

***P*-wave pentaquark and its decay in the quark model with instanton induced interaction**Tetsuya Shinozaki,<sup>1,\*</sup> Makoto Oka,<sup>1</sup> and Sachiko Takeuchi<sup>2</sup><sup>1</sup>*Department of Physics, H-27, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan*<sup>2</sup>*Japan College of Social Work, Kiyose 204-8555, Japan*

(Received 30 September 2005; published 21 September 2006)

*P*-wave pentaquarks with strangeness  $+1$ ,  $I = 0$ , and  $J^P = 1/2^+$  are studied in the nonrelativistic quark model with instanton induced interaction (III). We present their mass splittings and orbital-spin-isospin-color structures. It is found that decompositions of the wave functions are sensitive to III, while the mass splittings are insensitive. The decay of the lowest energy pentaquark,  $\Theta^+$ , is found to be suppressed when the contribution of III is increased. Spin structure of the dominant components of the wave function is studied.

DOI: [10.1103/PhysRevD.74.054029](https://doi.org/10.1103/PhysRevD.74.054029)

PACS numbers: 12.39.Mk, 12.38.Lg, 12.39.Jh, 14.20.-c

**I. INTRODUCTION**

The observation of a narrow resonance at 1540 MeV in the  $\gamma + {}^{12}\text{C}$  experiment by the LEPS Collaboration at SPring-8 was interpreted as the first positive evidence for the existence of the pentaquark  $\Theta^+(1540)$  [1]. Follow-up experiments and reanalyses of old data suggest the positive evidence of  $\Theta^+$ . On the other hand, there have appeared some experimental results which show no signs of  $\Theta^+$ . Until now, the existence of  $\Theta^+$  is controversial [2]. Peaks corresponding to  $\Phi^{--}$  and  $\Theta_c^+$  were also reported but still unconvincing. The experimental results of  $\Theta^+$  show that the mass is about 1540 MeV, the upper limit of the width is a few MeV, the isospin  $I = 0$ , the baryon number  $B = +1$  and the strangeness  $S = +1$ . Since the minimal quark component of  $\Theta^+$  is  $uudd\bar{s}$ ,  $\Theta^+$  is called ‘‘pentaquark.’’ The spin and the parity of  $\Theta^+$  have not been determined yet. In this work, we consider the case in which the spin and the parity of  $\Theta^+$  are  $J^P = 1/2^+$  [3,4].

The mass of  $\Theta^+$  has been investigated based on the quark model [5–14], the QCD sum rule [15–18] and the lattice QCD [19–25]. Most of them predict the mass is much larger than the observed one  $\approx 1540$  MeV. We, however, employ nonrelativistic quark model in this work and do not attempt to reproduce absolute masses of pentaquarks. In the nonrelativistic quark model, absolute masses of ordinary meson and baryons are often adjusted to the ground state. The mass splittings come mainly from the hyperfine interactions, e.g. one-gluon exchange interaction and instanton induced interaction (III).

The observed width of  $\Theta^+$  is unexpectedly narrow, considering that the decay of  $\Theta^+$  requires no pair creation. Attempts have been made to explain the width based on the quark model [26–28] and the QCD sum rule [29–32]. However, most of them suggest that the centrifugal barrier and the symmetry properties of the orbital-spin-flavor-color wave function can not make the width as narrow as a few MeV. Furthermore, such choices of the wave function, which are expected to give a small width, seem quite

artificial. In this work, we point out that the instanton plays an important role, and III explains why such choices are favorable. As a result, III is shown to make the width of  $\Theta^+$  narrower.

The instanton is a classical solution of the Yang-Mills equation in the Euclidean space [33], which is one of the most important nonperturbative effects in QCD. Importance of instantons in low-energy hadron phenomena can be seen from the  $U_A(1)$  symmetry breaking. It is well known that the large mass of  $\eta'$  indicates that the  $U_A(1)$  is broken due to anomaly but not by spontaneous symmetry breaking [34]. It is suggested that the anomaly comes from the instanton in the QCD vacuum [35,36]. Instanton induced interaction (III) is an effective interquark interaction through the zero modes of the light quarks around an instanton [37]. III makes the mass of  $\eta'$  heavy, thus reproduces the low-lying meson spectrum. Effects of III for the mass of  $\Theta^+$  have been investigated [9,18,38], but they study only limited cases. We treat all the *P*-wave states and study the decay of  $\Theta^+$  also.

In Sec. II, we show our quark model Hamiltonian and enumerate all the *P*-wave states and discuss the connections between the hyperfine interactions and the decay widths. In Sec. III, we show the results of the pentaquark masses and widths. In Sec. IV, we give a conclusion and an outlook.

**II. FORMALISM****A. Hamiltonian**

Hamiltonian of the nonrelativistic quark model [39] is given by

$$H = M_0 + \sum_i \frac{\vec{p}_i^2}{2m_i} + V_{\text{conf}} + H_{\text{HF}}, \quad (1)$$

where  $M_0$  is a constant term.  $m_i$  is the constituent mass of the *i*-th quark (340 MeV for *u*, *d*, and 500 MeV for *s*), and  $\vec{p}_i$  is the momentum of the *i*-th quark.  $V_{\text{conf}}$  is the confinement potential.  $H_{\text{HF}}$  is the hyperfine interaction.

\*Email: [shinozk@th.phys.titech.ac.jp](mailto:shinozk@th.phys.titech.ac.jp)

The confinement potential for the pentaquark configurations has been studied in the lattice QCD recently [40]. The result shows stringlike potential according to the color configurations. We employ a simpler two-body type potential because five-body potential is much harder to treat. We use the harmonic oscillator potential:

$$V_{\text{conf}} = \sum_{i < j} \frac{1}{2} K |\vec{r}_i - \vec{r}_j|^2, \quad (2)$$

where  $K$  is a constant.  $\vec{r}_i$  is the coordinate of the  $i$ -th quark. The harmonic oscillator potential can be used to reproduce the  $N = 1$  excited baryon spectrum, provided that the size parameter is adjusted [39,41]. We assume that the kinetic term is  $SU(3)_F$  symmetric for simplicity. As a result, the orbital wave functions, which are the  $1\hbar\omega$  states of the harmonic oscillator potential, are  $SU(3)_F$  symmetric. It is known that the  $SU(3)_F$  breaking effects from the kinetic term and the wave function are smaller than those from the hyperfine interactions [39]. The relative  $S$ -wave state of two-body is described by  $\pi^{-3/4} b^{-3/2} \exp(-\rho^2/2b^2)$ , where  $\rho$  is relative coordinate:  $\rho = |\vec{r}_1 - \vec{r}_2|/\sqrt{2}$  and  $b$  is the size parameter:  $b = (3Km)^{-1/4}$ .

