Dynamical realization of democratic Yukawa matrices and alignment of *A* terms

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We study realization of the democratic form of Yukawa matrices by infrared fixed points. We investigate renormalization-group flows of Yukawa couplings in models with a single Yukawa matrix for three families, and up and down-sector Yukawa matrices. It is found that each model has its certain pattern of renormalization-group flows of Yukawa matrices. We apply them to the charged lepton sector and quark sector, and examine in which situation our class of models can lead to realistic results for the mass ratios and mixing angles between the second and third families. We also study corresponding *A*-terms. The *A*-terms approach toward the universal form with no physical *CP*-violating phase. Thus, constraints due to various neutral flavor changing processes except for $\mu \rightarrow e\gamma$ are found to be satisfied by this dynamics. In order to suppress the electric dipole moments as well as $\mu \rightarrow e\gamma$ sufficiently, more alignment of the *A*-terms with some reason is required.

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I. INTRODUCTION

Understanding the origin of the hierarchical structure of quark/lepton masses and their mixing angles is one of many important issues in particle physics. Actually, several types of scenarios have been proposed so far. Among them, the democratic Ansatz on Yukawa matrices is an interesting approach [1–4], in particular, for the lepton sector.

The exactly democratic form of the Yukawa matrix can be obtained by a certain flavor symmetry, e.g. S_3 symmetry. However, the exact one by itself is not realistic, because it is a rank-one matrix. Thus, a certain pattern of small symmetry breaking terms are usually added by hand for the purpose to realize realistic Yukawa matrices. However, it seems that any comprehensive explanation leading to such pattern of symmetry breaking terms has not been given.

Recently, two of the authors have considered dynamical realization of the almost democratic Yukawa matrices [5],[1] that is, each entry of the Yukawa matrix has the same infrared (IR) fixed point and the almost democratic Yukawa matrix can be realized by dynamics at the IR region whatever their initial values are. Actually, these fixed points correspond to the so-called Pendelton-Ross fixed point [7]. The exact IR fixed point is not realistic, because we just obtain the exact rank-one Yukawa matrix. Some region close to the IR fixed point would be interesting, and deviations from the fixed point correspond to small breaking parameters in the flavor symmetry approach to the democratic Ansatz.

Thus, one of our purposes in this paper is to study more about renormalization-group (RG) flows of Yukawa couplings in models with the above IR fixed points leading to the democratic Yukawa matrices. We shall show that we obtain certain patterns of RG-flows of Yukawa matrices for models with a single Yukawa matrix and models with the up and down sectors of Yukawa matrices as well. We also show with which conditions realistic results for the second and third families, e.g. $(V_{MNS})_{23} \sim 2/\sqrt{6}$, $m_s/m_b \sim V_{cb} \sim m_\mu/m_\tau$ and $m_c/m_t \sim (m_s/m_b)^{3/2}$, are obtained, and which type of fine-tuning for initial conditions is required. In our models, the mass hierarchy between the first and the second families or their mixing angles are not obtained with natural initial conditions. It is beyond our present scope to explain the mass matrices of the first and the second families.

Supersymmetric extension is attractive as new physics beyond the standard model. Within the framework of supersymmetric standard models, a flavor mechanism, which derives realistic quark/lepton masses and their mixing angles, would affect somehow their superpartners, that is, soft supersymmetry (SUSY) breaking squark/slepton masses and trilinear couplings, the so-called *A*-terms. Although superpartners have not been detected yet, patterns of squark/slepton mass matrices and *A*-term matrices are strongly constrained by experiments on flavor changing neutral current (FCNC) processes [8].[2] That is, FCNCs require almost degenerate *A*-terms and soft scalar masses. In particular, the constraints due to the $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow e\gamma$ decay are severe. In addition, *CP*-violating phases of *A*-terms are also strongly constrained by experiments on electric dipole moments (EDMs). These are the SUSY flavor and *CP* problems. A solution for these problems is to realize the universal soft scalar masses and the universal *A*-terms, whose phases are aligned with the phase

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[1]See also [6].

[2]See also Ref. [9] and references therein.

of gaugino masses, e.g. the gluino mass. In this case, the A-term has no physical CP-violating phase. Thus, it is quite important to examine these constraints on any SUSY flavor mechanism.

So far various flavor symmetries have been also applied to suppress the flavor nonuniversality in SUSY breaking parameters, while the mass matrices are also explained by the same symmetries. As for the democratic mass matrices, the S_3 symmetry also constrains structure of the SUSY breaking parameters [10]. However, the FCNC or the CP problem is not solved by the symmetry alone. Besides, the breaking effects of the flavor symmetry on the SUSY breaking parameters are unclear.

It is known that when a model with one flavor has a Pendelton-Ross fixed point, the corresponding A-term A as well as the sum of soft scalar masses squared also has an IR fixed point like $A = -M$, where M is the gaugino mass [11,12]. It is expected that our model with three flavors has a similar fixed point, and A-terms are aligned dynamically at the IR region [13]. Indeed, we shall show IR fixed points of A-term matrices are universal $A_{ij} = -M$. That is a favorable aspect, but the exact IR fixed point is not realistic, because we just obtain the rank-one Yukawa matrix on the exact IR fixed point as said above. Some region close to the fixed point would be interesting. Hence, in this paper we study RG-flows of Yukawa couplings and A-terms. We evaluate how much A-term matrices differ from the universal form, when Yukawa matrices are somehow realistic.

This paper is organized as follows. In Sec. II, we review briefly the Pendelton-Ross fixed point of Yukawa coupling and the corresponding A-term in a SUSY one-flavor model. In Sec. III, we review briefly our three-flavor model, where all entries of a Yukawa matrix have fixed points. We study fixed points of the corresponding A-terms, A_{ij}. In Sec. IV, we study RG-flow of Yukawa couplings such as to obtain a certain pattern of the Yukawa matrix by natural initial conditions. We apply our results to the charged lepton sector. We also analyze RG-flows of A-terms and examine their nonuniversality. In Sec. V, we study RG-flows of up and down sector Yukawa matrices. We use $SU(5)' \times SU(5)''$ model as a concrete model. We show that a certain pattern of up and down sector Yukawa matrices are obtained. We also discuss RG-flows of A-terms and consider FCNC and CP-violation constraints. In Sec. VI, we comment on degeneracy of sfermion masses. Section VII is devoted to conclusion and discussion. In the appendix, we give examples with additional couplings, where values of fixed points are shifted significantly.

II. IR FIXED POINT IN ONE-FLAVOR MODEL

A. IR fixed point of Yukawa coupling

First, we briefly review on the Pendelton-Ross IR fixed point in the model with a simple gauge group and a single Yukawa coupling, which corresponds to the superpotential,

$$W = y\Phi_1\Phi_2\Phi_3, \tag{1}$$

where Φ_i's are chiral superfields. The one-loop RG equations for the gauge coupling g and the Yukawa coupling y are obtained as

$$\mu\frac{d\alpha_g}{d\mu} = -b\alpha_g^2, \tag{2}$$

$$\mu\frac{d\alpha_y}{d\mu} = (a\alpha_y - c\alpha_g)\alpha_y, \tag{3}$$

where $\alpha_g \equiv g^2/(8\pi^2)$, $\alpha_y \equiv y^2/(8\pi^2)$, b is the one-loop beta function coefficient of the gauge coupling, and a and c are group-theoretical constants and both are always positive. Now, let us consider the RG equation of the ratio $x = \alpha_y/\alpha_g$,

$$\mu\frac{dx}{d\mu} = [ax - (c-b)]\alpha_g x. \tag{4}$$

This equation has a nontrivial fixed point at

$$x^* = \frac{c-b}{a}. \tag{5}$$

The condition $c - b > 0$ should be satisfied such that this fixed point is physical, i.e. $x > 0$.

Actually, we can solve this equation exactly, and show this fixed point is IR attractive. However, analysis on linear perturbation around the fixed point $x = x^* + \Delta x$ would be useful for discussions in the following sections. Here we study the RG equation for Δx, which satisfies

$$\mu\frac{d\Delta x}{d\mu} = (c-b)\alpha_g\Delta x. \tag{6}$$

Thus, this fixed point is IR attractive when $c - b > 0$. Actually, its solution is obtained as

$$\frac{\Delta x(\mu)}{\Delta x(\Lambda)} = \left(\frac{\alpha_g(\mu)}{\alpha_g(\Lambda)}\right)^{(b-c)/b} = \left(\frac{\alpha_g(\mu)}{\alpha_g(\Lambda)}\right)^{-ax^*/b}. \tag{7}$$

Large values of α_g and $(c - b)$ lead to stronger convergence toward the IR fixed point. Asymptotically nonfree theories, i.e. $b < 0$, are favorable to realize strong convergence.

