Production of a $Q^2\bar{Q}^2$ state in $J/\psi \to \gamma\omega\phi$

Bing An Li

Department of Physics and Astronomy, University of Kentucky Lexington, Kentucky 40506, USA

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The possibility of the resonance discovered in $J/\psi \to \gamma\omega\phi$ by BESII as a candidate of 0^{++} $Q^2\bar{Q}^2$ state $f_0(1810)$ is investigated. Tests on the $Q^2\bar{Q}^2$ nature of $f_0(1810)$ are found: $f_0(1810) \to \omega\phi$, $K^*\bar{K}^*$ are the two dominant decay channels and $f_0(1810) \to K\bar{K}$, $\eta\eta$, $\eta\eta'$ are suppressed. The cross sections of $\gamma\gamma \to f_0(1810) \to \omega\phi$, $K^*\bar{K}^*$ are estimated.

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Hadron spectrum is always one of important topics of particle physics and the test ground of nonperturbative QCD. In recent few years both experimental and theoretical studies of hadron spectroscopy are very active. There are many new discoveries. Most recently BESII has reported a new resonance near the threshold of $\omega\phi$ in $J/\psi \to \gamma\omega\phi$ [1]. Preliminary fit with a s-wave Breit-Wigner formula leads to

$$m = 1810^{+19}_{-26} \pm 18\ \text{MeV}, \qquad \Gamma = 105 \pm 20 \pm 28\ \text{MeV},$$
$$BR(J/\psi \to \gamma f_0(1810), f_0(1810) \to \omega\phi) = (2.61 \pm 0.27 \pm 0.65) \times 10^{-4}. \qquad (1)$$

0^{++} are the favored quantum numbers. This resonance is labeled as $f_0(1810)$ in this paper. $\omega\phi$ is a very special decay mode and it has never been found before. It is natural that there are more possibilities in understanding this new resonance.

In the quark model the couplings between mesons of $u\bar{u} + d\bar{d}$ and $K\bar{K}$ and $K^*\bar{K}^*$ are OZI allowed and the couplings with $\phi\phi$, $K^*\bar{K}^*$, $K\bar{K}$ are allowed for $s\bar{s}$ mesons. According to the OZI rule the $\omega\phi$ decay channel for a $q\bar{q}$ is suppressed. It is known that the OZI rule can be explained by the N_C expansion of QCD. The OZI allowed processes are at the leading order in N_C expansion and the processes suppressed by the OZI rule are at higher order in N_C expansion. A $q\bar{q}$ meson can via rescattering of $q\bar{q} \to K\bar{K}$, $K_1\bar{K}$, $\bar{K}_1K$, $K^*\bar{K}^* \ldots \to \omega\phi$ decay to $\omega\phi$. The diagrams of OZI allowed processes of mesons are at the tree level, which are at $O(N_C)$ in N_C expansion [2]. The diagrams of the rescattering processes $q\bar{q} \to K\bar{K}$, $K_1\bar{K}$, $\bar{K}_1K$, $K^*\bar{K}^* \ldots \to \omega\phi$ are at one-loop level and at $O(1)$ in N_C expansion [2]. Therefore, the rescattering processes are suppressed. $\phi \to \rho\pi$ is an example. It is well known that $\phi \to \rho\pi$ is OZI suppressed. The diagrams of this decay are at one-loop level and at $O(1)$ in N_C expansion. $BR(\phi \to \rho\pi)$ is much smaller than $BR(\phi \to K\bar{K})$ which is at $O(N_C)$ [2]. $f_2(1950)(0^+(2^{++}))$ is another example. It has been discovered in 1991 [3]. $f_2(1950) \to K^*\bar{K}^*$, $K\bar{K}$ have been measured. $f_2(1950)$ can via the rescatterings $K\bar{K}$, $K^*\bar{K}^* \to \omega\phi$ decay to $\omega\phi$. However, the decay $f_2(1950) \to \omega\phi$ has not been reported. So far no experimental indication of large violation of the OZI rule in the decays of mesons has been reported.

