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Exotic tetraquark $ud\bar{s}\,\bar{s}$ of $J^P = 0^+$ in the QCD sum rule

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We study a QCD sum rule analysis for an exotic tetraquark $ud\bar{s}\,\bar{s}$ of $J^P = 0^+$ and $I = 1$. We construct $qq\bar{q}\,\bar{q}$ currents in a local product form and find that there are five independent currents for this channel. Because of the high dimensional nature of the current, it is not easy to form a good sum rule when using a single current. This means that we do not find any sum rule window to extract reliable results, due to the insufficient convergence of the operator product expansion and to the exceptional important role of QCD continuum. Then we examine sum rules by using currents of linear combinations of two currents among the independent ones. We find two reasonable cases that predict a mass of the tetraquark around 1.5 GeV.

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I. INTRODUCTION

The history of exotic hadrons is rather long. But the recent experimental observations have triggered a tremendous amount of research activities [1–3]. Among them the report on the pentaquark Θ^+ from the LEPS group in 2002 was the most influential one [4], partly because Θ^+ is a genuine exotic state of the quark content $uudd\bar{s}$. It also has unusual properties such as a light mass and a very narrow width. Its existence is, however, now questioned, which should be confirmed in the future experiments [5].

Turning to mesons, though not genuine exotic states, $X(3872)$ and $D_s(2317)$ are found to have properties that seem difficult to be explained by a conventional picture of $\bar{q}q$ [6–12]. Rather, they could be considered to have a significant amount of multiquark components. Historically, tetraquark mesons were investigated long ago as an attempt to explain relatively light masses and excess of states in scalar channels [13–17]. Just as in the exotic baryons, it is interesting to consider genuine exotic states in the meson sector whose minimal component is $qq\bar{q}\,\bar{q}$. Tetraquark states of $ud\bar{s}\,\bar{s}$ component have been studied as candidates of such exotic states. Since they may be obtained by replacing one of ud diquarks in Θ^+ by an $\bar{s}$ antiquark, similarities between Θ^+ and $ud\bar{s}\,\bar{s}$ have been discussed, though precise analogy is a dynamical question [18–20].

In the former studies, the tetraquark $ud\bar{s}\,\bar{s}$ of $J^P = 1^+$ was investigated in detail, where it was shown that the state has a relatively low mass and a narrow width decaying into K^*K in the flux tube model [21]. The narrow decay width is associated with the fact that the KK channel is forbidden due to the conservation of parity and angular momentum, which partly motivated the study of the 1^+ channel.

In principle, it is also possible to study other channels of the $ud\bar{s}\,\bar{s}$ tetraquarks [21–23]. From a naive point of view of mass, it is natural to investigate 0^+ scalar states. In contrast to $\bar{q}q$ mesons, the tetraquark does not need orbital excitation to form the quantum number 0^+, but all quarks may occupy the lowest state. In this case, it is shown that the tetraquark should have isospin one $I = 1$. This is the object that we would like to study in this paper.

We perform QCD sum rule analyses for the scalar ($J^P = 0^+$) and isovector ($I = 1$) exotic tetraquark $ud\bar{s}\,\bar{s}$. We attempt a rather comprehensive analysis in which we will pay special attention to the structure of the interpolating fields (currents). First, we find that there are five independent interpolating fields for the tetraquark. We show this by constructing the tetraquark currents in terms of diquark fields ($(qq)(\bar{q}\,\bar{q})$) and mesonic fields ($(\bar{q}q)(\bar{q}q)$), where $\bar{q}q$ can be both color singlet and octet. We then consider two-point correlation functions first by using a single current of various types. It turns out that many of them do not achieve a good sum rule. Therefore, we attempt linear combinations of two independent currents. This method was first proposed in Ref. [24]. We then find that there are several cases with good Borel stability, indicating the mass of the tetraquark around 1.5 GeV. We also investigate the reliability of the sum rule not only from the Borel stability but also from the dependence on the threshold value and the amount of the pole contribution in the total sum rule. We

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also mention the convergence of operator product expansion (OPE).

The difficulties to make a good sum rule for exotic particles of high dimensional operators were nicely discussed in a recent work by Kojo *et al.* [25]. They proposed a sum rule using a linear combination of two-point functions rather than currents in order, for instance, to suppress large contributions from low dimensional terms that are irrelevant to nonperturbative properties of hadrons. They have successfully achieved a good sum rule that satisfy the necessary requirements. In our present study, our strategy is different from theirs, but the consideration along their idea is certainly important in the discussion of the tetraquark also.

This paper is organized as follows. In Sec. II, we establish five independent currents in diquark-antidiquark and meson-meson (actually mesonlike) constructions. Some relations among various currents will be discussed. Section III is the main part of this paper, where we perform sum rule analyses using various tetraquark currents constructed in Sec. II. We study the sum rule of a single current and then consider linear combinations of currents. Section IV is devoted to the summary. In the Appendix, we discuss the equivalence and relations between the currents of diquark-antidiquark and meson-meson constructions.

II. INDEPENDENT CURRENTS

Let us consider currents for the tetraquark $ud\bar{s}\,\bar{s}$ having $J^P = 0^+$. Here we consider only local currents. To write a current, Lorentz and color indices are contracted with suitable coefficients ($L^{abcd}_{\mu\nu\rho\sigma}$) to provide necessary quantum numbers,

$$\eta = L^{abcd}_{\mu\nu\rho\sigma}\bar{s}^{\mu}_{a}\bar{s}^{\nu}_{b}u^{\rho}_{c}d^{\sigma}_{d}, \tag{1}$$

where the sum over repeated indices ($\mu, \nu, \ldots$ for Dirac spinor indices, and $a, b, \ldots$ for color indices) is taken.

For the Dirac spinor space, using possible diquark and antidiquark bilinears [26–29], there are five independent terms:

$$\begin{aligned} S_{abcd} &= (\bar{s}_a\gamma_5 C\bar{s}_b^T)(u_c^T C\gamma_5 d_d),\\ V_{abcd} &= (\bar{s}_a\gamma_\mu\gamma_5 C\bar{s}_b^T)(u_c^T C\gamma^\mu\gamma_5 d_d),\\ T_{abcd} &= (\bar{s}_a\sigma_{\mu\nu} C\bar{s}_b^T)(u_c^T C\sigma^{\mu\nu} d_d),\\ A_{abcd} &= (\bar{s}_a\gamma_\mu C\bar{s}_b^T)(u_c^T C\gamma^\mu d_d),\\ P_{abcd} &= (\bar{s}_a C\bar{s}_b^T)(u_c^T C d_d). \end{aligned} \tag{2}$$

Here, color indices are not yet specified. For the diquark and antidiquark pair, color structures providing a color-singlet tetraquark are $\mathbf{3}\otimes\bar{\mathbf{3}}$ and $\bar{\mathbf{6}}\otimes\mathbf{6}$, which we will denote by labels **3** and **6** for short.