B. Infrared fixed point of A-term

Next, we review on fixed points of A-terms. In general, softly broken SUSY models have trilinear couplings of scalar components, that is, the so-called A-terms. In the above model, we would have the soft SUSY breaking term,

$$h\phi_1\phi_2\phi_3, \tag{8}$$

where ϕ_i are scalar components of chiral superfields Φ_i. Here and hereafter, we use the notation of A-term $A = h/y$.

Now let us consider the RG equations of the gaugino mass M and the A-term. It is straightforward to obtain those RG equations in a simple model. However, it is convenient to use the spurion formalism to obtain RG equations of soft SUSY breaking terms in complicated models, which we shall discuss in the following sections. Such procedure is as follows. We replace α_g and α_y as [14]

$$\alpha_g \to \tilde{\alpha}_g = \alpha_g(1 + M\theta^2 + \bar{M}\bar{\theta}^2) + \cdots, \tag{9}$$

$$\alpha_y \to \tilde{\alpha}_y = \alpha_y(1 - A\theta^2 - \bar{A}\bar{\theta}^2) + \cdots, \tag{10}$$

where θ is the Grassmann coordinate of the superspace and the ellipsis denotes terms with $\theta^2\bar{\theta}^2$, which are irrelevant to the RG equation of the A-term, but relevant to soft scalar masses. The couplings, $\tilde{\alpha}_g$ and $\tilde{\alpha}_y$, satisfy the same RG equations as those of α_g and α_y. That is, the RG equations of the gaugino mass M and the A-term A corresponding to Eqs. (2) and (3) are written as

$$\mu\frac{dM}{d\mu} = -b\alpha_g M, \tag{11}$$

$$\mu\frac{dA}{d\mu} = a\alpha_y A + cM\alpha_g, \tag{12}$$

where a, b, and c are the same constants as Eqs. (2) and (3). We can find that $\Delta A = (A + M)$ has a fixed point in these equations. Actually, it satisfies the RG equation,

$$\mu\frac{d\Delta A}{d\mu} = -a\alpha_g M\Delta x + ax^*\alpha_g\Delta A. \tag{13}$$

At $x = x^*$, it reduces to

$$\mu\frac{d\Delta A}{d\mu} = ax^*\alpha_g\Delta A = (c - b)\alpha_g\Delta A. \tag{14}$$

Note that the constant a is always positive. Thus, when the Yukawa coupling has the Pendelton-Ross IR fixed point, the corresponding A-term always has the fixed point,

$$A = -M. \tag{15}$$

The deviation from the fixed point ΔA decreases in the same way as Δx satisfying Eq. (6). Note that the gaugino mass M and the A-term A are complex parameters. Thus, the above relation, $A = -M$, is realized including their CP phases, that is, the CP phase of the A-term is aligned with one of the gaugino mass at the IR fixed point.

III. INFRARED FIXED POINTS IN THREE-FLAVOR MODEL

A. Democratic fixed point

In this section, we briefly review on the model with three flavors [5], in which the democratic form of the Yukawa matrix is realized dynamically by IR fixed point.

We consider the SUSY model including three flavors of matter superfields F_i and f_i $(i = 1, 2, 3)$ and nine Higgs fields H_{ij} with their superpotential,

$$W = \sum_{i,j} y_{ij}F_i f_j H_{ij}. \tag{16}$$

Anomalous dimensions of these fields, γ_{F_i}, γ_{f_j}, and $\gamma_{H_{ij}}$, are written as

$$\gamma_{F_i} = a_F\sum_{k=1}^{3}\alpha_{y_{ik}} - c_F\alpha_g, \tag{17}$$

$$\gamma_{f_j} = a_f\sum_{k=1}^{3}\alpha_{y_{kj}} - c_f\alpha_g, \tag{18}$$

$$\gamma_{H_{ij}} = 3a_H\alpha_{y_{ij}} - c_H\alpha_g, \tag{19}$$

where $\alpha_{y_{ij}} = |y_{ij}|^2/(8\pi^2)$, $a_{F,f,H}$ are positive constants and $c_{F,f,H}$ are obtained as $c_{F,f,H} = 2C_2(R_{F,f,H})$ by the corresponding quadratic Casimir $C_2(R_{F,f,H})$. The RG equations of $\alpha_{y_{ij}}$ are given as

$$\mu\frac{d\alpha_{y_{ij}}}{d\mu} = (\gamma_{F_i} + \gamma_{f_j} + \gamma_{H_{ij}})\alpha_{y_{ij}}. \tag{20}$$

As the previous section, we define $x_{ij} = \alpha_{y_{ij}}/\alpha_g$. Then, it is found that the RG equations of x_{ij} have the fixed point,

$$x_{ij} = x^* = \frac{c - b}{3a}, \tag{21}$$

where $a = a_F + a_f + a_H$. To see whether this fixed point is IR attractive, we consider the linear perturbation around the fixed point, $x_{ij} = x^* + \Delta x_{ij}$. The RG equations of Δx_{ij} are obtained as

$$\mu\frac{d}{d\mu}\Delta x_{ij} = 3a_H\alpha_g x^*\Delta x_{ij} + \sum_k a_F\alpha_g x^*\Delta x_{ik} + \sum_k a_f\alpha_g x^*\Delta x_{kj}. \tag{22}$$

Alternatively, we can write more explicitly

$$\mu\frac{d}{d\mu}\begin{pmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{pmatrix} = \alpha_g x^*\begin{pmatrix}A & E & E\\ E & A & E\\ E & E & A\end{pmatrix}\begin{pmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{pmatrix}, \tag{23}$$

where $\Delta_i = (\Delta x_{i1}, \Delta x_{i2}, \Delta x_{i3})^T$ and

$$A = \begin{pmatrix}a' & a_F & a_F\\ a_F & a' & a_F\\ a_F & a_F & a'\end{pmatrix}, \qquad E = \begin{pmatrix}a_f & 0 & 0\\ 0 & a_f & 0\\ 0 & 0 & a_f\end{pmatrix}, \tag{24}$$

where $a' \equiv a_F + a_f + 3a_H$. This (9×9) matrix has the following eigenvalues,

$$\begin{gathered}3a_H, \quad 3a_H, \quad 3(a_F + a_H), \quad 3a_H, \quad 3a_H,\\ 3(a_F + a_H), \quad 3(a_f + a_H), \quad 3(a_f + a_H), \quad 3a.\end{gathered} \tag{25}$$

All of them are positive and it is found that the fixed point $x_{ij} = x^*$ is IR attractive. Hence, this model dynamically realizes the democratic form of the Yukawa matrix,

$$\alpha^*_{yij} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{26}$$

To obtain the democratic form of the fermion mass matrix, we have to assume all of the vacuum expectation values (VEVs) $\langle H_{ij} \rangle$ are the same. Alternatively, we assume mass terms among H_{ij} such that they lead to a single light mode $H = 1/3\sum_{ij} H_{ij}$ and the others have masses at a high energy scale. Here we take the latter scenario, and such mass terms have been shown in Refs. [5,6]. We denote their mass scale by M_H, and at M_H the RG-flows approaching toward their fixed points are terminated.

