Little type IIB matrix model

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(Received 18 May 2006; published 9 August 2006)

We study the zero-dimensional reduced model of $D = 6$ pure super Yang-Mills theory and argue that the large N limit describes the (2,0) Little String Theory. The one-loop effective action shows that the force exerted between two diagonal blocks of matrices behaves as $1/r^4$, implying a six-dimensional spacetime. We also observe that it is due to nongravitational interactions. We construct wave functions and vertex operators which realize the $D = 6$, (2,0) tensor representation. We also comment on other little analogues of the IIB matrix model and Matrix Theory with less supercharges.

DOI: 10.1103/PhysRevD.74.046003 PACS numbers: 11.25.Tq, 11.15.−q

I. INTRODUCTION

Realizing string theory as a matrix model is a powerful framework for understanding its nonperturbative aspects. The first successful example was the realization of $c < 1$ string theories in terms of zero-dimensional bosonic matrix models in double scaling limits [1]. More recently, among other approaches, the IIB matrix model [2] has been proposed as a nonperturbative formulation of type IIB string theory [3,4]. Basically, the IIB matrix model is defined as a zero-dimensional reduced model of $D = 10$ super Yang-Mills theory, which may also be viewed as an effective theory of D-instantons. See [5] for a review and further references.

The first link between the IIB matrix model and string theory is that the matrix model action can be regarded as that of a regularization of type IIB Green-Schwarz (GS) superstring in the Schild gauge. Since the GS superstrings [6] can be defined classically in $D = 6$, 4 and 3, and there also exist pure super Yang-Mills theories precisely in these dimensions, one may ask what kind of theories are described if one considers zero-dimensional reduced models of less supersymmetric Yang-Mills theories. Of course the GS superstrings in noncritical dimensions are known to suffer from Lorentz anomalies in the light-cone gauge quantization (See e.g. [7].). It is not clear, however, what is an obvious inconsistency in the reduced models because the identification can be made only by a classical argument. Therefore it is meaningful to ask what these theories are.

In this paper we study the model obtained by dimensionally reducing $D = 6$ (and also $D = 4$) pure super Yang-Mills theory to zero dimensions[1] with an emphasis on its string theory interpretation. The Witten indices of such models were computed in [8]. Using the topological formulation [9] some regularized correlation functions of certain operators were obtained [10] and the grand canonical partition function was shown to be a tau function of the KP hierarchy. Also, these models were explored numerically in [11]. We give evidence that the matrix model describes a six-dimensional (2,0) supersymmetric theory without gravity and argue that the large-N limit of the matrix model describes the (2,0) Little String Theory (LST) [12]. We should note that Matrix Theory descriptions of little string theories in the infinite-momentum frame have already been proposed and well-known [13]; our proposal is another different, manifestly Lorentz covariant one in terms of a zero-dimensional reduced model.

In Sec. II we first define our model, and compute the one-loop effective action. Unlike the maximally supersymmetric case, the force exerted between two diagonal blocks of matrices behaves as $1/r^4$, implying that it is a six-dimensional theory. We also observe that it is due to nongravitational interactions. We then construct vertex operators for this model, closely following [14,15]. In Sec. III we show that this "little" matrix model realizes the $D = 6$, (2,0) chiral supersymmetry by constructing wave functions transforming as a (2,0) tensor multiplet. In Sec. IV we derive vertex operators for those particles in the (2,0) tensor multiplet by expanding a supersymmetric Wilson loop operator. Finally in Sec. V we discuss relations between our model and Little String Theory. We also briefly comment on other little analogues of the IIB matrix model and Matrix Theory. The Appendix A summarizes the conventions of the $D = 6$ symplectic Weyl spinors.

II. THE MODEL

Our starting point is the following matrix model action

$$S = -\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\bar{\psi}^i\Gamma^\mu[A_\mu, \psi_i]\right), \tag{2.1}$$

where μ, $\nu = 0, 1, \ldots, 5$. This action can be obtained by dimensionally reducing the $D = 6$, $U(N)$ pure super Yang-Mills theory to zero dimensions, and as such is the same form as the ordinary IIB matrix model action except the

[1]We call this matrix model with half as many supersymmetries "little IIB matrix model" for an obvious reason, anticipating possible connections to Little String Theory.

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range of the space-time indices and the size of the gamma matrices. The matrices ψ_i $(i = 1, 2)$ are symplectic Majorana-Weyl spinors. The conventions used in this paper are summarized in Appendix.