The hyperfine interactions that we use are given by

$$H_{HF} = (1 - P_{\text{III}})H_{\text{OGE}} + P_{\text{III}}H_{\text{III}^{(2)}} + P_{\text{III}}H_{\text{III}^{(3)}}, \quad (3)$$

where  $H_{\text{OGE}}$  is the one-gluon-exchange interaction (OGE) [42].  $H_{\text{III}^{(2)}}$  and  $H_{\text{III}^{(3)}}$  are the two-body and three-body terms of III, respectively [43].  $P_{\text{III}}$  is a parameter which represents the portion of the hyperfine splittings originated from III. The hyperfine splittings come only from OGE at  $P_{\text{III}} = 0$ , while they come entirely from III at  $P_{\text{III}} = 1$ .

The spin-spin term of OGE, which is referred to simply as OGE in this work, is given by

$$H_{\text{OGE}} = \sum_{i < j} V_{ij} \delta^{(3)}(\vec{r}_{ij}) \sigma_i^a \lambda_i^b \sigma_j^a \lambda_j^b, \quad (4)$$

where  $V_{ij}$  is the strength of the interaction between the  $i$ -th quark and  $j$ -th quark,  $\propto (m_i m_j)^{-1}$ .  $\lambda_i^b$  is the color  $SU(3)$  Gell-Mann matrix for the  $i$ -th quark. For the antiquark, it means  $-\lambda_i^{b*}$ .  $\sigma_i^a$  is the spin  $SU(2)$  Pauli matrix for the  $i$ -th quark. Since OGE gives a contact interaction, only relative  $S$ -wave pairs are affected. OGE reproduces the baryon and the meson spectra except for the Goldstone bosons and  $\eta'$  meson. The effects of the pion exchange should also be included to reproduce the observed light mass of the positive-parity excited nucleon. We determine  $V_{ij}$  phenomenologically. However, it is known that  $V_{ij}$  is quite large if  $P_{\text{III}} = 0$  compared with that expected from QCD. It is favorable that III can reduce the contribution from OGE to the hyperfine interaction.

We introduce III so that the  $\eta - \eta'$  mass difference is reproduced. As a result, low-lying hadron spectrum is well reproduced. Since the average size of instantons,  $1/3$  fm, is smaller than that of the hadrons, we assume that III is approximately a contact interaction. III contains a three-body interaction and a two-body interaction, which are the determinant type in the flavor space. Thus, the three-body interaction of III affects only the systems of  $u$ ,  $d$ , and  $s$  quarks. The two-body  $qq$  and three-body  $qqq$  interactions of III are given [43] by

$$H_{\text{III}^{(2)}}^{qq} = \sum_{i,j=1,i < j}^4 V_{ij}^{(2)} \delta^{(3)}(r_{ij}) \left( \mathbf{1} + \frac{3}{32} \lambda_i^b \lambda_j^b + \frac{9}{32} \sigma_i^a \lambda_i^b \sigma_j^a \lambda_j^b \right), \quad (5)$$

$$\begin{aligned} H_{\text{III}^{(3)}}^{qqq} = & \sum_{i,j,k=1,i < j < k}^4 V_{ijk}^{(3)} \delta^{(3)}(r_{ij}) \delta^{(3)}(r_{ik}) \left( \mathbf{1} + \frac{3}{32} (\lambda_i^b \lambda_j^b + \lambda_j^b \lambda_k^b + \lambda_k^b \lambda_i^b) - \frac{9}{320} d_{abc} \lambda_i^a \lambda_j^b \lambda_k^c \right. \\ & + \frac{9}{32} (\sigma_i^a \lambda_i^b \sigma_j^a \lambda_j^b + \sigma_j^a \lambda_j^b \sigma_k^a \lambda_k^b + \sigma_k^a \lambda_k^b \sigma_i^a \lambda_i^b) + \frac{27}{320} d_{abc} \lambda_i^a \lambda_j^b \lambda_k^c (\sigma_i^a \sigma_j^a + \sigma_j^a \sigma_k^a + \sigma_k^a \sigma_i^a) \\ & \left. - \frac{9}{64} \epsilon_{\alpha\beta\gamma} f_{abc} \lambda_i^a \lambda_j^b \lambda_k^c \sigma_i^\alpha \sigma_j^\beta \sigma_k^\gamma \right). \end{aligned} \quad (6)$$

where  $V_{ij}^{(2)}$  is the strength of the two-body term between the  $i$ -th quark and  $j$ -th quark.  $V_{ijk}^{(3)}$  is the strength of the three-body term among the  $i$ -th,  $j$ -th and  $k$ -th quarks.  $f_{abc}$  is the  $SU(3)$  structure constant defined by  $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$ .  $d_{abc}$  is the constant defined by  $\{\lambda_a, \lambda_b\} = 4/3\delta_{ab}I + 2d_{abc}\lambda_c$ . The two-body term is obtained by contracting a quark pair from the three-body term as is illustrated in Fig. 1. We obtain the relations between  $V^{(2)}$  and  $V^{(3)}$  as

$$V_{us}^{(2)} = \langle \bar{u}u \rangle V_{uds}^{(3)}, \quad (7)$$

$$V_{ud}^{(2)} = V_{us}^{(2)} \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq V_{us}^{(2)} \frac{m_s}{m_u}, \quad (8)$$

where the  $m_u = m_d$  and  $m_s$  are constituent quark masses,  $m_u/m_s = 0.6$ , and  $\langle \bar{u}u \rangle$  and  $\langle \bar{s}s \rangle$  are the quark condensates of  $u$  and  $s$  quarks, respectively:  $\langle \bar{u}u \rangle = (-225 \text{ MeV})^3$ . Because the strength of the three-body term is repulsive and the quark condensate is negative, the strength of the two-body term is attractive. Since the three-body term of III is the three-body contact interaction, only relative

$S$ -wave pairs are affected. Moreover, this three-body term of III affects only the flavor-singlet stats for  $3q$ . and vanishes for the  $3q$  baryon.

The interaction between the  $\bar{s}$  quark and a  $u$  or  $d$  quark labeled by  $i$  or  $j$  can be obtained by the charge conjugation from Eqs. (5) and (6):

$$H_{\text{III}^{(2)}}^{q\bar{s}} = \sum_{i=1}^4 V_{is}^{(2)} \delta^{(3)}(r_{is}) \left( \mathbf{1} - \frac{3}{32} \lambda_i^b (-\lambda_s^{b*}) + \frac{9}{32} \sigma_i^a \lambda_i^b \sigma_s^a (-\lambda_s^{b*}) \right), \quad (9)$$

$$\begin{aligned} H_{\text{III}^{(3)}}^{qq\bar{s}} = & \sum_{i,j=1,i<j}^4 V_{ijs}^{(3)} \delta^{(3)}(r_{ij}) \delta^{(3)}(r_{js}) \left( \mathbf{1} + \frac{3}{32} (\lambda_i^b \lambda_j^b - \lambda_j^b (-\lambda_s^{b*}) - (-\lambda_s^{b*}) \lambda_i^b) + \frac{9}{320} d_{abc} \lambda_i^a \lambda_j^b (-\lambda_s^{*c}) \right. \\ & + \frac{9}{32} (\sigma_i^a \lambda_i^b \sigma_j^a \lambda_j^b + \sigma_j^a \lambda_j^b \sigma_s^a (-\lambda_s^{b*}) + \sigma_s^a (-\lambda_s^{b*}) \sigma_i^a \lambda_i^b) - \frac{27}{320} d_{abc} \lambda_i^a \lambda_j^b (-\lambda_s^{*c}) (\sigma_i^\alpha \sigma_j^\alpha - \sigma_j^\alpha \sigma_s^\alpha - \sigma_s^\alpha \sigma_i^\alpha) \\ & \left. - \frac{9}{64} \epsilon_{\alpha\beta\gamma} f_{abc} \lambda_i^a \lambda_j^b (-\lambda_s^{*c}) \sigma_i^\alpha \sigma_j^\beta \sigma_s^\gamma \right). \end{aligned} \quad (10)$$