B. Fixed points of A-terms

Here, we consider fixed points of A-terms. Using the spurion technique, we can write the RG equations of A_{ij},

$$\mu \frac{d}{d\mu} A_{ij} = a_F \sum_k A_{ik} \alpha_{yik} + a_f \sum_k A_{kj} \alpha_{ykj} + 3a_H A_{ij} \alpha_{yij} + cM\alpha_g. \tag{27}$$

It is straightforward to show that there is the fixed point $A_{ij} = -M$. Around the fixed points, we write the RG equations of $\Delta A_{ij} = A_{ij} + M$,

$$\mu \frac{d}{d\mu} \Delta A_{ij} = \alpha_g x^* \Big\{ a_F \sum_k \Delta A_{ik} + a_f \sum_k \Delta A_{kj} + 3a_H \Delta A_{ij} \Big\} - \alpha_g M \Big\{ a_F \sum_k \Delta x_{ik} + a_f \sum_k \Delta x_{kj} + 3a_H \Delta x_{ij} \Big\}. \tag{28}$$

When $\Delta x_{ij} = 0$, ΔA_{ij} satisfy the same RG equations as Δx_{ij}. Thus, it is found that when the point $x_{ij} = x^*$ is IR attractive, the corresponding A-terms have the IR attractive fixed point,

$$A_{ij} = -M \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{29}$$

This form of A-terms is quite important from the phenomenological viewpoint. Obviously, the A-terms are universal, that is, favorable from the FCNC constraints. Furthermore, the CP phases of the A-terms are aligned with the CP phase of the gaugino mass M, that is, there is no physical CP-violation phase in the A-terms. Therefore, this form of A-terms can avoid both SUSY flavor and CP problems. Constraints from FCNC processes as well as CP-violations are usually presented by using the mass insertion parameters, e.g. $(\delta^a_{LR})_{ij}$,

$$(\delta^a_{LR})_{ij} = \frac{y^a_{ij} A^a_{ij}}{m^2_{\mathrm{SUSY}}} v^a, \tag{30}$$

in the S-CKM basis, where $a = u, d, \ell$ and m_{SUSY} denotes the average of sfermion masses and v^a denotes the VEV of the corresponding Higgs field.

IV. REALISTIC FERMION MASSES

A. RG flow of Yukawa matrix

In the previous section, we show the democratic form of the Yukawa matrix is realized at the IR fixed point, and at the same time the universal A-terms are obtained. That is quite favorable from the viewpoint of SUSY flavor and CP problems. However, the Yukawa matrix at the exact fixed point is not realistic, because it is a rank-one matrix, and only one family becomes massive and the other two remain massless. A region deviated slightly from the fixed point would be interesting. Thus, in this section we study the RG flows of the Yukawa matrix to investigate which pattern of the Yukawa matrix is obtained through a finite range of running by generic initial conditions and examine the possibility for realistic Yukawa matrices. In particular, we apply our analysis to the charged lepton sector. In addition, in such a situation, the A-terms may also deviate from the universal form somehow. We study whether such deviation leads to dangerous FCNCs.

The democratic matrix can be diagonalized as follows,

$$\frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} U^T, \tag{31}$$

where[3]

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{32}$$

For example, if the lepton mixing matrix is obtained as $V_{MNS} \approx U^T$, that leads to almost realistic mixing angles, which are consistent with the neutrino oscillations [15], i.e.,

$$(V_{MNS})_{12} \approx 0.5, \qquad (V_{MNS})_{23} \approx 0.7, \qquad (V_{MNS})_{13} < 0.1. \tag{33}$$

That is the reason why the democratic form of the Yukawa matrix is interesting, in particular, in the lepton sector. That is, the Yukawa matrix of charged leptons may almost be democratic, and it determines the mixing angles dominantly while the contribution from the neutrino Yukawa matrix to the mixing angle may be small [5].

Sometimes this "hierarchical" basis is more convenient to discuss than the democratic basis. We denote the deviations of x_{ij} from the fixed point in the hierarchical basis by $\Delta\tilde{x}_{ij}$. Their RG equations are obtained as

[3]There is ambiguity rotating the first and second elements in this diagonalizing matrix.

$$\mu \frac{d}{d\mu}\Delta\tilde{x}_{ij}(\mu) = \alpha_g x^* \lambda_{ij}\Delta\tilde{x}_{ij}(\mu), \tag{34}$$

with

$$\lambda_{ij} = \begin{pmatrix} 3a_H & 3a_H & 3(a_Q + a_H) \\ 3a_H & 3a_H & 3(a_Q + a_H) \\ 3(a_u + a_H) & 3(a_u + a_H) & 3a \end{pmatrix}. \tag{35}$$

Thus, they are solved as

$$\Delta\tilde{x}_{ij}(\mu) = \Delta\tilde{x}_{ij}(\Lambda)\left(\frac{\alpha_g(\mu)}{\alpha_g(\Lambda)}\right)^{-(x^*/b)\lambda_{ij}}, \tag{36}$$

and the Yukawa matrix in the hierarchical basis, $\tilde{y}_{ij}$ is obtained as

$$\tilde{y}_{ij} \sim 3\sqrt{x^*}g(\mu)\begin{pmatrix} \frac{\Delta\tilde{x}_{11}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{12}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{13}(\Lambda)}{6x^*}\varepsilon(\mu)^{3(a_Q+a_H)} \\ \frac{\Delta\tilde{x}_{21}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{22}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{23}(\Lambda)}{6x^*}\varepsilon(\mu)^{3(a_Q+a_H)} \\ \frac{\Delta\tilde{x}_{31}(\Lambda)}{6x^*}\varepsilon(\mu)^{3(a_u+a_H)} & \frac{\Delta\tilde{x}_{32}(\Lambda)}{6x^*}\varepsilon(\mu)^{3(a_u+a_H)} & 1 + \frac{\Delta\tilde{x}_{33}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a} \end{pmatrix}, \tag{37}$$

where the suppression factor $\varepsilon(\mu)$ is obtained as

$$\varepsilon(\mu) \equiv \left(\frac{\alpha_g(\mu)}{\alpha_g(\Lambda)}\right)^{-x^*/b}. \tag{38}$$

Note that the powers of the suppression factor $\varepsilon(\mu)$ in the $(i, 3)$ and $(3, i)$ entries are larger than those in the (i, j) entries $(i, j = 1, 2)$. Hence, when initial values $\Delta\tilde{x}_{ij}(\Lambda)$ are of the same order, e.g. of $O(1)$, the $(i, 3)$ and $(3, i)$ entries are suppressed rapidly and we obtain the Yukawa matrix,

$$\tilde{y}_{ij} \sim 3\sqrt{x^*}g(\mu)\begin{pmatrix} \frac{\Delta\tilde{x}_{11}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{12}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & 0 \\ \frac{\Delta\tilde{x}_{21}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & \frac{\Delta\tilde{x}_{22}(\Lambda)}{6x^*}\varepsilon(\mu)^{3a_H} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{39}$$

Thus, this RG flow can explain why the third family is the heaviest.

To obtain numerically realistic result of the lepton sector, we need $\varepsilon(\mu)^{3a_H} \sim m_\mu/m_\tau$ when $\Delta\tilde{x}_{ij}/(6x^*) \sim 1$. Furthermore, the $(2, 3)$ mixing angle is almost determined by the (2,3) entry of U^T, because $\tilde{y}_{i3}$ are suppressed rapidly. Thus, we obtain

$$(V_{MNS})_{23} = \frac{2}{\sqrt{6}}\cos\theta_{12}, \tag{40}$$

up to a contribution from the neutrino sector, where θ_{12} is the mixing of left-handed leptons in the first (2×2) submatrix of Eq. (39). This prediction is good for $\cos\theta_{12} = O(1)$, e.g. $\theta_{12} \lesssim \pi/4$, when the contribution from the neutrino sector is not large [5].

On the other hand, the above form cannot explain the mass hierarchy between the first and second families when all of initial values $\Delta\tilde{x}_{ij}(\Lambda)$ are of the same order, e.g. of $O(1)$. If the initial values of the first and second families are chosen, e.g.,

$$\begin{pmatrix} \Delta\tilde{x}_{11}(\Lambda) & \Delta\tilde{x}_{12}(\Lambda) \\ \Delta\tilde{x}_{21}(\Lambda) & \Delta\tilde{x}_{22}(\Lambda) \end{pmatrix} = \Delta\tilde{x}(\Lambda)_{22}\begin{pmatrix} 0 & \sqrt{\frac{m_e}{m_\mu}} \\ \sqrt{\frac{m_e}{m_\mu}} & 1 \end{pmatrix}, \tag{41}$$

we obtain the correct mass ratio m_e/m_μ. In the democratic basis, these initial couplings are given as

$$\begin{pmatrix} \Delta x_{11}(\Lambda) & \Delta x_{12}(\Lambda) \\ \Delta x_{21}(\Lambda) & \Delta x_{22}(\Lambda) \end{pmatrix} = \Delta x(\Lambda)_{22} \times \begin{pmatrix} 1 - \sqrt{\frac{m_e}{m_\mu}} & 1 \\ 1 & 1 + \sqrt{\frac{m_e}{m_\mu}} \end{pmatrix}. \tag{42}$$

In addition, it also contributes to the mixing angle $(V_{MNS})_{12}$ as

$$(V_{MNS})_{12} \sim \frac{1}{\sqrt{2}}(1 - \sqrt{m_e/3m_\mu}), \tag{43}$$

up to the contribution from the neutrino sector. This small modification is rather preferable.

