

# Bounding the magnetic and electric dipole moments of $\nu_\tau$ from the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in $E_6$ superstring models

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(Received 24 May 2006; revised manuscript received 23 August 2006; published 8 September 2006)

We obtain bounds on the anomalous magnetic and electric dipole moments of the tau-neutrino through the reaction  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  at the  $Z_1$ -pole in the framework of a Left-Right symmetric model and a class of  $E_6$  inspired models with an additional neutral vector boson  $Z_\theta$ . We use the data collected by the L3 Collaboration at LEP. For the parameters of the  $E_6$  model we consider the mixing angle  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_\theta} = 7M_{Z_1}$ . We find that our bounds are of the same order of magnitude as those obtained in other extensions of the standard model.

DOI: [10.1103/PhysRevD.74.053002](https://doi.org/10.1103/PhysRevD.74.053002)

PACS numbers: 14.60.St, 12.15.Mm, 13.40.Em, 14.60.Fg

## I. INTRODUCTION

In the standard model (SM) [1] extended to contain right-handed neutrinos, the neutrino magnetic moment induced by radiative corrections is unobservably small,  $\mu_\nu \sim 3 \times 10^{-19}(m_\nu/1 \text{ eV})$  [2]. Current limits on these magnetic moments are several orders of magnitude larger, so that a magnetic moment close to these limits would indicate a window for probing effects induced by new physics beyond the SM [3]. Similarly, a neutrino electric dipole moment will point also to new physics and they will be of relevance in astrophysics and cosmology, as well as terrestrial neutrino experiments [4].

The existence of a heavy neutral ( $Z'$ ) vector boson is a feature of many extensions of the standard model. In particular, one (or more) additional  $U(1)'$  gauge factor provides one of the simplest extensions of the SM. Additional  $Z'$  gauge bosons appear in Grand Unified Theories (GUT's) [5], Superstring Theories [6], Left-Right Symmetric Models (LRSM) [2,7,8] and in other models such as models of composite gauge bosons [9]. The largest set of extended gauge theories are those which are based on GUT's. Popular examples are the groups  $SO(10)$  and  $E_6$ . Generically, additional  $Z$ -bosons originating from  $E_6$  grand unified theories are conveniently labeled in terms of the chain:  $E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \rightarrow SM \times U(1)_{\theta_{E_6}}$  where  $U(1)_{\theta_{E_6}}$  remains unbroken at low energies. Detailed discussions on GUTS can be found in the literature [5,6].

T. M. Gould and I. Z. Rothstein [10] reported in 1994 a bound on  $\mu_{\nu_\tau}$  obtained through the analysis of the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ , near the  $Z_1$ -resonance, with a massive neutrino and the SM  $Z_1 e^+e^-$  and  $Z_1 \nu\bar{\nu}$  couplings. In this process, the dependence of  $\mu_{\nu_\tau}$  and  $d_{\nu_\tau}$  comes from the radiation of the photon by the neutrino and antineutrino in the final state. The Feynman diagrams which give the most

important contribution to the cross-section are shown in Fig. 1. We stress here the importance of the final state radiation near the  $Z_1$  pole of a very energetic photon as compared to the conventional Bremsstrahlung. The study of the same process in the framework of a LRSM was reported recently [11]. It was found that the L3 data obtained at LEP [12] induce bounds on  $\mu_{\nu_\tau}$  and  $d_{\nu_\tau}$  which are almost independent of the mixing angle between  $Z_1$  and the new heavy  $Z_2$  gauge boson predicted in LRSM and  $M_{Z_2}$ .

Our aim in the present paper is to analyze the reaction  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  at the  $Z_1$  boson resonance and in the framework of a LRSM and in a class of  $E_6$  inspired models with a light additional neutral vector boson  $Z_\theta$  and we attribute an anomalous magnetic moment (MM) and an electric dipole moment (EDM) to a massive tau neutrino. Processes measured near the resonance serve to set limits on the tau-neutrino MM and EDM. In this paper, we take advantage of this fact to set bounds for  $\mu_{\nu_\tau}$  and  $d_{\nu_\tau}$  for various values of the mixing angle  $\phi$  of the LRSM and for  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_\theta} = 7M_{Z_1}$ , the parameters of a class of  $E_6$  inspired models, according to Ref. [13].

