Nonleptonic *B* decays to axial-vector mesons and factorization

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We present an analysis of two-body B decays with a pseudoscalar (P) and an axial-vector meson ($\mathcal{A}$) in the final state using factorization. We employ as inputs a limited number of experimental data, i.e. results for the $B \to K_1\gamma$, and $B \to K^*\gamma$ radiative decays and the branching ratios for $B \to \pi\rho$, πK^*, $K\rho$, $K\pi$ nonleptonic decays. In this way we are able to compare our predictions with recent data from the Belle and *BABAR* collaborations on $B \to a_1\pi$ and make predictions on several other $B \to P\mathcal{A}$ decay channels, which might be used as a guide for experimental researches and as tests of factorization.

I. INTRODUCTION

Recent observations of the decays $B^0 \to a_1^\pm(1260)\pi^\mp$ from the Belle [1] and the *BABAR* Collaborations [2] offer the possibility of new investigations for two-body decay channels of the B meson with an axial-vector meson in the final state. The *BABAR* result

$$\mathcal{B}(B^0 \to a_1^\pm(1260)\pi^\mp) \times \mathcal{B}(a_1^\pm(1260) \to \pi^\pm\pi^\pm\pi^\mp)$$
$$= (16.6 \pm 1.9 \pm 1.5) \times 10^{-6} \quad (1)$$

translates into

$$\mathcal{B}(B^0 \to a_1^\pm(1260)\pi^\mp) = (33.2 \pm 3.8 \pm 3.0) \times 10^{-6} \quad (2)$$

assuming [2] that a_1 only decays into three pions and an equal yield for $a_1^\pm(1260) \to \pi^\pm\pi^\pm\pi^\mp$ and for $a_1^\pm(1260) \to \pi^\pm\pi^0\pi^0$. On the other hand the Belle measurement gives:

$$\mathcal{B}(B^0 \to a_1^\pm(1260)\pi^\mp) = (48.6 \pm 4.1 \pm 3.9) \times 10^{-6}, \quad (3)$$

with an average of the two experiments

$$\mathcal{B}(B^0 \to a_1^\pm(1260)\pi^\mp) = (40.9 \pm 7.6) \times 10^{-6}. \quad (4)$$

In a recent paper [3], hereafter referred to as I, two of us have discussed other two-body nonleptonic decays of the B meson with an axial-vector meson in the final state and proposed some simple tests of factorization for them. The analysis of I was stimulated by the experimental results $\mathcal{B}(B^+ \to K_1^+(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$ and $\mathcal{B}(B^+ \to K_1^+(1400)\gamma) < 1.44 \times 10^{-5}$ (at 90% C.L.) from the Belle collaboration [4]. These numerical results are comparable with data for the analogous channels with a vector meson in the final state: $\mathcal{B}(B^+ \to K^{*+}\gamma) = (4.18 \pm 0.31) \times 10^{-5}$ and $\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.17 \pm 0.23) \times 10^{-5}$ (averages of [5–7]). Therefore they suggest an approxi-

mate equality between the form factors for $B \to$ vector and $B \to$ axial-vector transitions [3]. Using this simple observation, in I we have proposed several tests of factorization for the B decay channels with a strange axial-vector meson in the final state. In this paper we wish to reconsider this approach and to extend it to other decay channels with strange particles in the final state as well as final states with no strange particles. In particular we wish to compare theoretical expectations with the *BABAR* and Belle results (2) and (3), and to give predictions for several similar decay channels that have not been examined yet theoretically, but might be studied by the *BABAR* and other experimental collaborations. The study of two-body charmless B decays with a light pseudoscalar meson and an axial meson in the final states, besides providing us with information on the $B \to a_1$ and $B \to K_1$ transition form factors, could also tell us about the dynamics of these decays modes. Unlike the $B \to K\rho$ decays which is much suppressed because of the destructive interference of the O_4 and O_6 matrix elements, the decays $B \to a_1K$ could have a large branching ratio (*BR*), since the interference term becomes constructive and enhances the decay rates as in the $B \to K\pi$ decay. Therefore a large *BR* similar to the *BR* for $B \to K\pi$ would be a confirmation of a large $B \to a_1$ transition form factors and the penguin dominance of this decay.

Our approach is based on the idea that factorization, together with experimental data for the *BRs* of the decays $B \to K^*\pi$, $K\pi$, $\rho\pi$, ρK, can provide enough information to predict nonleptonic B-decays with one axial-vector meson in the final state. It is known that factorization holds only approximately and in some cases its predictions are at odds with experiment. In the last few years factorization has been proved to be a rigorous prediction of QCD in the infinite quark mass limit [8–10] and the naive factorization scheme has evolved into a more precise approach, using effective theories and an expansion in Λ_{QCD}/m_b. In this context it should be noted however that for B decays into

two light hadrons a proof at all orders is still missing; moreover, for charmless B decays with a strange light hadron in the final state, the tree-level O_1, O_2 operators are CKM suppressed compared with the O_4, O_6 matrix elements. This gives a numerically important contribution to the penguin-dominated decays since O_6 matrix elements are chirally-enhanced in naive and in QCD factorization, although power suppressed by inverse powers of m_b [10] if annihilation terms are neglected.

To our knowledge there is currently no extensive study of charmless B decays with a final axial-vector meson based on factorization. Therefore we feel it can be useful to collect predictions on these channels using the simple naive factorization approach, though we are conscious that these results should be interpreted with some care and used more as guidelines for experiment than as absolute predictions. The advantage over previous calculations of some related decay channels using factorization, e.g. [11–13], is the fact that we do not use predictions from theoretical models for the form factors. Therefore any discrepancy that might be found between our predictions and future data would point to a breakdown of naive factorization and suggest more refined treatments.

The plan of the paper is as follows. After a review of the approach in Sec. II, we apply this method to get predictions for $B \to K_1 \pi$ in Sec. III, for $B \to a_1 K$ and $B \to b_1 K$ in Sec. IV and for $B \to a_1 \pi$, $b_1 \pi$ in Sec. V. Section VI contains our conclusions, while in Appendix A we have collected some relevant formulae used in the main text.

II. METHOD AND DEFINITIONS

Let us start with some relations among the various form factors of the $V - A$ currents that will be used below. We use definitions of form factors as listed in Appendix A. The main idea of I was to use ratios of BRs to deduce ratios of form factors and, subsequently, to use this piece of information to predict decays of the B-meson into final states with an axial-vector meson. To this effect we will need below the ratios $\frac{A_0^{B\to V}}{F_{0,1}^{B\to P}}$, $\frac{F_0^{B\to K}}{F_0^{B\to \pi}}$ and $\frac{V_0^{B\to \mathcal{A}}}{F_0^{B\to V}}$ where P, V and $\mathcal{A}$ refer to pseudoscalar, vector and axial-vector meson. We will determine these quantities by factorization and using experimental data.