According to this general analysis, if $f_0(1810)$ is a $q\bar{q}$ meson, then $f_0(1810) \to \omega\phi$ is suppressed by the OZI rule. The coupling between $f_0(1810)$ and $\omega\phi$ should be very weak. On the other hand, the phase space of the decay $f_0(1810) \to \omega\phi$ is very small. These two factors lead to a very small $BR(J/\psi \to \gamma f_0(1810), f_0(1810) \to \omega\phi)$ for a $q\bar{q}(1810)$ state. $J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}$ is an OZI allowed decay, whose branching ratio is measured to be $(8.5^{+1.2}_{-0.9}) \times 10^{-4}$ [3]. Comparing with this decay and taking very small phase space of $f_0(1810) \to \omega\phi$ into account, the branching ratio of $J/\psi \to \gamma f_0(1810)$, $f_0(1810) \to \omega\phi$ (1) is not small. $\phi \to K\bar{K}$, $\rho\pi$ are OZI allowed and suppressed processes, respectively. They are taken as examples to estimate the strength of the coupling of an OZI suppressed process. The general Lagrangian of both processes are constructed as

$$\mathcal{L}_{\phi K\bar{K}} = g_1\phi_\mu\{K^+\partial_\mu K^- - K^-\partial_\mu K^+ + K^0\partial_\mu\bar{K}^0 - \bar{K}^0\partial_\mu K^0\},$$
$$\mathcal{L}_{\phi\rho\pi} = g_2\frac{2}{f_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\phi_\nu\partial_\alpha\rho^i_\beta\pi^i. \qquad (2)$$

As a matter of fact, $\mathcal{L}_{\phi K\bar{K}}$ has been derived in a chiral meson theory [2] and it has been determined

$$g_1 = \frac{\sqrt{2}}{g}\left\{1 + \frac{m_\phi^2}{2\pi^2 f_\pi^2}\left[\left(1 - \frac{2c}{g}\right)^2 - 4\pi^2c^2\right]\right.$$

where $c = \frac{f_\pi^2}{2gm_\rho^2}$. $g = 0.39$ is determined by the decay rate of $\rho \to ee^+$ and f_π is pion decay constant, $f_\pi = 0.184$ GeV.

$$g_1^2 = 21.8$$

is obtained and it agrees with the decay rate of $\phi \to K\bar{K}$ very well. $\mathcal{L}_{\phi\rho\pi}$ is the Wess-Zumino-Witten anomaly [4]. In a chiral meson theory the pion fields are always associated with the factor $\frac{2}{f_\pi}$. $\mathcal{L}_{\omega\rho\pi}$ is an example. It is derived from the Wess-Zumino-Witten Lagrangian [2]

$$\mathcal{L}_{\omega\rho\pi} = -\frac{N_C}{\pi^2g^2f_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\rho^i_\beta\pi^i.$$

In the chiral theory of mesons [2] $\mathcal{L}_{\omega\rho\pi}$ is obtained from the tree diagrams and at $O(N_C)$ in N_C expansion. $\mathcal{L}_{\phi\rho\pi}$ is obtained from one-loop diagrams of mesons and at $O(1)$ in N_C expansion. $\phi \to \rho\pi$ is a OZI suppressed process. Therefore, g_2 should be smaller than $\frac{N_C}{2\pi^2 g^2}$. The decay width of $\phi \to \rho\pi$ determines

$$g_2^2 = 0.0125.$$

The strength of the OZI suppressed channel is weaker than the strength of the OZI allowed by 3 order of magnitude. If $f_0(1810)$ is a $q\bar{q}$ meson and $f_0(1810) \to \omega\phi$ is OZI suppressed, much smaller $BR(J/\psi \to \gamma f_0(1810), f_0(1810) \to \omega\phi)$ should be expected. However, the data (1) shows this branching ratio is compatible with $BR(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K})$. Therefore, it is very difficult to fit $f_0(1810)$ into the spectrum of ordinary $q\bar{q}$ mesons.