Therefore, we have altogether ten terms of products

$$\{S\oplus V\oplus T\oplus A\oplus P\}_{\text{Lorentz}}\otimes\{\mathbf{3}\oplus\mathbf{6}\}_{\text{color}}. \tag{3}$$

However, half of them drop due to the Pauli principle. For instance,

$$P_3 \equiv P_{\text{Lorentz}}\otimes\mathbf{3}_{\text{color}} = \epsilon_{abc}(\bar{s}_b C\bar{s}_c^T)\epsilon_{ab'c'}(u_{b'}^T C d_{c'}) = 0. \tag{4}$$

Eventually, we end up with five independent currents:

$$\begin{aligned} S_6 &= (\bar{s}_a\gamma_5 C\bar{s}_b^T)(u_a^T C\gamma_5 d_b),\\ V_6 &= (\bar{s}_a\gamma_\mu\gamma_5 C\bar{s}_b^T)(u_a^T C\gamma^\mu\gamma_5 d_b),\\ T_3 &= (\bar{s}_a\sigma_{\mu\nu} C\bar{s}_b^T)(u_a^T C\sigma^{\mu\nu} d_b),\\ A_3 &= (\bar{s}_a\gamma_\mu C\bar{s}_b^T)(u_a^T C\gamma^\mu d_b),\\ P_6 &= (\bar{s}_a C\bar{s}_b^T)(u_a^T C d_b). \end{aligned} \tag{5}$$

In the nonrelativistic language, these five terms correspond to combinations of diquarks and antidiquarks:

$$[({}^1S_0)({}^1S_0)]_{0^+},\quad [({}^3S_1)({}^3S_1)]_{0^+},\quad [({}^1P_1)({}^1P_1)]_{0^+},\quad [({}^3P_0)({}^3P_0)]_{0^+},\quad [({}^3P_1)({}^3P_1)]_{0^+}. \tag{6}$$

Another possible piece of ${}^3P_2$ is irrelevant, since the five bilinear forms $q^T\Gamma q$ ($\Gamma = S, V, T, A, P$) can only have spin $j \leq 1$, while the ${}^3P_2$ diquark has $j = 2$.

Finally we consider the flavor structure. The $\bar{s}\,\bar{s}$ antidiquark is symmetric in flavor, and hence belongs to the symmetric representation $\bar{\mathbf{6}}_f$. If the other ud diquark belongs to $\bar{\mathbf{3}}_f$, and so isospin $I = 0$, the diquark and antidiquark will have different flavor symmetry. But they should have the same color and spin symmetries for composing a color-singlet scalar tetraquark. Considering the Pauli principle, they must have different parity, and hence their combination is a negative-parity scalar tetraquark. Accordingly, the other ud diquark also belongs to $\mathbf{6}_f$, and so isospin $I = 1$. Among the irreducible representations of the tetraquark

$$\bar{\mathbf{6}}\otimes\mathbf{6} = \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27},$$

$S = +2$ and $I = 1$ states are in the **27** representation of $SU(3)_f$, which is the flavor structure of the present tetraquark. As shown in Fig. 1, three isovector states of the $\mathbf{27}_f$ are $uu\bar{s}\,\bar{s}$, $1/\sqrt{2}(ud + du)\bar{s}\,\bar{s}$, and $dd\bar{s}\,\bar{s}$.

We have constructed five independent currents using diquark and antidiquark combination. We refer to this as the diquark construction. Similarly, we can also construct the tetraquark currents using $\bar{q}q$ combination (mesonic construction). Obviously, there are ten combinations of the Dirac (S, V, T, A, and P) and color (**1** and **8**) spaces:

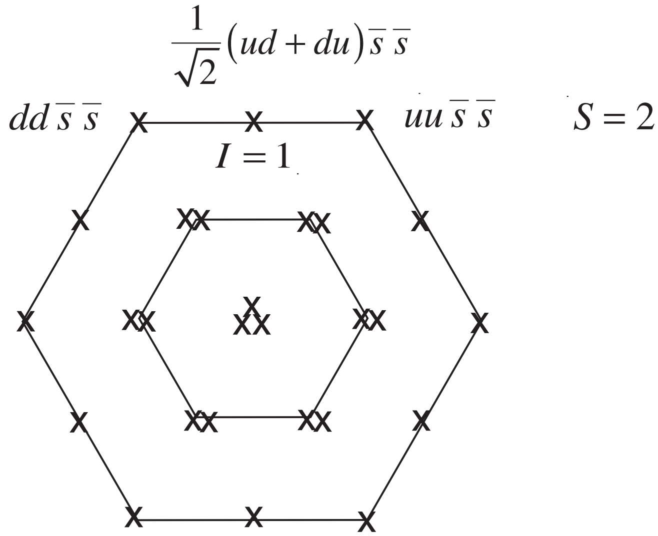

FIG. 1. SU(3) weight diagram for **27**, where the locations of three tetraquark components of $S = 2$ and $I = 1$ are shown.

$$
\begin{aligned}
S_1 &= (\bar{s}_a u_a)(\bar{s}_b d_b),\\
S_8 &= (\bar{s}_a \lambda^n_{ab} u_b)(\bar{s}_c \lambda^n_{cd} d_d),\\
V_1 &= (\bar{s}_a \gamma_\mu u_a)(\bar{s}_b \gamma^\mu d_b),\\
V_8 &= (\bar{s}_a \gamma_\mu \lambda^n_{ab} u_b)(\bar{s}_c \gamma^\mu \lambda^n_{cd} d_d),\\
T_1 &= (\bar{s}_a \sigma_{\mu\nu} u_a)(\bar{s}_b \sigma^{\mu\nu} d_b),\\
T_8 &= (\bar{s}_a \sigma_{\mu\nu} \lambda^n_{ab} u_b)(\bar{s}_c \sigma^{\mu\nu} \lambda^n_{cd} d_d),\\
A_1 &= (\bar{s}_a \gamma_\mu \gamma_5 u_a)(\bar{s}_b \gamma^\mu \gamma_5 d_b),\\
A_8 &= (\bar{s}_a \gamma_\mu \gamma_5 \lambda^n_{ab} u_b)(\bar{s}_c \gamma^\mu \gamma_5 \lambda^n_{cd} d_d),\\
P_1 &= (\bar{s}_a \gamma_5 u_a)(\bar{s}_b \gamma_5 d_b),\\
P_8 &= (\bar{s}_a \gamma_5 \lambda^n_{ab} u_b)(\bar{s}_c \gamma_5 \lambda^n_{cd} d_d),
\end{aligned}
\tag{7}
$$

where subscripts **1** and **8** denote color-singlet and octet representations, respectively. Unlike the diquark construction, all ten currents in Eq. (7) remain finite. However, it is possible to show only five of them (in fact any five of them) are independent. The proof of this and various relations among different currents are discussed in Appendix A.

III. QCD SUM RULES ANALYSIS

A. Formulas of QCD sum rule

For the past decades QCD sum rule has proven to be a very powerful and successful nonperturbative method [30,31]. In sum rule analyses, we consider two-point correlation functions:

$$\Pi(q^2) \equiv i\int d^4x e^{iqx}\langle 0|T\eta(x)\eta^\dagger(0)|0\rangle, \tag{8}$$

where η is an interpolating current for the tetraquark. We compute $\Pi(q^2)$ in the OPE of QCD up to certain order in the expansion, which is then matched with a hadronic parametrization to extract information of hadron properties. At the hadron level, we express the correlation function in the form of the dispersion relation with a spectral function:

$$\Pi(p) = \int_0^\infty \frac{\rho(s)}{s - p^2 - i\varepsilon} ds, \tag{9}$$

where

$$
\begin{aligned}
\rho(s) &\equiv \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle\\
&= f_X^2 \delta(s - M_X^2) + \text{higher states}.
\end{aligned}
\tag{10}
$$

For the second equation, as usual, we adopt a parametrization of one pole dominance for the ground state X and a continuum contribution. The sum rule analysis is then performed after the Borel transformation of the two expressions of the correlation function, (8) and (9),

$$\Pi^{(\mathrm{all})}(M_B^2) \equiv \mathcal{B}_{M_B^2}\Pi(p^2) = \int_0^\infty e^{-s/M_B^2}\rho(s)ds. \tag{11}$$

Assuming the contribution from the continuum states can be approximated well by the spectral density of OPE above a threshold value s_0 (duality), we arrive at the sum rule equation

$$\Pi(M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} = \int_0^{s_0} e^{-s/M_B^2}\rho(s)ds. \tag{12}$$

Differentiating Eq. (12) with respect to $\frac{1}{M_B^2}$ and dividing it by Eq. (12), finally we obtain

$$M_X^2 = \frac{\int_0^{s_0} e^{-s/M_B^2} s\rho(s)ds}{\int_0^{s_0} e^{-s/M_B^2}\rho(s)ds}. \tag{13}$$

In the following, we study both Eqs. (12) and (13) as functions of the parameters such as the Borel mass M_B and the threshold value s_0 for various combinations of the tetraquark currents.