The L3 Collaboration evaluated the selection efficiency using detector-simulated  $e^+e^- \rightarrow \nu\bar{\nu}\gamma(\gamma)$  events, random trigger events, and large-angle  $e^+e^- \rightarrow e^+e^-$  events. A total of 14 events were found by the selection. The distributions of the photon energy and the cosine of its polar angle are consistent with SM predictions.

This paper is organized as follows: In Sec. II we describe the neutral current couplings in  $E_6$ . In Sec. III we present the calculation of the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  with an extra  $Z_\theta$  boson. Finally, we present our results and conclusions in Sec. IV.

## II. NEUTRAL CURRENT COUPLINGS IN $E_6$

In this section we describe the neutral current couplings involved in the class of  $E_6$  inspired models we are inter-

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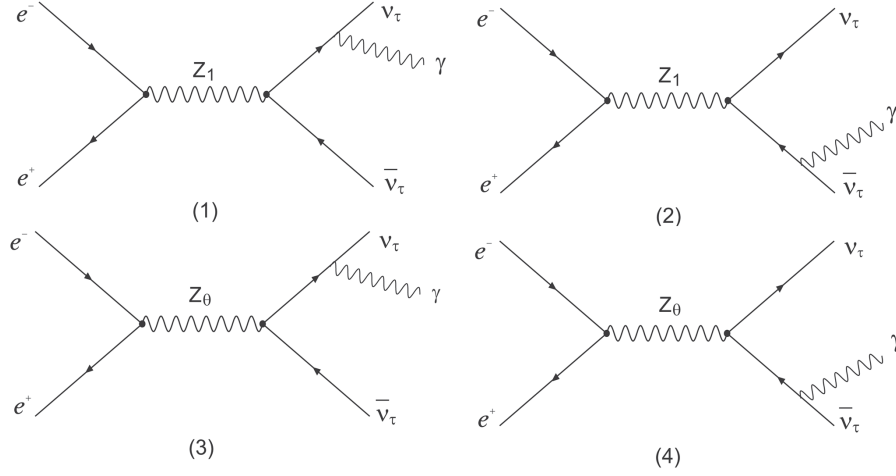


FIG. 1. The Feynman diagrams contributing to the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  in a left-right symmetric model, and in the  $E_6$  model.

ested in. Let us consider the following breakdown pattern in  $E_6$ :

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \\ &\rightarrow SM \times U(1)_{\theta_{E_6}}, \end{aligned} \quad (1)$$

where the  $SU(3)_C \times SU(2)_L \times U(1)_{Y_W}$  groups of the standard model are embedded in  $SU(5)$  of  $SO(10)$ . The couplings of the fermions to the standard model  $Z_1$  are given, as usual, by

$$Q = I_{3L} - Q_{em} \sin^2 \theta_W, \quad (2)$$

while the couplings to the  $Z_\theta$  are given by linear combinations of the  $U(1)_\chi$  and  $U(1)_\psi$  charges [14,15]:

$$\begin{aligned} Q' &= Q_\chi \cos \theta_{E_6} + Q_\psi \sin \theta_{E_6}, \\ Q'' &= -Q_\chi \sin \theta_{E_6} + Q_\psi \cos \theta_{E_6}, \end{aligned} \quad (3)$$

where the operators  $Q_\psi$  and  $Q_\chi$  are orthogonal to those of  $Q_{em}$  and that of the standard model  $Z_1$  and  $\theta_{E_6}$  is the  $Q_\chi - Q_\psi$  mixing angle in  $E_6$ .

With the extra  $Z_\theta$  neutral vector boson the neutral current Lagrangian is [16,17]

$$-\mathcal{L}_{NC} = eA^\mu J_{em\mu} + g_1 Z_1^\mu J_{Z_1\mu} + g_2 Z_\theta^\mu J_{Z_\theta\mu}, \quad (4)$$

where  $J_{em\mu}$ ,  $J_{Z_1\mu}$  and  $J_{Z_\theta\mu}$  are the electromagnetic current, the  $Z_1$  current of the standard model and the  $J_{Z_\theta\mu}$  current of the new boson, respectively, and are given by