As a matter of fact factorization predicts the following results

$$\frac{\mathcal{B}(B^+ \to K^0 \rho^+)}{\mathcal{B}(B^+ \to K^0 \pi^+)} \simeq \frac{4}{m_B^2} \left| \frac{W_8}{W_1} \right|^2 \left(\frac{A_0^{B\to\rho}(m_K^2)}{F_0^{B\to\pi}(m_K^2)} \right)^2 \frac{q_\rho^3}{q_\pi} \quad (5)$$

$$\frac{\mathcal{B}(B^0 \to K^+ \rho^-)}{\mathcal{B}(B^0 \to K^+ \pi^-)} \simeq \frac{4}{m_B^2} \left| \frac{W_6}{W_2} \right|^2 \left(\frac{A_0^{B\to\rho}(m_K^2)}{F_0^{B\to\pi}(m_K^2)} \right)^2 \frac{q_\rho^3}{q_\pi} \quad (6)$$

$$\frac{\mathcal{B}(B^0 \to K^0 \rho^0)}{\mathcal{B}(B^0 \to K^0 \pi^0)} \simeq \frac{4}{m_B^2} \left| \frac{W_8 \frac{A_0^{B\to\rho}(m_K^2)}{F_0^{B\to\pi}(m_K^2)} + W_7 \frac{f_\rho}{f_K} \frac{F_1^{B\to K}(m_\rho^2)}{F_0^{B\to\pi}(m_K^2)}}{W_1 + W_3 \frac{f_\pi}{f_K} \frac{F_0^{B\to K}(m_\pi^2)}{F_0^{B\to\pi}(m_K^2)}} \right|^2 \frac{q_\rho^3}{q_\pi} \quad (7)$$

$$\frac{\mathcal{B}(B^+ \to K^+ \rho^0)}{\mathcal{B}(B^+ \to K^+ \pi^0)} \simeq \frac{4}{m_B^2} \left| \frac{W_6 \frac{A_0^{B\to\rho}(m_K^2)}{F_0^{B\to\pi}(m_K^2)} + W_7 \frac{f_\rho}{f_K} \frac{F_1^{B\to K}(m_\rho^2)}{F_0^{B\to\pi}(m_K^2)}}{W_2 + W_3 \frac{f_\pi}{f_K} \frac{F_0^{B\to K}(m_\pi^2)}{F_0^{B\to\pi}(m_K^2)}} \right|^2 \frac{p_\rho^3}{p_\pi} \quad (8)$$

$$\frac{\mathcal{B}(B^+ \to K^{*0} \pi^+)}{\mathcal{B}(B^+ \to K^0 \pi^+)} \simeq \frac{4}{m_B^2} \left| \frac{W_4}{W_1} \right|^2 \left(\frac{f_{K^*}}{f_K} \right)^2 \frac{q_{K^*}^3}{q_K} \quad (9)$$

$$\frac{\mathcal{B}(B^0 \to K^{*+} \pi^-)}{\mathcal{B}(B^0 \to K^+ \pi^-)} \simeq \frac{4}{m_B^2} \left| \frac{W_5}{W_2} \right|^2 \left(\frac{f_{K^*}}{f_K} \right)^2 \frac{q_{K^*}^3}{q_K} \quad (10)$$

$$\frac{\mathcal{B}(B^0 \to K^{*0} \pi^0)}{\mathcal{B}(B^0 \to K^0 \pi^0)} \simeq \frac{4}{m_B^2} \left| \frac{W_3 \frac{f_\pi}{f_{K^*}} \frac{A_0^{B\to K^*}(m_\pi^2)}{F_1^{B\to\pi}(m_{K^*}^2)} + W_4}{W_3 \frac{f_\pi}{f_{K^*}} \frac{F_0^{B\to K}(m_\pi^2)}{F_1^{B\to\pi}(m_{K^*}^2)} + W_1 \frac{f_K}{f_{K^*}}} \right|^2 \frac{q_{K^*}^3}{q_K} \quad (11)$$

$$\frac{\mathcal{B}(B^+ \to K^{*+} \pi^0)}{\mathcal{B}(B^+ \to K^+ \pi^0)} \simeq \frac{4}{m_B^2} \left| \frac{W_3 \frac{f_\pi}{f_{K^*}} \frac{A_0^{B\to K^*}(m_\pi^2)}{F_1^{B\to\pi}(m_{K^*}^2)} + W_5}{W_3 \frac{f_\pi}{f_{K^*}} \frac{F_0^{B\to K}(m_\pi^2)}{F_1^{B\to\pi}(m_{K^*}^2)} + W_2 \frac{f_K}{f_{K^*}}} \right|^2 \frac{q_{K^*}^3}{q_K}. \quad (12)$$

We have indicated the squared meson mass in the argument of the form factors to keep track of the factorization procedure, but in the numerical computations all form factors are evaluated at $q^2 = 0$, which should be a rather good approximation; q_ρ, q_π, q_{K^*} and q_K are momenta in the B rest frame and W_i are combinations of Wilson coefficients and CKM matrix elements that can be found in the upper part of the Table reported in Appendix A. Using as inputs the experimental ratios for $\frac{\mathcal{B}(B^0\to K^+\rho^-)}{\mathcal{B}(B^0\to K^+\pi^-)}$, $\frac{\mathcal{B}(B^0\to K^0\rho^0)}{\mathcal{B}(B^0\to K^0\pi^0)}$, and $\frac{\mathcal{B}(B^0\to K^{*0}\pi^0)}{\mathcal{B}(B^0\to K^0\pi^0)}$, we have computed the entries in the first columns in Table I (the last column is obtained by the ratio of the first two data).

We also present a comparison with other theoretical approaches. We notice that our predictions for the ratios are in general higher than other methods. The Light Cone Sum Rules (LCSR) results of Refs. [14] are however the less distant from ours.

We can now use these data to compute the remaining *BRs* in Eqns. (5)–(12). The results are reported in Table II and can be considered as a consistence test for the method to be used in the subsequent Sections. In particular we note that the ratios $\frac{\mathcal{B}(B^+\to K^+\rho^0)}{\mathcal{B}(B^+\to K^+\pi^0)}$ and $\frac{\mathcal{B}(B^+\to K^{*+}\pi^0)}{\mathcal{B}(B^+\to K^+\pi^0)}$ agree with the experimental results. It can be also noticed that $\frac{\mathcal{B}(B^+\to K^{*0}\pi^+)}{\mathcal{B}(B^+\to K^0\pi^+)}$

TABLE I. Ratio of form factors involving *B* decays to negative parity mesons. The results of the present paper can be compared to the findings obtained by the Bauer-Stech-Model (BSW), the Heavy Meson Effective Lagrangian (HMEL), Light-Cone Sum Rules (LCSR) and the Covariant Light Front Approach (CLFA).