On the other hand, it is interesting to mention that the authors of Ref. [5] have studied the violation of the OZI rule by hadronic loops in a quark model. In Ref. [5] the authors have reported "...in the scalar sector, the cancellation mechanism breaks down spectacularly, leading us to predict very large effects which are expected to manifest themselves as large shifts in the masses and mixing angles of the scalar mesons...". This finding leads to a question whether the loop diagram of the rescatterings $K\bar{K}$, $K_1\bar{K}$, $\bar{K}_1 K$, $K^*\bar{K}^* \ldots \to \omega\phi$ make large contributions to the OZI suppressed decay $f_0(1810) \to \omega\phi$. A calculation of all possible loop diagrams is needed. However, the decay modes $f_0(1810) \to K\bar{K}$, $K_1\bar{K}$, $\bar{K}_1 K$, $K^*\bar{K}^*$ have not been found yet. For study of the effect of rescattering search for these decay mode is the first priority. This study is beyond the scope of this paper.

In this paper another possibility that $f_0(1810)$ is a $Q^2\bar{Q}^2$ is investigated. In Ref. [6] the spectrum and the properties of $Q^2\bar{Q}^2$ states have been studied in the MIT bag model. Some of the $Q^2\bar{Q}^2$ states decay to vector-vector mesons dominantly by "fall apart" and their masses are at the threshold of the two vector mesons. The authors of Ref. [7] present a lattice gauge calculation which shows that the light scalar mesons are $Q^2\bar{Q}^2$ states rather than $q\bar{q}$. Especially, some of the $Q^2\bar{Q}^2$ states have exotic quantum numbers [6], $I = 2$, which decay to $\rho\rho$ dominantly. In Ref. [8] the $Q^2\bar{Q}^2$ states have been used to study $\gamma\gamma \to \rho\rho$. Both 0^{++} and 2^{++} four quark states of $I = 0, 2$, whose masses are in the range of the mass of $\rho\rho$ [6], contribute to $\gamma\gamma \to \rho\rho$. For $\rho^0\rho^0$ the interference between $Q^2\bar{Q}^2$ states of $I = 0, 2$ is constructive and for $\rho^+\rho^-$ the interference is destructive. The data of $\sigma(\rho^0\rho^0) \gg \sigma(\rho^+\rho^-)$ are explained naturally. The large cross section of $\gamma\gamma \to \rho^0\rho^0$ and the position of the peak are explained pretty well in the model of four quark states. In Ref. [9] the model of 2^{++} $Q^2\bar{Q}^2$ states of $I = 0$, 1 successfully explains why $\sigma(\gamma\gamma \to K^{*0}\bar{K}^{*0}) \gg \sigma(\gamma\gamma \to K^{*+}K^{*-})$. In Ref. [10] the productions of $Q^2\bar{Q}^2$ in $J/\psi \to \gamma VV$ and hadron collisions have been studied.

The possibility of $f_0(1810)$ as a $Q^2\bar{Q}^2$ state is investigated in this paper. According to Ref. [6], there is a $C^s(\underline{9}^*)$ 0^{++} $Q^2\bar{Q}^2$ state whose mass has been computed to be 1.8 GeV [6]. The flavor wave function of $C^s(\underline{9}^*)$ is

$$C^s(\underline{9}^*) = \frac{1}{\sqrt{2}} K\bar{K} + \frac{1}{\sqrt{2}} \eta_0\eta_s, \tag{3}$$

where $\eta_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $\eta_s = s\bar{s}$, the mesons in Eq. (3) could be either pseudoscalars or vectors. The color wave function of $Q^2\bar{Q}^2$ state consists of color octet- color octet and color singlet-color singlet two parts. The recoupling coefficients of $C^s(\underline{9}^*)$ are shown in Table I [10]. The color octet is indicated by underline.