$$\begin{aligned} J_{Z_1\mu} &= \sum_f \bar{f} \gamma_\mu (C_V^1 + C_A^1 \gamma_5) f, \\ J_{Z_\theta\mu} &= \sum_f \bar{f} \gamma_\mu (C_V' + C_A' \gamma_5) f, \end{aligned} \quad (5)$$

where  $f$  represents fermions, while

$$g_1 = (g^2 + g'^2)^{1/2} = \frac{e}{2 \sin \theta_W \cos \theta_W}, \quad g_2 = g_\theta, \quad (6)$$

$$\begin{aligned} C_V^{1e} &= -\frac{1}{2} + 2 \sin^2 \theta_W, & C_A^{1e} &= \frac{1}{2}, \\ C_V^{1\nu} &= \frac{1}{2}, & C_A^{1\nu} &= \frac{1}{2}, \end{aligned} \quad (7)$$

$$\begin{aligned} C_V^{le} &= z^{1/2} \left( \frac{\cos \theta_{E_6}}{\sqrt{6}} + \frac{\sin \theta_{E_6}}{\sqrt{10}} \right), \\ C_A^{le} &= 2z^{1/2} \frac{\sin \theta_{E_6}}{\sqrt{10}}, \\ C_V^{l\nu} &= z^{1/2} \left( -\frac{\cos \theta_{E_6}}{\sqrt{6}} + \frac{3 \sin \theta_{E_6}}{\sqrt{10}} \right), \\ C_A^{l\nu} &= z^{1/2} \left( -\frac{\cos \theta_{E_6}}{\sqrt{6}} + \frac{3 \sin \theta_{E_6}}{\sqrt{10}} \right), \end{aligned} \quad (8)$$

with

$$z = \left( \frac{g_\theta^2}{g^2 + g'^2} \right) \left( \frac{M_{Z_1}}{M_{Z_\theta}} \right)^2, \quad (9)$$

a parameter that depends on the coupling constant  $g_\theta$  and  $M_{Z_\theta}$ .

The class of  $E_6$  models we shall be interested in arise with the following specific values for the mixing angle  $\theta_{E_6}$  [14]:

$$\begin{aligned} \theta_{E_6} = 0^\circ, & \quad Z_{\theta_{E_6}} \rightarrow Z_\psi, & \theta_{E_6} = 37.8^\circ, & \quad Z_{\theta_{E_6}} \rightarrow Z', \\ \theta_{E_6} = 90^\circ, & \quad Z_{\theta_{E_6}} \rightarrow Z_\chi, & \theta_{E_6} = 127.8^\circ, & \quad Z_{\theta_{E_6}} \rightarrow Z_I, \end{aligned} \quad (10)$$

where  $Z_\psi$  is the extra neutral gauge boson arising in  $E_6 \rightarrow SO(10) \times U(1)_\psi$ ,  $Z'$  corresponds to the respective neutral gauge boson obtained if  $E_6$  is broken down to a rank-5 group,  $Z_\chi$  is the neutral gauge boson involved in  $SO(10) \rightarrow SU(5) \times U(1)_\chi$ , and  $Z_I$  is the neutral gauge boson associated to the breaking of  $E_6$  via a non-Abelian discrete symmetry to a rank-5 group [14].

### III. THE TOTAL CROSS SECTION

We will take advantage of our previous work on the LRSM and we will calculate the total cross section for the reaction  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  using the transition amplitudes given in Eqs. (14) and (15) of Ref. [11] for the LRSM for diagrams 1 and 2 of Fig. 1. For the contribution coming from diagrams 3 and 4 of Fig. 1, we use Eqs. (5) and (8) given in Sec. II for the  $E_6$  model. The respective transition amplitudes are thus given by