B. RG flow of A-terms

Here, let us study the RG flow of the corresponding A-terms. In the hierarchical basis the RG equations (28) are written as

$$\mu\frac{d\Delta\tilde{A}_{ij}}{d\mu} = \alpha_g x^* \lambda_{ij}\Delta\tilde{A}_{ij} - M\alpha_g\lambda_{ij}\Delta\tilde{x}_{ij}. \tag{44}$$

Their solutions are obtained as

$$\Delta\tilde{A}_{ij}(\mu) = \left[\frac{\Delta\tilde{A}_{ij}(\Lambda)}{\Delta\tilde{x}_{ij}(\Lambda)} - \frac{\lambda_{ij}M(\Lambda)}{b}\left(1 - \frac{\alpha_g(\mu)}{\alpha_g(\Lambda)}\right)\right]\Delta\tilde{x}_{ij}(\mu). \tag{45}$$

Note that the damping factor is the same except the term including $\Delta\tilde{x}_{ij}(\mu)\alpha_g(\mu)/\alpha_g(\Lambda)$.

When we apply to the charged lepton sector, it is first noted that $\Delta\tilde{A}_{i3}$ and $\Delta\tilde{A}_{3i}$ for $i, j = 1, 2$ are strongly suppressed as $\tilde{y}_{ij}$ given in (39). Therefore, the branching ratios for the lepton flavor violating processes, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, turn out to be much smaller than the experimental bounds.

It is found that another lepton flavor violating process, $\mu \to e\gamma$, is also suppressed to some extent. Since it is expected that $\varepsilon(\mu)^{3a_H} \sim (m_\mu/m_\tau)$, we may evaluate

$$\Delta\tilde{A}_{ij}(\mu) \sim (m_\mu/m_\tau) F_{ij}(\Lambda), \tag{46}$$

$$F_{ij}(\Lambda) = \Delta\tilde{A}_{ij}(\Lambda) - \frac{\lambda_{ij} M(\lambda)}{b} \Delta\tilde{x}_{ij}(\Lambda), \tag{47}$$

for $i, j = 1, 2$. Hence, the mass insertion parameters $(\delta^\ell_{LR})_{ij}$ for $i, j = 1, 2$ are estimated as

$$(\delta^\ell_{LR})_{ij} \sim (m_\mu/m_\tau)(m_\tau/m_{\rm SUSY}), \tag{48}$$

where $m_{\rm SUSY}$ is the average mass of sleptons, and we have taken $F_{ij}(\Lambda) = m_{\rm SUSY}$. For example, in the case with $m_{\rm SUSY} = 100$ GeV, we obtain $(\delta^\ell_{LR})_{12} = 10^{-4}$. The second term in $F_{ij}(\Lambda)$ has more suppression factor like $O(10^{-1} - 10^{-2})$ when we take $\Delta\tilde{x}_{12}(\Lambda) = \sqrt{m_e/m_\mu}$ like Eq. (41). Furthermore, if $|b| = O(10)$, it would lead to a further suppression factor by $O(10^{-1})$. On the other hand, experimental bound on the $\mu \to e\gamma$ decay requires $(\delta^\ell_{LR})_{12} \leq 10^{-6}$. Furthermore, the experiment on the EDM of the electron requires $Im(\delta^\ell_{LR})_{11} \leq 10^{-7}$. If there is no suppression factor for the first term $\Delta\tilde{A}_{ij}(\Lambda)$ in the $F_{ij}(\Lambda)$, then these constraints are not satisfied. Conversely, if we have another mechanism to suppress $\Delta\tilde{A}_{ij}(\Lambda)$,[4] then the flavor nonuniversality generated through $\Delta\tilde{x}$ is sufficiently small.

V. REALISTIC QUARK MASSES

A. RG flow of Yukawa couplings

In the previous section, we studied a single sector of a Yukawa matrix, and apply to the charged lepton sector. In this section, we study quark masses. For quarks, both the up and down sectors of Yukawa coupling matrices should have democratic fixed points. Otherwise, we cannot realize small mixing angles. Thus, we have to extend the previous analysis to the case including the up and down sectors of Yukawa matrices.

Here, we consider the concrete model, which has been proposed in Ref. [5]. It is the $SU(5)' \times SU(5)''$ GUT model. We assume three families of quarks as well as leptons correspond to $\mathbf{10}_i$ and $\bar{\mathbf{5}}_i$ for $SU(5)'$ as the usual $SU(5)$ GUT, and they are singlets under $SU(5)''$. We denote the gauge couplings g' and g'' for $SU(5)'$ and $SU(5)''$, respectively. We assume that g' is strong, but g'' is weak. We also assume that $SU(5)' \times SU(5)''$ is broken into the diagonal group $SU(5)$ at/above the GUT scale. Then, we would obtain the usual $SU(5)$ GUT. The gauge coupling g of the diagonal group is obtained as $1/g^2 = 1/g'^2 + 1/g''^2$, and it is weak. The Pendelton-Ross IR fixed points can be realized for $SU(5)'$ within short running, because the gauge coupling g' is strong. That is one of the reasons why we extend the usual $SU(5)$ GUT to the $SU(5)' \times SU(5)''$ GUT.[5] We assume that the mass scale M_H is around the GUT scale.

Another reason to consider the $SU(5)' \times SU(5)''$ GUT is related with the SUSY breaking parameters. In the asymptotically nonfree gauge theories considered in the previous section, the running gaugino mass also decreases towards lower energy scale. Meanwhile the soft scalar masses are enhanced through the strong gauge interaction and become much larger than the gaugino mass. In the extended GUT, however, there are two gaugino masses M' and M'' for the distinct gauge sectors of $SU(5)'$ and $SU(5)''$ respectively, and the gaugino mass M obtained after the symmetry breaking is given by

$$\frac{M}{g^2} = \frac{M'}{g'^2} + \frac{M''}{g''^2}. \tag{49}$$

Since we suppose g' to be large, M is almost the same as M'', which is not reduced from its initial value due to smallness of the gauge coupling g''. Therefore, the soft scalar masses, which may be enhanced to be of the order of the initial M'', do not dominate over the gaugino mass M, as long as the gaugino masses M' and M'' are given with their initial values of the same order.

We introduce nine pairs of Higgs fields $H^u(\mathbf{5})_{ij}$ and $H^d(\bar{\mathbf{5}})_{ij}$, corresponding to the up and down sector Higgs fields. We have the following superpotential;

$$W = y^u_{ij} \mathbf{10}_i \mathbf{10}_j H^u(\mathbf{5})_{ij} + y^d_{ij} \mathbf{10}_i \bar{\mathbf{5}}_j H^d(\bar{\mathbf{5}})_{ij}. \tag{50}$$

The anomalous dimensions of matter fields and Higgs fields are obtained as

$$\gamma_{\mathbf{10}_i} = \sum_{k=1}^{3} \{a^u_Q(\alpha^u_{yik} + \alpha^u_{yki}) + a^d_Q \alpha^d_{yik}\} - a^u_Q \alpha^u_{yii} - c_Q \alpha'_g, \tag{51}$$

$$\gamma_{\bar{\mathbf{5}}_i} = a^d \sum_{k=1}^{3} \alpha^d_{ykj} - c_d \alpha'_g, \tag{52}$$

$$\gamma_{H^u_{ij}} = 3a^u_H \alpha^u_{yij} - c^u_H \alpha'_g, \tag{53}$$

$$\gamma_{H^d_{ij}} = 3a^d_H \alpha^d_{yij} - c^d_H \alpha'_g, \tag{54}$$

with $3a^u_H = 6$, $3a^d_H = 4$, $a^u_Q = 3$, $a^d_Q = 2$, $a^d = 4$, $c_Q = 36/5$, $c_d = c_H = 24/5$, where $\alpha^{u,d}_{y_{ij}} = |y^{u,d}_{ij}|^2/(8\pi^2)$ and $\alpha'_g = g'^2/(8\pi^2)$. Then, the RG equations of $\alpha^u_{y_{ij}}, \alpha^d_{y_{ij}}$ are obtained as

[4] For example, a certain class of (string-motivated) supergravity models leads to $A_{ij}(\Lambda) = -M(\Lambda)$ as initial conditions. See e.g. Ref. [16] and references therein.

