_{us}/χ_s on the aspect ratio, for $N_\tau = 4$ at $1.1T_c$. The volume dependence is expected to be even smaller as one goes further away from first order phase transition point. Also the agreement of our results with that of [13], where an aspect ratio of 4 have been used, shows that the these ratios have almost no volume dependence for $N_s \geq N_\tau + 2$.

While the closeness of C_{XU} and C_{XS} ($X = B, Q$) to their respective ideal gas values do support the notion of quasiparticle like excitations in QGP, a significant deviation of these ratios from their ideal gas values neither rule out the quasiparticle picture nor confirms the existence of the bound states proposed in [10]. Large contributions from the chemical potential dependence of the quasiparticle masses may lead to significant deviation of these ratios, especially in the vicinity of T_c. It has already been pointed out [7,18] that, near T_c, the chemical potential dependence of the quasiparticle masses becomes crucial for the baryonic susceptibilities.

Nevertheless, it may be interesting to compare our results with the predictions of the bound state model of [10]. Based on the model of [10] (and assuming that the mass formulae given in [10] hold right down to T_c) the predicted values of C_{BS} are approximately 0.62 at $1.5T_c$ [12], 0.11 at $1.25T_c$, and almost zero at $1.1T_c$ [29]. Clearly, as can be seen form Fig. 7 (bottom panel), these values are very much different from our continuum results. However, it has been argued in [18] that apart from all the bound states mentioned in [10], baryon like bound states may also exist in QGP. These baryons make large contributions to the baryonic susceptibilities, especially close to T_c [18]. Taking account of the contributions from the strange baryons may increase the value of C_{BS}. In [18] it has also been argued that for two light flavors if one considers the contributions of the baryons only then close to T_c the ratio of 2nd order isospin susceptibility (d_2^I) to the 2nd order baryonic susceptibility (d_2) is $d_2^I/d_2 = (\chi_u - \chi_{ud})/(\chi_u + \chi_{ud}) = 0.467$. Clearly this is inconsistent with our results since a value of $d_2^I/d_2 = 0.467$ gives a positive χ_{ud}/χ_u ($= 0.363$), whereas the lattice results for χ_{ud}/χ_u are negative and much smaller in magnitudes. This suggest that the contribution of the mesons (also possibly of the quarks, diquarks, and qg-states) are definitely important in the isospin susceptibility d_2^I. If one takes into account of the contributions of the mesons (pions and rhos) and assumes that the Boltzmann weight of the mesons are equal to that of the baryon, one gets a lower bound for d_2^I/d_2, namely $d_2^I/d_2 \geq 0.644$ [30]. But this lower bound gives $\chi_{ud}/\chi_u = 0.217$ and hence very far from our results. Moreover, very recently it has been argued [31] that one can carefully tune the densities of the baryon and meson like bound states in the model of Refs. [10,18] to reproduce the lattice results for off-diagonal QNS. But even those carefully tuned values fail to reproduce [31] the lattice results for higher order susceptibilities. In view of all this, the lattice results of [13] favor a quasiparticlelike picture of QGP, as opposed to the bound state model of [10,18]. The results of this paper show that these lattice results are really robust in the sense that they have very mild dependence on the lattice spacing and the sea quark content of the theory.

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[1] K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005); I. Arsene *et al.* (BRAHMS Collaboration), Nucl. Phys. **A757**, 1 (2005); B. B. Back *et al.* (PHOBOS Collaboration), Nucl. Phys. **A757**, 28 (2005); J. Adams *et al.* (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005).
[2] D. Teaney, Phys. Rev. C **68**, 034913 (2003).
[3] R. V. Gavai, S. Gupta, and S. Mukherjee, Proc. Sci., LAT2005 (2005) 173 [hep-lat/0506015].
[4] G. Boyd *et al.*, Phys. Rev. Lett. **75**, 4169 (1995); Nucl. Phys. **B469**, 419 (1996); F. Karsch, E. Laermann, and A. Peikert, Phys. Lett. B **478**, 447 (2000).
[5] A. Nakamura and S. Sakai, Phys. Rev. Lett. **94**, 072305 (2005).
[6] P. Levai and U. Heinz, Phys. Rev. C **57**, 1879 (1998); A. Peshier, Nucl. Phys. **A702**, 128 (2002).
[7] M. Bluhm, B. Kampfer, and G. Soff, Phys. Lett. B **620**, 131 (2005).
[8] P. Arnold and C. Zhai, Phys. Rev. D **50**, 7603 (1994); **51**, 1906 (1995); E. Braaten and A. Nieto, Phys. Rev. D **53**, 3421 (1996); J.-P. Blaizot, E. Iancu, and A. Rebhan, Phys. Rev. D **63**, 065003 (2001); K. Kajantie *et al.*, Phys. Rev. Lett. **86**, 10 (2001); J. High Energy Phys. 04 (2003) 036; J. O. Andersen *et al.*, Phys. Rev., D **66**, 085016 (2002).
[9] A. Dumitru, J. Lenaghan, and R. D. Pisarski, Phys. Rev. D **71**, 074004 (2005); C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73**, 014019 (2006).
[10] E. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004); Phys. Rev. C **70**, 021901 (2004).
[11] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004); S. Datta, F. Karsch, P. Petreczky, and I. Wetzorke, Phys. Rev. D **69**, 094507 (2004).
[12] V. Koch, A. Majumder, and J. Randrup, Phys. Rev. Lett. **95**, 182301 (2005).
[13] R. V. Gavai and S. Gupta, Phys. Rev. D **73**, 014004 (2006).
[14] S. Ejiri, F. Karsch, and K. Redlich, Phys. Lett. B **633**, 275 (2006).
[15] R. V. Gavai and S. Gupta, Phys. Rev. D **67**, 034501 (2003).
[16] R. V. Gavai, Nucl. Phys. B, Proc. Suppl. **119**, 529 (2003).
[17] J. P. Blaizot, E. Iancu, and A. Rebhan, Phys. Lett. B **523**, 143 (2001).
[18] J. Liao and E. Shuryak, Phys. Rev. D **73**, 014509 (2006).
[19] R. V. Gavai and S. Gupta, Phys. Rev. D **71**, 114014 (2005).
[20] S. Ejiri *et al.*, Phys. Rev. D **71**, 054508 (2005).
[21] M. Asakawa, U. Heinz, and B. Muller, Phys. Rev. Lett. **85**, 2072 (2000); S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).
[22] K. Adcox *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **89**, 082301 (2002); J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **68**, 044905 (2003).
[23] A. Wroblewski, Acta Phys. Pol. B **16**, 379 (1985).
[24] J. Rafelski and B. Muller, Phys. Rev. Lett. **48**, 1066 (1982); **56**, 2334 (1986).
[25] R. V. Gavai and S. Gupta, Phys. Rev. D **65**, 094515 (2002).
[26] S. Gupta and R. Ray, Phys. Rev. D **70**, 114015 (2004).
[27] J. Cleymans, J. Phys. G **28**, 1575 (2002).
[28] R. V. Gavai, S. Gupta, and P. Majumdar, Phys. Rev. D **65**, 054506 (2002).
[29] We are grateful to Abhijit Majumder for supplying us with these values.
[30] We thank Jinfeng Liao for pointing out this lower bound and for other useful correspondences.
[31] A. Majumder and B. Muller, nucl-th/0605079.