$$\begin{aligned} \mathcal{M}_1 &= \frac{-g^2}{8\cos^2\theta_W(l^2 - m_\nu^2)} [\bar{u}(p_3)\Gamma^\alpha(\not{l} + m_\nu) \\ &\times \gamma^\beta(a - b\gamma_5)v(p_4)] \frac{(g_{\alpha\beta} - P_\alpha P_\beta/M_{Z_1}^2)}{[(p_1 + p_2)^2 - M_{Z_1}^2 - i\Gamma_{Z_1}^2]} \\ &\times [\bar{u}(p_2)\gamma^\alpha(aC_V^{1e} - bC_A^{1e}\gamma_5)v(p_1)]\epsilon_\alpha^\lambda, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{M}_2 &= \frac{-g^2}{8\cos^2\theta_W(k^2 - m_\nu^2)} [\bar{u}(p_3)\gamma^\beta(a - b\gamma_5)(\not{k} + m_\nu) \\ &\times \Gamma^\alpha v(p_4)] \frac{(g_{\alpha\beta} - P_\alpha P_\beta/M_{Z_1}^2)}{[(p_1 + p_2)^2 - M_{Z_1}^2 - i\Gamma_{Z_1}^2]} \\ &\times [\bar{u}(p_2)\gamma^\alpha(aC_V^{1e} - bC_A^{1e}\gamma_5)v(p_1)]\epsilon_\alpha^\lambda, \end{aligned} \quad (12)$$

and for  $M'_1$  and  $M'_2$

$$M'_1 = M_1(a \rightarrow C'_V, b \rightarrow C'_A, M_{Z_1} \rightarrow M_{Z_0}), \quad (13)$$

$$M'_2 = M_2(a \rightarrow C'_V, b \rightarrow C'_A, M_{Z_1} \rightarrow M_{Z_0}), \quad (14)$$

where

$$\begin{aligned} \Gamma^\alpha &= eF_1(q^2)\gamma^\alpha + \frac{ie}{2m_\nu}F_2(q^2)\sigma^{\alpha\mu}q_\mu \\ &+ eF_3(q^2)\gamma_5\sigma^{\alpha\mu}q_\mu, \end{aligned} \quad (15)$$

is the neutrino electromagnetic vertex,  $e$  is the charge of the electron,  $q^\mu$  is the photon momentum and  $F_{1,2,3}(q^2)$  are the electromagnetic form factors of the neutrino, corresponding to charge radius, MM and EDM, respectively, at  $q^2 = 0$  [18,19], while  $\epsilon_\alpha^\lambda$  is the polarization vector of the photon.  $l$  ( $k$ ) stands for the momentum of the virtual neutrino (antineutrino), and the coupling constants  $a$  and  $b$  are given in Eq. (15) of Ref. [11], while  $C'_V$  and  $C'_A$  are given above in Eq. (8).

Using the same notation as in Ref. [10], we find that the MM, EDM, the mixing angle  $\phi$  of the LRSM as well as the mixing angle  $\theta_{E_6}$  and the mass of the additional neutral vector boson  $M_{Z_0}$  of the  $E_6$  model give a contribution to the differential cross section for the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  of the form:

$$\begin{aligned} \frac{d\sigma}{E_\gamma dE_\gamma d\cos\theta_\gamma} &= \frac{\alpha^2}{192\pi} [\mu_{\nu\tau}^2 + d_{\nu\tau}^2] [C[\phi, x_W]\mathcal{F}[\phi, s, E_\gamma, \cos\theta_\gamma] + C_1[\theta_{E_6}, M_{Z_0}, x_W]\mathcal{F}_1[M_{Z_0}, s, E_\gamma, \cos\theta_\gamma] \\ &+ 8f[M_{Z_1}, \Gamma_{Z_1}, M_{Z_0}, \Gamma_{Z_0}] \cdot \{C_2[\phi, \theta_{E_6}, M_{Z_0}, x_W]\mathcal{F}_2[s] + C_3[\phi, \theta_{E_6}, M_{Z_0}, x_W]\mathcal{F}_3[s, E_\gamma] \\ &+ C_4[\theta_{E_6}, M_{Z_0}]\mathcal{F}_4[E_\gamma] + C_5[\phi, \theta_{E_6}, M_{Z_0}, x_W]\mathcal{F}_5[s, E_\gamma, \cos\theta_\gamma] \\ &+ C_6[\phi, \theta_{E_6}, M_{Z_0}, x_W]\mathcal{F}_6[s, E_\gamma, \cos\theta_\gamma]\}] \end{aligned} \quad (16)$$

where  $E_\gamma, \cos\theta_\gamma$  are the energy and scattering angle of the photon.

