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Semirelativistic potential model for low-lying three-gluon glueballs

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The three-gluon glueball states are studied with the generalization of a semirelativistic potential model giving good results for two-gluon glueballs. The Hamiltonian depends only on 3 parameters fixed on two-gluon glueball spectra: the strong coupling constant, the string tension, and a gluon size which removes singularities in the potential. The Casimir scaling determines the structure of the confinement. Low-lying J^{PC} states are computed and compared with recent lattice calculations. A good agreement is found for 1^{--} and 3^{--} states, but our model predicts a 2^{--} state much higher in energy than the lattice result. The 0^{-+} mass is also computed.

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I. INTRODUCTION

The QCD theory allows the existence of bound states of gluons, called glueballs, but no firm experimental discovery of such states has been obtained yet. An important difficulty is that glueball states might possibly mix strongly with nearby meson states. Nevertheless, the computation of pure gluon glueballs remains an interesting task. This could guide experimental searches and provide some calibration for more realistic models of glueballs.

Lattice calculations are undoubtedly a powerful tool to investigate the structure of glueballs. A previous study [1] predicts the existence of a lot of resonances between 2 and 4 GeV. A recent update of this work [2] confirms the results already obtained.

The potential model, which is so successful to describe bound states of quarks, is also a possible approach to study glueballs [3–7]. In a recent paper [8], a semirelativistic Hamiltonian is used to compute two-gluon glueballs with masses in good agreement with those obtained by the lattice calculations of Ref. [1]. This Hamiltonian, the model III in Ref. [8], relies on the auxiliary fields formalism [9,10] and on a one-gluon exchange (OGE) interaction proposed in Ref. [3]. It depends only on three parameters: the strong coupling constant α_S, the string tension a, and a gluon size γ which removes singularities in the short-range part of the potential. The constituent gluon mass is dynamically generated and it is assumed that the Casimir scaling determines the color structure of the confinement. These two ingredients are actually necessary to obtain a good agreement between the results from a potential model and from lattice calculations.

The purpose of this paper is to check if the potential model built for two-gluon systems in Ref. [8] can be generalized to three-gluon systems. Compared to previous models [5,6], our approach is characterized by some improved features: semirelativistic kinematics, more realistic confinement, dynamical definition of the gluon mass, coherent treatment of the gluon size. These points will be detailed below. The masses of the lowest negative parity $L = 0$ glueballs are computed with a great accuracy and compared with lattice calculations [1,2]. In Sec. II, the three-gluon Hamiltonian is build, and the structure of the glueballs studied is presented in Sec. III. The three-gluon glueball spectrum is presented with the two-gluon glueball spectrum from Ref. [8] and is discussed in Sec. IV. Some concluding remarks are given in Sec. V.

II. HAMILTONIAN

A. Parameters

In Ref. [8], two sets of parameters, denoted A and B, were presented for the model III (see Table I). With the set A, it is possible to obtain glueball masses in agreement with the results of some experimental works [11,12]: the lowest 2^{++} state near 2 GeV, the lowest 0^{++} state near 1.5 GeV, and the lowest 0^{-+} state near 2.1 GeV. The values of a and α_S are close to the ones used in some recent

TABLE I. Parameters for models A and B ($\sigma = 3a/4$). For both models, the gluon current mass is zero and $f = 0.9515$.

	Model A	Model B
a	0.16 GeV^2	0.21 GeV^2
α_S	0.40	0.50
γ	0.504 GeV^{-1}	0.495 GeV^{-1}

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baryon calculations [13]. With the set B, glueball masses were computed in agreement with the results of the lattice calculations of Ref. [1]. If the absolute glueball masses found in Ref. [8] with both sets are strongly different, the rescaled spectra are nearly identical. As we use in this work a three-body generalization of the Hamiltonian model III of Ref. [8], the two sets will also be considered.

It is worth mentioning how the parameters have been determined in Ref. [8]. The mass of the lightest 2^{++} is nearly independent of the values of α_S and γ, but depends strongly on a. So, this last parameter has been determined with this 2^{++} state. The remaining parameters α_S and γ have then be computed in order to reproduce the lightest 0^{++} and 0^{-+} states. The three states 2^{++}, 0^{++}, and 0^{-+} have been chosen because they are possible experimental glueball candidates [11,12] and because they are computed with relatively small errors in lattice calculations [1,2].

