Black holes on Eguchi-Hanson space in five-dimensional Einstein-Maxwell theory

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In this paper, we construct a pair of black holes on the Eguchi-Hanson space as a solution in the five-dimensional Einstein-Maxwell theory. The metric and the gauge potential one-form are given by

$$ds^2 = -H^{-2}(r, \tilde{\theta})dT^2 + H(r, \tilde{\theta})ds^2_{\rm EH}, \tag{1}$$

$$\boldsymbol{A} = \pm \frac{\sqrt{3}}{2} H^{-1}(r, \tilde{\theta})dT, \tag{2}$$

with

$$ds^2_{\rm EH} = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + \frac{r^2}{4}\left[\left(1 - \frac{a^4}{r^4}\right)(d\tilde{\psi} + \cos\tilde{\theta} d\tilde{\phi})^2 + d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2\right], \tag{3}$$

$$H(r, \tilde{\theta}) = 1 + \frac{m_1}{r^2 - a^2\cos\tilde{\theta}} + \frac{m_2}{r^2 + a^2\cos\tilde{\theta}}, \tag{4}$$

where a and $m_j (j = 1, 2)$ are constants, $0 \le \tilde{\theta} \le \pi$, $0 \le \tilde{\phi} \le 2\pi/n$, ($n$: natural number) and $0 \le \tilde{\psi} \le 2\pi$.

Equation (3) is the metric form of the Eguchi-Hanson space [1]. The Eguchi-Hanson space has a S^2-bolt at $r = a$, where the Killing vector field $\partial/\partial\tilde{\psi}$ vanishes. The function $H(r, \tilde{\theta})$ is a harmonics on the Eguchi-Hanson space (3).

As is seen later, two black holes are located on the north pole ($\tilde{\theta} = 0$) and the south pole ($\tilde{\theta} = \pi$) on the S^2-bolt. The asymptotic behavior of the metric (1) near the spatial infinity $r \to \infty$ becomes

$$ds^2 \simeq -dT^2 + dr^2 + \frac{r^2}{4}\left[d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\psi}^2 + (d\tilde{\phi} + \cos\tilde{\theta} d\tilde{\psi})^2\right]. \tag{5}$$

Since the $T = \text{const}$ surface has the structure of lens space $L(2n; 1)$, this solution is asymptotically locally flat. The Komar mass, $M_{\rm Komar}$ and the total electric charge, Q at the spatial infinity are given by

$$M_{\rm Komar} = \frac{\sqrt{3}}{2}|Q| = \frac{3\pi(m_1 + m_2)}{8nG}, \tag{6}$$

where G is the five-dimensional gravitational constant.

In order to clarify the physical properties of the solution, we introduce the coordinates as follows [2],

$$R = a\sqrt{\frac{r^4}{a^4} - \sin^2\tilde{\theta}}, \qquad \tan\theta = \sqrt{1 - \frac{a^4}{r^4}}\tan\tilde{\theta},$$

$$\phi = \tilde{\psi}, \qquad \psi = 2\tilde{\phi}.$$

$$(0 \le \theta \le \pi, \quad 0 \le \phi \le 2\pi, \quad 0 \le \psi \le 4\pi/n) \tag{7}$$

Then, the metric takes the form of

$$ds^2 = -H^{-2}(R, \theta)dT^2 + H(R, \theta)ds^2_{\rm EH}, \tag{8}$$

with

$$ds^2_{\rm EH} = V^{-1}(R, \theta)[dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)] + V(R, \theta)\left(\frac{a}{8}d\psi + \omega_\phi d\phi\right)^2, \tag{9}$$

$$H(R, \theta) = 1 + \frac{m_1/a}{|\boldsymbol{R} - \boldsymbol{R}_1|} + \frac{m_2/a}{|\boldsymbol{R} - \boldsymbol{R}_2|}, \tag{10}$$

$$V^{-1}(R, \theta) = \frac{a/8}{|\boldsymbol{R} - \boldsymbol{R}_1|} + \frac{a/8}{|\boldsymbol{R} - \boldsymbol{R}_2|}, \tag{11}$$

$$\omega_\phi(R, \theta) = \frac{a}{8}\left(\frac{R\cos\theta - a}{\sqrt{R^2 + a^2 - 2aR\cos\theta}} + \frac{R\cos\theta + a}{\sqrt{R^2 + a^2 + 2aR\cos\theta}}\right), \tag{12}$$

where $\boldsymbol{R} = (x, y, z)$ is the position vector on the three-dimensional Euclid space and $\boldsymbol{R}_1 = (0, 0, a)$, $\boldsymbol{R}_2 = (0, 0, -a)$. The metric (9) is the Gibbons-Hawking two-center form of the Eguchi-Hanson space [2,3]. It is manifest in the coordinate that the space has two nut singularities at $\boldsymbol{R} = \boldsymbol{R}_j$ where the Killing vector field $\partial/\partial\psi$ vanishes.

The function $H(R, \theta)$ is the harmonics given by (4) on the Eguchi-Hanson space in the Gibbons-Hawking coor-

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dinates (9). The harmonics $H(R,\theta)$ converts nut singularities on the Eguchi-Hanson space to regular hypersurfaces in the total spacetime. Since each hypersurface $\boldsymbol{R} = \boldsymbol{R}_j$ becomes a Killing horizon with respect to the Killing vector field $\partial/\partial T$, and each three-dimensional section of them with $T = \text{const}$ has finite area, then the hypersurfaces $\boldsymbol{R} = \boldsymbol{R}_j$ are event horizons.

Since the Kretschmann invariant $R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}$ has a finite value on each horizon, we see that the geometry on the horizons is regular. Even if one of m_j (for an example m_2) vanishes, which corresponds to a single black hole and a naked nut charge with the value $a/8$, the horizon is regular. The spacetime is regular in the case of $n = 1$ but it has a nut singularity at $\boldsymbol{R} = \boldsymbol{R}_2$ in the case of $n \geq 2$.

The induced metric on the spatial cross section of the jth horizon is given by

$$ds^2_{\text{Horizon}} = \frac{m_j}{8}[d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + \cos\theta d\phi)^2], \quad (0 \leq \psi \leq 4\pi/n) \tag{13}$$

which is the lens space $L(n;1)$. The geometry near horizons of this solution is similar to the multi-black hole solutions on the Gibbons-Hawking multi-instanton space [4], but the asymptotic structures are different. Although both are asymptotically locally flat, the former is isotropic in four spatial dimensions while the latter has a compact dimension as same as Kaluza-Klein black holes discussed in Refs. [4,5].

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