The kinematics is contained in the functions

$$\begin{aligned} \mathcal{F}[\phi, s, E_\gamma, \cos\theta_\gamma] &\equiv \frac{(a^2 + b^2)(s - 2\sqrt{s}E_\gamma) + b^2E_\gamma^2\sin^2\theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2\Gamma_{Z_1}^2}, \\ \mathcal{F}_1[M_{Z_0}, s, E_\gamma, \cos\theta_\gamma] &\equiv \frac{6(s - 2\sqrt{s}E_\gamma) + 3E_\gamma^2\sin^2\theta_\gamma}{(s - M_{Z_0}^2)^2 + M_{Z_0}^2\Gamma_{Z_0}^2}, \quad \mathcal{F}_2[s] \equiv 4\sqrt{s}, \\ \mathcal{F}_3[s, E_\gamma] &\equiv 2\sqrt{s}E_\gamma, \quad \mathcal{F}_4[E_\gamma] \equiv \sqrt{15}E_\gamma, \quad \mathcal{F}_5[s, E_\gamma, \cos\theta_\gamma] \equiv -\left(s + \frac{1}{2}E_\gamma^2\sin^2\theta_\gamma\right), \\ \mathcal{F}_6[s, E_\gamma, \cos\theta_\gamma] &\equiv \left(s - E_\gamma^2 + \frac{1}{2}E_\gamma^2\sin^2\theta_\gamma\right), \end{aligned} \quad (17)$$

$$f(s, M_{Z_1}, \Gamma_{Z_1}, M_{Z_0}, \Gamma_{Z_0}) \equiv \frac{-2[(s - M_{Z_1}^2)(s - M_{Z_0}^2) + M_{Z_1}\Gamma_{Z_1}M_{Z_0}\Gamma_{Z_0}]}{[(s - M_{Z_1}^2)(s - M_{Z_0}^2) + M_{Z_1}\Gamma_{Z_1}M_{Z_0}\Gamma_{Z_0}]^2 + [(s - M_{Z_0}^2)M_{Z_1}\Gamma_{Z_1} - (s - M_{Z_1}^2)M_{Z_0}\Gamma_{Z_0}]^2}. \quad (18)$$

The coefficients  $C, C_1, \dots, C_6$  are given by

$$\begin{aligned}
\mathcal{C}[\phi, x_W] &\equiv \frac{[\frac{1}{2}(a^2 + b^2) - 4a^2x_W + 8a^2x_W^2]}{x_W^2(1 - x_W)^2}, & \mathcal{C}_1[\theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{(C_V^2 + C_A^2)}{x_W^2(1 - x_W)^2}, \\
\mathcal{C}_2[\phi, \theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{[2ax_W - \frac{1}{2}(a + b)][C_V^e C_A^e - (C_A^e)^2]}{x_W^2(1 - x_W)^2}, \\
\mathcal{C}_3[\phi, \theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{[2ax_W + \frac{1}{2}(a + b)][3(C_A^e)^2 - (C_V^e - C_A^e)^2]}{x_W^2(1 - x_W)^2} & \mathcal{C}_4[\theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{[3(C_A^e)^2 + (C_V^e - C_A^e)^2]}{x_W^2(1 - x_W)^2}, \\
\mathcal{C}_5[\phi, \theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{3(C_A^e)^2[2ax_W + \frac{1}{2}(a + b + 1)] + [4ax_W - (a + b)][C_V^e C_A^e - (C_A^e)^2]}{x_W^2(1 - x_W)^2}, \\
\mathcal{C}_6[\phi, \theta_{E_6}, M_{Z_\theta}, x_W] &\equiv \frac{[2ax_W - \frac{1}{2}a](C_V^e - C_A^e)^2}{x_W^2(1 - x_W)^2}, \tag{19}
\end{aligned}$$

where  $C_V$  and  $C_A$  are now given by:

$$\begin{aligned}
C_V &= z \left( \frac{\cos\theta_{E_6}}{\sqrt{6}} + \frac{\sin\theta_{E_6}}{\sqrt{10}} \right) \left( -\frac{\cos\theta_{E_6}}{\sqrt{6}} + \frac{3\sin\theta_{E_6}}{\sqrt{10}} \right), \\
C_A &= 2z \frac{\sin\theta_{E_6}}{\sqrt{10}} \left( -\frac{\cos\theta_{E_6}}{\sqrt{6}} + \frac{3\sin\theta_{E_6}}{\sqrt{10}} \right), \tag{20}
\end{aligned}$$

with  $x_W \equiv \sin^2\theta_W$ .