	$\frac{A_0^{B\to\rho}(0)}{F_1^{B\to\pi}(0)}$	$\frac{F_1^{B\to K}(0)}{F_1^{B\to\pi}(0)}$	$\frac{A_0^{B\to K^*}(0)}{F_1^{B\to\pi}(0)}$	$\frac{A_0^{B\to\rho}(0)}{F_1^{B\to K}(0)}$
BSW [11]	0.84	1.15	0.97	0.73
HMEL [13]	0.45 ± 0.56	0.92 ± 0.32	0.38 ± 0.46	0.49 ± 0.60
LCSR [14]	1.15	1.30	1.38	0.88
CLFA [15]	1.12	1.40	1.24	0.80
This work	1.63	1.56	1.98	1.02

and $\frac{\mathcal{B}(B^0\to K^{*+}\pi^-)}{\mathcal{B}(B^0\to K^+\pi^-)}$ in the present approximation are completely independent of form factors.

There are no data so far for the ratio $\frac{\mathcal{B}(B^+\to K^0\rho^+)}{\mathcal{B}(B^+\to K^0\pi^+)}$: since we will need below the *BR* for the decay $B^+ \to K^0\rho^+$, we will use the result that can be obtained from Table II, together with the experimental value $\mathcal{B}(B^+ \to K^0\pi^+) = (24.1 \pm 1.3) \times 10^{-6}$ [16], i.e we will take

$$\mathcal{B}(B^+ \to K^0\rho^+) \approx 0.44 \times 10^{-6}. \quad (13)$$

In order to apply this method to the decays with an axial-vector meson in the final state we need information on the corresponding form factors, whose definition is in the Appendix. In I we assumed that the effect of substituting K^* with K_1 is identical in the radiative and in the nonleptonic decay, in other words that each form factor for the $B \to K_1$ transition is given by the corresponding form factor for $B \to K^*$ multiplied by the same factor y, once the change of parity between the two strange mesons and the kinematical factors are taken into account. For our purposes only the form factor V_0 (see the Appendix) for the transition $B \to$ axial-vector meson is relevant. Using the above-mentioned assumption we get

$$V_0^{B\to K_1(1270)}(q^2) = hA_0^{B\to K^*}(q^2),$$
$$V_0^{B\to K_1(1400)}(q^2) = kA_0^{B\to K^*}(q^2) \quad (14)$$

with

$$\begin{pmatrix} h \\ k \end{pmatrix} = \frac{m_{K^*}}{m_{K_1}} \frac{m_B + m_{K_1} - (m_B - m_{K_1})z}{m_B + m_{K^*} - (m_B - m_{K^*})z} \begin{pmatrix} y \\ y' \end{pmatrix}, \quad (15)$$

where y and y' are defined in the Appendix, while the factor z is defined as

$$z = \frac{A_2^{B\to\rho}(0)}{A_1^{B\to\rho}(0)} \approx \frac{A_2^{B\to K^*}(0)}{A_1^{B\to K^*}(0)}. \quad (16)$$

We take the value $z = 0.93$, intermediate between the value predicted by light cone sum rules [14] ($z = 0.9$) and that given by the BSW model [11] ($z = 0.95$). In the following we will need also of the ratio $\frac{V_0^{B\to\mathcal{A}_1}}{A_0^{B\to\rho}}$ (with $\mathcal{A}_1 = a_1$ or b_1); we can predict it from the previous result:

$$\frac{V_0^{B\to a_1}(0)}{A_0^{B\to\rho}(0)} \approx \frac{V_0^{B\to K_{1A}}(0)}{A_0^{B\to K^*}(0)} = h\sin\theta + k\cos\theta, \quad (17)$$

$$\frac{V_0^{B\to b_1}(0)}{A_0^{B\to\rho}(0)} \approx \frac{V_0^{B\to K_{1B}}(0)}{A_0^{B\to K^*}(0)} = h\cos\theta - k\sin\theta. \quad (18)$$

In previous equations we assume that the ratios satisfy $SU(3)$ flavor symmetry to a good approximation since $SU(3)$ breaking terms tend to cancel out in the ratio, see e.g. [12]. In Eqs. (17) and (18) θ is the mixing angle between the octets $^3P_1$ and $^1P_1$ from which the states $K_1(1270)$ and $K_1(1400)$ result. To the former octet belong a_1 and the unmixed strange state K_{1A}; to the latter b_1 and K_{1B}, see the Appendix A for further details. The mixing scheme we adopt here is analogous to that based on the conventional quark model of Ref. [17]; θ is the mixing angle between two strange *P* wave axial meson; therefore, differently from, e.g. $\eta - \eta'$ mixing, it should not be affected by gluonium contributions. The phenomenological analysis [3,15,17] gives as possible results $\theta = 32^0$ or $58°$. In Table III we report our predictions for both values and a comparison with the result of Ref. [15].

III. $B \to K_1\pi$

These channels were already considered in I and we report them for completeness. If q_{K_1} and q_{K^*} are, respectively, the c.m. momenta of K_1 and K^* in the reactions $B \to K_1\pi$ and $B \to K^*\pi$, one gets, using factorization:

TABLE II. Ratios of Branching Ratios and their comparison with experiment.

Ratios	Th.	Exp. [16]	Ratios	Th.	Exp. [16]
$\frac{\mathcal{B}(B^0\to K^+\rho^-)}{\mathcal{B}(B^0\to K^+\pi^-)}$	input	0.54 ± 0.11	$\frac{\mathcal{B}(B^0\to K^{*+}\pi^-)}{\mathcal{B}(B^0\to K^+\pi^-)}$	0.55	0.69 ± 0.13
$\frac{\mathcal{B}(B^0\to K^0\rho^0)}{\mathcal{B}(B^0\to K^0\pi^0)}$	input	0.44 ± 0.17	$\frac{\mathcal{B}(B^0\to K^{*0}\pi^0)}{\mathcal{B}(B^0\to K^0\pi^0)}$	input	0.14 ± 0.08
$\frac{\mathcal{B}(B^+\to K^+\rho^0)}{\mathcal{B}(B^+\to K^+\pi^0)}$	0.42	0.42 ± 0.10	$\frac{\mathcal{B}(B^+\to K^{*+}\pi^0)}{\mathcal{B}(B^+\to K^+\pi^0)}$	0.71	0.57 ± 0.22
$\frac{\mathcal{B}(B^+\to K^0\rho^+)}{\mathcal{B}(B^+\to K^0\pi^+)}$	0.018	$\cdots$	$\frac{\mathcal{B}(B^+\to K^{*0}\pi^+)}{\mathcal{B}(B^+\to K^0\pi^+)}$	0.46	0.40 ± 0.07

TABLE III. Ratio of form factors for *B* decays to axial-vector mesons.