We point out that III contains the three-body term, which is absent in OGE. The three-body term of III does not change the spectrum of three-quark baryons. It is, however, expected that III is important in multi-quark systems with strange quarks, such as  $2\Lambda$  systems and pentaquarks [43–48], since they are sensitive to the three-body term of III. On the other hand, the effects of the two-body term of III are similar to that of OGE, since the spin dependent forces the last terms of Eqs. (5) and (9) are identical to that of OGE [49,50]. Thus, the spin-spin splittings in the baryon spectrum can be reproduced by any combinations of OGE and the two-body term of III. As a result, the spectrum in the ground-state baryon can be reproduced by any combinations. In contrast, the  $\eta - \eta'$  mass splitting is sensitive to III [51–53]. The  $\eta - \eta'$  mass splitting comes from the diagrams of the annihilation type of the two-body term of III, while such diagrams are absent in  $\Theta^+$ . Therefore, the origin of effects of III for the pentaquark is different from that for  $\eta$  and  $\eta'$ .

Finally, we mention the effects of III for the  $S$ -wave pentaquarks [14]. The antisymmetrization of  $4q$  leads to the unique spin,  $S_{4q} = 1$ . Thus, there are two possible states, a state with  $S_{5q} = 1/2$  and a state with  $S_{5q} = 3/2$ . Effect of III appears in their mass splitting. In fact, III reduces the mass splitting as  $P_{\text{III}}$  increases, given by  $225 -$

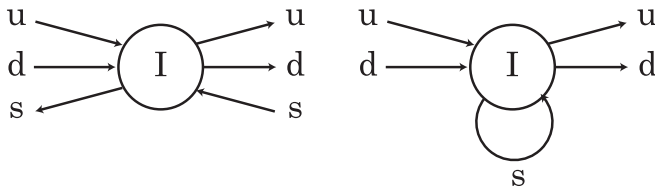


FIG. 1. The three-body interaction among  $u, d$ , and  $\bar{s}$  (left) and the two-body interaction between  $u, d$  (right). The latter is obtained by contracting  $s\bar{s}$  quark pair from the three-body interaction.

$203P_{\text{III}}$  MeV. Thus, III may favor a possible assignment that  $\Theta^+$  has  $J^P = 3/2^-$  [11,16,23,25].

### B. $P$ -wave states

The  $J^P = 1/2^+$  pentaquarks correspond to the  $P$ -wave states since the intrinsic parity of the  $\bar{s}$  quark is negative. In order to remove the center-of-mass motion and realize permutation symmetry of the orbital wave function, we use the harmonic oscillator wave function [12,39,41]. The classification of states is based on the permutation symmetry in the nonrelativistic quark model.  $\Theta^+$  consists of four  $u, d$  quarks ( $4q$ ) and an  $\bar{s}$  quark. The orbital-spin-isospin-color wave function of  $4q$  must be antisymmetrized. The wave function of the five quarks ( $5q$ ) must be color singlet and is assumed to be flavor antidecuplet.

There are two types of  $P$ -wave  $1\hbar\omega$  excited states. In the first type of states (I), the  $\bar{s}$  quark is excited, which is associated with the excitation of the center-of-mass of  $4q$  since the center-of-mass of  $5q$  must not be excited. The state (I) corresponds to the total symmetric states for the orbital wave function of  $4q$ . It is known that this state takes a unique spin  $S_{4q} = 1$  since the total wave function must be the antisymmetric for  $4q$  [6]. The second (II) type is the one without the  $\bar{s}$  quark excited, while an internal coordinate of  $4q$  is excited. The type (II) corresponds to the [1,3] symmetric states for the orbital wave function of  $4q$  [12]. The antisymmetrization for  $4q$  leads to no restrictions on  $S_{4q}$ .

We find that there are nine independent states with  $1\hbar\omega$  excitation, which consist of two type (I) states and seven type (II) states. We show the nine states in Tables I and II. They are further classified into five states with  $S_{5q} = 1/2$  and four states with  $S_{5q} = 3/2$ . They are classified by the spin-color  $SU(6)$  representation [54,55].

The first state in Table I is reduced to the Jaffe-Wilczek diquark state [4,56] when the internal degrees of freedom between  $q-q$  are suppressed. We call it the  $S0$  state, since

TABLE I. the five states with  $S_{5q} = 1/2$ .  $L_{4q}$  is the angular momentum of  $4q$  and means that the orbital wave function  $4q$  has a definite permutation symmetry.  $S_{4q}$ ,  $S_{5q}$ , and  $J_{5q}$  are the spin of  $4q$ ,  $5q$  and the total angular momentum of  $5q$ , respectively.  $S_{4q} \times C_{4q}$  is the symmetry of the spin-color  $SU(6)$  of  $4q$ . All states are identified by these quantum numbers.

	$L_{4q}$	$S_{4q}$	$S_{4q} \times C_{4q}$	$S_{5q}$	$J_{5q}$
$S0$	1	0		$\frac{1}{2}$	$\frac{1}{2}$
$S0^*$	1	0		$\frac{1}{2}$	$\frac{1}{2}$
$S1$	1	1		$\frac{1}{2}$	$\frac{1}{2}$
$S1^*$	1	1		$\frac{1}{2}$	$\frac{1}{2}$
$L0$	0	1		$\frac{1}{2}$	$\frac{1}{2}$

$S_{4q} = 0$ . The symmetry is shown in Appendix A. The second row is a state with  $S_{4q} = 0$ , which is in general heavier than the  $S0$  and we refer it to  $S0^*$ . The third row is reduced to the diquark-triquark state proposed by Karliner and Lipkin [5,8] when the internal degrees of freedom of diquark and triquark are suppressed and the  $\bar{s}$  quark is heavy. We call it the  $S1$  state in this work. The fourth state has  $S_{4q} = 1$ , which we refer to  $S1^*$ . The fifth row is the  $\bar{s}$ -excited state [type (I)], which we refer to  $L0$ .