B. Analysis of single diquark currents

In this subsection, we perform a QCD sum rule analysis using the five diquark currents, S_6, V_6, T_3, A_3, and P_6, separately. Let us first outline briefly how we performed the OPE calculation. For illustration, let us take P_6. Then

$$
\begin{aligned}
\Pi(q^2) &\equiv i\int d^4x e^{iqx}\langle 0|TP_6(x)P_6^\dagger(0)|0\rangle\\
&= \mathrm{Tr}[C(S_u^{aa'}(x))^T C S_d^{bb'}(x)]\\
&\quad\times \mathrm{Tr}[S_s^{a'a}(-x)C(S_s^{b'b}(-x))^T C]\\
&\quad+ \mathrm{Tr}[C(S_u^{aa'}(x))^T C S_d^{bb'}(x)]\\
&\quad\times \mathrm{Tr}[S_s^{b'a}(-x)C(S_s^{a'b}(-x))^T C].
\end{aligned}
\tag{14}
$$

For the quark propagator, we use

$$\begin{aligned} iS_q^{ab}(x) &\equiv \langle 0|T[q^a(x)\bar{q}^b(0)]|0\rangle \\ &= \frac{i\delta^{ab}}{2\pi^2 x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2} g_c G^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) \\ &\quad - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma Gq\rangle - \frac{\delta^{ab}m_q}{4\pi^2 x^2} \\ &\quad + \frac{i\delta^{ab}m_q}{48}\langle\bar{q}q\rangle\hat{x} + \frac{i\delta^{ab}m_q^2}{8\pi^2 x^2}\hat{x}. \end{aligned} \tag{15}$$

The two-point function is then divided into three parts:

(1) Terms proportional to δ^{ab} (a, b being color indices), where no soft gluon is emitted. The lowest term of this kind is the continuum term.

(2) Terms containing one λ_{ab} (color matrix), where one soft gluon is emitted. The lowest terms of this type contain condensates such as $\langle g\bar{q}\sigma Gq\rangle$ ($q = u$ and d) and $\langle g\bar{s}\sigma Gs\rangle$.

(3) Terms containing two λ_{ab}'s, where two soft gluons are emitted. The lowest terms of this type contain the condensate $\langle g^2G^2\rangle$.

We have performed the OPE calculation for the spectral function up to dimension eight, which is up to the constant (s^0) term of $\rho(s)$. Actual computation is very complicated. We have performed this calculation using MATHEMATICA with FEYNCALC [32]. MATHEMATICA programs are available from the authors. The results are

$$\begin{aligned} \rho_{S6}(s) &= \frac{s^4}{61\,440\pi^6} - \frac{{m_s}^2 s^3}{3072\pi^6} + \left(\frac{{m_s}^4}{256\pi^6} - \frac{m_s\langle\bar{s}s\rangle}{192\pi^4} - \frac{\langle g^2GG\rangle}{12\,288\pi^6}\right)s^2 + \left(-\frac{m_s^3\langle\bar{s}s\rangle}{32\pi^4} + \frac{m_s^2\langle g^2GG\rangle}{4096\pi^6} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{64\pi^4} + \frac{\langle\bar{q}q\rangle^2}{24\pi^2} \right. \\ &\quad \left. + \frac{\langle\bar{s}s\rangle^2}{24\pi^2}\right)s - \frac{m_s^2\langle\bar{q}q\rangle^2}{12\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{48\pi^2} + \frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma Gq\rangle}{24\pi^2} + \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{1536\pi^4} + \frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{24\pi^2} - \frac{m_s^4\langle g^2GG\rangle}{2048\pi^6}, \end{aligned} \tag{16}$$

$$\begin{aligned} \rho_{V6}(s) &= \frac{s^4}{15\,360\pi^6} - \frac{5{m_s}^2 s^3}{1536\pi^6} + \left(\frac{{m_s}^4}{64\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{24\pi^4} + \frac{5\langle g^2GG\rangle}{6144\pi^6}\right)s^2 + \left(-\frac{m_s^3\langle\bar{s}s\rangle}{8\pi^4} - \frac{11m_s^2\langle g^2GG\rangle}{2048\pi^6} + \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{32\pi^4} \right. \\ &\quad \left. - \frac{\langle\bar{q}q\rangle^2}{12\pi^2} - \frac{\langle\bar{s}s\rangle^2}{12\pi^2}\right)s + \frac{2m_s^2\langle\bar{q}q\rangle^2}{3\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma Gq\rangle}{12\pi^2} + \frac{7m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{768\pi^4} - \frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{12\pi^2}, \end{aligned} \tag{17}$$

$$\begin{aligned} \rho_{T3}(s) &= \frac{s^4}{5120\pi^6} - \frac{{m_s}^2 s^3}{128\pi^6} + \left(\frac{3{m_s}^4}{64\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{16\pi^4} + \frac{\langle g^2GG\rangle}{1536\pi^6}\right)s^2 + \left(-\frac{3m_s^3\langle\bar{s}s\rangle}{8\pi^4} - \frac{m_s^2\langle g^2GG\rangle}{256\pi^6}\right)s + \frac{m_s^2\langle\bar{q}q\rangle^2}{\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{4\pi^2} \\ &\quad + \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{192\pi^4} - \frac{m_s^4\langle g^2GG\rangle}{256\pi^6}, \end{aligned} \tag{18}$$

$$\begin{aligned} \rho_{A3}(s) &= \frac{s^4}{30\,720\pi^6} - \frac{{m_s}^2 s^3}{1024\pi^6} + \left(\frac{{m_s}^4}{128\pi^6} + \frac{\langle g^2GG\rangle}{6144\pi^6}\right)s^2 + \left(-\frac{m_s^3\langle\bar{s}s\rangle}{16\pi^4} - \frac{3m_s^2\langle g^2GG\rangle}{2048\pi^6} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{64\pi^4} + \frac{\langle\bar{q}q\rangle^2}{24\pi^2} + \frac{\langle\bar{s}s\rangle^2}{24\pi^2}\right)s \\ &\quad + \frac{m_s^2\langle\bar{s}s\rangle^2}{24\pi^2} + \frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma Gq\rangle}{24\pi^2} + \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{256\pi^4} + \frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{24\pi^2}, \end{aligned} \tag{19}$$

$$\begin{aligned} \rho_{P6}(s) &= \frac{s^4}{61\,440\pi^6} - \frac{{m_s}^2 s^3}{1024\pi^6} + \left(\frac{{m_s}^4}{256\pi^6} - \frac{m_s\langle\bar{s}s\rangle}{64\pi^4} - \frac{\langle g^2GG\rangle}{12\,288\pi^6}\right)s^2 + \left(-\frac{m_s^3\langle\bar{s}s\rangle}{32\pi^4} + \frac{3m_s^2\langle g^2GG\rangle}{4096\pi^6} + \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{64\pi^4} - \frac{\langle\bar{q}q\rangle^2}{24\pi^2} \right. \\ &\quad \left. - \frac{\langle\bar{s}s\rangle^2}{24\pi^2}\right)s + \frac{m_s^2\langle\bar{q}q\rangle^2}{4\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{48\pi^2} - \frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma Gq\rangle}{24\pi^2} - \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{512\pi^4} - \frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{24\pi^2} - \frac{m_s^4\langle g^2GG\rangle}{2048\pi^6}. \end{aligned} \tag{20}$$

In these equations, q represents a u or d quark, and s represents an s quark. $\langle\bar{q}q\rangle$ and $\langle\bar{s}s\rangle$ are dimension $D = 3$ quark condensates; $\langle g^2GG\rangle$ is a $D = 4$ gluon condensate; $\langle g\bar{q}\sigma Gq\rangle$ and $\langle g\bar{s}\sigma Gs\rangle$ are $D = 5$ mixed condensates. From these expressions, we observe the following:

(i) The coefficients of the lowest dimension, or of the leading term in powers of s, have the relations $c^{(4)}_{S6} = c^{(4)}_{P3}$ and $c^{(4)}_{A3} = 1/2c^{(4)}_{V6}$. These are the consequences of chiral symmetry at the perturbative level [33].