Besides $C^s(\underline{9}^*)$ another four quark state $C^s(\underline{9})$ of 2^{++} is studied in Ref. [6], whose mass is computed to be 1.95 GeV [6]. This state decays to $\omega\phi$ and $K^*\bar{K}^*$ only. The contributions of $C^s(\underline{9})$ in various processes have been studied [9,10]. It is possible that this state appears in the tail of BESII's data. $\omega\phi$ is an allowed decay channel of $C^s(\underline{9}^*)$ and there is no suppression. The mass, the quantum numbers, and the main decay channel of $f_0(1810)$ agree with 0^{++} $C^s(\underline{9}^*)$ very well. Therefore, it is very possible that $f_0(1810)$ is the 0^{++} $C^s(\underline{9}^*)$ $Q^2\bar{Q}^2$ state. The production of 0^{++} $C^s(\underline{9}^*)$ in $J/\psi \to \gamma\omega\phi$ is studied in this paper.

The decays of a four quark state are through a mechanism called "fall apart" [6]. This mechanism has been used in previous studies [7–10]. According to the mechanism of "fall apart", $\omega\phi$, $K^*\bar{K}^*$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ are allowed decay channels for $C^s(\underline{9}^*)$ and the amplitude of a decay of $C^s(\underline{9}^*)$ is proportional to the corresponding recoupling coefficient (see Table I) of VV or PP and is at s-wave. The matrix elements of $f_0(1810) \to \omega\phi$, $K^*\bar{K}^*$ are written as

$$\langle V_1 V_2|T|f_0(1810)\rangle = \frac{1}{\sqrt{2}} 0.644 a \epsilon^{\lambda_1} \cdot \epsilon^{\lambda_2}, \tag{4}$$

where a is taken as a constant in the range of $f_0(1810)$, ϵ^{λ_1} and ϵ^{λ_2} are the polarization vectors of the two vector mesons, respectively. For $f_0(1810) \to K\bar{K}$, $\eta\eta$, $\eta\eta'$

$$\begin{aligned} \langle K\bar{K}|T|f_0(1810)\rangle &= -\frac{1}{\sqrt{2}} 0.177 a, \\ \langle \eta\eta|T|f_0(1810)\rangle &= -\frac{1}{\sqrt{2}} 0.177 a\left(-\frac{\sqrt{2}}{3}\cos 2\theta - \frac{1}{6}\sin 2\theta\right), \\ \langle \eta\eta'|T|f_0(1810)\rangle &= -\frac{1}{\sqrt{2}} 0.177 a\left(-\frac{1}{3}\cos 2\theta + \frac{2\sqrt{2}}{3}\sin 2\theta\right), \end{aligned} \tag{5}$$

TABLE I. Recoupling Coefficients

	PP	VV	$\underline{P}\cdot\underline{P}$	$\underline{V}\cdot\underline{V}$
9^*	−0.177	0.644	0.623	0.407

where θ is the mixing angle of $\eta - \eta'$ and $\theta = -11^0$ is taken. The decay widths of various channels are obtained

$$\Gamma(f_0(1810)\to\omega\phi) = \left(\frac{1}{\sqrt{2}}0.644a\right)^2\frac{k_1}{8\pi q^2}\left\{2+\frac{1}{4m_\omega^2 m_\phi^2}(q^2-m_\omega^2-m_\phi^2)^2\right\},\qquad k_1^2=\frac{1}{4q^2}\left\{(q^2-m_\omega^2-m_\phi^2)^2-4m_\omega^2 m_\phi^2\right\},$$

$$\Gamma(f_0(1810)\to K^*\bar{K}^*) = \left(\frac{1}{\sqrt{2}}0.644a\right)^2\frac{k_2}{8\pi q^2}\left\{2+\frac{1}{4m_{K^*}^4}(q^2-2m_{K^*}^2)^2\right\},\qquad k_2^2=\frac{q^2}{4}-m_{K^*}^2,$$

$$\Gamma(f_0(1810)\to K\bar{K}) = \left(\frac{1}{\sqrt{2}}0.177a\right)^2\frac{k_3}{8\pi q^2},\qquad k_3^2=\frac{q^2}{4}-m_K^2,$$