B. Confinement potential

A good approximation of the confining interaction between a quark and an antiquark in a meson is given by the linear potential ar, where r is the distance between the two particles and where a is the string tension. In a baryon, lattice calculations and some theoretical considerations indicate that each quark generates a flux tube and that these flux tubes meet in a junction point $\boldsymbol{R}_0$ which minimizes the potential energy. Following this hypothesis, the confinement in a baryon could be simulated by the three-body interaction

$$V_{qqq} = a \sum_{i=1}^{3} |\boldsymbol{r}_i - \boldsymbol{R}_0|. \tag{1}$$

For such a potential, the point $\boldsymbol{R}_0$ minimizes also the length of the three flux tubes and is identified with the Toricelli point [14].

The energy density λ_c of a flux tube (string tension) can depend on the color charge c which generates it. Lattice calculations [15] and effective models of QCD [16] predict that the Casimir scaling hypothesis is well verified in QCD, that is to say that the energy density is proportional to the value of the quadratic Casimir operator $\hat{F}_c^2$ of the color source

$$\lambda_c = \hat{F}_c^2 \sigma. \tag{2}$$

We have then $\lambda_q = \lambda_{\bar{q}} = 4\sigma/3 = a$ and $\lambda_g = 3\sigma$. In this work we will assume that the confinement in a three-body color singlet is given by

$$V_{ccc} = \sigma \sum_{i=1}^{3} \hat{F}_i^2 |\boldsymbol{r}_i - \boldsymbol{R}_0|. \tag{3}$$

This potential can be considered as the three-body generalization of the confinement used in Ref. [8]. No constant potential is added, contrary to usual Hamiltonians in mesons and baryons [3,17]. Let us note that if the three color charges are not the same, $\boldsymbol{R}_0$ is no longer identified with the Toricelli point [18].

Interaction (3) is very difficult to use in a practical calculation. A good approximation can be obtained for three identical color charges by replacing $\boldsymbol{R}_0$ by the center of mass coordinate $\boldsymbol{R}_{\text{cm}}$ and by renormalizing the potential by a factor f which depends on the three-body system [14]. For three identical particles, the best value is $f = 0.9515$. We will use this approximation in the following, which seems more realistic than a confinement obtained by the sum of two-body forces [5,6].

As already mentioned, only the $L = 0$ states are studied in this paper. So, no spin-orbit correction to the confinement [8] is taken into account here.

In Refs. [3,5,6], the confinement potential saturates at large distances in order to simulate the breaking of the color flux tube between gluons due to color screening effects. An interaction of type (3) seems *a priori* inappropriate since the potential energy can grow without limit. But the phenomenon of flux tube breaking must only contribute to the masses of the highest glueball states. Moreover, it has been shown that the introduction of a saturation could not be the best procedure to simulate the breaking of a string joining two colored objects [19].

C. Dynamical constituent gluon mass

Within the auxiliary field formalism (also called einbein field formalism) [9], which can be considered as an approximate way to handle semirelativistic Hamiltonians [10,20], the effective QCD Hamiltonian has a kinetic part depending on the current particle masses m_i and the interaction is dominated by the confinement. A state-dependent constituent mass $\mu_i = \langle\sqrt{\boldsymbol{p}_i^2 + m_i^2}\rangle$ can be defined for each particle, and all relativistic corrections (spin, momentum, ...) to the static potentials are then expanded in powers of $1/\mu_i$. This approach has been used in Ref. [8] to build the two-gluon Hamiltonian. So, the same formalism will be applied also in this paper.

Taking into account the considerations of Sec. II B, the simplest generalization to a three-gluon system of the dominant part of the model III two-gluon Hamiltonian of Ref. [8] is

$$H_0 = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + f\sigma \sum_{i=1}^{3} \hat{F}_i^2 |\boldsymbol{r}_i - \boldsymbol{R}_{\text{cm}}|, \tag{4}$$

with the condition $\sum_{i=1}^{3} \boldsymbol{p}_i = 0$, since we work in the center of mass of the glueball. The gluons have vanishing current masses and their color is such that $\langle \hat{F}_i^2 \rangle = 3$. Contrary to some previous works [3,5,6], our Hamiltonian is a semirelativistic one. In Ref. [8], it has been shown that it is an important ingredient to obtain correct two-gluon glueball spectra.