The action is invariant under the $D = 6$, (2,0) supersymmetries

$$\bar{\epsilon}^{\,i} Q_i^{(1)} = i(\bar{\epsilon}^i \Gamma_\mu \psi_i) \frac{\delta}{\delta A_\mu} - \frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon_i \frac{\delta}{\delta \psi_i}, \qquad \bar{\xi}^i Q_i^{(2)} = \xi_i \frac{\delta}{\delta \psi_i}. \tag{2.2}$$

Again, all the spinor variables carry the symplectic Majorana index $i = 1, 2$, but except this, they are the same as the transformations in the original IIB matrix model. If the contracted indices are suppressed (according to the NW-SE rule ; for instance, $\bar{\epsilon}\Gamma_\mu \psi \equiv \bar{\epsilon}^i \Gamma_\mu \psi_i$) then the transformations look completely identical.

The one-loop effective action of the IIB matrix model was computed in the original IKKT paper. We can use their result for our little IIB matrix model with some trivial changes. That is, we expand the matrices variables around the backgrounds $A_\mu = p_\mu$ and $\psi_i = 0$ as

$$A_\mu = p_\mu + a_\mu, \qquad \psi_i = 0 + \chi_i \tag{2.3}$$

and integrate out the fluctuations a_μ and χ_i. Then we obtain the one-loop effective action [2]

$$W = \frac{1}{2}\operatorname{tr}\log(P_\lambda^2 \delta_\mu{}^\nu - 2iF_\mu{}^\nu) - \frac{1}{2}\operatorname{tr}\log\left(\left(P_\lambda^2 - \frac{i}{2}F_{\mu\nu}\Gamma^{\mu\nu}\right) \times \frac{1+\Gamma_7}{2}\right) - \operatorname{tr}\log P_\lambda^2, \tag{2.4}$$

where P_μ denotes the adjoint representation matrix of p_μ, and similarly $F_{\mu\nu}$ does that of $f_{\mu\nu} = i[p_\mu, p_\nu]$. $\frac{1+\Gamma_7}{2}$ is the projection operator onto the complex four-dimensional space of Weyl spinors of positive chirality in six dimensions. The leading term of the $1/P^2$ expansion of W is

$$W = \frac{1}{2}\operatorname{tr}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F^{\nu\mu} + O(P^{-4}). \tag{2.5}$$

As in [2,14] we assume that the background p_μ be in the block-diagonal form, and let $p_\mu^{(i)}$ and $f_{\mu\nu}^{(i)}$ be the ith block of p_μ and $f_{\mu\nu}$, respectively. In this notation we can write the contribution $W^{(i,j)}$ from the ij-th block as [2,14]

$$W^{(i,j)} = \frac{1}{2r^4}(\operatorname{tr} f_{\mu\nu}^{(i)} f^{(i)\nu\mu} \operatorname{tr} 1^{(j)} + (i \leftrightarrow j) - 2\operatorname{tr} f_{\mu\nu}^{(i)} \operatorname{tr} f^{(j)\nu\mu}) + \cdots, \tag{2.6}$$

where $r = |d_\mu^{(i)} - d_\mu^{(j)}|$ ($d_\mu^{(i)}$ is the "center-of-mass" of the ith block: $p_\mu^{(i)} = d_\mu^{(i)} 1^{(i)}$ + traceless part.) is interpreted as the distance between the two diagonal blocks.[2]

Unlike the maximally supersymmetric IIB matrix model, the expansion of W starts with the quadratic term in $F_{\mu\nu}$, and consequently the force exerted from one block to another behaves like $1/r^4$, implying that the model describes an interaction in a six-dimensional spacetime. Moreover, we can see from the tensor structure that the first line can be regarded as a scalar-scalar interaction, while the second line corresponds to a force due to exchanges of 2-form fields; there are no gravitational interactions to this order. Thus we conclude that the lowest excitation of this matrix model is a $D = 6$ tensor multiplet.