[5] In extra dimensional models convergence toward IR fixed points may be sufficiently rapid [17] in a single $SU(5)$ model. Besides, the A-terms as well as the soft scalar masses are strongly aligned into the universal for asymptotically free gauge theories in the extra dimensions [18].

$$\mu \frac{d\alpha^u_{y_{ij}}}{d\mu} = (\gamma_{\mathbf{10}_i} + \gamma_{\mathbf{10}_j} + \gamma_{H^u_{ij}})\alpha^u_{y_{ij}}, \tag{55}$$

$$\mu \frac{d\alpha^d_{y_{ij}}}{d\mu} = (\gamma_{\mathbf{10}_i} + \gamma_{\bar{\mathbf{5}}_j} + \gamma_{H^d_{ij}})\alpha^d_{y_{ij}}. \tag{56}$$

We consider the RG equations of $x^{u,d}_{ij} = \alpha^{u,d}_{yij}/\alpha'_g$, that is,

$$\mu \frac{d}{d\mu} x^u_{ij} = \Big\{(b' - 2c_Q - c^u_H) + 3a^u_H x^u_{ij} + \sum_{k=1}^{3} a^u_Q(x^u_{ik} + x^u_{ki} + x^u_{kj} + x^u_{jk}) + \sum_{k=1}^{3} a^d_Q(x^d_{ik} + x^d_{jk}) - a^u_Q x^u_{ii} - a^u_Q x^u_{jj}\Big\}\alpha'_g x^u_{ij}, \tag{57}$$

$$\mu \frac{d}{d\mu} x^d_{ij} = \Big\{(b' - c_Q - c_d - c^d_H) + 3a^d_H x^d_{ij} + \sum_{k=1}^{3} a^u_Q(x^u_{ik} + x^u_{ki}) + \sum_{k=1}^{3} a^d_Q(x^d_{ik}) + \sum_{k=1}^{3} a^d x^d_{kj} - a^u_Q x^u_{ii}\Big\}\alpha'_g x^d_{ij}, \tag{58}$$

where b' is the one-loop beta function coefficient of g'. It is straightforward to show these RG equations have the following fixed points,

$$x^u_{ij} = x^{u*} = \frac{X(a^d_Q + a^d_H + a^d) - 2a^d_Q Y}{10a^u_Q(a^d_H + a^d) + 3a^u_H(a^d_Q + a^d_H + a^d)}, \tag{59}$$

$$x^d_{ij} = x^{d*} = \frac{5a^u_Q X - (3a^u_H + 10a^u_Q)Y}{30a^d_Q a^u_Q - 3(3a^u_H + 10a^u_Q)(a^d_H + a^d_Q + a^d)}, \tag{60}$$

where $X = 2c_Q + c^u_H - b'$ and $Y = c_Q + c_d + c^d_H - b'$. We consider perturbations around these fixed points, $\Delta^u_{ij} = x^u_{ij} - x^{u*}$ and $\Delta^d_{ij} = x^d_{ij} - x^{d*}$. It is convenient to define

$$\Delta^S_{ij} = \Delta^u_{ij} + \Delta^u_{ji}, \qquad \Delta^A_{ij} = \Delta^u_{ij} - \Delta^u_{ji}, \tag{61}$$

to solve their RG equations. Actually, the RG equations of $\Delta^{S,A,d}_{ij}$ are written as

$$\mu \frac{d}{d\mu} \Delta^A_{ij} = 3a^u_H x^{u*}\alpha'_g \Delta^A_{ij}, \tag{62}$$

$$\mu \frac{d}{d\mu} \Delta^S_{ij} = \Big\{\sum_{k=1}^{3}(2a^u_Q(\Delta^S_{ik} + \Delta^S_{kj}) + 2a^d_Q(\Delta^d_{ik} + \Delta^d_{jk})) + 3a^u_H\Delta^S_{ij} - a^u_Q\Delta^S_{ii} - a^u_Q\Delta^S_{jj}\Big\}\alpha'_g x^{u*}, \tag{63}$$

$$\mu \frac{d}{d\mu} \Delta^d_{ij} = \Big\{\sum_{k=1}^{3}(a^u_Q\Delta^S_{ik} + a^d_Q\Delta^d_{kj} + a^d\Delta^d_{kj}) + 3a^d_H\Delta^d_{ij} - \frac{a^u_Q}{2}\Delta^S_{ii}\Big\}\alpha'_g x^{d*}. \tag{64}$$

Easily, we can solve the RG equations of Δ^A_{ij} as

$$\Delta^A_{ij}(\mu) = \Delta^A_{ij}(\Lambda)\left(\frac{\alpha'_g(\mu)}{\alpha'_g(\Lambda)}\right)^{-3a^u_H x^{u*}/b'}. \tag{65}$$

The other elements are mixed in the above equations. Here, we use the hierarchical basis. Through longsome but simple algebraic calculations, we find that the RG equations of $\tilde{\Delta}^S_{ij}$, $\tilde{\Delta}^d_{ij}$ and $\tilde{\Delta}^d_{3i}$ for $i, j = 1, 2$ are not mixed. Then we can solve them,

$$\tilde{\Delta}^S_{ij}(\mu) = \tilde{\Delta}^S_{ij}(\Lambda)\left(\frac{\alpha'_g(\mu)}{\alpha'_g(\Lambda)}\right)^{-3a^u_H x^{u*}/b'}, \tag{66}$$

$$\tilde{\Delta}^d_{ij}(\mu) = \tilde{\Delta}^d_{ij}(\Lambda)\left(\frac{\alpha'_g(\mu)}{\alpha'_g(\Lambda)}\right)^{-3a^d_H x^{d*}/b'}, \tag{67}$$

$$\tilde{\Delta}^d_{3i}(\mu) = \tilde{\Delta}^d_{3i}(\Lambda)\left(\frac{\alpha'_g(\mu)}{\alpha'_g(\Lambda)}\right)^{-3(a^d + a^d_H)x^{d*}/b'}, \tag{68}$$

for $i, j = 1, 2$. However, the other elements $\tilde{\Delta}^S_{i3}$ and $\tilde{\Delta}^d_{i3}$ for $i = 1, 2, 3$ are mixed as

$$-b'\alpha'_g \frac{d}{d\alpha'_g}\begin{pmatrix}\tilde{\Delta}^S_{13}\\ \tilde{\Delta}^d_{13}\end{pmatrix} = \begin{pmatrix}x^{u*}(3a^u_H + 4a^u_Q) & 6a^d_Q x^{d*}\\ 2x^{u*}a^u_Q & 3x^{d*}(a^d_Q + a^d_H)\end{pmatrix} \times \begin{pmatrix}\tilde{\Delta}^S_{13}\\ \tilde{\Delta}^d_{13}\end{pmatrix} - \frac{a^u_Q x^{u*}}{\sqrt{2}}\begin{pmatrix}2\\ 1\end{pmatrix}\tilde{\Delta}^S_{12}, \tag{69}$$

$$-b'\alpha'_g \frac{d}{d\alpha'_g}\begin{pmatrix}\tilde{\Delta}^S_{23}\\ \tilde{\Delta}^d_{23}\end{pmatrix} = \begin{pmatrix}x^{u*}(3a^u_H + 4a^u_Q) & 6a^d_Q x^{d*}\\ 2x^{u*}a^u_Q & 3x^{d*}(a^d_Q + a^d_H)\end{pmatrix} \times \begin{pmatrix}\tilde{\Delta}^S_{23}\\ \tilde{\Delta}^d_{23}\end{pmatrix} - \frac{a^u_Q x^{u*}}{2\sqrt{2}}\begin{pmatrix}2\\ 1\end{pmatrix}(\tilde{\Delta}^S_{11} - \tilde{\Delta}^S_{22}), \tag{70}$$

$$-b'\alpha'_g \frac{d}{d\alpha'_g}\begin{pmatrix}\tilde{\Delta}^S_{33}\\ \tilde{\Delta}^d_{33}\end{pmatrix} = \begin{pmatrix}x^{u*}(3a^u_H + 10a^u_Q) & 12a^d_Q x^{d*}\\ \frac{5}{2}x^{u*}a^u_Q & 3x^{d*}(a^d_Q + A^d + a^d_H)\end{pmatrix}\begin{pmatrix}\tilde{\Delta}^S_{33}\\ \tilde{\Delta}^d_{33}\end{pmatrix} - \frac{a^u_Q x^{u*}}{2}\begin{pmatrix}4\\ 1\end{pmatrix}(\tilde{\Delta}^S_{11} + \tilde{\Delta}^S_{22}). \tag{71}$$

