Entropy of the microwave background radiation in the observable universe

Yun-Song Piao

College of Physical Sciences, Graduate School of Chinese Academy of Sciences, YuQuan Road 19A, Beijing 100049, China

We show that the cosmological constant at late-time places a bound on the entropy of microwave background radiation deposited in the future event horizon of a given observer, $S \leq S_{\Lambda_0}^{3/4}$. This bound is independent of the energy scale of reheating and the FRW evolution after reheating. We also discuss why the entropy of microwave background in our observable universe has its present value.

Recent observations have indicated that the universe is accelerating, which suggests that the universe might be being dominated by a dark energy component with state equation $\kappa = p/\rho < -1/3$, see Refs. [1–5] for reviews on various aspects. The simplest form of dark energy is a small and positive cosmological constant, which can fit data very nicely and is phenomenologically one of the most appealing choices so far. The universe with a cosmological constant will generally asymptotically approach a dS equilibrium and any observer in it will eventually be surrounded by an event horizon. This limits the region of universe accessible to any given observer, see Refs. [6–10] for some discussions on interesting issues.

Recently, Fischler *et al.* [11] have pointed out that in a universe dominated by a positive cosmological constant at late time there is a bound on the total entropy of radiation in the entire universe, whose scales like $S_{\max} \sim \Lambda_0^{-3/4}$, where Λ_0 is the value of cosmological constant observed. This is parametrically smaller than the entropy of dS, $S_{\Lambda_0} \sim \Lambda_0^{-1}$. Further Banks and Fischler [12] argued that this entropy bound places a upper limit on the total efoldings number $\mathcal{N}_{\text{tot}}$ of inflation that can be described by a conventional quantum field theory, see also related Refs. [13]. However, in subsequent discussion in Ref. [14] based on the covariant entropy bounds [15], see also Refs. [16], it has been shown that no bound on the total $\mathcal{N}_{\text{tot}}$ was found. The reason is that the existence of a horizon only constrains the entropy visible to a given observer on any homogeneous spacelike slice to be less than the area of its interaction with the past light cone of the observer, but can not restrict the entropy outside of the causal future of this given observer. However, there is actually a bound $\mathcal{N}_{\text{obs}}$ on the efoldings number that will ever be observable to a given observer at late time [17]. If the observable universe accelerates forever, the observer will never see more efoldings number than $\mathcal{N}_{\text{obs}}$. Thus similarly, it may be expected that there should also be a bound on the entropy of microwave background radiation deposited in the future event horizon of a given observer. In this brief report, we will show that this bound is $S \leq S_{\Lambda_0}^{3/4}$, independent of the energy scale of reheating and the FRW evolution after reheating.

We assume that the inflation ended at some time t_e. The efoldings number $\mathcal{N}$ corresponding to the present Hubble scale is given by

$$\mathcal{N} \equiv \ln\left(\frac{a_e h_e}{a_k h_k}\right) \equiv \ln\left(\frac{a_e h_e}{a_0 h_0}\right) \tag{1}$$

where $h \equiv \dot{a}/a$, and the subscript 'k' labels the time of inflation corresponding the present Hubble scale and "0" is the present time, In an universe without the future event horizon, a patient observer would be able to see arbitrarily far back to inflation, since $a_0 h_0$ can be made arbitrarily small at late time. However, the existence of event horizon $1/h_0$ constrains the size r of the reheating surface visible to any given observer at late time. Taking (effectively) $a_0 \simeq 1/h_0$, we can obtain this size $r = a_e \simeq h_e^{-1} e^{\mathcal{N}}$. Thus in Planck unit, the entropy S released into this region of the reheating surface is simply the product of the entropy density and the volume,

$$S \simeq \sigma r^3 \simeq \rho_e^{1/(1+\kappa)} h_e^{-3} e^{3\mathcal{N}} \tag{2}$$

where we have assume that the observable universe is filled with fluid $p = \kappa\rho$ after reheating and $\sigma \sim \rho^{1/(1+\kappa)}$ has been used. We also assume that the reheating after inflation is perfectly efficient, thus the reheating energy scale ρ_e can approximately equal to the Hubble scale $\sqrt{h_e}$ during inflation, and also hereafter there are not other reheating processes bringing a large number of entropy, such as the decay of some extra productions. During following FRW evolution, if the observable universe is still dominated by this kind of fluid, we have $\rho \sim 1/a^{3(1+\kappa)}$. Thus from (1), we obtain the efoldings number