(ii) As empirically known, the terms of quark condensates have important contributions to the sum rule.

For numerical calculations, we use the following values of condensates [34–40]:

$$\begin{aligned} \langle\bar{q}q\rangle &= -(0.240\text{ GeV})^3, \\ \langle\bar{s}s\rangle &= -(0.8 \pm 0.1) \times (0.240\text{ GeV})^3, \\ \langle g_s^2GG\rangle &= (0.48 \pm 0.14)\text{ GeV}^4, \\ m_s(2\text{ GeV}) &= 0.11\text{ GeV}, \\ \langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle, \\ M_0^2 &= (0.8 \pm 0.2)\text{ GeV}^2. \end{aligned} \tag{21}$$

As usual we assume the vacuum saturation for higher dimensional operators such as $\langle 0|\bar{q}q\bar{q}q|0\rangle \sim \langle 0|\bar{q}q|0\rangle \times \langle 0|\bar{q}q|0\rangle$. There is a minus sign in the definition of the mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$, which is different with some other QCD sum rule calculation. This is just because the definition of coupling constant g_s is different [34,41].

In Fig. 2, we show all five Borel transformed correlation functions $\Pi(M_B^2)$ [the left-hand side (lhs) of Eq. (12)] as functions of Borel mass square for threshold value $s_0 = 3\ \mathrm{GeV}^2$. From the definition of (10), the lhs should be positive definite quantities. In practical calculations, however, the positivity may not be necessarily realized, if the OPE up to finite terms does not work due to insufficient convergence of the OPE. In the present analysis, we find that among the five cases, two functions of V_6 and P_6 currents show such a bad behavior. Therefore, the QCD sum rules for these two currents are not physically acceptable. The correlation functions of A_3 and S_6 change the sign from negative to positive values. But the sum rule values take positive values for $M_B^2 \sim$ several GeV^2.

The tetraquark currents S_6 and A_3 are constructed by diquark fields which correspond to $^1S_0$ and $^3S_1$ in the nonrelativistic language, where the two quarks can be in the ground state s-orbit. In contrast, the currents V_6 and P_6 correspond to linear combinations of $^3P_1$, and $^3P_0$, respectively, where one of the two quarks is in an excited p-orbit. The T_3 current is a linear combination of $^3S_1$ and $^1P_1$. Therefore, we verify an empirical fact that the sum rule constructed by currents having the s-wave components in the nonrelativistic limit works better than those dominated by p-wave components. For completeness, we show the lhs with numerical coefficients for the three better cases A_3, T_3, and S_6:

$$\begin{aligned}\Pi_{A3}^{(\mathrm{all})} &= 8.2\times 10^{-7}M_B^{10} - 7.4\times 10^{-8}M_B^8 + 1.6\\ &\quad\times 10^{-7}M_B^6 + 1.8\times 10^{-6}M_B^4 - 1.1\times 10^{-6}M_B^2,\\ \Pi_{T3}^{(\mathrm{all})} &= 4.8\times 10^{-6}M_B^{10} - 5.9\times 10^{-7}M_B^8 - 9.1\\ &\quad\times 10^{-7}M_B^6 + 3.4\times 10^{-8}M_B^4 + 2.4\times 10^{-7}M_B^2,\\ \Pi_{S6}^{(\mathrm{all})} &= 4.1\times 10^{-7}M_B^{10} - 2.5\times 10^{-8}M_B^8 + 5.1\\ &\quad\times 10^{-8}M_B^6 + 1.8\times 10^{-6}M_B^4 - 1.1\times 10^{-6}M_B^2.\end{aligned} \tag{22}$$

From these expressions, we observe that the convergence of the current T_3 seems better, while the convergence of the currents A_3 and S_6 is not very good in the region $1 < M_B^2 < 2\ \mathrm{GeV}^2$. They can only converge at $M_B^2 \sim 3\ \mathrm{GeV}^2$.

To determine the mass, we need to fix the two parameters: the threshold value s_0 and the Borel mass square M_B^2. For a good sum rule, the predicted masses should not depend on these two parameters strongly with sizable pole contribution (Borel window). In Fig. 3, we show the masses of the tetraquark as functions of the Borel mass for several threshold values s_0 (Borel curves). We observe that the Borel mass dependence is somewhat strong for the currents S_6 and A_3 in the region $1 < M_B^2 < 2\ \mathrm{GeV}^2$, which is expected to be a reasonable choice of the Borel mass. For these currents S_6 and A_3, however, we see that the minimum occurs at around $3\ \mathrm{GeV}^2$ when s_0 is varied in the region $M_B^2 \gtrsim 1.5\ \mathrm{GeV}^2$. (For the current S_6, the mass of $s_0 = 2\ \mathrm{GeV}^2$ is far above the region shown in the figure.) For this reason, we consider that $s_0 = 3\ \mathrm{GeV}^2$ is a reasonable choice which we will mainly use for the estimation of the mass of the tetraquark in the following sum rule analyses. At this s_0 value, the mass of the tetraquark turns out to be about 1.6 GeV. For the T_3 current, the Borel stability

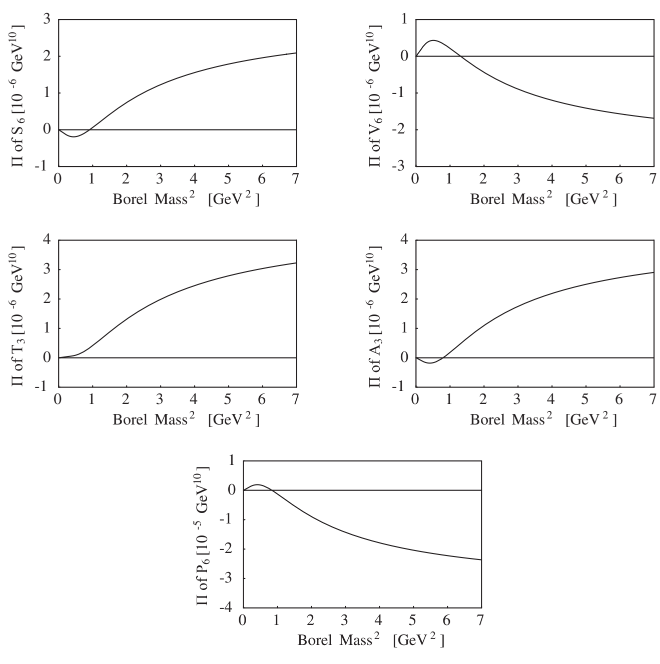

FIG. 2. Borel transformed correlation functions Π_{S6}, Π_{V6}, Π_{T3}, Π_{A3}, and Π_{P6} as functions of Borel mass square, in units of GeV^{10}, for threshold value $s_0 = 3\ \mathrm{GeV}^2$.