	$\frac{V_0^{B\to K_{1A}}(0)}{A_0^{B\to K^*}(0)}$	$\frac{V_0^{B\to K_{1B}}(0)}{A_0^{B\to K^*}(0)}$
This work ($\theta = 32^o$)	0.64	0.78
This work ($\theta = 58^o$)	1.02	0.26
CLFA [15]	0.45	1.32

$$\frac{\mathcal{B}(B^+ \to K_1^0 \pi^+)_{\text{fact.}}}{\mathcal{B}(B^+ \to K^{*0} \pi^+)_{\text{fact.}}} = \frac{\mathcal{B}(B^0 \to K_1^+ \pi^-)_{\text{fact.}}}{\mathcal{B}(B^0 \to K^{*+} \pi^-)_{\text{fact.}}}$$

$$= \left(\frac{q_{K_1}}{q_{K^*}}\right)^3 \left(\frac{F_1^{B\to\pi}(m_{K_1}^2) f_{K_1}}{F_1^{B\to\pi}(m_{K^*}^2) f_{K^*}}\right)^2, \quad (19)$$

where the subscript means that we consider only factorizable contributions. Therefore, using f_{K_1} from τ decays (see the Appendix) one can predict $\mathcal{B}(B^+ \to K_1^0 \pi^+)$ and $\mathcal{B}(B^0 \to K_1^+ \pi^-)$ for both $K_1(1270)$ and $K_1(1400)$ from the knowledge of $\mathcal{B}(B^+ \to K^{*0}\pi^+)$ and $\mathcal{B}(B^0 \to K^{*+}\pi^-)$ [18].

The reactions with a π^0 in the final state: $B^+ \to K_1^+ \pi^0$, $B^0 \to K_1^0 \pi^0$ involve three form factors F_1, A_0 and V_0 and different combinations of Wilson coefficients and CKM matrix elements. One gets ($s = h$, k, see Eq. (15), for $K_1(1270)$ and $K_1(1400)$ respectively):

$$\frac{\mathcal{B}(B^+ \to K_1^+ \pi^0)_{\text{fact.}}}{\mathcal{B}(B^+ \to K^{*+}\pi^0)_{\text{fact.}}} = \left(\frac{q_{K_1}}{q_{K^*}}\right)^3 \left| \frac{\frac{W_5}{W_3} \frac{f_{K_1}}{f_\pi} \frac{F_1^{B\to\pi}(m_{K_1}^2)}{A_0^{B\to K^*}(m_\pi^2)} + s}{\frac{W_5}{W_3} \frac{f_{K^*}}{f_\pi} \frac{F_1^{B\to\pi}(m_{K^*}^2)}{A_0^{B\to K^*}(m_\pi^2)} + 1} \right|^2 \quad (20)$$

$$\frac{\mathcal{B}(B^0 \to K_1^0 \pi^0)_{\text{fact.}}}{\mathcal{B}(B^0 \to K^{*0}\pi^0)_{\text{fact.}}} = \left(\frac{q_{K_1}}{q_{K^*}}\right)^3 \left| \frac{\frac{W_4}{W_3} \frac{f_{K_1}}{f_\pi} \frac{F_1^{B\to\pi}(m_{K_1}^2)}{A_0^{B\to K^*}(m_\pi^2)} + s}{\frac{W_4}{W_3} \frac{f_{K^*}}{f_\pi} \frac{F_1^{B\to\pi}(m_{K^*}^2)}{A_0^{B\to K^*}(m_\pi^2)} + 1} \right|^2; \quad (21)$$

where W_i are listed in Appendix. The result of this analysis is in Table IV. For the form factors ratios, that we have considered at $q^2 = 0$, we have used the values in Table I.

IV. $B \to \mathcal{A}_1 K$

In this section we consider the decays $B \to a_1 K$, $b_1 K$. Also in this case we have some clear predictions based on factorization for the decays with a charged axial-vector meson in the final state [3]:

$$\frac{\mathcal{B}(B^+ \to \mathcal{A}_1^+ K^0)_{\text{fact.}}}{\mathcal{B}(B^+ \to \rho^+ K^0)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{W_1}{W_8} \frac{V_0^{B\to\mathcal{A}_1}(m_K^2)}{A_0^{B\to\rho}(m_K^2)} \right|^2, \quad (22)$$

$$\frac{\mathcal{B}(B^0 \to \mathcal{A}_1^- K^+)_{\text{fact.}}}{\mathcal{B}(B^0 \to \rho^- K^+)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{W_2}{W_6} \frac{V_0^{B\to\mathcal{A}_1}(m_K^2)}{A_0^{B\to\rho}(m_K^2)} \right|^2, \quad (23)$$

where $\frac{V_0^{B\to\mathcal{A}_1}(m_K^2)}{A_0^{B\to\rho}(m_K^2)}$ is given by Eq. (17) or Eq. (18) for $\mathcal{A}_1 = a_1$ or b_1 respectively.

Similar predictions can be given also for the channels with a neutral axial-vector meson in the final state, ie the decay channels $B^+ \to a_1^0 K^+$, $B^+ \to b_1^0 K^+$, $B^0 \to a_1^0 K^0$ and $B^0 \to b_1^0 K^0$, though the corresponding formulae are more involved. In fact we have

$$\frac{\mathcal{B}(B^+ \to \mathcal{A}_1^0 K^+)_{\text{fact.}}}{\mathcal{B}(B^+ \to \rho^0 K^+)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{\frac{V_0^{B\to\mathcal{A}_1}(m_K^2)}{A_0^{B\to\rho}(m_K^2)} W_2 + W_3 \frac{f_{\mathcal{A}_1}}{f_K} \frac{F_1^{B\to K}(m_{\mathcal{A}_1}^2)}{A_0^{B\to\rho}(m_K^2)}}{W_6 + W_7 \frac{f_\rho}{f_K} \frac{F_1^{B\to K}(m_\rho^2)}{A_0^{B\to\rho}(m_K^2)}} \right|^2, \quad (24)$$

$$\frac{\mathcal{B}(B^0 \to \mathcal{A}_1^0 K^0)_{\text{fact.}}}{\mathcal{B}(B^0 \to \rho^0 K^0)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{\frac{V_0^{B\to\mathcal{A}_1}(m_K^2)}{A_0^{B\to\rho}(m_K^2)} W_1 + W_3 \frac{f_{\mathcal{A}_1}}{f_K} \frac{F_1^{B\to K}(m_{\mathcal{A}_1}^2)}{A_0^{B\to\rho}(m_K^2)}}{W_8 + W_7 \frac{f_\rho}{f_K} \frac{F_1^{B\to K}(m_\rho^2)}{A_0^{B\to\rho}(m_K^2)}} \right|^2; \quad (25)$$

TABLE IV. Theoretical branching ratios for B decays into a strange axial-vector meson and a pion. Units 10^{-6}.