We only use the spin-spin terms of the hyperfine interactions for simplicity because the observed  $LS$  splitting is small, and because the  $LS$  terms as well as tensor terms of OGE and III tend to cancel each other [57]. We point out that the spin-spin terms of the hyperfine interactions do

TABLE II. the four states with  $S_{5q} = 3/2$ . The spin-spin terms do not couple the states with  $S_{5q} = 1/2$  and the states with  $S_{5q} = 3/2$ .

$L_{4q}$	$S_{4q}$	$S_{4q} \times C_{4q}$	$S_{5q}$	$J_{5q}$
1	1		$\frac{3}{2}$	$\frac{1}{2}$
1	1		$\frac{3}{2}$	$\frac{1}{2}$
1	2		$\frac{3}{2}$	$\frac{1}{2}$
0	1		$\frac{3}{2}$	$\frac{1}{2}$

not couple the states with  $S_{5q} = 1/2$  and the states with  $S_{5q} = 3/2$ .

### C. Decay

Decays of the pentaquarks goes through a fall-apart process [26,27,54,55], which does not require  $q\bar{q}$  pair creation. It is usually expected that the decay widths for the fall-apart decay are much larger than ordinary decays with  $q\bar{q}$  creation.

A measure that is often used to estimate the fall-apart width is the  $KN$ -overlap [5,27,58]. We define the  $KN$ -overlap by a projection operator for relative  $P$ -wave  $KN$  states,

$$\mathcal{O}_{KN} = \mathcal{S}_{\text{orb}}^{123} \mathcal{S}_{\text{orb}}^{4\bar{s}} \mathcal{A}_{\text{color}}^{123} \mathcal{A}_{\text{color}}^{4\bar{s}} \mathcal{M}_{\text{spin}}^{123} \mathcal{A}_{\text{spin}}^{4\bar{s}}, \quad (11)$$

where  $\mathcal{S}_{\text{orb}}$  is the projection operator to the ground state  $K$  or  $N$ .  $\mathcal{A}_{\text{color}}$ ,  $\mathcal{M}_{\text{spin}}$ , and  $\mathcal{A}_{\text{spin}}$  are the projection operators to color-singlet, the spin  $1/2$  and the spin 0, respectively. Note that the matrix elements of the operator,  $\mathcal{O}_{KN}$ , correspond to the absolute square of the  $KN$ -overlap.

For the four states with  $S_{5q} = 3/2$ , the matrix elements of  $\mathcal{O}_{KN}$  are zero since the total spin is different from  $KN$ . Thus, they can not decay to  $KN$  unless a tensor-type interaction is strong. We obtain the  $KN$ -overlaps for the five states with  $S_{5q} = 1/2$  in the bases of Table I,

$$\langle \mathcal{O}_{KN} \rangle = \begin{pmatrix} \frac{5}{192} & \frac{5}{192} & \frac{5\sqrt{3}}{192} & -\frac{5}{192} & -\frac{\sqrt{10}}{64} \\ \frac{5}{192} & \frac{5}{192} & \frac{5\sqrt{3}}{192} & -\frac{5}{192} & -\frac{\sqrt{10}}{64} \\ \frac{5\sqrt{3}}{192} & \frac{5\sqrt{3}}{192} & \frac{5}{64} & -\frac{5\sqrt{3}}{192} & -\frac{\sqrt{30}}{64} \\ -\frac{5}{192} & -\frac{5}{192} & -\frac{5\sqrt{3}}{192} & \frac{5}{192} & \frac{\sqrt{10}}{64} \\ -\frac{\sqrt{10}}{64} & -\frac{\sqrt{10}}{64} & -\frac{\sqrt{30}}{64} & \frac{\sqrt{10}}{64} & \frac{3}{32} \end{pmatrix}. \quad (12)$$

We diagonalize this matrix with a unitary matrix  $U$ :

$$\langle \mathcal{O}_{KN} \rangle = U^{-1} \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U. \quad (13)$$

We find that a state, which we call “ $KN$  state,” has nonzero  $KN$ -overlap, while the other four states, called “confined states,” have no  $KN$ -overlaps. The “confined states” can not directly fall apart to  $KN$ , while the “ $KN$  state” is expected to couple strongly to the  $KN$  scattering state. (A model conclusion of the decay width according to the fall-apart process is achieved by using the meson-quark-quark vertex [27].) The “ $KN$  state” is given by an eigenvector of Eq. (12):

$$|KN\rangle = \left( \frac{\sqrt{15}}{12}, \frac{\sqrt{15}}{12}, \frac{3\sqrt{5}}{12}, -\frac{\sqrt{15}}{12}, -\frac{3\sqrt{6}}{12} \right). \quad (14)$$

This state corresponds to the antisymmetrized  $K$  and  $N$

with  $1\hbar\omega$  excitation:  $|KN\rangle = \mathcal{A}^{1234}(N_{123} \otimes K_{4\bar{3}})$ , where  $\mathcal{A}^{1234}$  is antisymmetrizer of  $4q$ .

The couplings between the “ $KN$  state” and the “confined states” can be described by channel coupling scattering formalism [26,59]. We assume that the pentaquark states are given only by the “confined states,” i.e. the pentaquark states are eigenstates of the Hamiltonian within the subspace of the “confined states.” We have to diagonalize only the hyperfine interaction in the subspace of the “confined states,” resulting in

$$\langle H_{HF} \rangle = \begin{pmatrix} E_{KN} & a_{KN\Theta^+} & a_{KN\Theta_1^*} & a_{KN\Theta_2^*} & a_{KN\Theta_3^*} \\ a_{KN\Theta^+} & E_{\Theta^+} & 0 & 0 & 0 \\ a_{KN\Theta_1^*} & 0 & E_{\Theta_1^*} & 0 & 0 \\ a_{KN\Theta_2^*} & 0 & 0 & E_{\Theta_2^*} & 0 \\ a_{KN\Theta_3^*} & 0 & 0 & 0 & E_{\Theta_3^*} \end{pmatrix}, \quad (15)$$

where all the values are functions of  $P_{III}$ .  $E_{\Theta}$ 's are the eigenvalues after the diagonalization in the “confined space.” The off-diagonal elements,  $a_{KN\Theta}$ , correspond to couplings between the “ $KN$  state” and “confined states.” Note that in the present model, no other terms in the Hamiltonian have off-diagonal matrix elements. We assign the lowest state in the “confined states” to  $\Theta^+$ . Then, the narrow width of the pentaquarks can be attributed to a small  $a_{KN\Theta^+}$ . The other seven states,  $\Theta_1^*$ ,  $\Theta_2^*$ ,  $\Theta_3^*$ , and the four states with  $S_{5q} = 3/2$ , are regarded as excited states of  $\Theta^+$ . All the states have  $J^P = 1/2^+$ .