$$e^{\mathcal{N}} = \frac{a_e h_e}{a_0 h_0} = \left(\frac{\Lambda_0}{\rho_e}\right)^{1/3(1+\kappa)}\left(\frac{\rho_e}{\Lambda_0}\right)^{1/2} = \left(\frac{\rho_e}{\Lambda_0}\right)^{(1+3\kappa)/6(1+\kappa)} \tag{3}$$

where $\rho_0 \simeq \Lambda_0$ has been taken. Thus instituting it into (2), the entropy in observable universe can be given by

$$S \simeq \Lambda_0^{-(1+3\kappa)/2(1+\kappa)} \tag{4}$$

Thus it seems that the capacity of store information in the event horizon at late time only depends on the state equation of fluid filling it. For the state equation $\kappa = 1$, such as

black hole gas [18], $S \simeq \Lambda_0^{-1} \simeq S_{\Lambda_0}$. For the radiation $\kappa = 1/3$, the entropy is

$$S \simeq \Lambda_0^{-3/4} \simeq S_{\Lambda_0}^{3/4} \qquad (5)$$

In Ref. [12], it was pointed out that to avoid a big crunch the total entropy of microwave background radiation in a system should be bounded from above by (5). However, here we point out that what (5) bound is only the entropy deposited in the region of event horizon at late time, which is accessible to any given observer in future event horizon. Taking the present observed value of cosmological constant, $\Lambda_0 \sim 10^{-123}$, we have $S \simeq 10^{91}$. This is the maximal entropy which the microwave background radiation deposited in the event horizon potentially approaches.

We discuss some interesting cases for further arguments in the following. The observable universe actually consists of radiation and matter. The energy density of matter will exceed that of radiation at some time $t_{\rm eq}$. From (1) and (3), we have

$$e^{\mathcal{N}} = \frac{a_e h_e}{a_{\rm eq} h_{\rm eq}} \cdot \frac{a_{\rm eq} h_{\rm eq}}{a_0 h_0} = \left(\frac{\rho_e}{\rho_{\rm eq}}\right)^{1/4} \left(\frac{\rho_{\rm eq}}{\Lambda_0}\right)^{1/6} \qquad (6)$$

where the subscript "eq" labels the time of matter-radiation equality. Thus from (2), we can obtain

$$S \simeq \rho_e^{3/4} \cdot \left(\frac{\rho_e}{\rho_{\rm eq}}\right)^{3/4} \left(\frac{\rho_{\rm eq}}{\Lambda_0}\right)^{1/2} / \rho_e^{3/2} \simeq S_{\Lambda_0}^{3/4} \left(\frac{\Lambda_0}{\rho_{\rm eq}}\right)^{1/4} \le S_{\Lambda_0}^{3/4} \qquad (7)$$

In our observable universe, $\Lambda_0/\rho_e \sim 10^{-12}$, thus we have $S \sim 10^{88}$, which is just the present entropy of microwave background radiation. This gives a simple explanation why the entropy of microwave background in our observable universe is several orders of magnitude lower but not far lower than the entropy bound (5). The reason is that the matter-dominated universe only began in the not far past. The higher the energy density of matter is, the earlier it will dominate the universe and thus the larger $\rho_{\rm eq}$ is. Thus the entropy of microwave background in observable universe will be lower. The limit case is $\rho_{\rm eq} \simeq \rho_e$, i.e. the universe just entered into the matter-dominated phase shortly after reheating, in which the observable universe will has lowest radiation entropy deposited in the future event horizon of a given observer. Whereas the higher the energy density of radiation is, the smaller $\rho_{\rm eq}$ will be, thus the higher the radiation entropy will be. The limit case is $\rho_{\rm eq} \simeq \Lambda_0$, i.e. the radiation still dominated the universe up to date, which corresponds to saturate the entropy bound (5), i.e. $S \sim 10^{91}$.