Using the technics of Ref. [21], it is possible to obtain an analytical approximate formula giving the glueball mass

M_0 and the constituent gluon mass μ_0 (the three constituent gluon masses are the same since the wave function is completely symmetrized, see Sec. III) for the $L = 0$ eigenstates of Hamiltonian H_0

$$M_0 \approx 6\mu_0 \quad \text{with} \quad \mu_0 \approx 2\sqrt{\frac{6f\sigma}{\pi}}\left[\frac{2}{3\times 5^{1/3}} + \frac{8}{9\times 5^{5/6}}\left(n+\frac{1}{2}\right)\right]^{3/4}. \tag{5}$$

The accuracy of these formulas, which is around 5% for the ground state ($n = 0$), deteriorates with increasing values of n. With a value of $a = 4\sigma/3$ around 0.2 GeV^2, the smallest gluon constituent mass is around 600 MeV. It is then relevant to use an expansion in powers of $1/\mu_0$. Such a value of the gluon mass is in agreement with the values used in Refs. [3,5,6], but here the constituent mass is dynamically generated.

Instead of using the auxiliary field formalism, it is possible to consider relativistic corrections which are expanded in powers of $1/E_i(\boldsymbol{p}_i)$ where $E_i(\boldsymbol{p}_i) = \sqrt{\boldsymbol{p}_i^2 + m_i^2}$ (see for instance Ref. [22]). But, this leads to very complicated non local potentials which are difficult to handle.

D. Short-range potential

The Hamiltonian H_0 (4) gives the main features of the three-gluon glueball spectra, but the introduction of a short-range potential is necessary to achieve a detailed study. In Ref. [8], a OGE interaction between two gluons, coming from Ref. [3], has been considered. It is not possible to use it directly for a three-gluon glueball because the color structure of the interaction is different. So, we use here the last version of a OGE interaction between two gluons developed specifically for three-gluon glueballs [5,6]. Its explicit form, which is very similar to the form of the OGE interaction for two-gluon glueballs, is given below.

This interaction contains a tensor part and a spin-orbit part. Both are neglected in this paper since only $L = 0$ states are studied. Moreover, it has been shown that the tensor interaction between two gluons is small in two-gluon glueballs [8].

The OGE two-gluon potential has *a priori* a very serious flaw: depending on the spin state, the short-range singular part of the potential may be attractive and leads to a Hamiltonian unbounded from below [6]. This problem is solved, as in Ref. [8], by giving a finite size to the gluon (see Sec. II E).

The OGE two-gluon potential depends on the gluon constituent mass. To determine it, we follow the procedure proposed in Ref. [8]. For a given set of quantum numbers $\{\alpha\}$, the eigenstate $|\phi_\alpha\rangle$ of the Hamiltonian H_0 is computed. With this state, a constituent gluon mass is computed $\mu_\alpha = \langle\phi_\alpha|\sqrt{\boldsymbol{p}_1^2}|\phi_\alpha\rangle$. This value of μ_α is then used in the complete Hamiltonian (see Sec. II F) to compute its eigenstate with quantum numbers $\{\alpha\}$. It is worth noting that, with this procedure, two states which differ only by the radial quantum number are not orthogonal since they are eigenstates of two different Hamiltonians which differ by the value of μ. It is shown in Ref. [10] that this problem is not serious, the overlap of these states being generally weak.

E. Gluon size

In potential models, the gluon is considered as an effective degree of freedom with a constituent mass. Within this framework, it is natural to assume that a gluon is not a pure pointlike particle but an object dressed by a gluon and quark-antiquark pair cloud. Such an hypothesis for quarks leads to very good results in meson [23] and baryon [24] sectors. As in Ref. [8], we assume here a Yukawa color charge density for the gluon

$$\rho(\boldsymbol{u}) = \frac{1}{4\pi\gamma^2}\frac{e^{-u/\gamma}}{u}, \tag{6}$$

where γ is the gluon size parameter. The interactions between gluons are then modified by this density, a bare potential being transformed into a dressed one.