III. WAVE FUNCTIONS

In the previous section we have seen that the reduced model (2.1) of $D = 6$ pure super Yang-Mills theory is naturally regarded as describing some $D = 6$, (2,0) supersymmetric theory without gravity. In this and the next sections we will construct vertex operators for the particles in the (2,0) tensor multiplet. To this end we consider the following supersymmetric Wilson loop operator [16]

$$w(\lambda, k) = \operatorname{tr} e^{\bar{\lambda}^i Q_i^{(1)}} e^{ik_\mu A^\mu} e^{-\bar{\lambda}^i Q_i^{(1)}}. \tag{3.1}$$

We assume that $k^2 = 0$. λ_i $(i = 1, 2)$ are symplectic Majorana Weyl spinors satisfying

$$\Gamma_7 \lambda_i = +\lambda_i. \tag{3.2}$$

After a straightforward calculation we end up with the commutation relations

$$[\bar{\epsilon}_1^i Q_i^{(1)}, \bar{\epsilon}_2^j Q_j^{(1)}] = [(2\bar{\epsilon}_1^i \Gamma^\mu \epsilon_{2i})A_\mu, A_\nu]\frac{\delta}{\delta A_\nu} + [(2\bar{\epsilon}_1^i \Gamma^\mu \epsilon_{2i})A_\mu, \psi_j]\frac{\delta}{\delta \psi_j}, \qquad [\bar{\epsilon}^i Q_i^{(1)}, \bar{\xi}^j Q_j^{(2)}] = -i\bar{\epsilon}^i \Gamma_\mu \xi_i \frac{\delta}{\delta A_\mu} \tag{3.3}$$

up to the equations of motion. The right hand side of the first line is a gauge transformation and hence vanishes on any gauge invariant operator. Using these relations we obtain

$$e^{\bar{\epsilon}^i Q_i^{(1)}} w(\lambda, k) e^{-\bar{\epsilon}^i Q_i^{(1)}} = w(\lambda + \epsilon, k), \qquad e^{\bar{\xi}^i Q_i^{(2)}} w(\lambda, k) e^{-\bar{\xi}^i Q_i^{(2)}} = e^{(\bar{\xi}^i \Gamma_\mu \lambda_i)k^\mu} w(\lambda, k), \tag{3.4}$$

or in their infinitesimal forms

$$[\bar{\epsilon}^i Q_i^{(1)}, w(\lambda, k)] = \epsilon_i \frac{\partial}{\partial \lambda_i} w(\lambda, k), \tag{3.5}$$

[2]In order for $\operatorname{tr} f_{\mu\nu}^{(i)}$ to be nonvanishing the sizes of the blocks must be infinite (c.f. the static D-string solutions [2]).

$$[\bar{\xi}^i Q_i^{(2)}, w(\lambda, k)] = (\bar{\xi}^i \not{k}\lambda_i) w(\lambda, k). \tag{3.6}$$

The parameter λ_i may be thought of as an isolated eigenvalue of the matrix ψ_i representing the whole effect of the background as a mean field [15] (See also [17].); k_μ is the Fourier transform of the similarly isolated eigenvalue of A_μ.

We would like to have a vertex operator V_f for a particle of wave function f satisfy [18,19]

$$\delta V_f = V_{\delta f} \tag{3.7}$$

under the supersymmetry. Therefore we first construct a representation of the $D = 6$, (2,0) superalgebra

$$\delta^{(1)} f(\lambda, k) = \epsilon_i \frac{\partial}{\partial \lambda_i} f(\lambda, k) = \bar{\epsilon}^i \frac{\partial}{\partial \bar{\lambda}^i} f(\lambda, k), \tag{3.8}$$

$$\delta^{(2)} f(\lambda, k) = (\bar{\xi}^i \not{k}\lambda_i) f(\lambda, k), = -(\bar{\lambda}^i \not{k}\xi_i) f(\lambda, k) \tag{3.9}$$

in the space of polynomials of λ_i and find wave functions of the supermultiplet. Then if we expand the Wilson loop operator $w(\lambda, k)$ (3.1) in terms of those wave functions we can (in principle) automatically obtain desired vertex operators as their coefficients.

Let us start from the scalar wave function $\Phi = 1$. Applying $\delta^{(2)}$ to it, we have

$$\delta^{(2)}\Phi = \bar{\xi}^i \not{k}\lambda_i, \tag{3.10}$$

so we define the spinor wave function Ψ_i as

$$\Psi_i = \frac{1}{2}\not{k}\lambda_i. \tag{3.11}$$

Next we apply $\delta^{(2)}$ to Ψ_i to find

$$\begin{aligned} \delta^{(2)}\Psi_i &= -(\bar{\lambda}^j \not{k}\xi_j) \cdot \frac{1}{2}\not{k}\lambda_i \\ &= -\frac{1}{32} k^\sigma k_\rho (\lambda^j \Gamma_{\mu\nu\sigma}\lambda_j)\Gamma^{\mu\nu\rho}\xi_i + \frac{1}{4}(\bar{\lambda}^j \not{k}\lambda_i)\not{k}\xi_j \end{aligned} \tag{3.12}$$