We may suppose that the observable universe can be filled with some other fluid after reheating and will be still dominated by it before entering into the radiation-dominated phase. This corresponds to

$$e^{\mathcal{N}} = \frac{a_e h_e}{a_{\rm eq'} h_{\rm eq'}} \cdot \frac{a_{\rm eq'} h_{\rm eq'}}{a_{\rm eq} h_{\rm eq}} \cdot \frac{a_{\rm eq} h_{\rm eq}}{a_0 h_0}$$

$$= \left(\frac{\rho_e}{\rho_{\rm eq'}}\right)^{(1+3\kappa)/6(1+\kappa)} \left(\frac{\rho_{\rm eq'}}{\rho_{\rm eq}}\right)^{1/4} \left(\frac{\rho_{\rm eq}}{\Lambda_0}\right)^{1/6} \qquad (8)$$

where the subscript 'eq'' labels the time of fluid-radiation equality. Note that at time $t_{\rm eq'}$ the radiation begins to dominate the universe and have the energy density $\rho_{\rm eq'}$. Thus we can back to the reheating surface and obtain an equivalent energy density ρ_r of radiation at the time t_e

$$\frac{\rho_r}{\rho_{\rm eq'}} \simeq \left(\frac{\rho_e}{\rho_{\rm eq'}}\right)^{4/3(1+\kappa)} \qquad (9)$$

Thus instituting (8) and (9) into (2), we can obtain

$$S \simeq \rho_r^{3/4} \cdot \left(\frac{\rho_e}{\rho_{\rm eq'}}\right)^{(1+3\kappa)/2(1+\kappa)} \left(\frac{\rho_{\rm eq'}}{\rho_{\rm eq}}\right)^{3/4} \left(\frac{\rho_{\rm eq}}{\Lambda_0}\right)^{1/2} / \rho_e^{3/2}$$

$$\simeq S_{\Lambda_0}^{3/4} \left(\frac{\Lambda_0}{\rho_{\rm eq}}\right)^{1/4} \qquad (10)$$

This is the same as (7). Thus the intervening of other fluid phase before the radiation-dominated phase does not affect our result.

We can also forward the entropy of microwave background radiation to the future and note that after the time t_0, the observable universe will enter into a dS phase, thus we have

$$e^{\mathcal{N}'} = \frac{a_e h_e}{a_{\rm eq} h_{\rm eq}} \cdot \frac{a_{\rm eq} h_{\rm eq}}{a_0 h_0} \cdot \frac{a_0 h_0}{a h_0} = e^{\mathcal{N} - \mathcal{N}(t)} \qquad (11)$$

where (6) has been used and $\mathcal{N}(t) = h_0(t - t_0)$. We can see that the comoving Hubble scale ah begins to grow, as in early inflation, which results in the decrease of the effective efoldings number $\mathcal{N}'$, which is the efoldings number visible to any given observable at late time of t_0 [17], and thus the entropy S of microwave background radiation. Let us see this case further. Note that at the time $t_f - t_0 = \mathcal{N}/h_0$, the comoving Hubble scale equals its value at reheating, thus we have $\mathcal{N}' \simeq 0$. This means that all perturbations including very last perturbation generated during inflation will be pushed back out of the horizon again. Instituting $\mathcal{N}'$ at the time t_f into (2), we obtain $S \simeq \rho_e^{-3/4}$, in which $\kappa = 1/3$ has been taken. This is just $S_{\rm inf}^{3/4}$ of dS entropy during inflation. Note that

$$S \sim h_0^{-3} T^3 \qquad (12)$$

we can obtain the temperature $T_f \sim h_0 \rho_e^{-1/4}$ of microwave background at the time t_f. This temperature is still far larger than the characteristic temperature $T_h = h_0/2\pi$ of event horizon [19]. However, since the observable universe will accelerate forever, the temperature of microwave background can be eventually redshift to a point where $T \sim h_0$ and the noise of Hawking radiation will begin to overwhelm the microwave background. From (12), we

have the eventual radiation entropy $S \sim 1$. This means that after this time it would be impossible to extract any information about early universe from the microwave background.

The above discussions also apply to the case that the current and future evolution is dominated by phantom, in which $\kappa < -1$, and thus a well-defined event horizon exists. During phantom-dominated the energy density of phantom is increased with the time. This leads that the scale of event horizon decreases continuously, and from (5), thus the value of entropy bound on the microwave background radiation deposited in the event horizon. This result is different from the constant entropy bound with the cosmological constant. The entropy bound varies monotonously makes us able to expect that there should be a maximal entropy bound, which can be seen as follows. The phantom-dominated evolution only begins at present, and its energy density is determined by current observations and is approximately $\Lambda_0 \sim 10^{-123}$. Thus from (5), we have the entropy bound $S \sim 10^{91}$ on the microwave background radiation deposited in "present" event horizon. Now we forward the entropy bound to the future and have

$$e^{\mathcal{N}'} = \frac{a_e h_e}{a_{\text{eq}} h_{\text{eq}}} \cdot \frac{a_{\text{eq}} h_{\text{eq}}}{a_0 h_0} \cdot \frac{a_0 h_0}{ah} = e^{\mathcal{N} - \mathcal{N}(t)} \quad (13)$$