In the above expressions, the function  $\mathcal{F}$  includes the contribution coming from the exchange of the SM/LRSM  $Z_1$  gauge boson,  $\mathcal{F}_1$  includes the contribution arising from the exchange of the heavy gauge boson  $Z_\theta$ , while the function  $f$  contains the interference coming from both exchanges. Taking the limit when  $M_{Z_\theta} \rightarrow \infty$  and the mixing angle  $\phi = 0$ , the expressions for  $C_V^{e,\nu}$  and  $C_A^{e,\nu}$  reduce to  $C_V^{e,\nu} = C_A^{e,\nu} = C_V = C_A = 0$  and Eq. (16) reduces to the expression (3) given in Ref. [10] for the SM. On the other hand, taking the limit when  $M_{Z_\theta} \rightarrow \infty$ , Eq. (16) reduces to the expressions (25) given in Ref. [11] for the LRSM. Finally, if the mixing angle is taken as  $\phi = 0$  Eq. (16) reduces to the expression (24) given in Ref. [13] for the  $E_6$  model.

#### IV. RESULTS AND CONCLUSIONS

In order to evaluate the integral of the total cross section as a function of the parameters of the LRSM- $E_6$  models, that is to say,  $\phi$ ,  $M_{Z_\theta}$  and the mixing angle  $\theta_{E_6}$ , we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over  $\theta_\gamma$  from  $44.5^\circ$  to  $135.5^\circ$  and  $E_\gamma$  from 15 GeV to 100 GeV for various fixed values of the mixing angle  $\phi = -0.009, -0.004, 0, 0.004$  and for  $\theta_{E_6} = 37.8^\circ$  (which corresponds to  $Z_\theta \rightarrow Z'$ ) and  $M_{Z_\theta} = 7M_{Z_1}$  according to Ref. [13]. Using the following numerical values:  $\sin^2\theta_W = 0.2314$ ,  $M_{Z_1} = 91.18$  GeV,  $\Gamma_{Z_\theta} = \Gamma_{Z_1} = 2.49$  GeV and  $z = (\frac{3}{5}\sin^2\theta_W) \times (\frac{M_{Z_1}}{M_{Z_\theta}})^2$  we obtain the cross section  $\sigma = \sigma(\phi, \theta_{E_6}, M_{Z_\theta}, \mu_{\nu_\tau}, d_{\nu_\tau})$ .

For the mixing angle  $\phi$  between  $Z_1$  and  $Z_2$  of the LRSM, we use the reported data of Maya *et al.* [20]:

$$-9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3}, \tag{21}$$

with a 90% C.L.

Since we have calculated the cross-section at the  $Z_1$  pole, i.e. at  $s = M_{Z_1}^2$ , the value of  $\sin^2\theta_W$  is not affected by the  $Z_\theta$  physics [21,22]. Variation of the  $\Gamma_{Z_\theta}$  is taken in the range from 0.15 to 2.0 times  $\Gamma_{Z_1}$  in the results of the CDF Collaboration [23]. So we take  $\Gamma_{Z_\theta} = \Gamma_{Z_1}$  as a special case of this variation.

As was discussed in Ref. [10],  $N \approx \sigma(\phi, \theta_{E_6}, M_{Z_\theta}, \mu_{\nu_\tau}, d_{\nu_\tau}) \mathcal{L}$ . Using the Poisson statistic [12,24], we require that  $N \approx \sigma(\phi, \theta_{E_6}, M_{Z_\theta}, \mu_{\nu_\tau}, d_{\nu_\tau}) \mathcal{L}$  be less than 14, with  $\mathcal{L} = 137$  pb $^{-1}$ , according to the data reported by the L3 Collaboration Ref. [12] and references therein. Taking this into consideration, we can get a bound for the tau-neutrino magnetic moment as a function of  $\phi$ ,  $\theta_{E_6}$  and  $M_{Z_\theta}$  with  $d_{\nu_\tau} = 0$ . The values obtained for this bound for several values of  $\phi$  with  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_\theta} = 7M_{Z_1}$  are included in Table I.