Process	$\mathcal{B}$ (Th.)	$\mathcal{B}$(Exp.) [18]
$B^+ \to \pi^+ K_1^0(1270)$	5.8	$\cdots$
$B^+ \to \pi^0 K_1^+(1270)$	4.9	$\cdots$
$B^0 \to \pi^0 K_1^0(1270)$	0.4	$\cdots$
$B^0 \to \pi^- K_1^+(1270)$	7.6	$\cdots$
$B^+ \to \pi^+ K_1^0(1400)$	3.0	<260
$B^+ \to \pi^0 K_1^+(1400)$	1.0 ($\theta = 32^0$) 1.4 ($\theta = 58^0$)	$\cdots$
$B^0 \to \pi^0 K_1^0(1400)$	3.0 ($\theta = 32^0$) 1.7 ($\theta = 58^0$)	$\cdots$
$B^0 \to \pi^- K_1^+(1400)$	4.0	<1100

the ratio $\frac{F_1^{B\to K}}{A_0^{B\to\rho}}$ can be computed at $q^2 = 0$, as reported in Table I; the coefficients W_i are in the Appendix A. The results obtained are reported in Table V. We have used the experimental BR for $B \to \rho^- K^+$ as given by the HFAG group [16]. For the BR of the channel $B^+ \to \rho^+ K^0$ only an upper limit 4.8×10^{-5} is available [18]. Therefore we have determined this BR from the channel $B \to \rho^- K^+$, applying also in this case factorization, ie we have used the prediction in Eq. (13).

We are aware that in some cases the assumptions we make might be flawed. For example it is known that naive factorization gives a small contribution to the $B^0 \to \rho^- K^+$ channel. The experimental result $\mathcal{B}(B^0 \to \rho^- K^+) = (9.9^{+1.6}_{-1.5}) \times 10^{-6}$ [16] is larger by 1 order of magnitude than theoretical predictions based on naive factorization

TABLE V. Theoretical branching ratios for *B* decays into one nonstrange axial-vector meson and a kaon for two different values of the mixing angle. Units 10^{-5}.

Process	$\mathcal{B}(\theta = 32^o)$	$\mathcal{B}(\theta = 58^o)$
$B^+ \rightarrow K^+ a_1^0$	1.4	2.8
$B^+ \rightarrow K^0 a_1^+$	2.1	5.4
$B^0 \rightarrow K^0 a_1^0$	0.5	2.5
$B^0 \rightarrow K^+ a_1^-$	1.6	4.1
$B^+ \rightarrow K^+ b_1^0$	1.1	0.05
$B^+ \rightarrow K^0 b_1^+$	3.2	0.3
$B^0 \rightarrow K^0 b_1^0$	2.7	0.6
$B^0 \rightarrow K^+ b_1^-$	2.4	0.2

and reasonable models for the form factors [12], which is due to large cancellations between the penguin contributions appearing in W_i. An enhancement with respect to naive factorization can be due to various reasons. For example one can mention $\mathcal{O}(\alpha_s)$ corrections to the matrix elements. Moreover long-distance nonfactorizable contributions, that are power suppressed, such as the charming penguin contributions [19,20] are expected to play a role

[21], as well as other power corrections terms in QCD factorization [22]. Finally including final state interactions requires both perturbative corrections at leading power, as well as power corrections. The phenomenology due to these effects has been studied in detail in [9,10]. Because of these uncertainties the results in Table V should be interpreted more as tests of the factorization model than as absolute predictions and are based on the expectations that, large as they can be, long-distance effects, e.g. those described by final state interactions, cancel out in the ratios. In any case, to increase our confidence in the method, we use a different approach to get predictions for these channels, i.e we consider the ratio of $\mathcal{B}(B \rightarrow \mathcal{A}_1 K)$ to $\mathcal{B}(B \rightarrow \pi K)$. In this case factorization predicts

$$\frac{\mathcal{B}(B^+ \rightarrow K^0 \mathcal{A}_1^+)_{\text{fact.}}}{\mathcal{B}(B^+ \rightarrow K^0 \pi^+)_{\text{fact.}}} = \frac{\mathcal{B}(B^0 \rightarrow K^+ \mathcal{A}_1^-)_{\text{fact.}}}{\mathcal{B}(B^0 \rightarrow K^+ \pi^-)_{\text{fact.}}}$$

$$\simeq \frac{4}{m_B^2} \left(\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_K^2)}{F_0^{B \rightarrow \pi}(m_K^2)} \right)^2 \frac{q_{\mathcal{A}_1}^3}{q_\pi}, \quad (26)$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \mathcal{A}_1^0)_{\text{fact.}}}{\mathcal{B}(B^+ \rightarrow K^+ \pi^0)_{\text{fact.}}} \simeq \frac{4}{m_B^2} \left| \frac{\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_K^2)}{F_0^{B \rightarrow \pi}(m_K^2)} + \frac{W_3}{W_2} \frac{f_{\mathcal{A}_1}}{f_K} \frac{F_1^{B \rightarrow K}(m_{\mathcal{A}_1}^2)}{F_0^{B \rightarrow \pi}(m_{\mathcal{A}_1}^2)}}{1 + \frac{W_3}{W_2} \frac{f_\pi}{f_K} \frac{F_0^{B \rightarrow K}(m_\pi^2)}{F_0^{B \rightarrow \pi}(m_K^2)}} \right|^2 \frac{q_{\mathcal{A}_1}^3}{q_\pi}, \quad (27)$$

$$\frac{\mathcal{B}(B^0 \rightarrow K^0 \mathcal{A}_1^0)_{\text{fact.}}}{\mathcal{B}(B^0 \rightarrow K^0 \pi^0)_{\text{fact.}}} \simeq \frac{4}{m_B^2} \left| \frac{\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_K^2)}{F_0^{B \rightarrow \pi}(m_K^2)} + \frac{W_3}{W_1} \frac{f_{\mathcal{A}_1}}{f_K} \frac{F_1^{B \rightarrow K}(m_{\mathcal{A}_1}^2)}{F_0^{B \rightarrow \pi}(m_{\mathcal{A}_1}^2)}}{1 + \frac{W_3}{W_1} \frac{f_\pi}{f_K} \frac{F_0^{B \rightarrow K}(m_\pi^2)}{F_0^{B \rightarrow \pi}(m_K^2)}} \right|^2 \frac{q_{\mathcal{A}_1}^3}{q_\pi}; \quad (28)$$

the parameters W_j are in the Table of the Appendix A, while for $\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_K^2)}{F_0^{B \rightarrow \pi}(m_K^2)}$ we use

$$\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_K^2)}{F_0^{B \rightarrow \pi}(m_K^2)} \approx \frac{V_0^{B \rightarrow \mathcal{A}_1}(0)}{F_0^{B \rightarrow \pi}(0)} = \frac{V_0^{B \rightarrow \mathcal{A}_1}(0)}{A_0^{B \rightarrow \rho}(0)} \frac{A_0^{B \rightarrow \rho}(0)}{F_0^{B \rightarrow \pi}(0)}. \quad (29)$$

We can now compute again the entries of Table V using these formulae and the experimental *BRs* for $B \rightarrow K\pi$. The interesting fact is that we obtain results that differ a few percent at most from those found using the ratios to the decay channels $B \rightarrow K\rho$. Therefore we assume this as a rough estimate of the theoretical uncertainty associated with the present procedure.