$$\Gamma(f_0(1810)\to\eta\eta) = \left(\frac{1}{\sqrt{2}}0.177a\right)^2\frac{k_4}{4\pi q^2}\left(-\frac{\sqrt{2}}{3}\cos2\theta-\frac{1}{6}\sin2\theta\right)^2,\qquad k_4^2=\frac{q^2}{4}-m_\eta^2,$$

$$\Gamma(f_0(1810)\to\eta\eta') = \left(\frac{1}{\sqrt{2}}0.177a\right)^2\frac{k_5}{8\pi q^2}\left(-\frac{1}{3}\cos2\theta+\frac{2\sqrt{2}}{3}\sin2\theta\right)^2,\qquad k_5^2=\frac{1}{4q^2}\{(q^2-m_\eta^2-m_{\eta'}^2)^2-4m_\eta^2 m_{\eta'}^2\},$$

where q is the momentum of $f_0(1810)$. Adding up all the widths, the total width of $f_0(1810)$ is obtained. Using the value of $\Gamma(1)$, the parameter a is determined to be $7.1(1 \pm 0.11)$ GeV. The ratios of the widths are determined

$$\frac{\Gamma(f_0(1810)\to K^*\bar{K}^*)}{\Gamma(f_0(1810)\to\omega\phi)}=1.83,\quad \frac{\Gamma(f_0(1810)\to K\bar{K})}{\Gamma(f_0(1810)\to\omega\phi)}=0.22,\quad \frac{\Gamma(f_0(1810)\to\eta\eta)}{\Gamma(f_0(1810)\to\omega\phi)}=0.059,\quad \frac{\Gamma(f_0(1810)\to\eta\eta')}{\Gamma(f_0(1810)\to\omega\phi)}=0.062. \tag{7}$$

These ratios are independent of the parameter a. The model of $Q^2\bar{Q}^2$ predicts that $K^*\bar{K}^*$ and $\omega\phi$ are the two dominant decay channels of $f_0(1810)$, whose decay rates are at the same order of magnitude. If $f_0(1810)$ is a $q\bar{q}$ meson, $K^*\bar{K}^*$ and $\omega\phi$ are OZI allowed and suppressed decay channels, respectively, and $\Gamma(f_0(1810)\to K^*\bar{K}^*) \gg \Gamma(f_0(1810)\to\omega\phi)$ should be expected. Therefore, the prediction of the ratio of the decay widths of $K^*\bar{K}^*$ and $\omega\phi$ channels (7) is a very important test on the $Q^2\bar{Q}^2$ nature of $f_0(1810)$. The rate of $K\bar{K}$ channel is less than $\omega\phi$ by a factor 4.5, and the decay rates of $\eta\eta$, $\eta\eta'$ are very small. The dominance of the VV channels is resulted in three factors: the ratio of recoupling coefficients $(0.644/0.177)^2$, s-wave, and three independent directions of polarizations of vector meson. Of course, for $K\bar{K}$ channel the phase space is larger. The effects of the mixing angle between $\eta - \eta'$ make the decay rates of $\eta\eta$, $\eta\eta'$ channels smaller.

In QCD J/ψ radiative decays are described as $J/\psi \to \gamma gg$, $gg \to$ hadrons. The 0^{++} four quark state $C^s(\underline{9}^*)$ can via the $\underline{\omega}\cdot\underline{\phi}$ component (see Table I) couple to two gluons and decays to $\omega\phi$. The process $J/\psi\to\gamma\omega\phi$ is described as $J/\psi\to\gamma gg$, $gg(\underline{V}\,\underline{V})\to Q^2\bar{Q}^2\to\omega\phi$. As a matter of fact, the process $gg\to Q^2\bar{Q}^2\to VV$ has been used in the studies of the productions of $Q^2\bar{Q}^2$ states in hadron collisions and J/ψ radiative decays [10]. $gg(\underline{V}\,\underline{V})\to Q^2\bar{Q}^2$ is at $O(\alpha_s)$ and $Q^2\bar{Q}^2\to VV$ at $O(\alpha_s^0)$. Therefore, $J/\psi\to\gamma gg$, $gg(\underline{V}\,\underline{V})\to Q^2\bar{Q}^2\to VV$ is at the same order as $J/\psi\to\gamma+\text{hadrons}(q\bar{q})$.

