Nonlinear realization and Weyl scale invariant $p = 2$ brane

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The action of Weyl scale invariant $p = 2$ brane which breaks the target super-Weyl scale symmetry in the $N = 1$, $D = 4$ superspace down to the lower dimensional Weyl symmetry $W(1, 2)$ is derived by the approach of nonlinear realization. The dual form action for the Weyl scale invariant supersymmetric D2 brane is also constructed. The interactions of localized matter fields on the brane with the Nambu-Goldstone fields associated with the breaking of the symmetries in the superspace and one spatial translation directions are obtained through the Cartan one-forms of the Coset structures. The covariant derivatives for the localized matter fields are also obtained by introducing Weyl gauge field as the compensating field corresponding to the local scale transformation on the brane world volume.

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I. INTRODUCTION

Nonlinear realization of a compact Lie group can be realized on the Nambu-Goldstone fields related to the broken symmetry generators and it becomes linear when restricted to the given subgroup [1]. In Ref. [2], based on nonlinear transformation of a spinor field, which played a role of Goldstino field, the nonlinear realization method was extended to include fermion like generators. The resulted degeneracy of vacuum gave rise to spontaneous broken of supersymmetry. The approach of nonlinear realization of SUSY, besides the Goldstone field,

was generalized to matter fields as well as gauge fields [3,4], with a formalism of effective couplings to the Goldstino field. In Ref. [5], one can find applications of nonlinear realization to branes of *M* theory, and a general description is given to derive the dynamics of the branes. There, it is restricted to group *G* whose generators can be divided into two subgroups with one (such as Lorentz group) is the automorphism group of another (whose generators associated with (super)spacetime positions). The transformation of the group *G* with respect to the coset of the unbroken automorphism generators group would give us a description of the embedded submanifold, which has the dimensions of the coset space of the unbroken automorphism generators group with respect to the unbroken subgroup.

As presented in [6–10], the approach of nonlinear realization was extensively used to describe the spontaneous partial breaking of (extended) supersymmetry and construct actions of (super)brane dynamics. On the other hand, when considering conformal transformation, in Ref. [11] the dynamics of conformally invariant *p*-branes was introduced. In [12,13], it was further extended to describe Weyl invariant *D* *p*-brane and superconformal supermembrane. It is our purpose of this paper to introduce a Weyl scale (due to dilatation operator *D*) invariant $p = 2$ brane which embedded in the target $N = 1$, $D = 4$ superspace defined by $\{x_\mu, \theta, \bar{\theta}\}$. Considering the unbroken subgroup $W(1, 2)$ (Weyl group) of the super-Weyl group *G* and the coset with respect to the unbroken automorphism group of the unbroken subgroup, the symmetry *G* can be realized on the nonlinear transformation of collective coordinates fields which is a result of acting a group element of *G* on the coset representative element Ω. When applied to brane theory, it is illustrated that for such a brane that breaks the supersymmetry and one spatial translation symmetry, its dynamics is described by the low energy oscillations of the Nambu-Goldstone modes associated with these broken symmetries. Accordingly, the invariant action of the brane can be obtained by using vielbein and connection one forms on the submanifold after constructing Cartan one-forms from $\Omega^{-1}d\Omega$.

In this paper, we start from introducing the super-Weyl scale group and its automorphism subgroup, then use the above stated formalism of the nonlinear realization approach to find the fluctuations modes of Goldstone bosons (Goldstino fermions) associated with spacetime coordinates (Grassmann coordinates) of the broken symmetry (supersymmetry). A Weyl scale invariant $p = 2$ brane will be given when the target $D = 4$ superspace $\{x_\mu, \theta, \bar{\theta}\}$ broken down to $D = 3$ spacetime world volume described by parameters $\{x_0, x_1, x_2\}$ in the static gauge with Weyl scale (dilatation) symmetry kept, which becomes a local symmetry on the $p = 2$ brane world volume. The dual form non-BPS Weyl scale invariant D2 brane supersymmetric Born-Infeld action is also obtained.

Finally, in addition to the massless Nambu-Goldstone fields of the $p = 2$ brane oscillations, we also consider matter fields degree of freedom localized on the domain wall brane. The Weyl scale

invariant actions of these matter fields are constructed by using Weyl gauge field and spin connections. The latter gives interactions of the matter fields with the Nambu-Goldstone fields.

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II. WEYL SCALE INVARIANT $p = 2$ BRANE