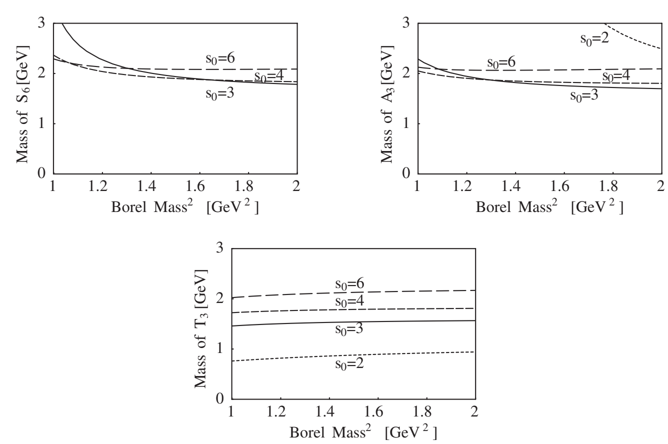

FIG. 3. Mass of the tetraquark calculated by the three currents S_6, A_3, and T_3, as functions of the Borel mass square M_B^2, for several threshold values $s_0 = 2$, 3, 4, and $6\ \mathrm{GeV}^2$.

TABLE I. Pole contributions of various currents. The threshold value $s_0 = 3\ \mathrm{GeV}^2$ is used.

	Diquark current			Mesonic current		Mixed current	
M_B^2	A_3	T_3	S_6	V_8	T_8	η_1	η_2
0.7 GeV^2	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.60	0.49
1 GeV^2	0.17	0.11	0.10	0.54	0.23	0.30	0.22
2 GeV^2	0.04	0.01	0.05	0.09	0.02	0.03	0.02

seems better. The result, however, depends on the threshold value s_0 to some extent. However, it is interesting to see that the mass of the tetraquark is about 1.6 GeV when $s_0 \sim 3\ \mathrm{GeV}^2$.

To see the amount of the pole contribution, we define the quantity

$$\text{pole contribution} \equiv \frac{\int_{4m_s^2}^{s_0} e^{-s/M_B^2}\rho(s)ds}{\int_{4m_s^2}^{\infty} e^{-s/M_B^2}\rho(s)ds}. \tag{23}$$

As shown in Table I, the pole contribution of the diquark currents A_3, T_3, and S_6 is not very large; at $M_B^2 \sim 1\ \mathrm{GeV}^2$ they are of order 10%. This is a general problem of the QCD sum rule when multiquark currents are used. Therefore the results so far might be doubtful.

From the analysis of the single current of the diquark construction, we expect that the mass of the tetraquark is about 1.6 GeV, although the stability against the variation of both the Borel mass and the threshold parameter is not simultaneously achieved. Furthermore, the pole contribution is rather small. As we will see, however, a suitable linear combination will improve them.

C. Analysis of single mesonic currents

In this subsection, we perform QCD sum rule analysis using the ten mesonic currents, $S_{1,8}$, $V_{1,8}$, $T_{1,8}$, $A_{1,8}$, and $P_{1,8}$, separately. Here we only show two important spectral densities:

$$\begin{aligned}\rho_{V8}(s) = {}& \frac{s^4}{110\,592\pi^6} - \frac{19{m_s}^2 s^3}{55\,296\pi^6} + \left(\frac{5{m_s}^4}{2304\pi^6} - \frac{m_s\langle\bar{q}q\rangle}{432\pi^4} + \frac{m_s\langle\bar{s}s\rangle}{432\pi^4} + \frac{17\langle g^2GG\rangle}{221\,184\pi^6}\right)s^2 \\ &+ \left(\frac{m_s^3\langle\bar{q}q\rangle}{72\pi^4} - \frac{5m_s^3\langle\bar{s}s\rangle}{288\pi^4} - \frac{13m_s^2\langle g^2GG\rangle}{24\,576\pi^6} + \frac{m_s\langle g\bar{q}Gq\rangle}{2304\pi^4} - \frac{5m_s\langle g\bar{s}Gs\rangle}{4608\pi^4} + \frac{\langle\bar{q}q\rangle^2}{432\pi^2} + \frac{\langle\bar{s}s\rangle^2}{432\pi^2} + \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{54\pi^2}\right)s \\ &+ \frac{m_s^2\langle\bar{q}q\rangle^2}{27\pi^2} + \frac{5m_s^2\langle\bar{s}s\rangle^2}{432\pi^2} - \frac{m_s\langle\bar{q}q\rangle\langle g^2GG\rangle}{6912\pi^4} + \frac{5\langle\bar{q}q\rangle\langle g\bar{q}Gq\rangle}{1728\pi^2} + \frac{m_s^3\langle g\bar{q}Gq\rangle}{144\pi^4} - \frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{18\pi^2} - \frac{\langle g\bar{q}Gq\rangle\langle\bar{s}s\rangle}{864\pi^2} \\ &+ \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{1024\pi^4} - \frac{\langle\bar{q}q\rangle\langle g\bar{s}Gs\rangle}{864\pi^2} + \frac{5\langle\bar{s}s\rangle\langle g\bar{s}Gs\rangle}{1728\pi^2} - \frac{m_s^4\langle g^2GG\rangle}{9216\pi^6},\end{aligned} \tag{24}$$

$$\begin{aligned}\rho_{T8}(s) = {}& \frac{s^4}{18\,432\pi^6} - \frac{5{m_s}^2 s^3}{2304\pi^6} + \left(\frac{5{m_s}^4}{384\pi^6} + \frac{5m_s\langle\bar{s}s\rangle}{288\pi^4} + \frac{31\langle g^2GG\rangle}{55\,296\pi^6}\right)s^2 + \left(-\frac{5m_s^3\langle\bar{s}s\rangle}{48\pi^4} - \frac{31m_s^2\langle g^2GG\rangle}{9216\pi^6}\right)s + \frac{5m_s^2\langle\bar{q}q\rangle^2}{18\pi^2} \\ &+ \frac{5m_s^2\langle\bar{s}s\rangle^2}{72\pi^2} + \frac{31m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{6912\pi^4} - \frac{13m_s^4\langle g^2GG\rangle}{9216\pi^6}.\end{aligned} \tag{25}$$

As shown in Fig. 4, we find that, among the ten correlation functions, only two correlation functions for the currents V_8 and T_8 show good behavior with having positive values.

The currents V_1, V_8, P_1, and P_8 are constructed by mesonic fields (either color singlet or color octet) which correspond to ${}^3S_1$ and ${}^1S_0$ in the nonrelativistic language, where two quark-antiquark pairs can be in the ground state s-orbit. Their spectral densities then show similar behavior to S_6 and A_3 in the previous subsection. In contrast, S_1, S_8, A_1, and A_8 correspond to linear combinations of ${}^3P_0$ and ${}^3P_1$, respectively; T_1 and T_8 currents are the combinations of ${}^3S_1$ and ${}^1P_1$.