after some Fierz rearrangements summarized in the Appendix A. Thus we have the wave functions of the 2-form field

$$B_{\mu\nu} = \frac{1}{2} b_{\mu\nu}, \qquad b_{\mu\nu} = k^\rho \bar{\lambda}^i \Gamma_{\mu\nu\rho}\lambda_i, \tag{3.13}$$

and another set of scalars

$$\Phi^i{}_j = \bar{\lambda}^i \not{k}\lambda_j. \tag{3.14}$$

The field strength $H_{\mu\nu\rho}$ of $B_{\mu\nu}$ is manifestly self-dual:

$$-\frac{1}{6}\epsilon^{\mu\nu\rho\sigma\tau\lambda} H_{\sigma\tau\lambda} = H^{\mu\nu\rho} \tag{3.15}$$

since one can write it as

$$H_{\mu\nu\rho} = 3ik_{[\mu}B_{\nu\rho]} = \frac{3}{2} i\bar{\lambda}^i \not{k}\Gamma_{\mu\nu\rho}\not{k}\lambda_i. \tag{3.16}$$

$B_{\mu\nu}$ is further transformed as

$$\delta^{(2)} B_{\mu\nu} = -\frac{1}{3}(\bar{\epsilon}^i \Gamma_{\mu\nu}\not{k}\lambda_j)(\bar{\lambda}^j \not{k}\lambda_i), \tag{3.17}$$

which leads us to the definition of the conjugate spinors

$$\Psi_i^c = \frac{1}{3}\not{k}\lambda_j(\bar{\lambda}^j \not{k}\lambda_i). \tag{3.18}$$

Finally we choose

$$\Phi^c = \frac{1}{3}(\bar{\lambda}^i \not{k}\lambda_j)(\bar{\lambda}^j \not{k}\lambda_i) \tag{3.19}$$

as the conjugate scalar wave function.

With these definitions one may check that these wave functions satisfy the following $D = 6$, (2,0) superalgebra [20]

$$\delta B_{\mu\nu} = -\bar{\epsilon}^I \gamma_{\mu\nu}\psi_I, \tag{3.20}$$

$$\delta\psi_I = +\frac{i}{48} H^+_{\mu\nu\rho}\Gamma^{\mu\nu\rho}\epsilon_I + \frac{i}{4}\not{\partial}\phi_I{}^J\epsilon_J, \tag{3.21}$$

$$\delta\phi^{IJ} = -4\bar{\epsilon}^{[I}\psi^{J]} - \Omega^{IJ}\bar{\epsilon}^K\psi_K \tag{3.22}$$

if they are identified with the fields $B_{\mu\nu}$, ψ^I and ϕ^{IJ} as

$$\begin{aligned} &B_{\mu\nu} = \frac{1}{2} b_{\mu\nu}, \qquad \psi_I = (\psi_i, \psi_{i'}) = (\Psi_i^c, \Psi_{i'}), \\ &\phi^{12} = \phi_{1'2'} = \Phi, \qquad \phi^{i'}{}_{j'} = \Phi^{i'}{}_{j'}, \end{aligned} \tag{3.23}$$

$$\phi^i{}_j = -\Phi^i{}_j, \tag{3.24}$$

$$\phi^{1'2'} = \phi_{12} = -\Phi^c. \tag{3.25}$$

The identifications of the supersymmetry parameters are

$$\epsilon^I = (\xi^i, -2\epsilon^{i'}), \qquad \bar{\epsilon}^I = (\bar{\xi}^i, -2\bar{\epsilon}^{i'}). \tag{3.26}$$

$I = (i, i')$, $J = (j, j')$, … $(i, j = 1, 2; i', j' = 1, 2)$ are the $USp(4)$ indices. They are raised and lowered by multiplications of

$$\Omega^{IJ} = \begin{pmatrix} 0 & \varepsilon^{ij'} \\ \varepsilon^{i'j} & 0 \end{pmatrix}, \qquad \Omega_{IJ} = \begin{pmatrix} 0 & \varepsilon_{ij'} \\ \varepsilon_{i'j} & 0 \end{pmatrix} \tag{3.27}$$

as

$$\epsilon^I = \Omega^{IJ}\epsilon_J, \qquad \epsilon_I = \epsilon^J\Omega_{JI}. \tag{3.28}$$

The Majorana condition for the $USp(4)$ spinor λ_I is

$$(\bar{\lambda}^I)^T = C\lambda^I. \tag{3.29}$$

These rules are consistent with the definitions (3.27) and the identifications (3.26). Because of (A16) and (A17), ϕ^{IJ} satisfy the constraints

$$\phi^{IJ}\Omega_{IJ} = 0, \qquad \phi^{IJ} = -\phi^{JI} \tag{3.30}$$

so there are only five independent scalars.