At lower energies the electromagnetic interactions of hadrons are determined by the Vector Meson Dominance (VMD) [11]. The substitutions of the VMD for ω and ϕ mesons are expressed as

$$\omega\to\frac{1}{6}egA,\qquad \phi\to-\frac{\sqrt{2}}{6}egA, \tag{8}$$

where $g = 0.39$ is determined in Ref. [2]. Using Eqs. (8), a vector meson can be replaced by a photon and corresponding Lagrangian of electromagnetic interactions of hadrons is obtained. It is similar to the VMD (8) the substitutions between a gluon and color octet vectors are obtained in Ref. [10] (different notations are used in this paper)

$$\underline{\omega}^a\to\frac{1}{2}g_s g_{\underline{V}} g^a,\qquad \underline{\phi}^a\to\frac{\sqrt{2}}{2}g_s g_{\underline{V}} g^a, \tag{9}$$

where $g_{\underline{V}}^2=\frac{2}{3}g^2$ is determined and g_s is the coupling constant of QCD. Using Eqs. (9),

$$\mathcal{L}_{f_0(1810)gg}=0.407a\frac{1}{4}g_s^2 g_{\underline{v}}^2 f_0 g_\mu^a g_\mu^a \tag{10}$$

is determined. In order to satisfy gauge invariance in the momentum picture the tensor $g_{\mu\nu}$ of $g_{\mu\nu}g_\mu^a g_\nu^a$ is replaced by $(-g_{\mu\sigma}+\frac{k_{1\mu}k_{1\sigma}}{k_1^2})(-g_{\nu\sigma}+\frac{k_{2\mu}k_{1\sigma}}{k_1^2})$, where $k_{1,2}$ are the momentum of two gluons, respectively. Instead direct calculation of the matrix element of $J/\psi\to\gamma gg$ the approach of effective Lagrangian is exploited. The simplest effective Lagrangian of $J/\psi\to\gamma gg$ is constructed as

$$\mathcal{L}_{J\gamma gg}=\frac{eA}{m_{J/\psi}^4}(\partial_\mu J_\nu-\partial_\nu J_\mu)(\partial_\mu A_\nu-\partial_\nu A_\mu)\times(\partial_\lambda g_\sigma^a-\partial_\sigma g_\lambda^a)(\partial_\lambda g_\sigma^a-\partial_\sigma g_\lambda^a), \tag{11}$$

where A is taken as a parameter. This effective Lagrangian is gauge invariant.

Using Eqs. (10) and (11), the decay rate of $J/\psi \to \gamma f_0(1810)$, $f_0(1810) \to f$ is derived

$$\frac{d\Gamma}{dq^2} = \alpha \frac{96}{\pi} \frac{A^2}{m_{J/\psi}^{11}} \left(0.407a \frac{1}{4} g_s^2 g_{\underline{V}}^2\right)^2 \times \sqrt{q^2}(m_{J/\psi}^2 - q^2)^3 f^2(q^2) \frac{\Gamma_{f_0(1810)\to f}(q^2)}{(q^2 - m_{f_0}^2)^2 + q^2\Gamma^2},$$

$$f(q^2) = -\frac{1}{(2\pi)^3} \int_0^{q^2} dx(q^2 + 2x)(q^2 - x) \times \int_0^{\pi/2} d\phi \frac{\sin^2\phi}{(q^2 - x)^2 + 4q^2 x\cos^2\phi}, \tag{12}$$