### III. RESULTS

We investigate spectrum of the  $P$ -wave pentaquarks with  $I = 0$  and  $J^P = 1/2^+$  in the nonrelativistic quark model with instanton induced interaction (III). We assume that the kinetic term and the orbital wave functions are  $SU(3)_F$  symmetric for simplicity. Furthermore, we neglect the  $LS$  terms and the tensor terms.

Then, the pentaquarks are obtained by diagonalizing the hyperfine interaction in the subspace of the “confined states.” Both the masses and the couplings are obtained simultaneously. Because the five-quark confinement may allow an extra constant, we have discuss only the excitation energies and the structures of the obtained states. Thus, we set the mass of  $\Theta^+$  to 1540 MeV. It should be noted that if we use the confinement potential derived from the baryon spectrum without adjusting a constant term, then the absolute mass of  $\Theta^+$  in this model is about 2 GeV.  $P_{III}$  can be determined from the  $\eta - \eta'$  mass splitting, which gives  $P_{III} = 0.3-0.5$  [46]. However, the nonrelativistic quark model gives large ambiguities for the pseudoscalar mesons. Thus, we here treat  $P_{III}$  as a free parameter. The size parameter,  $b$ , is unknown for the pentaquarks. We use  $b \approx 0.5$  fm, which is taken from that of the nucleon. It is known that the radii of the  $S$ -wave pentaquarks are as small as that

of the nucleon if III is introduced [48]. The strengths of OGE and the two-body term of III are fixed phenomenologically from the  $N - \Delta$  mass splitting so that they reproduce the baryon and meson spectra except for the pseudoscalar mesons. The strength of the three-body term of III is determined by Eq. (7). They give

$$\begin{aligned} \langle V_{ud} \rangle &= -19 \text{ MeV}, & \langle V_{us} \rangle &= -11 \text{ MeV}, \\ \langle V_{ud}^{(2)} \rangle &= -67 \text{ MeV}, & \langle V_{us}^{(2)} \rangle &= -40 \text{ MeV}, \\ \langle V_{uds}^{(3)} \rangle &= +20 \text{ MeV}. \end{aligned} \quad (16)$$

The matrix elements of the hyperfine interaction,  $H_{HF}$ , for the nine states in Tables I and II are given in Appendix B. Since the other terms of the Hamiltonian, Eq. (1), are identical for the nine states, the spectrum is determined only by the hyperfine interaction. We find that both OGE and the two-body term of III play similar roles in the spectrum. A small difference comes from the first and second terms in Eqs. (5) and (9). Therefore, the hyperfine interaction of Eq. (3) is approximately described as  $H_{HF} = H^{(2)} + P_{III}H_{III^{(3)}}$ , where  $H^{(2)}$  contains both OGE and the two-body term of III and is approximately independent of  $P_{III}$ . Thus, the effects of III are attributed to the three-body term. We find that the contribution of the three-body term of III is repulsive and smaller than those of  $H^{(2)}$  for the pentaquarks.

#### A. Spectrum

We show the masses of pentaquarks with  $S_{5q} = 1/2$ , in Fig. 2. Since the three-body term of III is weakly repulsive, the hyperfine splittings are reduced as  $P_{III}$  increases. We set the mass of  $\Theta^+$  to 1540 MeV. Note that the “ $KN$  state” is always above the  $KN$  threshold. Thus, the “ $KN$  state”

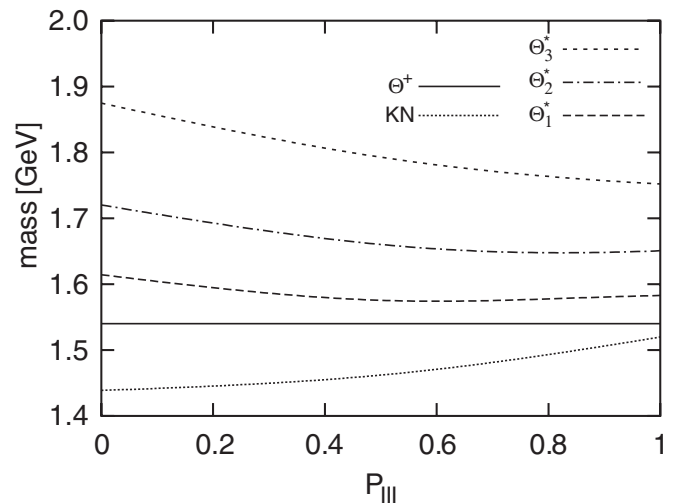


FIG. 2. The masses of the pentaquarks with  $S_{5q} = 1/2$ . The mass of  $\Theta^+$  is set to 1540 MeV.  $\Theta_1^*$ ,  $\Theta_2^*$ , and  $\Theta_3^*$  are excited states of  $\Theta^+$ .  $KN$  is the “ $KN$  state,” which is very broad.

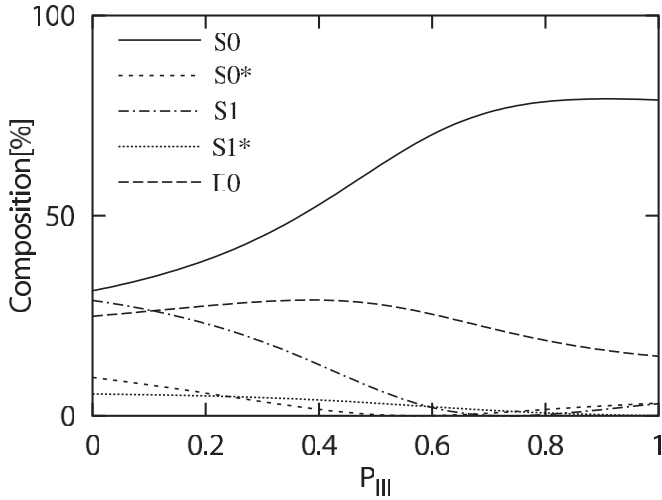


FIG. 3. The composition of  $\Theta^+$  in the bases of Table I. The solid line is the  $S0$  state. The  $S0$  state is dominant for large  $P_{\text{III}}$ , while the  $S1$  state is suppressed.

may not form a resonance. The excited states of  $\Theta^+$  lie at about 1.6 GeV, 1.7 GeV, and 1.8 GeV. If the widths are narrow, those states should be observed.

Figure 3 shows the decomposition of the lowest  $\Theta^+$  state in the bases of Table I. We find that the  $S0$  state is dominant at large  $P_{\text{III}}$ , while the  $S1$  state is suppressed. The contributions from the heavier states,  $S0^*$  and  $S1^*$ , are very small. The  $P_{\text{III}}$  dependence can be understood as follows. OGE and the two-body term of III are more attractive for both the  $S0$  state and the  $S1$  state. However, the three-body term of III for the  $S0$  state is less repulsive than that for the  $S1$  state. The triquark ( $ud\bar{s}$ ) of the  $S1$  state is strongly affected by the three-body term of III, since the  $u$ ,  $d$ , and  $\bar{s}$  quarks are in relative  $S$ -wave states. In contrast, the  $S0$  state is less sensitive since the  $\bar{s}$  quark is separated from the  $u$ ,  $d$  quarks. At large  $P_{\text{III}}$ ,  $\Theta^+$  attains nearly the maximum  $S0$  component. Thus, we find that the  $S0$  state is a favorable state with respect to III.