The main purpose of the gluon dressing is to remove all singularities in the short-range part of the interaction [24]. But, for consistency, the same regularization is applied to the confinement potential, although no singularity is present in this case. We think that the definition of a gluon size, which has a clear physical meaning, is preferable to the use of a smearing function only for potentials with singularity [4,6].

A one-body potential, like $|\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm}|$, is dressed by a simple convolution over the density of the interacting gluon and the potential and the corresponding potential is given by

$$V(\boldsymbol{r})^* = \int d\boldsymbol{r}' V(\boldsymbol{r}')\rho(\boldsymbol{r}-\boldsymbol{r}'). \tag{7}$$

A dressed two-body potential, depending on $|\boldsymbol{r}_i - \boldsymbol{r}_j|$, is obtained by a double convolution. This procedure is equivalent to the following calculation [25]

$$V(\boldsymbol{r})^{**} = \int d\boldsymbol{r}' V(\boldsymbol{r}')\Gamma(\boldsymbol{r}-\boldsymbol{r}') \quad \text{with} \quad \Gamma(\boldsymbol{u}) = \frac{1}{8\pi\gamma^3}e^{-u/\gamma}. \tag{8}$$

F. Total Hamiltonian

To obtain the total Hamiltonian for three-gluon glueballs which is the simplest generalization of the Hamiltonian for two-gluon glueballs from Ref. [8], we take the Hamiltonian H_0 given by the relation (4); we add the OGE interactions coming from Ref. [6] (without spin-orbit and tensor parts); and we dress all the potentials with the gluon color density (6). This gives the following Hamiltonian

$$H = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + V_{\mathrm{OGE}}^{**} + V_{\mathrm{Conf}}^{*} \quad \text{with} \tag{9a}$$

$$V_{\mathrm{OGE}}^{**} = \alpha_S \sum_{i<j=1}^{3} \hat{F}_i \cdot \hat{F}_j \left[\left(\frac{1}{4} + \frac{1}{3} \vec{S}_{ij}^2 \right) U(r_{ij})^{**} - \frac{\pi}{\mu^2} \delta(\boldsymbol{r}_{ij})^{**} \left(\beta + \frac{5}{6} \vec{S}_{ij}^2 \right) \right], \tag{9b}$$

$$U(r)^{**} = \frac{1}{(\mu^2\gamma^2 - 1)^2} \left(\frac{e^{-\mu r}}{r} - \frac{e^{-r/\gamma}}{r} \right) + \frac{e^{-r/\gamma}}{2\gamma(\mu^2\gamma^2 - 1)} \quad \text{with} \quad U(r) = \frac{e^{-\mu r}}{r}, \tag{9c}$$

$$\delta(\boldsymbol{r})^{**} = \frac{1}{8\pi\gamma^3} e^{-r/\gamma}, \tag{9d}$$

$$V_{\mathrm{Conf}}^{*} = f\sigma \sum_{i=1}^{3} \hat{F}_i^2 |\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}}|^{*} \quad \text{with} \quad r^{*} = r + 2\gamma^2 \frac{1 - e^{-r/\gamma}}{r}, \tag{9e}$$

where $\sigma = 3a/4$, $\sum_{i=1}^{3} \boldsymbol{p}_i = 0$ and $\vec{S}_{ij} = \vec{S}_i + \vec{S}_j$. $\beta = +1\ (-1)$ for a gluon pair in color octet antisymmetrical (symmetrical) state. The constituent state-dependent gluon mass μ is computed in advance with a solution of the Hamiltonian H_0.

III. WAVE FUNCTIONS

A gluon is a $I(J^P) = 0(1^-)$ color octet state. Two different three-gluon color singlet states exist [5], which are completely symmetrical or completely antisymmetrical ($\langle \hat{F}_i \cdot \hat{F}_j \rangle = 3$ for such states). The total isospin state of a glueball is an isosinglet and is completely symmetrical. Different total spin states are allowed with different properties of symmetry. They are presented in Table II. As gluons are bosons, the total wave function must be completely symmetrical. Its parity is the opposite of the spatial parity, and its C-parity is positive for color antisymmetrical state and negative for color symmetrical state. Let us note that a two-gluon glueball has always a positive C-parity.