Some interesting predictions can be read from Table V. For $\theta = 32^o$, for all the decay channels, with the exception of $a_1^0 K^0$, we predict *BRs* of the order of 10^{-5}; for $\theta = 58^o$ we have *BRs* of similar sizes only for $B \rightarrow a_1 K$. Summing up one can say that nonleptonic *B* decays with a kaon and a light nonstrange axial-vector meson in the final state represent interesting decay channels with generally large *BRs*.

V. $B \rightarrow \mathcal{A}_1 \pi$

In this section we consider the decays $B \rightarrow a_1 \pi, b_1 \pi$. To start with, we consider the channel with at least one neutral particle in final state. We get the following results for the ratios $\frac{\mathcal{B}(B \rightarrow \mathcal{A}_1 \pi)}{\mathcal{B}(B \rightarrow \rho \pi)}$:

$$\frac{\mathcal{B}(B^+ \rightarrow \mathcal{A}_1^0 \pi^+)_{\text{fact.}}}{\mathcal{B}(B^+ \rightarrow \rho^0 \pi^+)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho} \right)^3 \left| \frac{\frac{V_0^{B \rightarrow \mathcal{A}_1}(m_\pi^2)}{A_0^{B \rightarrow \rho}(m_\pi^2)} w_1 + w_2 \frac{f_{\mathcal{A}_1}}{f_\pi} \frac{F_1^{B \rightarrow \pi}(m_{\mathcal{A}_1}^2)}{A_0^{B \rightarrow \rho}(m_\pi^2)}}{w_5 + w_6 \frac{f_\rho}{f_\pi} \frac{F_1^{B \rightarrow \pi}(m_\rho^2)}{A_0^{B \rightarrow \rho}(m_\pi^2)}} \right|^2, \quad (30)$$

$$\frac{\mathcal{B}(B^+ \to \mathcal{A}_1^+ \pi^0)_{\text{fact.}}}{\mathcal{B}(B^+ \to \rho^+ \pi^0)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{\frac{V_0^{B \to \mathcal{A}_1}(m_\pi^2)}{A_0^{B \to \rho}(m_\pi^2)} w_3 + w_4 \frac{f_{\mathcal{A}_1}}{f_\pi} \frac{F_1^{B \to \pi}(m_{\mathcal{A}_1}^2)}{A_0^{B \to \rho}(m_\pi^2)}}{w_7 + w_3 \frac{f_\rho}{f_\pi} \frac{F_1^{B \to \pi}(m_\rho^2)}{A_0^{B \to \rho}(m_\pi^2)}} \right|^2, \tag{31}$$

$$\frac{\mathcal{B}(B^0 \to \mathcal{A}_1^0 \pi^0)_{\text{fact.}}}{\mathcal{B}(B^0 \to \rho^0 \pi^0)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \left| \frac{\frac{V_0^{B \to \mathcal{A}_1}(m_\pi^2)}{A_0^{B \to \rho}(m_\pi^2)} w_4 + w_2 \frac{f_{\mathcal{A}_1}}{f_\pi} \frac{F_1^{B \to \pi}(m_{\mathcal{A}_1}^2)}{A_0^{B \to \rho}(m_\pi^2)}}{w_7 + w_6 \frac{f_\rho}{f_\pi} \frac{F_1^{B \to \pi}(m_\rho^2)}{A_0^{B \to \rho}(m_\pi^2)}} \right|^2, \tag{32}$$

We use the ratios in Table I and the experimental *BRs* for $B \to \rho\pi$ as given by the HFAG group [16]. The results are reported in Table VI.

Let us now consider the channels having only charged mesons in the final state. In order to use the same method employed in the previous sections we would need the *BRs* from the decays $B^0 \to \rho^+ \pi^-$ and $B^0 \to \rho^- \pi^+$. Only their sum: $\mathcal{B}(B^0 \to \rho^\pm \pi^\mp) = \mathcal{B}(B^0 \to \rho^+ \pi^-) + \mathcal{B}(B^0 \to \rho^- \pi^+)$ is at the moment known $\mathcal{B}(B^0 \to \rho^\pm \pi^\mp) = (24.0 \pm 2.5) \times 10^{-6}$ [16], therefore we consider the following ratios ($\mathcal{A}_1 = a_1, b_1$):

$$\frac{\mathcal{B}(B^0 \to \pi^+ \mathcal{A}_1^-)_{\text{fact.}}}{\mathcal{B}(B^0 \to \pi^\pm \rho^\mp)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \frac{\left|\frac{w_1}{w_5} \frac{V_0^{B \to \mathcal{A}_1}(m_\pi^2)}{A_0^{B \to \rho}(m_\pi^2)}\right|^2}{1 + \left|\frac{w_3}{w_5} \frac{f_\rho}{f_\pi} \frac{F_1^{B \to \pi}(m_\rho^2)}{A_0^{B \to \rho}(m_\pi^2)}\right|^2}, \tag{33}$$

$$\frac{\mathcal{B}(B^0 \to \pi^- \mathcal{A}_1^+)_{\text{fact.}}}{\mathcal{B}(B^0 \to \pi^\pm \rho^\mp)_{\text{fact.}}} = \left(\frac{q_{\mathcal{A}_1}}{q_\rho}\right)^3 \frac{\left|\frac{w_3}{w_5} \frac{f_{\mathcal{A}_1}}{f_\pi} \frac{F_1^{B \to \pi}(m_{\mathcal{A}_1}^2)}{A_0^{B \to \rho}(m_\pi^2)}\right|^2}{1 + \left|\frac{w_3}{w_5} \frac{f_\rho}{f_\pi} \frac{F_1^{B \to \pi}(m_\rho^2)}{A_0^{B \to \rho}(m_\pi^2)}\right|^2}, \tag{34}$$

where the parameters w_k are defined in the Appendix A. In this way we can complete the inputs of Table VI. There is an independent analysis, given by Höcker *et al.* [23], which extracts from $B^0 \to \rho^\pm \pi^\mp$ the values of the single channels with the result $\mathcal{B}(B^0 \to \rho^+ \pi^-) = (15.3^{+3.7}_{-3.3}) \times 10^{-6}$ and $\mathcal{B}(B^0 \to \rho^- \pi^+) = (14.5^{+4.1}_{-3.6}) \times 10^{-6}$. Using these