The results obtained in Table I are in agreement with the literature [10,12,18,25–28]. However, if the photon angle and energy are  $0 \leq \theta_\gamma \leq \pi$  and  $15$  GeV  $\leq E_\gamma \leq 100$  GeV with  $\phi = -0.009, -0.004, 0, 0.004$ ,  $\theta_{E_6} = 37.8^\circ$ ,  $M_{Z_\theta} = 7M_{Z_1}$ ,  $N = 14$  and  $\mathcal{L} = 48$  pb $^{-1}$ , we obtained the results given in Table II.

TABLE I. Bounds on the  $\mu_{\nu_\tau}$  magnetic moment and  $d_{\nu_\tau}$  electric dipole moment for different values of the mixing angle  $\phi$  with  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_\theta} = 7M_{Z_1}$ . We have applied the cuts used by L3 for the photon angle and energy.

$\phi$	$\mu_{\nu_\tau}$ ( $10^{-6} \mu_B$ )	$d_{\nu_\tau}$ ( $10^{-17} e$ cm)
-0.009	3.37	6.50
-0.004	3.33	6.43
0	3.31	6.40
0.004	3.30	6.33

TABLE II. Bounds on the  $\mu_{\nu_\tau}$  magnetic moment and  $d_{\nu_\tau}$  electric dipole moment for different values of the mixing angle  $\phi$  with  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_0} = 7M_{Z_1}$ . In this case, we did not use cuts for the photon angle and energy.

$\phi$	$\mu_{\nu_\tau}$ ( $10^{-6} \mu_B$ )	$d_{\nu_\tau}$ ( $10^{-17} e$ cm)
-0.009	1.85	3.58
-0.004	1.84	3.55
0	1.83	3.53
0.004	1.82	3.52

The previous analysis and comments can readily be translated to the EDM of the  $\tau$ -neutrino with  $\mu_{\nu_\tau} = 0$ . The resulting bounds for the EDM as a function of  $\phi$ ,  $\theta_{E_6}$  and  $M_{Z_0}$  are shown in Tables I and II.

We plot the total cross section in Fig. 2 as a function of the mixing angle  $\phi$  for the bounds of the magnetic moment given in Tables I and II with  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_0} = 7M_{Z_1}$ . We reproduce Fig. 2 of Ref. [11].

We have determined bounds on the magnetic moment and the electric dipole moment of a massive tau neutrino in the framework of a LRSM and a class of  $E_6$  inspired models with a light additional neutral vector boson, as a function of  $\phi$ ,  $M_{Z_0}$  and the mixing angle  $\theta_{E_6}$ , as shown in Tables I and II.

In a previous paper [11] we estimated bounds on the anomalous magnetic moment and the electric dipole moment of the tau neutrino through the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  in the context of a LRSM at the  $Z_1$  pole. We found that the bounds are almost independent of the mixing angle  $\phi$  of the model. In the present paper we reproduce these bounds for  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_0} = 7M_{Z_1}$ , corresponding to the  $E_6$  superstring models. In Ref. [13], Aydemir *et al.* analyzed the same process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  also in  $E_6$  models. Their

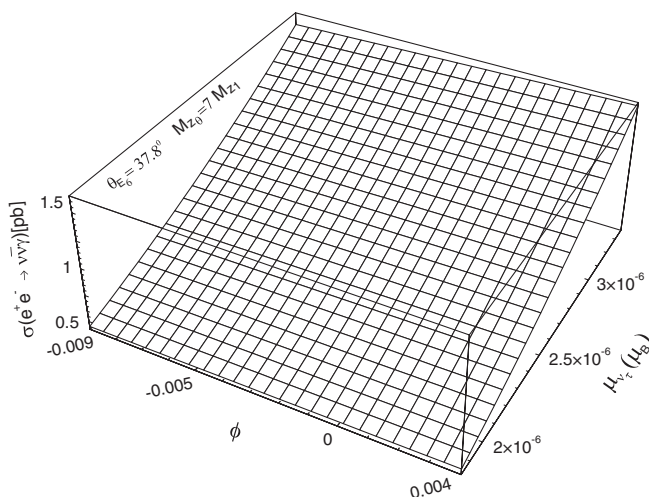


FIG. 2. The total cross section for  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  as a function of  $\phi$  and  $\mu_{\nu_\tau}$  (Tables I and II), with  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_0} = 7M_{Z_1}$ .

analytical and numerical results for MM are similar than ours, and we are able to reproduce their limits for  $\phi = 0$ ,  $0 \leq \theta_\gamma \leq \pi$  and  $\mathcal{L} = 48 \text{ pb}^{-1}$ . These results are in agreement with the bounds obtained in previous studies [27–29], but are well above the SM effects induced by one-loop diagrams [26].