In this work, we will mainly consider glueballs with the lowest masses. These states are characterized by a vanishing total orbital angular momentum $L = 0$ and by a spatial wave function completely symmetrical with a positive parity. This immediately implies that the lowest glueballs are states with J^{PC} equal to 0^{-+}, 1^{--}, and 3^{--} [5,6]. In order to reach a good accuracy, the trial spatial wave functions are expanded in large gaussian function bases [26]. With more than 10 gaussian functions for each color/isospin/spin channels, we have checked that the numerical errors on masses presented are around or less than 1 MeV.

Using the value of σ from models A and B, the $L = 0$ ground state masses of the Hamiltonian H_0 (4) are presented in Table III for all possible J^{PC} quantum numbers. The 0^{-+}, 1^{--}, and 3^{--} glueballs have clearly the lowest masses. In this table, they are degenerate since the Hamiltonian H_0 is spin-independent. For each state, the corresponding constituent gluon masses μ_0 is indicated. It is used to define the complete Hamiltonian H (9).

IV. RESULTS

We present here the three-gluon glueball masses obtained with the complete Hamiltonian H (9) together with the two-gluon glueball masses computed in Ref. [8] (see Table IV). These masses are compared with the results obtained by the lattice calculations of Ref. [2]. This work is an update of a previous study [1]. So, a state not computed in Ref. [2] but presented in Ref. [1] is also considered here. As it can be seen on Fig. 1, with the set B of parameters, our masses are in quite good agreement with the results of the lattice calculations, except for one exception discussed below. Unfortunately, the masses predicted for the 0^{++}, 2^{++}, and 0^{+-} two-gluon glueballs are larger than some possible experimental candidates [11,12]. A best agreement with these data can be achieved with the set A of

TABLE II. Characteristics of three-gluon spin functions with total spin S, intermediate couplings S_{int}, and symmetry properties which can be obtained by coupling (A: Antisymmetrical, S: Symmetrical, MS: Mixed symmetry).

S	S_{int}	Symmetry
0	0	1 A
1	0, 1, 2	1 S, 2 MS
2	1, 2	2 MS
3	2	1 S

TABLE III. $L = 0$ ground state masses M_0 of the Hamiltonian H_0 (4) as a function of the J^{PC} quantum numbers. The corresponding constituent gluon masses μ_0 are also given. Values in MeV are computed with the value of σ from models A/B.

J^{PC}	M_0	μ_0	J^{PC}	M_0	μ_0
0^{--}	5574/6385	929/1064	0^{-+}	3211/3679	535/613
1^{--}	3211/3679	535/613	1^{-+}	4156/4761	693/794
2^{--}	4156/4761	693/794	2^{-+}	4156/4761	693/794
3^{--}	3211/3679	535/613	3^{-+}	5574/6385	929/1064

TABLE IV. Glueball masses in MeV and (glueball mass ratios normalized to lightest 2^{++}). The two-gluon masses are taken from Ref. [8]. The error bars for lattice mass ratios are computed without the normalization error on the masses. The lightest 0^{++}, 2^{++}, and 0^{-+} states are taken as inputs to fix the parameters. The first column indicates the valence gluon content as predicted by our model.

	J^{PC}	Lattice [Ref.]	Model A	Model B
gg	0^{++}	$1710 \pm 50 \pm 80$ (0.72 ± 0.03) [2]	1604 (0.78)	1855 (0.78)
		$2670 \pm 180 \pm 130$ (1.12 ± 0.09) [1]	2592 (1.26)	2992 (1.26)
	2^{++}	$2390 \pm 30 \pm 120$ (1.00 ± 0.03) [2]	2051 (1.00)	2384 (1.00)
	0^{-+}	$2560 \pm 35 \pm 120$ (1.07 ± 0.03) [2]	2172 (1.06)	2492 (1.05)
		$3640 \pm 60 \pm 180$ (1.52 ± 0.04) [1]	3228 (1.57)	3714 (1.56)
	2^{-+}	$3040 \pm 40 \pm 150$ (1.27 ± 0.03) [2]	2573 (1.25)	2984 (1.25)
		$3890 \pm 40 \pm 190$ (1.63 ± 0.04) [1]	3345 (1.63)	3862 (1.62)
	3^{++}	$3670 \pm 50 \pm 180$ (1.54 ± 0.04) [2]	3132 (1.53)	3611 (1.51)
ggg	1^{--}	$3830 \pm 40 \pm 190$ (1.60 ± 0.04) [2]	3433 (1.67)	3999 (1.68)
	2^{--}	$4010 \pm 45 \pm 200$ (1.68 ± 0.04) [2]	4422 (2.16)	5133 (2.15)
	3^{--}	$4200 \pm 45 \pm 200$ (1.76 ± 0.04) [2]	3569 (1.74)	4167 (1.75)
	0^{-+}		3688 (1.80)	4325 (1.81)