IV. VERTEX OPERATORS

In the previous section we obtained wave functions of the particles in the (2,0) tensor multiplet. In this section we construct vertex operators for these particles by expanding the supersymmetric Wilson loop in terms of wave functions. Vertex operators are given as coefficients in this expansion:

$$w(\lambda, k) = \Phi V_\Phi + \Phi_i V_{\Phi_i} + B_{\mu\nu} V_{B_{\mu\nu}} + \Phi^i{}_j V_{\Phi^i{}_j} + \Psi_i^c V_{\Psi_i^c} + \Phi^c V_{\Phi^c}. \tag{4.1}$$

We begin by rewriting the Wilson loop operator as follows:

$$w(\lambda, k) = \mathrm{tr}\, e^{\bar\lambda^i Q_i^{(1)}} e^{ik_\mu A^\mu} e^{-\bar\lambda^i Q_i^{(1)}} = \mathrm{tr}\, e^{\sum_{n=0}^4 G_n}, \tag{4.2}$$

where n

$$G_n = \frac{1}{n!}\overbrace{[\bar\lambda Q^{(1)}, \cdots [\bar\lambda Q^{(1)}, ikA]]}. \tag{4.3}$$

In (4.3) and below we suppress the indices contracted according to the NW-SE rule. Note that G_n contains n λ's. The sum in Eq. (4.1) terminates at fourth order because the on-shell λ's have 4 independent components. Each G_n can be evaluated as follows:

$$\begin{aligned} &G_0 = ikA, \qquad G_1 = -(\bar\lambda \not k \psi),\\ &G_2 = \frac{i}{4}[A_\mu, A_\nu], \qquad G_3 = -\frac{1}{3!} b^{\mu\nu}[\bar\lambda\Gamma_\mu \psi, A_\mu],\\ &G_4 = \frac{1}{4}\Big(\frac{1}{2} b^{\mu\nu}(\bar\lambda\Gamma_{\mu\rho\sigma}\lambda)[[A^\rho, A^\sigma], A_\nu] - ib^{\mu\nu}[\bar\lambda\Gamma_\mu\psi, \lambda\Gamma_\nu\psi]\Big),\\ &G_n = 0 \qquad (n \geq 5). \end{aligned} \tag{4.4}$$

Expanding the exponential of Eq. (4.1) and collecting the terms with the same power of λ, we can read off vertex operators.

The leading order term, which has no λ, is $\mathrm{tr}\, e^{ikA}$. This should equal Φ vertex operator multiplied by Φ wave function, thus we obtain Φ vertex operator

$$V_\Phi = \mathrm{tr}\, e^{ikA}. \tag{4.5}$$

The first order term gives the Ψ_i vertex operator V_{Ψ_i} as

$$\mathrm{tr}\ e^{ikA} G_1 = \mathrm{tr}\, e^{ikA}(\bar\psi \not k \lambda) = V_{\Psi_i}\Psi_i. \tag{4.6}$$

Hence

$$V_{\Psi_i} = \mathrm{tr}\, e^{ikA} 2\bar\psi^i. \tag{4.7}$$

The second order terms can be evaluated as follows:

$$\begin{aligned} \mathrm{Str}\, e^{ikA}\Big(\frac{1}{2}G_1^2 + G_2\Big) &= \mathrm{Str}\, e^{ikA}\Big(\Big(-\frac{1}{32}k^\rho(\bar\psi \cdot \Gamma_{\rho\mu\nu}\psi) + \frac{i}{4}[A_\mu, A_\nu]\Big)b^{\mu\nu} + \frac{1}{4}(\bar\psi^i \cdot \not k\psi_j)(\bar\lambda^j \not k \lambda_i)\Big)\\ &= V_{B_{\mu\nu}} B_{\mu\nu} + V_{\Phi^j{}_i}\Phi^j{}_i, \end{aligned} \tag{4.8}$$

where "Str" is the symmetrized trace (See [15] for its definition and some properties.) and $\cdot$ means that the operators are symmetrized. Thus we have the vertex operator for $B_{\mu\nu}$

$$V_{B_{\mu\nu}} = \mathrm{Str}\, e^{ikA}\Big(-\frac{1}{16}k_\rho \bar\psi \cdot \Gamma^{\mu\nu\rho}\psi + \frac{i}{2}[A_\mu, A_\nu]\Big) \tag{4.9}$$

and for $\Phi^j{}_i$

$$V_{\Phi^j{}_i} = \mathrm{Str}\, e^{ikA}\frac{1}{4}\bar\psi^i \cdot \not k \psi_j. \tag{4.10}$$