values one would estimate the *BRs* of $B^0 \to \mathcal{A}_1^+ \pi^-$ and $B^0 \to \mathcal{A}_1^- \pi^+$ with results $\sim 20\%$ greater than those in Table VI, i.e within our estimated theoretical error. A greater confidence can be obtained using a slightly different approach. One might note that the ratio $\frac{\mathcal{B}(B^0 \to \mathcal{A}_1^- \pi^+)}{\mathcal{B}(B^0 \to \pi^- \pi^+)}$ is independent of the Wilson coefficient and CKM matrix elements. Using as an experimental input $\mathcal{B}(B^0 \to \pi^- \pi^+) = (4.5 \pm 0.4) \times 10^{-6}$ [16], one gets values for $\mathcal{B}(B^0 \to \mathcal{A}_1^- \pi^+)$ in agreement with Table VI within $\sim 10\%$. We note that the prediction for $B^0 \to \pi^\mp a_1^\pm$ is somewhat smaller than the result from the Belle [1] and *BABAR* Collaborations [2], although the value 58° for the mixing angle offers a better agreement. This is an indication that this value of the angle is to be preferred. If subsequent analyses would lead one to prefer the solution $\theta = 32^o$, this would mean either a failure of some of our assumptions or that there are nonresonant effects, not included in the theoretical analysis, and implicitly taken into account in the data. This might happen because, for nonresonant diagrams, some particles in the final state might fall in the same kinematical range as the a_1 state, with an effect similar to what discussed for $B \to 3\pi$ in Ref. [24].

VI. CONCLUSIONS

In conclusions we have presented predictions for the nonleptonic *B*-meson decay channel with one axial-vector meson in the final state. We have used uniquely experimental data, e.g. the decay rates for $B \to K^* \pi$, $K\pi$, $\rho\pi$, ρK and, as a theoretical input, the assumption of naive factorization. Our results may provide a useful benchmark for the future searches of the decay channels $B \to K_1 \pi$, $B \to a_1 K$, $B \to b_1 K$, $B \to a_1 \pi$, $B \to b_1 \pi$ that might be investigated by the experimental collaborations.

APPENDIX

In this appendix we list the values of the Wilson coefficients and the CKM matrix elements we have used in the main text.

TABLE VI. Theoretical branching ratios for *B* decays into one nonstrange axial-vector meson and a pion for two different values of the mixing angle. Units are 10^{-6}.

Process	$\mathcal{B}(\theta = 32^o)$	$\mathcal{B}(\theta = 58^o)$	Exp.
$B^+ \to \pi^+ a_1^0$	3.9	8.8	<900 [18]
$B^+ \to \pi^0 a_1^+$	10.3	12.3	<1700 [18]
$B^0 \to \pi^0 a_1^0$	1.1	1.7	<1100 [18]
$B^0 \to \pi^+ a_1^-$	4.7	11.8	40.9 ± 7.6 [1,2]
$B^0 \to \pi^- a_1^+$	11.1	12.3	
$B^+ \to \pi^+ b_1^0$	4.5	0.4	$\cdots$
$B^+ \to \pi^0 b_1^+$	1.7	0.1	$\cdots$
$B^0 \to \pi^0 b_1^0$	0.5	0.01	$\cdots$
$B^0 \to \pi^+ b_1^-$	6.9	0.7	$\cdots$
$B^0 \to \pi^- b_1^+$	1.3	0.2	$\cdots$

Wilson coefficients (using the results of [25] for $\Lambda_{\overline{MS}}^{(5)} =$ 225 MeV in the HV scheme) and current quark masses:

$$\{a_1, a_2\} = \{1.029, 0.140\},$$

$$\{a_3, \cdots a_{10}\} = \{33, -246, -10, -300, 2, 4.8,$$

$$- 93, -12\} \times 10^{-4},$$

$$\{m_u, m_d, m_s, m_b\} = \{4, 8, 150, 4600\} \text{ MeV}. \tag{A1}$$

CKM matrix elements:

$$V_{ud} = 0.97, V_{us} = 0.22, \qquad V_{ub} = 0.0018 - 0.0032i,$$

$$V_{td} = 0.0074 - 0.0031i, \qquad V_{ts} = -0.04 - 0.00072i,$$

$$V_{tb} \simeq 1. \tag{A2}$$

In the text we use some combinations of Wilson coefficients and CKM matrix elements as reported in Table VII. One may note the correct treatment of isospin invariance [26] in these results.

We have used the following definitions for the form factors. If $|V\rangle$ is a vector meson state (ρ, K^*) and $|\mathcal{A}\rangle$ an axial-vector meson state (i.e one of the states K_{1A}, K_{1B}, a_1, b_1) we use

$$\langle V(\epsilon, p')|V^\mu - A^\mu|B(p)\rangle = -i(m_B + m_V)\epsilon^{*\mu}A_1(q^2)$$

$$+ i\frac{(\epsilon^* \cdot q)}{m_B + m_V}(p + p')^\mu A_2(q^2)$$

$$+ i(\epsilon^* \cdot q)\frac{2m_V}{q^2}q^\mu$$

$$\times [A_3(q^2) - A_0(q^2)]$$

$$+ \frac{2V(q^2)}{m_B + m_V}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu p_\alpha p'_\beta \tag{A3}$$

$$\langle \mathcal{A}(\epsilon, p')|V^\mu - A^\mu|B(p)\rangle = +i(m_B + m_\mathcal{A})\epsilon^{*\mu}V_1(q^2)$$

$$- i\frac{(\epsilon^* \cdot q)}{m_B + m_\mathcal{A}}(p + p')^\mu V_2(q^2)$$

$$- i(\epsilon^* \cdot q)\frac{2m_\mathcal{A}}{q^2}q^\mu$$

$$\times [V_3(q^2) - V_0(q^2)]$$

$$- \frac{2A(q^2)}{m_B + m_\mathcal{A}}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu p_\alpha p'_\beta. \tag{A4}$$

TABLE VII. Parameters used in the paper.