where the function $f(q^2)$ represents the effects of gluons in the process $J/\psi \to \gamma gg$, $gg \to f_0(1810)$ and $\Gamma_{f_0(1810)\to f}$ is shown in Eqs. (6). $\alpha_s = \frac{g_s^2}{4\pi} = 0.3$ is taken. The ratios of the branching ratios of various channels of $J/\psi \to \gamma f_0(1810)$, $f_0(1810) \to f$ are obtained from Eq. (12)

$$\frac{BR(K^*\bar{K}^*)}{BR(\omega\phi)} = 1.44, \quad \frac{BR(K\bar{K})}{BR(\omega\phi)} = 0.18,$$
$$\frac{BR(\eta\eta)}{BR(\omega\phi)} = 4.8 \times 10^{-2}, \quad \frac{BR(\eta\eta')}{BR(\omega\phi)} = 7.5 \times 10^{-2},$$
$$BR(K^*\bar{K}^*) = 3.76(1 \pm 0.27) \times 10^{-4}, \tag{13}$$
$$BR(K\bar{K}) = 0.47(1 \pm 0.27) \times 10^{-4},$$
$$BR(\eta\eta) = 1.26(1 \pm 0.27) \times 10^{-5},$$
$$BR(\eta\eta') = 1.97(1 \pm 0.27) \times 10^{-5}.$$

The ratios are independent of both parameters A and a. Combining the five decay modes and using Eq. (1) and (13), a lower bound is obtained

$$BR(J/\psi \to \gamma + f_0(1810)) > 7.15 \times 10^{-4}. \tag{14}$$

Presumably this lower bound is nearly saturated if $f_0(1810)$ is a $C^s(\underline{9}^*)$. The upper limit of the integrals over q^2 is taken as $(1.81 + 0.105)^2$ GeV^2 and the lower limits are $(m_\omega + m_\phi)^2$, $4m_{K^*}^2$, and $(1.81 - 0.105)^2$ for $\omega\phi$, $K^*\bar{K}^*$, and PP channels, respectively. Because of the dependence on q^2 and the choices of the lower limits of $\frac{d\Gamma}{dq^2} \times (J/\psi \to \gamma f_0(1810), f_0(1810) \to f)$ the ratios shown in Eqs. (13) are not the same as shown in Eqs. (7). However, the same predictions are made by the model of $Q^2\bar{Q}^2$ that $K^*\bar{K}^*$ and $\omega\phi$ are two dominant channels and the $K\bar{K}$, $\eta\eta$, $\eta\eta'$ channels are suppressed. The branching ratio of $J/\psi \to \gamma K^*\bar{K}^*$ has been measured [12]. The data were fitted by simple Breit-Wigner resonances of 0^{-+}, 2^{++}, and 2^{-+}. A broad 0^- at 1800 ± 100 MeV with width at 500 ± 200 MeV is revealed. The branching ratio of $J/\psi \to \gamma(K^*\bar{K}^*)_{0^+}$ predicted in this paper (13) is less than the branching ratios of 0^{-+}, 2^{++}, and 2^{-+} [12]. Therefore, a smaller 0^{++} component is not ruled out.

From Eq. (12)

$$BR(J/\psi \to \gamma\omega\phi) = 1.06 \times 10^{-3} A^2 \tag{15}$$

is computed. Inputting the data (1), $A^2 = 0.25(1 \pm 0.27)$ is determined in the region of the mass of $f_0(1810)$.

As a $Q^2\bar{Q}^2$ state $f_0(1810)$ can be produced in $\gamma\gamma$ collisions. Using the VMD (9),

$$\mathcal{L}_{f_0(1810)\gamma\gamma} = -\frac{1}{36} e^2 g^2 0.644 a f_0 A_\mu A_\mu \tag{16}$$

is obtained. The problem of gauge invariance of Eq. (16) can be solved by the same way as in the case of Eq. (10). As a matter of fact, when the two photons are on mass shell the extra factor for gauge invariance does not contribute. From Eqs. (4) and (16) the cross section of $\gamma\gamma \to f_0(1810) \to f$ is derived

$$\sigma = 32\pi^2\alpha^2(0.0179ag^2)^2 \frac{1}{\sqrt{q^2}} \frac{\Gamma(q^2)_{f_0(1810)\to f}}{(q^2 - m_{f_0}^2)^2 + q^2\Gamma^2}. \tag{17}$$