Other upper limits on the tau-neutrino magnetic moment reported in the literature are  $\mu_{\nu_\tau} < 3.3 \times 10^{-6} \mu_B$  (90% C.L.) from a sample of  $e^+e^-$  annihilation events collected with the L3 detector at the  $Z_1$  resonance corresponding to an integrated luminosity of  $137 \text{ pb}^{-1}$  [12];  $\mu_{\nu_\tau} \leq 2.7 \times 10^{-6} \mu_B$  (95% C.L.) at  $q^2 = M_{Z_1}^2$  from measurements of the  $Z_1$  invisible width at LEP [18];  $\mu_{\nu_\tau} \leq 2.62 \times 10^{-6}$  in the effective Lagrangian approach at the  $Z_1$  pole [29];  $\mu_{\nu_\tau} < 1.83 \times 10^{-6} \mu_B$  (90% C.L.) from the analysis of  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  at the  $Z_1$ -pole, in a class of  $E_6$  inspired models with a light additional neutral vector boson [13]; from the order of  $\mu_{\nu_\tau} < O(1.1 \times 10^{-6} \mu_B)$  Keiichi Akama *et al.* derive and apply model-independent bounds on the anomalous magnetic moments and the electric dipole moments of leptons and quarks due to new physics [30]. However, the limits obtained in Ref. [30] are for the tau neutrino with an upper bound of  $m_\tau < 18.2 \text{ MeV}$  which is the current experimental limit. It was pointed out in Ref. [30] however, that the upper limit on the mass of the electron neutrino and data from various neutrino oscillation experiments together imply that none of the active neutrino mass eigenstates is heavier than approximately 3 eV. In this case, the limits given in Ref. [30] are improved by 7 orders of magnitude. The limit  $\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$  (90% C.L.) is obtained at  $q^2 = 0$  from a beam-dump experiment with assumptions on the  $D_s$  production cross section and its branching ratio into  $\tau\nu_\tau$  [31], thus severely restricting the cosmological annihilation scenario [32]. Our results in Tables I and II for  $\phi = -0.009, -0.004, 0, 0.004$ ,  $\theta_{E_6} = 37.8^\circ$  and  $M_{Z_0} = 7M_{Z_1}$  confirm the bound obtained by the L3 Collaboration [12] as well as the bound obtained in Ref. [13].

In the case of the electric dipole moment, other upper limits reported in the literature are [18,30]:

$$|d(\nu_\tau)| \leq 5.2 \times 10^{-17} e \text{ cm} \quad 95\% \text{C.L.}, \quad (22)$$

$$|d(\nu_\tau)| < O(2 \times 10^{-17} e \text{ cm}). \quad (23)$$

Our bounds for the EMD given in Table II compare favorably with the limits given in Eqs. (22) and (23). On the other hand, it seems that in order to improve these limits it might be necessary to study direct  $CP$ -violating effects [33].

In summary, we conclude that the estimated bounds for the tau-neutrino magnetic and electric dipole moments are almost independent of the experimental allowed values of the  $\phi$  parameter of the LRSM. In the limit  $\phi = 0$  and  $M_{Z_0} \rightarrow \infty$ , our bound takes the value previously reported in

Ref. [10] for the SM. The bounds in the MM and the EDM are not affected for the additional neutral vector boson  $Z_\theta$  since its mass is higher than  $Z_1$  at  $\sqrt{s} = M_{Z_1}$ . But at higher center-of-mass energies  $\sqrt{s} \sim M_{Z_\theta}$ , the  $Z_\theta$  contribution to the cross section becomes comparable with  $Z_1$ . In addition, the analytical and numerical results for the total cross-section have never been reported in the literature before

and could be of some practical use for the scientific community.

## ACKNOWLEDGMENTS

We acknowledge support from CONACyT and SNI (México).

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