The Ψ_i^c vertex operator can be obtained from the third order terms

$$\begin{aligned} &\mathrm{Str}\ e^{ikA}\Big(\frac{1}{3!}G_1 \cdot G_1 \cdot G_1 + G_1 \cdot G_2 + G_3\Big)\\ &= \mathrm{Str}\, e^{ikA}\Big(\frac{1}{9}(\bar\psi^i \cdot \not k\psi_j)(\bar\psi^j \not k \lambda_l)(\bar\lambda^l \not k \lambda) - \frac{i}{6}\bar\psi^i\Gamma_{\mu\nu}[A^\mu, A^\nu]\not k \lambda_j(\bar\lambda^j \not k \lambda_i)\Big)\\ &= V_{\Psi_i^c}\Psi_i^c. \end{aligned} \tag{4.11}$$

After some Fierz rearrangement and with a help of formulas for the symmetrized trace [15] we find

$$V_{\Psi_i^c} = \mathrm{Str}\, e^{ikA}\Big(\frac{1}{3}(\bar\psi^i \cdot \not k\psi_j) \cdot \bar\psi^j - \frac{i}{2}[A^\mu, A^\nu] \cdot \bar\psi^i\Gamma_{\mu\nu}\Big). \tag{4.12}$$

The computation becomes more complicated as the order of λ becomes higher. Although the vertex operator for Φ^c could also be read off from the fourth order terms, we use the following shortcut method: We first notice from (2.2) and (3.9) that multiplying $\xi \not k \lambda$ to $w(\lambda, k)$ is equivalent to replacing every ψ_i with ξ_i. Since we have already computed what becomes of each wave function after the operation of $\delta^{(2)}$, we can use it to guess what the next-order vertex operator is, up to ψ_i-independent terms. The latter can also be determined by e.g. expanding the Wilson loop as above. In this way we finally find the expression for the conjugate scalar vertex operator

$$V_{\Phi^c} = \mathrm{Str}\, e^{ikA}\Big(\frac{1}{48}(\bar{\psi}^i \cdot \not{k}\psi_j)(\bar{\psi}^j \cdot \not{k}\psi_i) - \frac{i}{16}[A_\mu, A_\nu] \cdot k^\rho \bar{\psi} \cdot \Gamma_{\mu\nu\rho}\psi + \frac{1}{8}[A_\mu, A_\nu]\cdot[A^\nu, A^\mu]\Big). \tag{4.13}$$

V. CONCLUSIONS AND DISCUSSION

We have seen that the reduced model of $D = 6$ super Yang-Mills theory appears to describe a theory with (1) a six-dimensional spacetime, (2) $D = 6$, (2,0) chiral supersymmetry, (3) a coupling to a self-dual 2-form field and (4) no massless gravitons. We have also consistently constructed wave functions and vertex operators transforming as a (2,0) tensor multiplet, which we expect to describe emissions of the particles responsible for the above non-gravitational interactions. Technically the method we have described at the end of Sec. IV can save much labor in computing vertex operators, and we expect that we can use it to derive the complete forms of vertex operators in the full IIB matrix model.

It seems that maximal supersymmetry is essential to include gravity in matrix models. On the other hand, the items (1) ~ (4) are the common features shared by the (2,0) little string theory (LST) (See [21,22] for reviews; also [23] for more recent discussions.). Basically a LST is defined as a decoupling limit of (5 + 1)-dimensional world-volume theory on a stack of NS5-branes. Since the supersymmetry is (2,0)((1,1)) for type IIA(IIB) 5-branes [24], the former is of our interest. It is believed to allow a holographic dual description in terms of strings on a linear-dilaton background [25].