### B. Decay

Figure 4 shows the couplings between the “ $KN$  state” and the pentaquarks with  $S_{5q} = 1/2$ . The coupling of  $\Theta^+$  is very large in the case without III,  $P_{\text{III}} = 0$ , while the coupling becomes much weaker as  $P_{\text{III}}$  increases. In the relevant range of  $P_{\text{III}}$ , the coupling is about 1/3 of that at  $P_{\text{III}} = 0$ . It hits zero at  $P_{\text{III}} = 0.61$ , where  $\Theta^+$  does not couple to the “ $KN$  state,” that is,  $\Theta^+$  becomes stable against the decay to  $KN$  in the present model. At  $P_{\text{III}} = 0.81$ , the second resonance  $\Theta_2^*$  becomes stable. The other states,  $\Theta_1^*$  and  $\Theta_3^*$ , do not become stable within  $0 \leq P_{\text{III}} \leq 1$ . We point out that the couplings are nonzero in most range of  $P_{\text{III}}$ , and therefore no more than one pentaquark becomes simultaneously stable. This may explain why only one pentaquark has been seen.

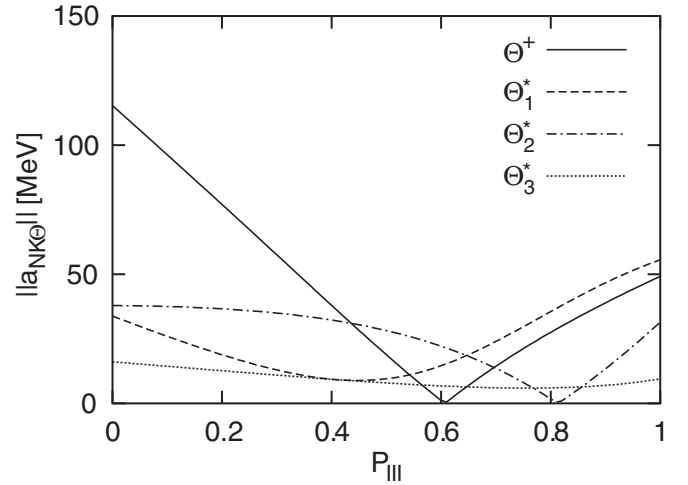


FIG. 4. The couplings between the “ $KN$  state” and the pentaquarks with  $S_{5q} = 1/2$ . The coupling of  $\Theta^+$  is zero at  $P_{\text{III}} = 0.61$ . At the zero point,  $\Theta^+$  can not decay to  $KN$ . The  $\Theta_2^*$  is stable at  $P_{\text{III}} = 0.81$ .

### C. The states with $S_{5q} = 3/2$

The spin-spin terms of the hyperfine interactions do not change the total spin and therefore do not mix  $S_{5q} = 1/2$  and  $S_{5q} = 3/2$ . In Fig. 5, the masses of the  $S_{5q} = 3/2$  states are plotted. They lie at about 1.6 GeV, 1.65 GeV, 1.7 GeV, and above 1.8 GeV. OGE and the two-body term of III for the pentaquarks with  $S_{5q} = 1/2$  are more attractive than those with  $S_{5q} = 3/2$ . Thus, the lowest state,  $\Theta^+$ , has always  $S_{5q} = 1/2$ . It is also seen that the effects of the three-body term of III are strong in  $S_{5q} = 3/2$ . One sees several level-crossings when  $P_{\text{III}}$  goes from 0 to 1. The state which rises with  $P_{\text{III}}$  corresponds to the most repulsive eigen-state of the three-body term of III. Since the spin

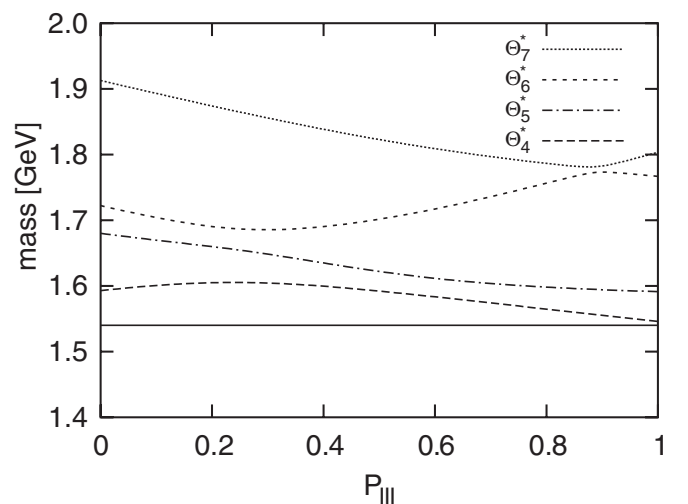


FIG. 5. The masses of the pentaquarks with  $S_{5q} = 3/2$ , which are excited states of  $\Theta^+$ . The solid line is  $\Theta^+$ , which is 1540 MeV. The pentaquarks lie above  $\Theta^+$ .

is different from the “ $KN$  states,” the  $S_{5q} = 3/2$  states decay to  $KN$  only through a tensor force, which is expected to be weak in the present quark model.

#### IV. CONCLUSION

We have investigated the spectrum of the  $P$ -wave pentaquarks with  $I = 0$  and  $J^P = 1/2^+$  in the nonrelativistic quark model with instanton induced interaction (III). We expect that effects of the two-body term of III should appear in the  $I = 0$  meson channels, while the effects of the three-body III may be important in multi-quark systems with strange quarks.

We find that there are nine  $1\hbar\omega$  states in the harmonic oscillator potential. We find that one of them, the “ $KN$  state,” has a finite overlap with  $KN$ , while eight other “confined states” have no  $KN$ -overlaps. The pentaquarks can decay to  $KN$  only through the “ $KN$  state.” We assign the lowest energy eigen-state in the “confined states” to  $\Theta^+$ . We point out that the decay is not only a fall-apart process, and introduce channel coupling scattering between the “ $KN$  state” and the “confined states,” where the couplings are determined only by the hyperfine interaction under our assumptions. Since the fall-apart leads to the large widths, the narrow widths of the pentaquarks are explained by small couplings of the channel coupling scattering.

We find that III reduces the coupling between  $\Theta^+$  and the “ $KN$  state.” As the contributions of III become larger,  $\Theta^+$  becomes significantly stable and the dominant component of  $\Theta^+$  becomes the  $S0$  state. We therefore give a qualitative explanation of the narrow width of  $\Theta^+$  and we find the dynamics that the  $S0$  state becomes naturally the dominant component. III changes the composition of  $\Theta^+$ . In other words, III leads to a cancellation of the coupling between  $\Theta^+$  and  $KN$ , where OGE, the two-body term of III and the three-body term of III are canceled out each other.