parameters [8]. But the situation is not simple, since recent works [27,28] suggest that the new observed resonance $f_0(1810)$ reported by the BES collaboration [29] could be a 0^{++} glueball. Such a data is then compatible with the set B of parameters.

As the lattice results can suffer from quite large error of normalization, it is then interesting to present a rescaled spectra. In Fig. 2, the ratios of two- and three-gluon glueball masses on the lowest 2^{++} mass are presented. At this scale, results coming from sets A and B of parameters cannot be distinguished.

Let us first point that the 1^{--} and 3^{--} states occur only in three-gluon glueballs, whereas the 0^{-+} state is available for two- and three-gluon systems. The mixing between these two channels is ignored here. Our results share some similarities with other potential models. For instance, in Ref. [5], the three lowest three-gluon glueballs are those with J^{PC} equal to 0^{-+}, 1^{--}, and 3^{--}, but they are found around 2400 MeV and the mass splitting between theses states is around 50 MeV. In Ref. [6], the same three lowest states are found: the 1^{--} glueball is predicted in the range 3500–3700 MeV and the mass difference between these three sates is around 100 MeV.

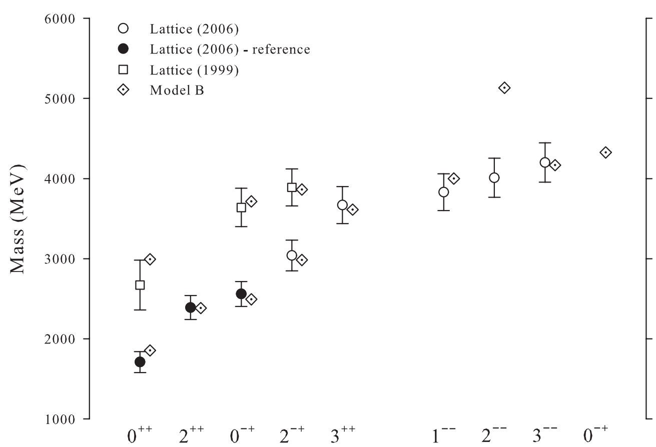

FIG. 1. Glueball masses given in MeV. Dotted diamonds: Results from model B (two-gluon masses are taken from Ref. [8]); Black and white circles: Lattice results from Ref. [1]; White squares: Lattice results from Ref. [2]. Black circles indicate the reference states taken as inputs to fix the parameters. The error bars for lattice results are computed by summing the two uncertainties (see Table IV).

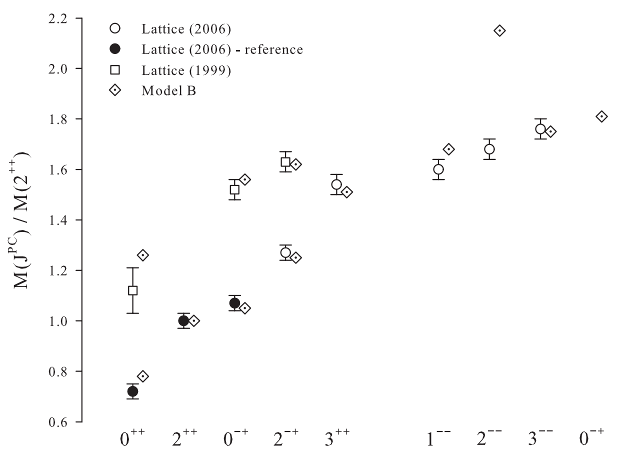

FIG. 2. Glueball mass ratios normalized to the lightest 2^{++} state (see Table IV). Dotted diamonds: Results from model B (two-gluon masses are taken from Ref. [8]); Black and white circles: Lattice results from Ref. [1]; White squares: Lattice results from Ref. [2]. Black circles indicate the reference states taken as inputs to fix the parameters. The error bars for lattice results are computed without the normalization error on glueball masses. Results from models A and B cannot be distinguished on this graphic.