Consider super-Weyl group G, whose generators include $N = 1$, $D = 4$ super-Poincare generators (P_μ, $M_{\mu\nu}$ two Weyl spinor supersymmetry charges Q_α, $\bar{Q}_{\dot\alpha}$) and the Weyl scale (dilatation) generator D. It has the following (anti)-commutation relations

$$\begin{aligned}
&\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu\\
&[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\mu\rho})\\
&[Q_\alpha, M_{\mu\nu}] = i\frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta\\
&[\bar{Q}^{\dot\alpha}, M_{\mu\nu}] = i\frac{1}{2}(\bar\sigma_{\mu\nu})^{\dot\alpha}_{\dot\beta}\bar{Q}^{\dot\beta}
\end{aligned} \tag{1}$$

and

$$[D, M_{\mu\nu}] = 0, \qquad [D, D] = 0, \qquad [D, P_\mu] = -iP_\mu \tag{2}$$

$$[D, Q_\alpha] = -\frac{1}{2}iQ_\alpha, \qquad [D, \bar{Q}_{\dot\alpha}] = -\frac{1}{2}i\bar{Q}_{\dot\alpha} \tag{3}$$

where the dilatation operator $D = -ix^\mu\frac{\partial}{\partial x^\mu}$ in x-representation. From Eqs. (1)–(3), one can find generators $\{M_{\mu\nu}, D\}$ form a subgroup H' which is the automorphism group of another subgroup, i.e. super spacetime group by the set of charges of $\{Q_\alpha, \bar{Q}_{\dot\alpha}, P_\mu\}$. In the case when the G group symmetry is broken to the $1 + 2$ dimensional Weyl $W(1, 2)$ symmetry [14], whose unbroken generators, for example, in the static gauge, are $\{M_{ij}, D, P_i\}$, where the index $i = 0, 1, 2$ and the spontaneously broken automorphism generators are M_{i3}. In such a case, we have a two dimensional brane which is embedded in the superspace and breaks down the target space super-Weyl invariance to a lower dimensional Weyl group symmetry $W(1, 2)$. Besides M_{i3}, the broken generators in superspace are the generators Q_α, $\bar{Q}_{\dot\alpha}$ in the Grassmann coordinate directions $\{x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha}\}$ and the translation generator P_3 transverse to the brane.

Consider the coset G/H, where H has unbroken antomorphism generators $\{M_{ij}, D\}$. We hence have a $p = 2$ brane, which has a $W(1, 2)$ symmetry, moving through the coset space G/H' with tangent group H'. It sweeps out a submanifold that has the dimensions of the coset space G'/H with tangent group H, where G' is spanned by the unbroken automorphism generators $\{M_{ij}, D\}$ and the unbroken spacetime generators P_i. Since we will work on a $D = 3$ manifold, it is more convenient to express $N = 1$, $D = 4$ super-Weyl algebra in terms of $D = 3$ Lorenz group indices with new defined generators $M^m = \frac{1}{2}\varepsilon^{mnr}M_{nr}$ and $K^m = M^{m3}$, where $m = 0, 1, 2$. In $N = 2$, $D = 3$ supersymmetry theory, there are four supercharges which are the same number as for $N = 1$ supersymmetry in $D = 4$. In fact, $N = 2$, $D = 3$ SUSY algebra [15–17] can be obtained by dimensionally reducing $N = 1$ supersymmetry in four dimensions. From the $D = 3$ standpoint the $N = 1$, $D = 4$ SUSY algebra is a central-charged extended $N = 2$ Poincare superalgebra, with one $D = 4$ translation generator becomes the central charge generator Z_0 [8,10,15]. Taking $Z_0 = -P_3 = -Z$, with supercharges Q_α, $\bar{Q}_{\dot\beta}$ becomes two complex conjugate spinors Q_α, $\bar{Q}_\beta$ in three dimensions, the $N = 1$, $D = 4$ Supersymmetry reduces to $N = 2$, $D = 3$ extended Supersymmetry with relations (See Appendix A for derivation and notations):

$$\begin{aligned}
&[M^m, M^n] = -i\varepsilon^{mnr}M_r, \qquad [M^m, K^n] = -i\varepsilon^{mnr}K_r,\\
&[K^m, K^n] = i\varepsilon^{mnr}M_r \qquad [M^m, P^n] = -i\varepsilon^{mnr}P_r,\\
&[M^m, Z] = 0, \qquad [K^m, P^n] = +i\eta^{mn}Z,\\
&[K^m, Z] = +iP^m \qquad [Z, P_\mu] = 0, \qquad [D, q_i] = -\frac{1}{2}iq_i,\\
&[D, s_i] = -\frac{1}{2}is_i \qquad \{q_i, q_j\} = 2(\gamma^m C)_{ij}P_m,\\
&\{s_i, s_j\} = 2(\gamma^m C)_{ij}P_m, \qquad \{q_i, s_j\} = -2iC_{ij}P_3
\end{aligned} \tag{4}$$

where q, s are extended supercharges in three dimensions. The unbroken automorphism generators forms group H. With R symmetry suppressed, an exponential description of the Coset G/H representative element is

$$\Omega = e^{i\xi^m p_m}e^{i[\phi(\xi)Z+\bar\theta_i(\xi)q_i+\bar\lambda_i(\xi)s_i]}e^{iu^m(\xi)K_m}, \tag{5}$$

in which variables ξ parameterizes the embedded submanifold described by the $p = 2$ brane, and $\phi(\xi)$, $\theta(\xi)$, $\lambda(\xi)$, $u(\xi)$ are the Nambu-Goldstone fields that depend on variables ξ. The dynamics can be constructed about the brane which describe a broken symmetry in the z, θ, $\bar\theta$ superspace coordinates directions and whose long wave length excitation modes are described by these Nambu-Goldstone fields associated with these broken symmetries. By using reparameterization invariance, we choose static gauge $x^m = \xi^m$ for space time coordinates x^m lying in directions of the brane. Then it becomes

$$\Omega = e^{ix^m p_m}e^{i[\phi(x)Z+\bar\theta_i(x)q_i+\bar\lambda_i(x)s_i]}e^{iu^m(x)K_m}. \tag{6}$$