From the above argument, we might expect that six currents, V_1, V_8, P_1, P_8, T_1, and T_8 would work. However, we found that the Borel transformed correlation functions calculated by the currents V_1, P_1, P_8, and T_1 take negative values and therefore, they must be abandoned. Now there remain only two better currents V_8 and T_8 in the mesonic construction. This is the reason why we have shown their spectral densities in (24) and (25). Using the numerical values of various condensates (21), we find the Borel transformed correlation functions

$$\begin{aligned}\Pi_{V8}^{(\mathrm{all})} &= 2.3\times10^{-7}M_B^{10} - 2.6\times10^{-8}M_B^8 + 9.1 \\ &\quad\times 10^{-8}M_B^6 + 3.5\times10^{-7}M_B^4 - 4.9\times10^{-8}M_B^2, \\ \Pi_{T8}^{(\mathrm{all})} &= 1.4\times10^{-6}M_B^{10} - 1.7\times10^{-7}M_B^8 + 1.2 \\ &\quad\times 10^{-7}M_B^6 - 4.3\times10^{-9}M_B^4 + 4.9\times10^{-8}M_B^2.\end{aligned} \tag{26}$$

From these equations, we find that better convergence is achieved for T_8 than for V_8 in the region $1 \lesssim M_B^2 \lesssim 2\ \mathrm{GeV}^2$. The pole contributions are significantly improved as shown in Table I.

In Fig. 5, we show the masses of the tetraquark currents V_8 and T_8 as functions of the Borel mass for several

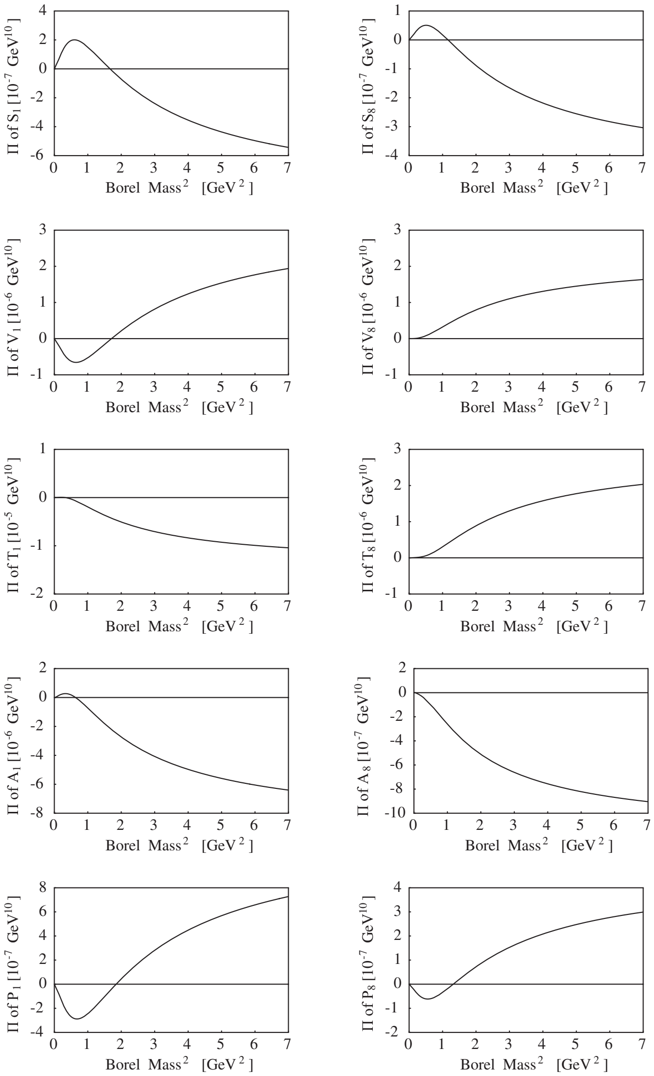

FIG. 4. Borel transformed correlation functions Π_{S1}, Π_{S8}, Π_{V1}, Π_{V8}, Π_{T1}, Π_{T8}, Π_{A1}, Π_{A8}, Π_{P1}, and Π_{P8} as functions of Borel mass square, in units of GeV^{10}, for threshold value $s_0 = 3\ \mathrm{GeV}^2$.

threshold values s_0 (Borel curves). As in the case of T_3 current, the Borel stability seems good but the result depends on the threshold value s_0. However, once again, if we take the threshold value at $s_0 \sim 3\ \mathrm{GeV}^2$, the mass of the tetraquark turns out to be reasonable, though the precise

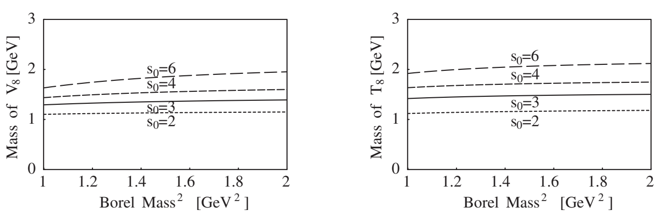

FIG. 5. Mass of the tetraquark calculated by the currents V_8 (left) and T_8 (right), as functions of the Borel mass square M_B^2, for several threshold values $s_0 = 2$, 3, 4, and 6 GeV^2.

values are slightly smaller: the mass of $T_8 \sim 1.5$ GeV and the mass of $V_8 \sim 1.4$ GeV.

D. Analysis of mixed currents

In order to improve the sum rule, we attempt to make linear combinations of independent currents for both diquark and mesonic currents. Since linear combinations of five currents contain ten mixing angles, the full consideration with these ten parameters is rather cumbersome. Instead, we make a linear combination of two currents J_1 and J_2 (any two from the independent currents), $\eta = \cos\theta J_1 + \sin\theta J_2$, where θ is a mixing angle. Then the correlation functions are written as

$$\langle \eta\eta^\dagger \rangle = \cos^2\theta \langle J_1 J_1^\dagger \rangle + \sin^2\theta \langle J_2 J_2^\dagger \rangle + \cos\theta \sin\theta \langle J_1 J_2^\dagger \rangle + \cos\theta \sin\theta \langle J_2 J_1^\dagger \rangle. \quad (27)$$

The mixing is chosen with the following requirements:

(1) The OPE has a good convergence as going to terms of higher dimensional operators.

(2) The spectral density becomes positive for all (or almost all) s values, and then $\Pi(M_B^2)$ becomes positive for all Borel mass and threshold values.

(3) Pole contribution is sufficiently large.

We have tried various combinations of two currents to realize good sum rules. While doing so, we have realized that the diquark currents are more independent than the mesonic currents. This means that the cross terms of (27) have only a minor contribution for diquark currents, while they have a large contribution for mesonic currents.

According to the requirement (1), we would like to make a linear combination such that the highest dimensional (eight) term is suppressed. For diquark currents, we find it convenient to take two combinations:

$$\eta = \cos\theta A_3 + \sin\theta V_6, \quad (28)$$

$$\xi = \cos\theta P_6 + \sin\theta S_6. \quad (29)$$

By choosing $\cot\theta \sim \sqrt{2}$, we find that the term of dimension eight of (28) is suppressed, while for $\cot\theta \sim 1$, the term of dimension eight of (29) is suppressed. The Borel transformed correlation function of (29) $\Pi_\xi(M_B^2)$, however, takes negative values. Therefore, this current should be rejected for the sum rule analysis. In this way we are led to the current η of (28). From now on, we will denote $\eta \rightarrow \eta_1$.

For the mesonic case, it turns out that the cross term contributions are large. Accordingly, we attempt a complex angle to improve the sum rule analysis. By choosing $t_1 = 0.91$, $t_2 = -0.41$, we construct a current:

$$\eta_2 = S_1 + (t_1 + \mathbf{i}t_2)P_1. \quad (30)$$

The numerical Borel transformed correlation functions are

$$\Pi_1^{(\mathrm{all})} = 1.1 \times 10^{-6} M_B^{10} - 1.3 \times 10^{-7} M_B^8 + 4.8 \times 10^{-7} M_B^6 - 2.0 \times 10^{-8} M_B^4 + 5.2 \times 10^{-9} M_B^2,$$
$$\Pi_2^{(\mathrm{all})} = 5.0 \times 10^{-7} M_B^{10} - 6.0 \times 10^{-8} M_B^8 + 8.4 \times 10^{-8} M_B^6 - 2.2 \times 10^{-8} M_B^4 + 8.3 \times 10^{-9} M_B^2, \quad (31)$$

which may be compared with the previous results of (22) and (26). Here the convergence of the series is improved significantly. Therefore, we can choose a smaller Borel mass square down to $M_B^2 \gtrsim 0.7\ \mathrm{GeV}^2$, where the pole contribution will be further increased up to around 50%, and the convergence is still maintained.