Coefficient	Formula
W_1	$V_{tb}^* V_{ts}(a_4 - \frac{1}{2}a_{10} + \frac{2m_K^2}{(m_b - m_d)(m_d + m_s)}(a_6 - \frac{1}{2}a_8))$
W_2	$V_{ub}^* V_{us}a_1 - V_{tb}^* V_{ts}(a_4 + a_{10} + \frac{2m_K^2}{(m_b - m_u)(m_u + m_s)}(a_6 + a_8))$
W_3	$V_{ub}^* V_{us}a_2 - V_{tb}^* V_{ts}\frac{3}{2}(a_9 - a_7)$
W_4	$V_{tb}^* V_{ts}(a_4 - \frac{1}{2}a_{10})$
W_5	$V_{ub}^* V_{us}a_1 - V_{tb}^* V_{ts}(a_4 + a_{10})$
W_6	$V_{ub}^* V_{us}a_1 - V_{tb}^* V_{ts}(a_4 + a_{10} - \frac{2m_K^2}{(m_b + m_u)(m_u + m_s)}(a_6 + a_8))$
W_7	$V_{ub}^* V_{us}a_2 - V_{tb}^* V_{ts}\frac{3}{2}(a_7 + a_9)$
W_8	$V_{tb}^* V_{ts}(a_4 - \frac{1}{2}a_{10} - \frac{2m_K^2}{(m_b + m_d)(m_d + m_s)}(a_6 - \frac{1}{2}a_8))$
w_1	$V_{ub}^* V_{ud}a_1 - V_{tb}^* V_{td}(a_4 + a_{10} + \frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)}(a_6 + a_8))$
w_2	$V_{ub}^* V_{ud}a_2 - V_{tb}^* V_{td}(-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9))$
w_3	$V_{ub}^* V_{ud}a_1 - V_{tb}^* V_{td}(a_4 + a_{10})$
w_4	$V_{ub}^* V_{ud}a_2 - V_{tb}^* V_{td}(-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9)) + \frac{2m_\pi^2}{(m_b - m_d)(m_u + m_d)}(a_6 - \frac{1}{2}a_8))$
w_5	$V_{ub}^* V_{ud}a_1 - V_{tb}^* V_{td}(a_4 + a_{10} - \frac{2m_\pi^2}{(m_b + m_u)(m_u + m_d)}(a_6 + a_8))$
w_6	$V_{ub}^* V_{ud}a_2 - V_{tb}^* V_{td}(-a_4 + \frac{1}{2}a_{10} + \frac{3}{2}(a_7 + a_9))$
w_7	$V_{ub}^* V_{ud}a_2 - V_{tb}^* V_{td}(-a_4 + \frac{1}{2}a_{10} + \frac{3}{2}(a_9 - a_7)) - \frac{2m_\pi^2}{(m_b + m_d)(m_u + m_d)}(-a_6 + \frac{1}{2}a_8))$

In these equations

$$A_3(q^2) = \frac{m_V - m_B}{2m_V} A_2(q^2) + \frac{m_V + m_B}{2m_V} A_1(q^2),$$

$$V_3(q^2) = \frac{m_{\mathcal{A}} - m_B}{2m_{\mathcal{A}}} V_2(q^2) + \frac{m_{\mathcal{A}} + m_B}{2m_{\mathcal{A}}} V_1(q^2) \quad \text{(A5)}$$

with $V_3(0) = V_0(0)$ and $A_3(0) = A_0(0)$,

If P, P' are pseudoscalar mesons, we have used

$$\langle P'(p')|V_\mu|P(p)\rangle = F_1(q^2)\left[(p_\mu + p'_\mu) - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu \right]$$

$$+ F_0(q^2)\frac{m_P^2 - m_{P'}^2}{q^2} q_\mu. \quad \text{(A6)}$$

We do not make assumptions on the q^2 behavior of the $F_1^{B\to\pi}$ form factor as we only need its value at $q^2 = 0$. Finally we have used the following definitions for the leptonic decay constants

$$\langle 0|A_\mu|P(p)\rangle = if_P p_\mu, \qquad \langle V(\varepsilon, p)|V_\mu|0\rangle = f_V m_V \varepsilon_\mu^*,$$

$$\langle \mathcal{A}(\varepsilon, p)|A_\mu|0\rangle = f_{\mathcal{A}} m_{\mathcal{A}} \varepsilon_\mu^*, \quad \text{(A7)}$$

with the following numerical values $(f_{\pi^+}, f_K, f_{\rho^+}, f_{K^*}) = (132, 161, 210, 210)$ MeV, and, from τ decays, $(f_{K_1(1270)}, f_{K_1(1400)}) = (171, 126)$ MeV [3].

For the determination of the analogous constants for the a_1 and b_1 nonstrange axial-vector mesons one has to take into account that the strange axial-vector mesons $K_1(1270)$ and $K_1(1400)$ are the result of the mixing of $^3P_1$ and $^1P_1$ states. Denoting by K_{1A} and K_{1B} the $^3P_1$ and $^1P_1$ states of K_1 one has

$$K_1(1270) = K_{1A} \sin\theta + K_{1B} \cos\theta,$$

$$K_1(1400) = K_{1A} \cos\theta - K_{1B} \sin\theta. \quad \text{(A8)}$$

K_{1B} belongs to the same nonet as the states $b_1(1235)$,

$h_1(1170)$ and $h_1(1380)$; K_{1A}, $a_1(1260)$, $f_1(1285)$ and $f_1(1400)$ are also in one nonet. The mixing angle θ and the masses of the K_1 states have been determined in [3] (but see also [15,17]) up to a twofold ambiguity

Sol.[a]: $\theta = 32^o,$ $(m_{K_{1B}}, m_{K_{1A}}) = (1310, 1367)$ MeV,

Sol.[b]: $\theta = 58^o,$ $(m_{K_{1B}}, m_{K_{1A}}) = (1367, 1310)$ MeV.

(A9)

Using this result and $SU(3)$ symmetry one gets [3]:

Sol.[a]$(\theta = 32^o)$: $(f_{b_1}, f_{a_1}) = (74, 215)$ MeV,

Sol.[b]$(\theta = 58^o)$: $(f_{b_1}, f_{a_1}) = (-28, 223)$ MeV. (A10)

We have also used the matrix element describing radiative transitions:

$$\langle K_1(p', \epsilon)|\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b|B(p)\rangle$$

$$= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p^\rho p'^\sigma 2T_1(q^2) + \left[\epsilon_\mu^*(m_B^2 - m_{K_1}^2)\right.$$

$$- (\epsilon^* \cdot q)(p + p')_\mu]T_2(q^2)$$

$$+ \left[(\epsilon^* \cdot q)q_\mu - \frac{q^2}{m_B^2 - m_{K_1}^2}(p + p')_\mu \right]T_3(q^2),$$

(A11)

with $T_1(0) = T_2(0)$ (T_3 does not contribute to the radiative decay). For $B \to K^*$ an analogous formula can be written. From experiment one has [3],

$$y = \frac{T_1^{B\to K_1(1270)}(0)}{T_1^{B\to K^*}(0)} \approx 1.06,$$

$$y' = \frac{T_1^{B\to K_1(1400)}(0)}{T_1^{B\to K^*}(0)} \approx \begin{cases} 0.14(\theta = 32^o) \\ 0.35(\theta = 58^o) \end{cases}. \quad \text{(A12)}$$

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