We find that at  $P_{\text{III}} = 0.61$ ,  $\Theta^+$  becomes stable within our approach. This value of  $P_{\text{III}}$  is close to the relevant range,  $P_{\text{III}} = 0.3\text{--}0.5$ , which is consistent with the  $\eta - \eta'$  mass splitting.

We point out that the effects of III strongly depend on the flavor part of the system. For example a pentaquark with  $I = 2$ ,  $uuuu\bar{s}$ , is not affected by the three-body term of III. We conjecture that the flavor dependence is a reason why no stable pentaquark is seen in other channels. In order to make the predictions more realistic, the  $SU(3)_F$  breaking effects of the kinetic term and the orbital wave function must be taken into account. The other pentaquarks in the  $\overline{10}_F$  representation,  $N_{\overline{10}}$ ,  $\Sigma_{\overline{10}}$ , and  $\Phi$ , may be affected by the  $SU(3)_F$  breaking effects. Again it is probable that those pentaquarks do not become stable simultaneously, which may explain why the other members of the  $\overline{10}_F$  representation is not observed. It is also noted that the mixing between  $\overline{10}_F$  and  $8_F$  representations is also sensitive to

the two-body term of III, since the mixing matrix elements include flavor-singlet  $q\bar{q}$  contributions.

Further effects of III will appear in diquark correlations in the pentaquarks, which are not included in the present harmonic oscillator wave functions. We expect that III makes the diquarks compact. The three-body term of III gives a repulsion between a diquark and the  $\bar{s}$  quark. Thus, we expect that the  $KN$ -overlap is reduced and the  $S0$  state is more favorable.

The tensor terms of OGE and III must be taken into account in order to evaluate the mixings of  $S_{5q} = 1/2$  and  $S_{5q} = 3/2$ . Such mixing is favorable for a small width since the  $S_{5q} = 3/2$  states does not couple directly to  $KN$ . Predictions and searches of the  $LS$  partner of  $\Theta^+$  are important. If the  $LS$  terms for the pentaquarks are significantly weak similarly to that for the  $P$ -wave baryons, the  $LS$  partner must be observed.

Finally, we conclude that III affects significantly not only  $\eta$  and  $\eta'$ , but also pentaquarks. Further studies of the pentaquarks should help a deeper understanding of dynamics of QCD.

#### ACKNOWLEDGMENTS

We would like to thank A. Hosaka, K. Shimizu, N. Ishii, and V. Dmitrasinovic for useful discussions. M.O. acknowledges supports from the Grants-in-Aid for Scientific Research in Priority Areas 1707002, and (B) 15340072. S.T. is supported by the Grants-in-Aid (c) 15540289. T.S. is supported by the 21st Century COE Program at Tokyo Tech “Nanometer-Scale Quantum Physics” by the Ministry of Education, Culture, Sports, Science, and Technology.

#### APPENDIX A: SYMMETRY OF THE FOUR-QUARK STATE

The Jaffe and Wilczek (JW) [4] proposed a picture of the pentaquark, a resonance of two scalar-isoscalar diquarks and an  $\bar{s}$  quark. We here construct a four-quark state corresponding to the JW picture.

The diquark consists of  $u$  and  $d$  quarks ( $2q$ ), which form a relative  $S$ -wave pair with the spin  $S_{2q} = 0$ , the isospin  $I_{2q} = 0$  and the color  $C_{2q} = \bar{3}$ . Possible symmetries in the spin  $\times$  color space of  $4q$  are

$$\begin{array}{c} \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}, \\ S_{4q} = 0 \quad C_{4q} = 3 \end{array} \quad (A1)$$

while possible symmetries in the spin  $\times$  color ( $SC$ ) space of the two diquarks are

$$\begin{array}{c} \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \otimes \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}. \\ SC_{2q} \quad SC_{2q} \end{array} \quad (A2)$$

Thus the symmetry of spin  $\times$  color of  $4q$  is uniquely determined:

$$\begin{array}{c} \square \square \\ \square \end{array} \times \begin{array}{c} \square \square \\ \square \end{array} \rightarrow \begin{array}{c} \square \square \square \\ \square \end{array} \quad (A3)$$

$$S_{4q} = 0 \quad C_{4q} = 3$$

When the isospin symmetry is combined, we have spin  $\times$  color  $\times$  isospin symmetries of  $4q$ :

$$\begin{array}{c} \square \square \square \\ \square \end{array} \times \begin{array}{c} \square \square \\ \square \end{array} = \begin{array}{c} \square \square \square \square \\ \square \end{array} + \begin{array}{c} \square \square \square \\ \square \end{array} \quad (A4)$$

$$SC_{4q} \quad I_{4q} = 0$$

On the other hand, possible symmetries of the orbital wave function ( $O$ ) of the two diquarks are

$$\square \square \otimes \square \square = \square \square \square \square + \begin{array}{c} \square \square \square \\ \square \end{array} + \begin{array}{c} \square \square \\ \square \end{array} \quad (A5)$$

$$O_{2q} \quad O_{2q}$$

Since the symmetry of the orbital wave function must be the conjugate to that of the spin  $\times$  color  $\times$  isospin, it is uniquely determined:

$$\square \square \otimes \square \square \rightarrow \begin{array}{c} \square \square \square \\ \square \end{array} \quad (A6)$$

$$O_{2q} \quad O_{2q}$$

Finally, we obtain

$$\begin{array}{c} \square \square \square \\ \square \end{array} \begin{array}{c} \square \square \\ \square \end{array} \begin{array}{c} \square \square \\ \square \end{array} \begin{array}{c} \square \square \square \\ \square \end{array} \begin{array}{c} \square \square \square \\ \square \end{array} \quad (A7)$$

$$O_{4q} \quad S_{4q} \quad I_{4q} \quad C_{4q} \quad SC_{4q}$$

It is noted that this state does not belong to a definite symmetry in the spin  $\times$  flavor ( $SI_{4q}$ ) space. The five-quark state with the same symmetry structure is labeled  $S0$  in the text of this paper.

## APPENDIX B: THE MATRIX ELEMENTS

We show the matrix elements of OGE and III for the states with  $S_{5q} = 1/2$  (Table I) and the states with  $S_{5q} = 3/2$  (Table II). The matrix elements depend on the orbital wave functions. We assume that the orbital wave functions are the  $1\hbar\omega$  states of the harmonic oscillator potential and  $SU(3)_F$  symmetric.