In Fig. 6, we show the mass calculated from η_1 and η_2 as functions of the Borel mass square for several threshold values s_0. The Borel stability is improved from the cases of the single currents.

In these figures, we may think that there is still a big threshold value s_0 dependence. However, this dependence will be largely reduced if we choose a small Borel mass, where the pole contribution is large enough. In Fig. 7, we show the mass calculated from η_1 and η_2 as functions of the threshold value for several Borel mass. When $M_B^2 = 0.7\ \mathrm{GeV}^2$, the curve is very stable. Moreover, the pole contribution is around 50%, and the convergence is still maintained. Therefore, we obtain a very good sum rule, where we find the mass calculated from the two currents η_1 and η_2 is about 1.5 GeV. As the Borel mass increases, the pole contribution decreases, therefore, the threshold dependence becomes bigger.

Finally, in order to summarize our analysis, we show in Fig. 8 masses of the tetraquark calculated by several rea-

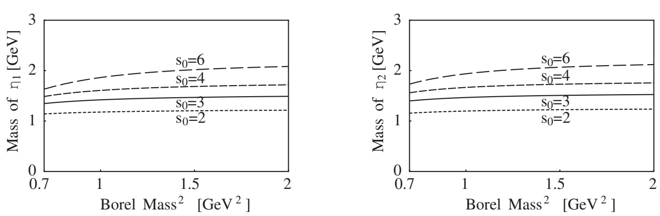

FIG. 6. Mass of the tetraquark calculated by the mixed currents η_1 (left) and η_2 (right), as functions of the Borel mass square M_B^2 for several threshold values $s_0 = 2$, 3, 4, and 6 GeV^2.

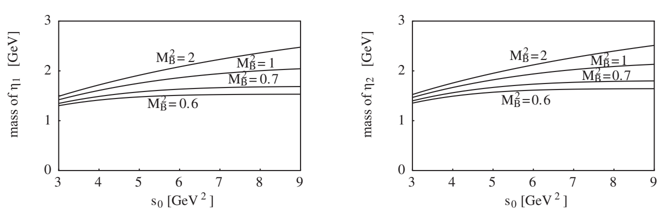

FIG. 7. Mass of the tetraquark calculated by the mixed currents η_1 (left) and η_2 (right), as functions of the threshold value s_0 for several Borel mass square $M_B^2 = 0.6$, 0.7, 1, and 2 GeV^2.

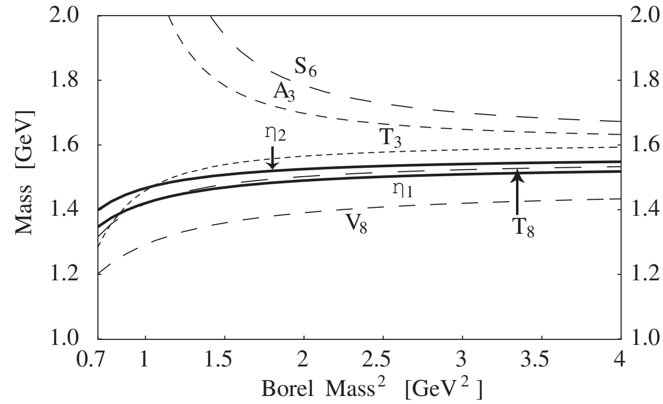

FIG. 8. Mass of the tetraquark calculated by the currents η_1, η_2, A_3, S_6, T_3, V_8, and T_8, as functions of the Borel mass square M_B^2 in the region $0.7 < M_B^2 < 4\ \mathrm{GeV}^2$, for threshold value $s_0 = 3\ \mathrm{GeV}^2$.

sonable currents used in the present study as functions of the Borel mass square at $s_0 = 3\ \mathrm{GeV}^2$. They are S_6, A_3, and T_3 for the diquark construction, T_8 and V_8 for the mesonic construction, and η_1 and η_2 for the mixed currents. The plots are extended to a wider region of M_B^2 up to 4 GeV^2, where the masses predicted by different currents tend to a same value. We verify once again a good Borel mass stability for the mixed currents, while some of the single currents show good stability also (T_3, T_8, and V_8). The mass values varies slightly, while we expect the mass of the tetraquark around 1.5 GeV.

IV. SUMMARY

We have presented a QCD sum rule study of the $ud\bar{s}\bar{s}$ tetraquark of $J^P = 0^+$ and $I = 1$, both in the diquark $[(\bar{q}\bar{q})(qq)]$ and mesonic $[(\bar{q}q)(\bar{q}q)]$ constructions. We have found that in this channel of tetraquark, there are five independent currents, which is shown both in the diquark and mesonic constructions. For each single current, we have tested the sum rule analysis, but it is found that not all of them provide a good stability.

As an attempt to improve the stability of the sum rule, we have considered linear combinations of independent currents. In order to simplify the analysis, we took a superposition of various combinations of two currents. Among them, we have found two cases that lead to good sum rules, where we investigated s_0 (threshold value) and M_B (Borel mass) dependence, and convergence of OPE. A good Borel stability is achieved in the region $0.7 \lesssim M_B^2 \lesssim 4\ \mathrm{GeV}^2$. In order to obtain a large enough pole contribution (50%) and reduce the threshold value dependence, we have to reduce the Borel mass. However, to maintain the convergence of OPE, we cannot reduce it too largely. When Borel mass square M_B^2 is around 0.7 GeV^2, we get a very good QCD sum rule, where the mass of the tetraquark turns out to be around 1.5 GeV.

Despite the seemingly good Borel mass stability, we think that we should investigate the following points more carefully. For instance, estimation of higher dimensional terms of $\mathcal{O}(1/s)$ could be important. Although we are able to construct the two mixed currents such that the higher order contributions (in the present calculation of OPE) of dimension six and eight terms are suppressed, the question still remains concerning even higher order contributions. Another question is the contribution of KK scattering states, since the mass of the tetraquark is around 1.5 GeV, and it can fall apart into the KK states. Such a contribution can be estimated by using the method proposed in Refs. [42,43]. These will be further investigated in the future work.

ACKNOWLEDGMENTS

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APPENDIX A: FIVE INDEPENDENT CURRENTS IN $(\bar{q}q)(\bar{q}q)$ BASIS