The solutions $\tilde{\Delta}^S_{23}(\mu)$ and $\tilde{\Delta}^d_{23}(\mu)$ are obtained as

$$\begin{pmatrix}\tilde{\Delta}^S_{23}(\mu)/\tilde{\Delta}^S_{23}(\Lambda)\\ \tilde{\Delta}^d_{23}(\mu)/\tilde{\Delta}^d_{23}(\Lambda)\end{pmatrix} = \begin{pmatrix}R_2 & Q_2\\ R_2 r_+ & Q_2 r_-\end{pmatrix}\begin{pmatrix}(\alpha'_g(\mu)/\alpha'_g(\Lambda))^{-n_+/b'}\\ (\alpha'_g(\mu)/\alpha'_g(\Lambda)))^{-n_-/b'}\end{pmatrix} + \begin{pmatrix}k^2_1\\ k^2_2\end{pmatrix}(\alpha'_g(\mu)/\alpha'_g(\Lambda))^{-3a^u_H x^{u*}/b'}, \tag{72}$$

where $n_\pm$ are obtained as

$$n_\pm = \frac{D + A \pm \sqrt{(D-A)^2 + 4BC}}{2}, \tag{73}$$

with

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} x^{u*}(3a_H^u + 4a_Q^u) & 6a_Q^d x^{d*} \\ 2x^{u*}a_Q^u & 3x^{d*}(a_Q^d + a_H^d) \end{pmatrix}, \tag{74}$$

and similarly $r_\pm$ are obtained as

$$r_\pm = \frac{D - A \pm \sqrt{(D-A)^2 + 4BC}}{2B(-b')}. \tag{75}$$

In addition, R_i and Q_i are integral constants determined by initial conditions. Furthermore, the constants k_1^2 and k_2^2 are determined by solving the following equation;

$$\begin{pmatrix} l - A & -B \\ -C & l - D \end{pmatrix}\begin{pmatrix} k_1^2 \\ k_2^2 \end{pmatrix} = -\frac{a_Q^u x^{u*}}{2\sqrt{2}(-b')}\begin{pmatrix} 2 \\ 1 \end{pmatrix}(\tilde{\Delta}_{11}^S - \tilde{\Delta}_{22}^S), \tag{76}$$

with $l = -3a_H^u x^{u*}/b'$. Substituting explicit values, we obtain

$$n_\pm = 9x^{u*} + 5x^{d*} \pm \sqrt{(9x^{u*} - 5x^{d*})^2 + 72x^{u*}x^{d*}}. \tag{77}$$

It is found that both $n_\pm$ are positive, and obviously $n_+ > n_-$. The solutions $\tilde{\Delta}_{13}^S(\mu)$ and $\tilde{\Delta}_{13}^d(\mu)$ are the same as $\tilde{\Delta}_{23}^S(\mu)$ and $\tilde{\Delta}_{23}^d(\mu)$ except by replacing

$$(\tilde{\Delta}_{11}^S - \tilde{\Delta}_{22}^S) \to \tilde{\Delta}_{12}^S. \tag{78}$$

Similarly we can solve $\tilde{\Delta}_{33}^S(\mu)$ and $\tilde{\Delta}_{33}^d(\mu)$, and they have rapidly damping behavior. Since their explicit forms are irrelevant to our discussions, we omit them.

The solutions $\tilde{\Delta}_{i3}^S(\mu)$ and $\tilde{\Delta}_{i3}^d(\mu)$ for $i = 1, 2$ include three damping factors. The most slowly damping term is important. Since we always have $n_- < 6(= 3a_H^u)$, we find that $\tilde{\Delta}_{i3}^S(\mu)$ and $\tilde{\Delta}_{i3}^d(\mu)$ behave like $(\alpha_g')^{-n_-/b'}$. Thus, the up and down-sectors of Yukawa matrices behave totally as

$$y_{ij}^u \sim \begin{pmatrix} \varepsilon'(\mu)^{3a_H^u x^{u*}} & \varepsilon'(\mu)^{3a_H^u x^{u*}} & \varepsilon'(\mu)^{n_-} \\ \varepsilon'(\mu)^{3a_H^u x^{u*}} & \varepsilon'(\mu)^{3a_H^u x^{u*}} & \varepsilon'(\mu)^{n_-} \\ \varepsilon'(\mu)^{n_-} & \varepsilon'(\mu)^{n_-} & 1 \end{pmatrix}, \tag{79}$$

$$y_{ij}^d \sim \begin{pmatrix} \varepsilon'(\mu)^{3a_H^d x^{d*}} & \varepsilon'(\mu)^{3a_H^d x^{d*}} & \varepsilon'(\mu)^{n_-} \\ \varepsilon'(\mu)^{3a_H^d x^{d*}} & \varepsilon'(\mu)^{3a_H^d x^{d*}} & \varepsilon'(\mu)^{n_-} \\ 0 & 0 & 1 \end{pmatrix}, \tag{80}$$

where

$$\varepsilon'(\mu) = \left(\frac{\alpha_g'(\mu)}{\alpha_g'(\Lambda)}\right)^{-1/b'}. \tag{81}$$

We have omitted coefficients. Also we have omitted (3,1) and (3,2) entries in the down-sector Yukawa matrix, because they are damping more rapidly than the other entries in the down-sector Yukawa matrix. That is the same behavior as in the case with a single Yukawa matrix, which discussed in the previous section. However, (1,3) and (2,3) entries are damping not rapidly in the case including both the up and down-sector Yukawa matrices. That would be important to derive the mixing angle V_{cb} in the quark sector.

Recall that $3a_H^u = 6$ and $3a_H^d = 4$. Explicit values of n_- are $n_-/x^{d*} = 3.5$, 4.6 and 5.3 for $x^{u*} = 0.5x^{d*}$, x^{d*} and $2x^{d*}$, respectively. Thus, when $x^{u*} \sim x^{d*}$, we can obtain the realistic relations,

$$\frac{m_c}{m_t} \sim \left(\frac{m_s}{m_b}\right)^{3/2}, \qquad V_{cb} \sim \frac{m_s}{m_b}. \tag{82}$$

The model which we are discussing leads to

$$x^{u*} = \frac{92}{255} - \frac{5}{306}b', \qquad x^{d*} = \frac{44}{85} - \frac{7}{204}b'. \tag{83}$$

Thus, it leads to the good relation, $\frac{m_s}{m_b} \sim V_{cb}$. However, this model predicts rather a large ratio of m_c/m_t like $m_c/m_t \sim m_s/m_b$.

Let us discuss extended models including other couplings. Fixed point values x^{u*} and x^{d*} change significantly when we include other couplings. In the appendix, we give examples where fixed points are shifted by additional couplings. However, in such a class of models, the above analysis is the same except by using new fixed point values x^{u*} and x^{d*}. That is, this class of models, in general, lead to the forms of Yukawa matrices (79) and (80) for the up and down sectors when additional couplings are close to their fixed points. Thus, we could obtain models with several couplings leading to the realistic mass relations of the second and third families, e.g. the model leading to $x^{u*} \sim x^{d*}$. Here we do not study explicitly a concrete model with values of x^{u*} and x^{d*} leading to realistic results. We use x^{u*} and x^{d*} as free parameters to present generic models with several couplings and effects due to such additional couplings.