A detailed comparison between the results of our potential model and the lattice calculations of Ref. [1] have been performed in Ref. [8] for the two-gluon glueballs. The conclusions of this work are not changed by the new lattice predictions obtained in Ref. [2]. Let us then focus our attention on the three-gluon glueball spectrum obtained with the set B of parameters. Without any new parameters, the 1^{--} and 3^{--} glueballs are in good agreement with the lattice predictions of Ref. [2] (see Table IV and Fig. 1). The agreement is slightly less good for the relative spectra (see Fig. 2) because the error is smaller for lattice mass ratios. Our results also suggest the existence of a three-gluon 0^{-+} glueball near these two last states. The others 0^{-+} states already computed by the lattice calculations can be identified as two-gluon systems by our model.

The lattice results predict a 2^{--} state at 4010 MeV near the 1^{--} and 3^{--} states. With our Hamiltonian, a mass more than 1 GeV above is computed. It is unavoidable in our model, since a spin 2 function has a mixed symmetry which implies a mixed symmetry for the space function and then a greater mass for the corresponding glueball, in agreement with the results of Refs. [5,6]. It has been checked that the spatial wave function of the 2^{--} state is dominated by a configuration in which each internal variable is characterized by one unit of angular momentum.

We have no firm explanation for such a discrepancy. This problem could arise because the gluon has a constituent mass within our formalism. So, it possesses a spin as any massive particle, that is to say three states of polarization. In lattice calculations, the gluon is a massless particle with a definite helicity and then only two states of polarization. The same phenomenon could be at the origin of another puzzling—to some extent opposite—situation in the two-gluon glueball sector: the existence of 1^{PC} states with low masses in our model [8], which are actually not predicted below 4 GeV by lattice calculations [1]. Further studies are necessary to clarify the situation.

The lattice results predict also several three-gluon J^{+-} states in the range 2980–4780 MeV. These states with positive parity have a negative spatial parity and then are expected to have masses larger than the lowest three-gluon J^{--} states. This is manifestly not the case in these lattice calculations. In Ref. [8], it is shown that the structure of some two-gluon glueballs can be explained with our potential model by the existence of strong spin-orbit forces coming from the OGE interaction and the confinement. It is possible that similar forces act in three-gluon system to lower the masses of some of these states.

V. CONCLUSION

The masses of pure three-gluon glueballs have been studied with the generalization of a semirelativistic potential model [8] which gives pure two-gluon glueball spectra in good agreement with lattice calculations [1,2]. The short-range part of the potential is the sum of two-body OGE interactions. For the confinement, a potential simulating a genuine Y-junction is used and it is assumed that the Casimir scaling hypothesis is well verified. The gluon is massless but the OGE interaction is expressed in terms of a state-dependent constituent mass. The Hamiltonian depends only on 3 parameters fixed in Ref. [8]: the strong coupling constant, the string tension, and a gluon size. All masses have been accurately computed with an expansion of trial states in gaussian functions [26].

In this work, only the negative (natural) parity $L = 0$ three-gluon glueballs are studied. The masses of the lowest 1^{--} and 3^{--} glueballs predicted by our potential model are in agreement with the results of a recent lattice calculations [2], but the lowest 2^{--} state is found at higher energy in agreement with other potential models [5,6]. The origin of such a discrepancy between both approaches is not known. It could be due to the fact that gluons have constituent non vanishing masses in our approach. They are then characterized by a spin, and not by a helicity as it could be expected for particles with a vanishing current mass.

Other positive parity three-gluon glueballs predicted by lattice calculations are not considered here [2]. We think that their properties could be explained by the action of strong spin-orbit forces, similar to those present into two-gluon glueballs. To take into account these interaction, it is necessary to consider the spin-orbit forces arising from OGE interactions but also from the Y-junction confinement. Such a work is in progress.

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