We attempt to write a diquark current of (5) as a sum of $(\bar{q}q)$ mesonic pairs ($q = u, d, s$),

$$L_{\mu\nu\rho\sigma} \times \bar{s}_a^\mu \bar{s}_b^\nu u_c^\rho d_d^\sigma = \sum_{n,i,j} C_i^1 C_j^2 (\bar{s}_b \lambda^n \Gamma_i d_d)(\bar{s}_a \lambda^n \Gamma_j u_c), \tag{A1}$$

where Γ_i are the five Dirac matrices and λ^n ($n = 1, \ldots, 8$) are color matrices forming color-singlet and octet states out of $\mathbf{3} \times \bar{\mathbf{3}}$. Therefore, in (A1), the sum runs over ten terms of five Γ_i matrices and two λ^n combinations. They are

$$\begin{aligned} &S_1 = (\bar{s}_a u_a)(\bar{s}_b d_b), \quad S_8 = (\bar{s}_a \lambda^n_{ab} u_b)(\bar{s}_c \lambda^n_{cd} d_d), \quad V_1 = (\bar{s}_a \gamma_\mu u_a)(\bar{s}_b \gamma^\mu d_b), \quad V_8 = (\bar{s}_a \gamma_\mu \lambda^n_{ab} u_b)(\bar{s}_c \gamma^\mu \lambda^n_{cd} d_d), \\ &T_1 = (\bar{s}_a \sigma_{\mu\nu} u_a)(\bar{s}_b \sigma^{\mu\nu} d_b), \quad T_8 = (\bar{s}_a \sigma_{\mu\nu} \lambda^n_{ab} u_b)(\bar{s}_c \sigma^{\mu\nu} \lambda^n_{cd} d_d), \quad A_1 = (\bar{s}_a \gamma_\mu \gamma_5 u_a)(\bar{s}_b \gamma^\mu \gamma_5 d_b), \\ &A_8 = (\bar{s}_a \gamma_\mu \gamma_5 \lambda^n_{ab} u_b)(\bar{s}_c \gamma^\mu \gamma_5 \lambda^n_{cd} d_d), \quad P_1 = (\bar{s}_a \gamma_5 u_a)(\bar{s}_b \gamma_5 d_b), \quad P_8 = (\bar{s}_a \gamma_5 \lambda^n_{ab} u_b)(\bar{s}_c \gamma_5 \lambda^n_{cd} d_d), \end{aligned} \tag{A2}$$

where in the octet representation inner product of λ^n ($n = 1, \ldots, 8$) is taken. The quark-antiquark pairs in different currents have different properties:

$$\begin{aligned} &S_1: (J^P = 0^+, 8_f, 1_c), \quad S_8: (J^P = 0^+, 8_f, 8_c), \quad V_1: (J^P = 1^-, 8_f, 1_c), \quad V_8: (J^P = 1^-, 8_f, 8_c), \\ &T_1: (J^P = 1^+ \& 1^-, 8_f, 1_c), \quad T_8: (J^P = 1^+ \& 1^-, 8_f, 8_c), \quad A_1: (J^P = 1^+, 8_f, 1_c), \quad A_8: (J^P = 1^+, 8_f, 8_c), \\ &P_1: (J^P = 0^-, 8_f, 1_c), \quad P_8: (J^P = 0^-, 8_f, 8_c). \end{aligned}$$

In order to establish the five independent currents, first we change their color structures:

$$\begin{aligned} (\bar{s}_a u_b)(\bar{s}_b d_a) &= \frac{1}{3}(\bar{s}_a u_a)(\bar{s}_b d_b) + \frac{1}{2}(\bar{s}_a u_b)(\bar{s}_c d_d)\lambda_{ab}\lambda_{cd}, \\ (\bar{s}_a u_d)(\bar{s}_c d_b)\lambda_{ab}\lambda_{cd} &= \frac{16}{9}(\bar{s}_a u_a)(\bar{s}_b d_b) - \frac{1}{3}(\bar{s}_a u_b)(\bar{s}_c d_d)\lambda_{ab}\lambda_{cd}. \end{aligned} \tag{A3}$$

Then we use the Fierz transformation [44]:

$$\begin{aligned} &\tfrac{1}{3}(\bar{s}_a u_a)(\bar{s}_b d_b) + \tfrac{1}{2}(\bar{s}_a u_b)(\bar{s}_c d_d)\lambda_{ab}\lambda_{cd} \\ &= (\bar{s}_a u_b)(\bar{s}_b d_a) \\ &= -\tfrac{1}{4}\{(\bar{s}_a u_a)(\bar{s}_b d_b) + (\bar{s}_a \gamma_\mu u_a)(\bar{s}_b \gamma^\mu d_b) \\ &\quad + \tfrac{1}{2}(\bar{s}_a \sigma_{\mu\nu} u_a)(\bar{s}_b \sigma^{\mu\nu} d_b) - (\bar{s}_a \gamma_\mu \gamma_5 u_a)(\bar{s}_b \gamma^\mu \gamma_5 d_b) \\ &\quad + (\bar{s}_a \gamma_5 u_a)(\bar{s}_b \gamma_5 d_b)\}. \end{aligned} \tag{A4}$$

We obtain 10 equations in all:

$$\begin{aligned}
\frac{1}{3}S_1+\frac{1}{2}S_8&=-\frac{1}{4}\left\{S_1+V_1+\frac{1}{2}T_1-A_1+P_1\right\},\\
\frac{16}{9}S_1-\frac{1}{3}S_8&=-\frac{1}{4}\left\{S_8+V_8+\frac{1}{2}T_8-A_8+P_8\right\},\\
\frac{1}{3}V_1+\frac{1}{2}V_8&=-\frac{1}{4}\{4S_1-2V_1-2A_1-4P_1\},\\
\frac{16}{9}V_1-\frac{1}{3}V_8&=-\frac{1}{4}\{4S_8-2V_8-2A_8-4P_8\},\\
\frac{1}{3}T_1+\frac{1}{2}T_8&=-\frac{1}{4}\{12S_1-2T_1+12P_1\},\\
\frac{16}{9}T_1-\frac{1}{3}T_8&=-\frac{1}{4}\{12S_8-2T_8+12P_8\},\\
\frac{1}{3}A_1+\frac{1}{2}A_8&=-\frac{1}{4}\{-4S_1-2V_1-2A_1+4P_1\},\\
\frac{16}{9}A_1-\frac{1}{3}A_8&=-\frac{1}{4}\{-4S_8-2V_8-2A_8+4P_8\},\\
\frac{1}{3}P_1+\frac{1}{2}P_8&=-\frac{1}{4}\left\{S_1-V_1+\frac{1}{2}T_1+A_1+P_1\right\},\\
\frac{16}{9}P_1-\frac{1}{3}P_8&=-\frac{1}{4}\left\{S_8-V_8+\frac{1}{2}T_8+A_8+P_8\right\}.
\end{aligned}\tag{A5}$$

Solving these linear equations, we find that there are five independent currents. In other words, the rank of the 10×10 coefficient matrix is five. Any five currents among (A1) are independent and can be expressed by the other five currents. For instance, we have the relations as

$$\begin{aligned}
S_8&=-\tfrac{7}{6}S_1-\tfrac{1}{2}V_1-\tfrac{1}{4}T_1+\tfrac{1}{2}A_1-\tfrac{1}{2}P_1,\\
V_8&=-2S_1+\tfrac{1}{3}V_1+A_1+2P_1,\\
T_8&=-6S_1+\tfrac{1}{3}T_1-6P_1,\\
A_8&=2S_1+V_1+\tfrac{1}{3}A_1-2P_1,\\
P_8&=-\tfrac{1}{2}S_1+\tfrac{1}{2}V_1-\tfrac{1}{4}T_1-\tfrac{1}{2}A_1-\tfrac{7}{6}P_1.
\end{aligned}\tag{A6}$$

Finally, we establish the relations between the diquark currents and the mesonic currents. For instance, we can verify the relations

$$\begin{aligned}
S_6&=-\tfrac{1}{4}S_1-\tfrac{1}{4}V_1+\tfrac{1}{8}T_1-\tfrac{1}{4}A_1-\tfrac{1}{4}P_1,\\
V_6&=S_1-\tfrac{1}{2}V_1+\tfrac{1}{2}A_1-P_1,\\
T_3&=3S_1+\tfrac{1}{2}T_1+3P_1,\\
A_3&=S_1+\tfrac{1}{2}V_1-\tfrac{1}{2}A_1-P_1,\\
P_6&=-\tfrac{1}{4}S_1+\tfrac{1}{4}V_1+\tfrac{1}{8}T_1+\tfrac{1}{4}A_1-\tfrac{1}{4}P_1.
\end{aligned}\tag{A7}$$

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