We obtain the matrix elements of OGE in Eq. (4):

$$\langle H_{\text{OGE}}^{S_{5q}=1/2} \rangle = \langle V_{ud} \rangle \times \begin{pmatrix} 13 & 3 & \frac{20}{\sqrt{3}}\xi & \xi & \frac{-15}{\sqrt{10}}\xi \\ 3 & \frac{7}{3} & \frac{-4}{\sqrt{3}}\xi & \frac{-19}{3}\xi & \frac{-35}{3\sqrt{10}}\xi \\ \frac{20}{\sqrt{3}}\xi & \frac{-4}{\sqrt{3}}\xi & \frac{46+63\xi}{6} & \frac{-10+5\xi}{2\sqrt{3}} & \frac{-25}{\sqrt{30}}\xi \\ \xi & \frac{-19}{3}\xi & \frac{-10+5\xi}{2\sqrt{3}} & \frac{-6-19\xi}{6} & \frac{7\sqrt{10}}{6}\xi \\ \frac{-15}{\sqrt{10}}\xi & \frac{-35}{3\sqrt{10}}\xi & \frac{-25}{\sqrt{30}}\xi & \frac{7\sqrt{10}}{6}\xi & \frac{16+15\xi}{3} \end{pmatrix}, \quad (B1)$$

$$\langle H_{\text{OGE}}^{S_{5q}=3/2} \rangle = \langle V_{ud} \rangle \times \begin{pmatrix} \frac{92-63\xi}{12} & \frac{-20-5\xi}{4\sqrt{3}} & \frac{\sqrt{30}}{12}\xi & \frac{5\sqrt{30}}{12}\xi \\ \frac{-20-5\xi}{4\sqrt{3}} & \frac{-12+19\xi}{12} & \frac{-47\sqrt{10}}{12}\xi & \frac{-7\sqrt{10}}{12}\xi \\ \frac{\sqrt{30}}{12}\xi & \frac{-47\sqrt{10}}{12}\xi & \frac{-28+34\xi}{6} & -\frac{5}{6}\xi \\ \frac{5\sqrt{30}}{12}\xi & \frac{-7\sqrt{10}}{12}\xi & -\frac{5}{6}\xi & \frac{32-15\xi}{6} \end{pmatrix}, \quad (B2)$$

where  $\xi = \langle V_{us} \rangle / \langle V_{ud} \rangle = m_u / m_s$ .  $\langle V_{ud} \rangle = \langle V_{ij} \delta^{(3)}(\vec{r}_{ij}) \rangle_{\text{orb}} = V_{ud} / \sqrt{2\pi b^2 3}$  contains the spatial integration between  $u$  and  $d$  quarks, which depends on the sizes of the pentaquarks.  $\langle V_{us} \rangle$  is similarly given.

We obtain the matrix elements of the two-body term of III in Eqs. (5) and (9):

$$\langle H_{\text{III}^{(2)}}^{S_{5q}=1/2} \rangle = \langle V_{ud}^{(2)} \rangle \times \begin{pmatrix} \frac{444+249\xi}{64} & \frac{36-9\xi}{64} & \frac{15\sqrt{3}}{8}\xi & \frac{9}{32}\xi & \frac{-135}{32\sqrt{10}}\xi \\ \frac{36-9\xi}{64} & \frac{252+249\xi}{64} & -\frac{3\sqrt{3}}{8}\xi & -\frac{57}{32}\xi & \frac{-105}{32\sqrt{10}}\xi \\ \frac{15\sqrt{3}}{8}\xi & -\frac{3\sqrt{3}}{8}\xi & \frac{180+219\xi}{32} & \frac{-36+27\xi}{32\sqrt{3}} & \frac{-135}{16\sqrt{30}}\xi \\ \frac{9}{32}\xi & -\frac{57}{32}\xi & \frac{-36+27\xi}{32\sqrt{3}} & \frac{108+99\xi}{32} & \frac{15}{4\sqrt{10}}\xi \\ \frac{-135}{32\sqrt{10}}\xi & \frac{-105}{32\sqrt{10}}\xi & \frac{15}{16\sqrt{30}}\xi & \frac{15}{4\sqrt{10}}\xi & \frac{216+99\xi}{32} \end{pmatrix}, \quad (B3)$$



$$\langle H_{III^{(2)}}^{S_{5q}=3/2} \rangle = \langle V_{ud}^{(2)} \rangle \times \begin{pmatrix} \frac{720+309\xi}{128} & \frac{-144-27\xi}{128\sqrt{3}} & \frac{3\sqrt{30}}{128} \xi & \frac{9\sqrt{30}}{128} \xi \\ \frac{-144-27\xi}{128\sqrt{3}} & \frac{432+567\xi}{128} & \frac{-141\sqrt{10}}{128} \xi & \frac{-15\sqrt{10}}{128} \xi \\ \frac{3\sqrt{30}}{128} \xi & \frac{-141\sqrt{10}}{128} \xi & \frac{144+411\xi}{64} & \frac{-15}{64} \xi \\ \frac{9\sqrt{30}}{128} \xi & \frac{-15\sqrt{10}}{128} \xi & \frac{-15}{64} \xi & \frac{432+63\xi}{64} \end{pmatrix}. \quad (\text{B4})$$

We obtain the matrix elements of the three-body term of III in Eq. (10):

$$\langle H_{III^{(3)}}^{S_{5q}=1/2} \rangle = \langle V_{uds}^{(3)} \rangle \times \begin{pmatrix} \frac{69}{8} & 0 & \frac{39\sqrt{3}}{8} & 0 & \frac{-45}{4\sqrt{10}} \\ 0 & \frac{27}{8} & \frac{-27}{8\sqrt{3}} & 0 & 0 \\ \frac{39\sqrt{3}}{8} & \frac{-27}{8\sqrt{3}} & 12 & 0 & \frac{-45}{4\sqrt{30}} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{-45}{4\sqrt{10}} & 0 & \frac{-45}{4\sqrt{30}} & 0 & \frac{9}{4} \end{pmatrix}, \quad (\text{B5})$$

$$\langle H_{III^{(3)}}^{S_{5q}=3/2} \rangle = \langle V_{uds}^{(3)} \rangle \times \begin{pmatrix} \frac{69}{32} & \frac{-27\sqrt{3}}{32} & \frac{9\sqrt{30}}{32} & \frac{3\sqrt{30}}{16} \\ \frac{-27\sqrt{3}}{32} & \frac{243}{32} & \frac{-81\sqrt{10}}{32} & 0 \\ \frac{9\sqrt{30}}{32} & \frac{-81\sqrt{10}}{32} & \frac{135}{16} & 0 \\ \frac{3\sqrt{30}}{16} & 0 & 0 & \frac{9}{16} \end{pmatrix}, \quad (\text{B6})$$

where  $\langle V_{uds} \rangle = \langle V_{ijs}^{(3)} \delta^{(3)}(\vec{r}_{ij}) \delta^{(3)}(\vec{r}_{js}) \rangle_{\text{orb}} = V_{uds} / \sqrt{3\pi^2 b^4} 3$  contains the spatial integration among  $u$ ,  $d$  quarks and  $\bar{s}$  quark.

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