We have shown that we can realize the quark mass ratios and the mixing angles between the second and third families when fixed point values x^{u*} and x^{d*} are in the proper region. However, only the RG dynamics cannot lead to the realistic mass hierarchy between the first and second families. That is the same as the situation in the previous section. Thus, we need fine-tuning of initial conditions. For example, if the initial conditions are chosen, e.g.,

$$\begin{pmatrix} \tilde{\Delta}_{11}^u & \tilde{\Delta}_{12}^u \\ \tilde{\Delta}_{21}^u & \tilde{\Delta}_{22}^u \end{pmatrix} = \tilde{\Delta}^u(\Lambda)_{22}\begin{pmatrix} 0 & \sqrt{\frac{m_u}{m_c}} \\ \sqrt{\frac{m_u}{m_c}} & 1 \end{pmatrix}, \tag{84}$$

$$\begin{pmatrix} \tilde{\Delta}_{11}^d & \tilde{\Delta}_{12}^d \\ \tilde{\Delta}_{21}^d & \tilde{\Delta}_{22}^d \end{pmatrix} = \tilde{\Delta}^d(\Lambda)_{22}\begin{pmatrix} 0 & \sqrt{\frac{m_d}{m_s}} \\ \sqrt{\frac{m_d}{m_s}} & 1 \end{pmatrix}, \tag{85}$$

we obtain the correct mass ratios m_u/m_c and m_d/m_s, and the mixing angle $V_{us} \sim m_d/m_s$. These initial conditions are also consistent with those in the lepton sector, which was discussed in the previous section, because $m_e/m_\mu \sim m_d/m_s$.

B. A-terms

Here, we study the corresponding A-terms. By using the spurion technique, we can obtain the RG equations of $A_{ij}^{u,d}$ and find they have the fixed points,

$$A_{ij}^{u,d} \to A^* = -M. \tag{86}$$

We expand $A_{ij}^{u,d}$ around the fixed point as $A_{ij}^{u,d} = A^* + \Delta A_{ij}^{u,d}$, and the deviations $\Delta A_{ij}^{u,d}$ satisfy the following RG equations;

$$\begin{aligned}\mu \frac{d}{d\mu}\Delta A_{ij}^S = \alpha_g'\Big[& 3a_H^u x^{u*}\Delta A_{ij}^S - a_Q^u x^{u*}\Delta A_{ii}^S \\ & - a_Q^u x^{u*}\Delta A_{jj}^S + \sum_{k=1}^{3}\{2a_Q^u x^{u*}(\Delta A_{ik}^S + \Delta A_{kj}^S) \\ & + 2a_Q^d x^{d*}(\Delta A_{ik}^d + \Delta A_{jk}^d)\}\Big] - \alpha_g' M\Big[3a_H^u \Delta_{ij}^S \\ & - a_Q^u \Delta_{ii}^S - a_Q^u \Delta_{jj}^S + \sum_{k=1}^{3}\{2a_Q^u(\Delta_{ik}^S + \Delta_{jk}^S) \\ & + 2a_Q^d(\Delta_{ik}^d + \Delta_{jk}^d)\}\Big],\end{aligned} \tag{87}$$

$$\begin{aligned}\mu \frac{d}{d\mu}\Delta A_{ij}^d = \alpha_g'\Big[& 3a_H^d x^{d*}\Delta A_{ij}^d - \frac{a_Q^u}{2} x^{u*}\Delta A_{ii}^S \\ & + \sum_{k=1}^{3}\{a_Q^u x^{u*}\Delta A_{ik}^S + (a_Q^d x^{d*}\Delta A_{ik}^d \\ & + a^d x^{d*}\Delta A_{kj}^d)\}\Big] - \alpha_g' M\Big\{3a_H^u \Delta_{ij}^d - \frac{a_Q^u}{2}\Delta_{ii}^S \\ & + \sum_{k=1}^{3}(a_Q^u \Delta_{ik}^S + a_Q^d \Delta_{ik}^d + A^d \Delta_{jk}^d)\Big\},\end{aligned} \tag{88}$$

$$\mu \frac{d}{d\mu}\Delta A_{ij}^A = 3a_H^u x^{u*}\Delta A_{ij}^A \alpha_g' - \alpha_g' M 3a_H^u \Delta_{ij}^A, \tag{89}$$

where $\Delta A_{ij}^S = \Delta A_{ij}^u + \Delta A_{ji}^u$ and $\Delta A_{ij}^A = \Delta A_{ij}^u - \Delta A_{ji}^u$. Thus, we see that ΔA_{ij} behave like $\Delta A_{ij} \sim \Delta_{ij}$, and that is similar to the model in the previous section.

The experimental bounds for ΔM_K, ΔM_B and the branching ratio of $b \to s\gamma$ restrict the mass insertion parameters as $(\delta_{LR}^d)_{12} < 4.4 \times 10^{-3}$, $(\delta_{LR}^d)_{13} < 3.3 \times 10^{-2}$ and $(\delta_{LR}^d)_{23} < 1.6 \times 10^{-2}$ respectively. On the other hand, we can estimate the mass insertion parameters of the present model $(\delta_{LR}^{u,d})_{ij}$ for $i, j = 1, 2$ as

$$(\delta_{LR}^{u,d})_{ij} \sim (m_s/m_{\rm SUSY}), \tag{90}$$

where we have taken $\Delta A_{ij}(\Lambda) = m_{\rm SUSY}$. Then we obtain $(\delta_{LR}^{u,d})_{12} = 10^{-4}$, which is consistent with the experiment on ΔM_K. We have taken $m_{\rm SUSY} = 500$ GeV. The mass insertion parameters $(\delta_{LR}^d)_{i3}$ for $i, j = 1, 2$ are also found to be comparable with $(\delta_{LR}^d)_{12}$, since the parameter n_- introduced in the previous subsection is not so different from $3a_H^d x^{d*}$. Therefore, the bounds from ΔM_B and $b \to s\gamma$ are also satisfied.

Moreover, the EDM of the neutron requires $\mathrm{Im}((\delta_{LR}^{u,d})_{11}) = 10^{-6}$, which seems to be rather severe in general. However, this condition may be also explained, when $|A_{ij}| \sim M'$ is smaller than $M'' \sim M$ by one or two orders at the GUT scale. This is because the A-terms become almost flavor universal at the weak scale due to corrections through the MSSM gauge interactions, which dominate over the initial values. Otherwise, we need fine-tuning of initial conditions $\Delta\tilde{A}_{ij}(\Lambda)$ or some mechanism to lead to such required initial conditions.

VI. COMMENT ON SFERMION MASSES

In the previous sections we have concentrated on constraints on the A-terms. Indeed, we have stronger constraints on the A-terms from FCNC and CP processes than soft sfermion masses. Here we give a comment on sfermion masses. From the viewpoint of experiments on FCNC processes, degenerate sfermion masses are favorable.

The dynamics due to IR fixed points leads to the certain relation among soft scalar masses. When the Yukawa coupling $y_{ij}F_i f_j H_{ij}$ has an IR fixed point, the sum of soft scalar masses squared also has the IR fixed point,

$$m_{\tilde{F}_i}^2 + m_{\tilde{f}_j}^2 + m_{H_{ij}}^2 = |M|^2. \tag{91}$$

However, these relations are not enough to lead to degenerate sfermion masses among flavors when there are nine Higgs fields H_{ij}. If there is any symmetry relating the soft scalar masses as well as the Yukawa couplings of the nine Higgs fields, then the above relations would lead to degenerate sfermion masses. An example is the model with A_4 symmetry [19]. In this model, three families correspond to triplets under A_4, and four pairs of Higgs fields are introduced, and they correspond to A_4 singlet and triplet. That is, three H_{ii} are identified, i.e., $H_{11} = H_{22} = H_{33}$, and H_{ij} for $i \neq j$ is identified with H_{ji}, $H_{ij} = H_{ji}$, in words of our model with nine Higgs fields. In such model, degeneracy of sfermion masses can be realized as IR fixed points [20]. However, in this model entries of the Yukawa matrix are also identified by the A_4 symmetry, that is, all of diagonal entries y_{ii} are always the same, i.e., $y_{11} = y_{22} = y_{33}$, and all of off-diagonal entries y_{ij} $(i \neq j)$ are always the same, i.e., $y_{ij} = y_{ji}$, even away IR fixed points. We would obtain realistic results from such model when $y_{ii} = y_{ij}$ for $i \neq j$ and VEVs of four pairs of Higgs fields are fine-tuned in a proper way [19,21]. Because that is a scenario different from the scenario in this paper, we do not study it here, but it may be another interesting scenario.