Since matrix models in general are naturally expected to define (in the sense of t'Hooft) string theories in the large N limit, it is tempting to conjecture that our model at large N is a description of the (2,0) LST. In this picture the number of 5-branes k will correspond to the number of diagonal blocks, each size of which goes to infinity in the limit. In support of this conjecture we note that, in addition to their common features they share as above, both our matrix model and (non-double-scaled [26]) LST have a single dimensionful parameter but no other dimensionless one. It is also consistent that we have successfully obtained a set of vertex operators for the (2,0) tensor multiplet, but the ones for the $D = 6$ gravity multiplet cannot be constructed in this framework. Although all the evidence we have so far is still only a circumstantial one, the features are suggestive and worth to be explored.[3]

In this paper we have studied the reduced model of $D = 6$ super Yang-Mills, but the model reduced from $D = 4$ is also interesting. In four dimensions there are also both the (classical) Green-Schwarz superstring and pure super Yang-Mills. Following the route of the original (or the little) IIB matrix model, one can similarly obtain its action and $D = 4$, $\mathcal{N} = 2$ supersymmetry. We encounter the following two puzzles, however.

One of them is the fact that the one-loop effective action similar to (2.5) starts with, again, the quadratic term in $F_{\mu\nu}$ with the r^{-4} factor, which would mean that the model lives in a six-dimensional spacetime. The other is that one cannot realize a $D = 4$, $\mathcal{N} = 2$ supermultiplet in the polynomial space of $D = 4$ Majorana spinors because it has only half as many degrees of freedom of what are needed. This would mean that this matrix model simply describes the initial $D = 4$, $\mathcal{N} = 1$ gauge theory, although there appears to be $\mathcal{N} = 2$ supersymmetry. A better understanding of this 1/4-supercharge model is an interesting problem for future investigations.

In order to further examine the relation between the little IIB matrix models and LSTs, it will be useful to consider the problem in the dual linear-dilaton [25] and/or the cigar $SL(2, R)/U(1)$ CFTs [27–34]. In particular, it was recently shown [35,36] that the world-volume theory of D-branes in a certain cigar CFT background is a lower-dimensional pure super Yang-Mills theory. Therefore, in view of the established role of $D = 10$ super Yang-Mills theory in the critical superstring theories, it seems consistent that the little IIB matrix model, which is defined as a reduced model of a lower-dimensional theory, describes some non-critical string theory. It would be interesting if one can directly compare the correlation functions of the corresponding vertex operators in the little matrix models and their dual CFTs.

Finally, we will now briefly comment on little analogues of Matrix Theory [37] (See [38] for a review.) with less supercharges. These models were studied in e.g. [39,40]. Let us consider matrix quantum mechanics obtained by reducing, again, the $D = 6$ and 4 pure super Yang-Mills theories to one dimension, and compute one-loop effective actions around a two-particle background in the standard eikonal approximation [37,41]. Namely, we set

$$B^1 = \frac{i}{2}\begin{pmatrix} vt & 0 \\ 0 & -vt \end{pmatrix}, \qquad B^2 = \frac{i}{2}\begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix}, \qquad B^3 = \cdots = B^{D-1} = 0, \tag{5.1}$$

where B^i $(i = 1, \ldots, D-1)$ are the backgrounds of the matrix variables X^i $(i = 1, \ldots, D-1)$ of the little Matrix Theory; they are the spacelike components of the D-dimensional gauge field A_μ. The computation of the one-loop effective action is completely the same as [41], except for the numbers of various types of fields appearing in the action. The result is

$$W \equiv \prod \log\det(\partial_\tau^2 + \mathrm{mass}^2) = -\int_0^\infty \frac{ds}{s} e^{-sb^2} \frac{1}{\sinh sv}\Big(c\cosh sv + b\cosh 2sv + \frac{a}{2}\Big), \tag{5.2}$$

[3]It is believed that LST for a single ($k = 1$) 5-brane is a free theory; it is not contradictory because our one-loop analysis only computes two-point correlators.

TABLE I. The numbers of fields having dierent masses in the one-dimensional reduced models of the $D = 10$, 6 and 4 pure super Yang-Mills theories.

Number of fields	$D = 10$	$D = 6$	$D = 4$
a	-6	-2	0
b	-1	-1	-1
c	$+4$	$+2$	$+1$

where a, b and c are shown in Table I. (We also list the original Matrix Theory case ($D = 10$) for comparison.) Using these data, we find $W = \int_{-\infty}^{\infty} d\tau x \frac{v^2}{r^3} + O(\frac{v^4}{r^7})$ with $x = \frac{1}{2}$ ($D = 6$) and $x = \frac{3}{4}$ ($D = 4$).