where (6) has been used. Reconsidering Eq. (3), we have

$$e^{\mathcal{N}(t)} = \frac{ah}{a_0 h_0} = \left(\frac{\rho}{\Lambda_0}\right)^{(1+3\kappa)/6(1+\kappa)} \quad (14)$$

where ρ is the energy density of phantom. We can see that

since $\kappa < -1$ and $\rho > \Lambda_0$, $\mathcal{N}(t)$ is always positive. Thus instituting $\mathcal{N}'$ of (13) to (2), we can find that the bound of entropy on the microwave background radiation deposited in the event horizon is decreased in the future, and thus the bound at present is the maximal entropy bound.

In summary, we have shown that the cosmological constant at late-time places a bound on the entropy of microwave background radiation deposited in the future event horizon of a given observer, $S \le S_{\Lambda_0}^{3/4} \sim 10^{91}$, which is independent of the energy scale of reheating and the FRW evolution after reheating. However, this dose not means that there is a limit on the total radiation entropy generated after inflation, since if inflation lasts long enough, the total entropy may exceed greatly the above bound. In fact due to the presence of cosmological constant, not all regions of reheating surface lie inside the causal patch of a given late-time observer, thus the entropy deposited in the future event horizon of this observer is only a portion of total entropy, which it is that obeys our bound. The entropy of microwave background in our observable universe is $S_{\Lambda_0}^{3/4}(\Lambda_0/\rho_{\text{eq}})^{1/4} \sim 10^{88}$ since $\Lambda_0 \sim 10^{-12}\rho_{\text{eq}}$. The reason that it seems not too far lower than the bound value is that the matter-dominated phase only began in the not too far past. This work might bring a litter insight why our observable universe look like so.

[1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[2] S. M. Carroll, astro-ph/0310342.

[3] V. Sahni, Lect. Notes Phys. **653**, 141 (2004).

[4] X. Zhang, hep-ph/0510072; B. Feng, astro-ph/0602156.

[5] E. J. Copeland, M. Sami, and S. Tsujikawa, hep-th/0603057.

[6] S. Hellerman, N. Kaloper, and L. Susskind, J. High Energy Phys. 06 (2001) 003; W. Fischler, A. Kashani-Poor, R. McNees, and S. Paban, J. High Energy Phys. 07 (2001) 003.

[7] T. Banks and W. Fischler, hep-th/0102077; T. Banks, hep-th/0503066.

[8] L. Dyson, M. Kleban, and L. Susskind, J. High Energy Phys. 10 (2002) 011; T. Banks, W. Fischler, and S. Paban, J. High Energy Phys. 12 (2002) 062; N. Goheer, M. Kleban, and L. Susskind, J. High Energy Phys. 07 (2003) 056.

[9] S. Dutta and T. Vachaspati, Phys. Rev. D **71**, 083507 (2005); S. Dutta, Phys. Rev. D **73**, 063524 (2006).

[10] Y. S. Piao, Phys. Rev. D **72**, 103513 (2005); gr-qc/0512161.

[11] W. Fischler, A. Loewy, and S. Paban, J. High Energy Phys. 09 (2003) 024.

[12] T. Banks and W. Fischler, astro-ph/0307459.

[13] R. G. Cai, J. Cosmol. Astropart. Phys. 02 (2004) 007; B. Wang and E. Abdalla, Phys. Rev. D **69**, 104014 (2004).

[14] D. A. Lowe and D. Marolf, Phys. Rev. D **70**, 026001 (2004).

[15] R. Bousso, J. High Energy Phys. 07 (1999) 004; J. High Energy Phys. 11 (2000) 038.

[16] R. Easther and D. A. Lowe, Phys. Rev. Lett. **82**, 4967 (1999); G. Veneziano, Phys. Lett. B **454**, 22 (1999); N. Kaloper and A. D. Linde, Phys. Rev. D **60**, 103509 (1999); R. Brustein and G. Veneziano, Phys. Rev. Lett. **84**, 5695 (2000); D. Bak and S. J. Rey, Class. Quant. Grav. **17**, L83 (2000).

[17] N. Kaloper, M. Kleban, and L. Sorbo, Phys. Lett. B **600**, 7 (2004).

[18] T. Banks and W. Fischler, hep-th/0212113; hep-th/0310288.

[19] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).