At the peak $q^2 = m_{f_0}^2$

$$\sigma(\gamma\gamma \to f_0(1810) \to \omega\phi) = 1.24nb,$$
$$\sigma(\gamma\gamma \to f_0(1810) \to K^*\bar{K}^*) = 2.24nb. \tag{18}$$

These results (18) are parameter independent. In Ref. [9] the states of 2^{++} $Q^2\bar{Q}^2$ of $I = 0, 1$ have been used to study $\gamma\gamma \to K^*\bar{K}^*$ in the range of 1.7–2.7 GeV. This study (18) shows that the contribution of 0^{++} $Q^2\bar{Q}^2$ of $I = 0$ to $\gamma\gamma \to K^*\bar{K}^*$ is small. However, the measurements of $\sigma(\gamma\gamma \to \omega\phi)$ is very interesting.

On the other hand, if $f_0(1810)$ is a $C^s(\underline{9}^*)$ state this $\omega\phi$ resonance can be produced in other processes without OZI suppression. The Table I shows $f_0(1810)(C^s(\underline{9}^*))$ has stronger couplings with both $\omega\phi$ and $K^*\bar{K}^*$. Therefore, it can be produced not only in $gg \to f_0(1810) \to f$ but in the processes $\omega\phi(K^*\bar{K}^*) \to f_0(1810) \to f$ and $\gamma\omega(\phi) \to f_0(1810) \to f$ too. As a matter of fact, the amplitude of the process $\gamma\gamma \to f_0(1810) \to \omega\phi(K^*\bar{K}^*)$ studied in this paper is obtained via the VMD from $\omega\phi \to f_0(1810) \to \omega\phi(K^*\bar{K}^*)$. $\omega\phi$ and $K^*\bar{K}^*$ of $I(J^{PC}) = 0(0^{++})$ can be produced in hadron collisions, photo-productions, decays of B mesons, and J/ψ hadronic decays. The study of these processes are beyond the scope of this paper.

In summary, because the decay mode $q\bar{q} \to \omega\phi$ is suppressed by the OZI rule the decay $f_0(1810) \to \omega\phi$ with not small branching ratio is very difficult to be understood if $f_0(1810)$ is a $q\bar{q}$ meson. On the other hand, the mass, quantum number, and the decay $J/\psi \to \gamma f_0(1810)$, $f_0(1810) \to \omega\phi$ can be explained by assigning $f_0(1810)$ to a $Q^2\bar{Q}^2$ state, $C^s(\underline{9}^*)$ [6]. There is no suppression for $C^s(\underline{9}^*) \to \omega\phi$. The model of $Q^2\bar{Q}^2$ predicts: (i) besides the $\omega\phi$ channel there is another dominant decay channel, $K^*\bar{K}^*$; (ii) the $Q^2\bar{Q}^2$ $C^s(\underline{9}^*)$ predicts that the decay rates of $K^*\bar{K}^*$ and $\omega\phi$ channels are at the same order of magnitude; (iii) the decay channels $K\bar{K}$, $\eta\eta$, $\eta\eta'$ channels are suppressed; (iv) the cross sections of the productions of $f_0(1810)$ in two photons collisions are estimated; (v) as $C^s(\underline{9}^*)$ $f_0(1810)$ can be produced either in two gluon fusion $gg \to f_0(1810) \to \omega\phi(K^*\bar{K}^*)$ or the fusion of two vector mesons, $VV(\omega\phi, K^*\bar{K}^*) \to f_0(1810) \to \omega\phi(K^*\bar{K}^*)$. These properties make $f_0(1810)$ a strong candidate of the four quark state $C^s(\underline{9}^*)$ after $f_0(980)$ and $a_0(980)$ [6].

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