VII. CONCLUSION AND DISCUSSION

We have considered the model to realize the democratic form of Yukawa matrices by use of fixed points. We have studied the RG flows of Yukawa couplings in detail and obtained their specific patterns of Yukawa matrices. That is, the model with a single Yukawa matrix leads to the RG flow (39), and the models with the up and down sectors of Yukawa matrices lead to the forms (79) and (80). These are of our main results. By use of these results, the mass hierarchy and the mixing angles between the second and third families can be realized. For a single sector of the Yukawa matrix, the mixing angle between the second and third families is determined almost by U^T_{23}. However, for the up and down sectors of Yukawa matrices the mixing angle between the second and third families can be obtained as a value similar to m_s/m_b, i.e., $m_s/m_b \sim V_{cb}$. These aspects are quite interesting. Also, we can obtain the mass hierarchy, $m_s/m_b \sim m_\mu/m_\tau$ and $m_c/m_t \sim (m_s/m_b)^{3/2}$.

The corresponding A-term couplings also have the universal IR fixed point. That is also important in order to avoid SUSY FCNC and CP problems. In particular, there are strong constraints from the Kaon system, the $\mu \to e\gamma$ and EDMs. FCNC constraints can be relaxed in the parameter region realizing realistic values of quark mass hierarchies, except for the $\mu \to e\gamma$ process. Constraints due to CP violations can be ameliorated, but that may not be sufficient. We would need fine-tuning of the initial conditions on the A-terms. Alternatively, we may need some mechanism to realize such fine-tuning.

Our model does not intend to explain the mass hierarchy and mixing angle between the first and second families. In order to do this, we need to choose proper initial conditions for them, or find some mechanism to realize such initial conditions. For example, we may consider to achieve dynamical alignment of Yukawa couplings so that the large mass hierarchy like m_u/m_t can be generated [5]. This can be realized by considering a strongly coupled GUT. Moreover, it was shown in Ref. [5] that the mass matrix close to the Fritzsch-type can be obtained, if one of the Yukawa couplings is given with a relatively small initial value. In such a situation, the gauge dynamics aligns the A-terms strongly enough, and the problematic processes of FCNC and CP violations may be automatically suppressed well below their experimental bounds. That is beyond the scope of this paper and we would study it elsewhere.

Within the framework of our model with nine pairs of Higgs fields, we cannot control squark/slepton masses only by fixed point dynamics such that they are degenerate to avoid FCNC constraints.

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APPENDIX

Here we give two examples of models with additional couplings, which shift values of fixed points.

First, we consider the model, which has the coupling among $H^{u,d}_{ij}$ and the adjoint field **24**. The relevant superpotential is written as

$$W = y^u_{ij}\mathbf{10}^i\mathbf{10}^j H^u(\mathbf{5})_{ij} + y^d_{ij}\mathbf{10}^i\bar{\mathbf{5}}^j H^d(\bar{\mathbf{5}})_{ij} + \kappa_{ij,kl}H^u(\mathbf{5})^{ij}H^d(\bar{\mathbf{5}})^{kl}\mathbf{24}. \tag{A1}$$

There may be other couplings as well as other fields. However, we assume that only the above couplings are around their fixed points, and we neglect other couplings. In this model, anomalous dimensions are obtained as

$$\gamma_{\mathbf{10}_i} = \sum_{k=1}^{3}\{a^u_Q(\alpha^u_{y_{ik}} + \alpha^u_{y_{ki}}) + a^d_Q\alpha^d_{y_{ik}}\} - a^u_Q\alpha^u_{y_{ii}} - c_Q\alpha'_g, \tag{A2}$$

$$\gamma_{\bar{\mathbf{5}}_i} = a^d\sum_{k=1}^{3}\alpha^d_{y_{kj}} - c_d\alpha'_g, \tag{A3}$$

$$\gamma_{H^u_{ij}} = 3a^u_H\alpha^u_{y_{ij}} + d_H\sum_{k,l}\alpha_{ij,kl} - c^u_H\alpha'_g, \tag{A4}$$

$$\gamma_{H^d_{ij}} = 3a^d_H\alpha^d_{y_{ij}} + d_H\sum_{k,l}\alpha_{kl,ij} - c^d_H\alpha'_g, \tag{A5}$$

$$\gamma_{\mathbf{24}} = a_{\mathbf{24}}\sum_{ij,kl}\alpha_{ij,kl} - c_{\mathbf{24}}\alpha'_g, \tag{A6}$$

where $a_{\mathbf{24}} = 1$, $c_{\mathbf{24}} = 10$, $d_H = 5$ and $\alpha_{ij,kl} = |\kappa_{ij,kl}|^2/(8\pi^2)$. The RG equations of Yukawa couplings are written as

$$\mu\frac{d}{d\mu}\alpha^u_{y_{ij}} = (\gamma_{\mathbf{10}_i} + \gamma_{\mathbf{10}_j} + \gamma_{H^u_{ij}})\alpha^u_{y_{ij}}, \tag{A7}$$

$$\mu\frac{d}{d\mu}\alpha^d_{y_{ij}} = (\gamma_{\mathbf{10}_i} + \gamma_{\bar{\mathbf{5}}_j} + \gamma_{H^d_{ij}})\alpha^d_{y_{ij}}, \tag{A8}$$

$$\mu\frac{d}{d\mu}\alpha_{ij,kl} = (\gamma_{H^u_{ij}} + \gamma_{H^d_{kl}} + \gamma_{\mathbf{24}})\alpha_{ij,kl}. \tag{A9}$$

This model has the IR fixed points,

$$x^{u*} = \frac{3959 - 175b'}{13635} = 0.29 - 0.013b', \quad \text{(A10)}$$

$$x^{d*} = \frac{3361 - 245b'}{9090} = 0.37 - 0.027b', \quad \text{(A11)}$$

Next we consider the model, which includes extra fields $\mathbf{5}'$ and it couples with H^d_{ij} and the adjoint field. Other extra fields are added such that this model is anomaly-free. The relevant superpotential is written as

$$W = y^u_{ij}\mathbf{10}^i\mathbf{10}^j H^u(\mathbf{5})_{ij} + y^d_{ij}\mathbf{10}^i\bar{\mathbf{5}}^j H^d(\bar{\mathbf{5}})_{ij} + \kappa'_{kl}\mathbf{5}' H^d(\bar{\mathbf{5}})^{kl}\mathbf{24}. \quad \text{(A12)}$$

The anomalous dimensions are obtained as

$$\gamma_{\mathbf{10}_i} = \sum_{k=1}^{3}\{a^u_Q(\alpha^u_{y_{ik}} + \alpha^u_{y_{ki}}) + a^d_Q\alpha^d_{y_{ik}}\} - a^u_Q\alpha^u_{y_{ii}} - c_Q\alpha'_g, \quad \text{(A13)}$$

$$\gamma_{\bar{\mathbf{5}}_i} = a^d\sum_{k=1}^{3}\alpha^d_{y_{kj}} - c_d\alpha'_g, \quad \text{(A14)}$$

$$\gamma_{H^u_{ij}} = 3a^u_H\alpha^u_{y_{ij}} - c^u_H\alpha'_g, \quad \text{(A15)}$$

$$\gamma_{H^d_{ij}} = 3a^d_H\alpha^d_{yij} + d_H\alpha'_{ij} - c^d_H\alpha'_g \quad \text{(A16)}$$

$$\gamma_{\mathbf{24}} = d_{\mathbf{24}}\sum_{ij}\alpha'_{ij} - c_{\mathbf{24}}\alpha'_g, \quad \text{(A17)}$$

$$\gamma_{\mathbf{5}'} = d_H\sum_{ij}\alpha'_{ij} - c_H\alpha'_g, \quad \text{(A18)}$$

where $\alpha'_{ij} = |\kappa_{ij}|^2/(8\pi^2)$. We can obtain the RG equations of Yukawa couplings. Then we find that they have the IR fixed points,

$$x^u = \frac{12828 - 575b'}{27270} = 0.47 - 0.021b', \quad \text{(A19)}$$

$$x^d = \frac{3432 - 365b'}{18180} = 0.19 - 0.020b'. \quad \text{(A20)}$$

Thus, as seen in these two models, values of fixed points x^{u*} and x^{d*} are shifted significantly.

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