Note that the systematics of the expansions [42] in terms of v and b are valid in $D = 6$ or 4 without any change; v^2/r^3 is the generic leading behavior of the potential and the above computations simply confirm that they do not vanish accidentally in the less supersymmetric cases. This is a similar phenomenon to the divergence structure of super Yang-Mills theory [43]. For the $D = 6$ case, according to the conventional interpretation, it suggests of some theory with *seven* dimensional spacetime. One naturally thinks of it as describing "little m theory" advocated in [44] in the infinite-momentum frame, while a different interpretation of this model has been given in [45]. For the $D = 4$ case, the long-range force again suggests of a seven-dimensional one rather than five. It will be interesting to compute brane charges [46] for these little matrix models.

ACKNOWLEDGMENTS

We thank H. Fuji, M. Hatsuda, S. Iso, H. Kunitomo, J. Nishimura, Y. Okawa, H. Suzuki, H. Umetsu and K. Yoshida for useful discussions. Y. K. and S. M. are supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

APPENDIX

The conventions of the gamma matrices are

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}, \qquad \eta^{\mu\nu} = \mathrm{diag}(-1, +1, \ldots, +1). \tag{A1}$$

$$\Gamma^{0\dagger} = -\Gamma^0, \qquad \Gamma^{i\dagger} = +\Gamma^i (i = 1, \ldots, 5). \tag{A2}$$

$$\Gamma_7 = \Gamma_0\Gamma_1 \cdots \Gamma_5 = -\Gamma^0\Gamma^1 \cdots \Gamma^5. \tag{A3}$$

The charge conjugation matrix C satisfies

$$C\Gamma^\mu C^{-1} = -\Gamma^{\mu T}, \qquad C^T = +C. \tag{A4}$$

Let $B = C\Gamma^0$, then

$$B\Gamma^\mu B^{-1} = +\Gamma^{\mu *} \qquad \text{(complex conjugate).} \tag{A5}$$

Let λ be a complex Weyl spinor with chirality

$$\Gamma_7\lambda = +\lambda. \tag{A6}$$

$$\bar{\lambda} = \lambda^\dagger\Gamma^0. \tag{A7}$$

Any complex spinor λ can be written as a sum of symplectic Majorana spinors

$$\lambda = \lambda_1 + \lambda_2, \tag{A8}$$

where

$$\lambda_1 = \frac{1}{2}(\lambda + B^{-1}\lambda^*), \tag{A9}$$

$$\lambda_2 = \frac{1}{2}(\lambda - B^{-1}\lambda^*). \tag{A10}$$

Then

$$B\lambda_1 = \lambda_2^*, \qquad B\lambda_2 = -\lambda_1^*. \tag{A11}$$

Since B commutes with Γ_7, this decomposition can be done in the subspace of spinors with definite chirality. It is conventional to define

$$\bar{\lambda}^i = \lambda_i^\dagger\Gamma^0 \qquad (i = 1, 2). \tag{A12}$$

Note the positions of the indices. In this notation we have

$$(\bar{\lambda}^i)^T = C\lambda^i, \tag{A13}$$

where

$$\lambda^i = \varepsilon^{ij}\lambda_j, \qquad \lambda_j = \lambda^i\varepsilon_{ij}, \qquad \varepsilon^{12} = \varepsilon_{12} = +1. \tag{A14}$$

The indices are raised (lower) by contracting ε^{ij} (ε_{ij}) according to the NW-SE rule. Similarly decomposing another complex spinor ϵ, we obtain the relation

$$\frac{1}{2}(\bar{\epsilon}\not{k}\lambda - \bar{\lambda}\not{k}\epsilon) = \bar{\epsilon}^i\not{k}\lambda_i = -\bar{\lambda}^i\not{k}\epsilon_i. \tag{A15}$$

The following relations are useful:

$$\bar{\lambda}^i\Gamma^\mu\lambda_i = 0, \tag{A16}$$

$$\bar{\lambda}^i\Gamma^\mu\lambda_j = +\bar{\lambda}_j\Gamma^\mu\lambda^i, \tag{A17}$$

$$\bar{\lambda}^i\Gamma^{\mu\nu\rho}\lambda_j = \frac{1}{2}\delta^i_j\bar{\lambda}^k\Gamma^{\mu\nu\rho}\lambda_k. \tag{A18}$$

For symplectic Majorana *Weyl* spinors λ_i, ψ_j with the same chirality, the Fierz rearrangement formula reads

$$\lambda_j\bar{\psi}^i = -\frac{1}{4}\Gamma^\mu(\bar{\psi}^i\Gamma_\mu\lambda_j) + \frac{1}{48}\Gamma^{\mu\nu\rho}(\bar{\psi}^i\Gamma_{\mu\nu\rho}\lambda_j). \tag{A19}$$

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