The elements of group G can be decomposed uniquely into a product form of Coset representative element Ω and subgroup element of H. In some neighborhood of the identity of group G, its element is parameterized as

$$g = e^{i[a^m p_m+\bar\xi q+\bar\eta s+zZ+b^mK_m+\alpha^mM_m+dD]} \tag{7}$$

Under a right group transformation g, the Coset element Ω transforms to $\Omega' = e^{ix'^m p_m}e^{i[\phi'(x')Z+\bar\theta'_i(x')q_i+\bar\lambda'_i(x')s_i]}e^{iu'^m(x')K_m}$ with the following relation

$$g\Omega = \Omega'h, \tag{8}$$

where h stands for the subgroup element. The field $\phi(x)$, which transforms linearly under rigid g transformations, i.e. $\phi'(x) = g\phi(x)$, could be re-expressed as field $\tilde{\phi}(x)$ through $\Omega^{-1}\phi(x) = \tilde{\phi}(x)$ from which the massless Goldstone mode has been eliminated. Therefore, when the symmetry group G is broken to subgroup H, from Eq. (8) one finds that $\phi'(x) = \Omega'\tilde{\phi}'(x) = g\phi(x) = g\Omega\tilde{\phi}(x) = \Omega' h\tilde{\phi}(x)$. Then the original G transformations is rewritten as the transformation depending on $\tilde{\phi}'(x) = h\tilde{\phi}(x)$ under the unbroken subgroup H, which is used as basic formalism to construct invariant Lagrangian when considering localized matter fields on the brane [18]. Applying Eqs. (B1)–(B3), it can be found the transformations of the space coordinates as well as the Nambu-Goldstone fields induced by the infinitesimal transformation of group g:

$$\begin{aligned}
x'^m &= x^m + dx^m + a^m - i(\bar{\xi}\gamma^m\theta + \bar{\eta}\gamma^m\lambda) \\
&\quad - \phi b^m + \varepsilon^{mnr}\alpha_n x_r, \\
\phi'(x') - \phi(x) &= \Delta\phi = z + d\phi + (\xi\gamma^0\lambda - \theta\gamma^0\eta) \\
&\quad - b^m x_m, \\
\theta'(x') - \theta(x) &= \Delta\theta_i = \xi_i + \frac{1}{2}d\theta_i + \frac{i}{2}b_m(\gamma^m\lambda)_i \\
&\quad - \frac{i}{2}\alpha_m(\gamma^m\theta)_i, \\
\lambda'(x') - \lambda(x) &= \Delta\lambda_i = \eta_i + \frac{1}{2}d\lambda_i - \frac{i}{2}b_m(\gamma^m\theta)_i \\
&\quad - \frac{i}{2}\alpha_m(\gamma^m\lambda)_i, \\
u'(x') - u(x) &= \Delta u^m = \frac{\sqrt{u^2}}{\tanh\sqrt{u^2}}\left(b^m - \frac{u^r b_r u^m}{u^2}\right) \\
&\quad + \frac{u^r b_r u^m}{u^2} + \varepsilon^{mnr}\alpha_n u_r,
\end{aligned} \tag{9}$$

where the linear terms of d represent the Weyl scale transformations of each collective coordinates. The element h is given by

$$h = e^{i[\alpha^m M_m - (1/2)(\tanh(\sqrt{u^2}/2)/(\sqrt{u^2}/2))b_n u_r \varepsilon^{nrm} M_m + dD)]} \tag{10}$$

From above, it can be found the spacetime coordinates have a field dependent transformation as a result of the nonlinear realization of group G. In this case, there are broken symmetries of Q_α, $\bar{Q}_{\dot{\alpha}}$, Z and rotation generators M^{m3} related to the z direction. For the breaking symmetry of spacetime, the only Nambu-Goldstone fields are those associated to the broken (super)translations [19], and the superfluous Nambu-Goldstone fields u^m can be eliminated by imposing invariant conditions on the Cartan differential forms [20] (see Eq. (19)).

The G symmetry is represented by transformation properties of the field $\tilde{\phi}(x)$ under the unbroken subgroup H. Considering incorporation of the dynamics of the field $\tilde{\phi}(x)$ with that of the brane, we work on dreibein basis in the local tangent space of the submanifold swept out by the $p = 2$ brane. The interval $ds^2 = g_{mn}dx^m dx^n$ has the form $ds^2 = \eta_{ab}dx^a dx^b$ in the tangent space, with relations $ds^2 = g_{mn}dx^m dx^n = \eta_{ab}e_m{}^a e_n{}^b dx^m dx^n$ and $dx^a = e_m{}^a dx^m$. The metric tensor is related to the dreibein through

$$g_{mn} = e_m{}^a e_n{}^b \eta_{ab} \tag{11}$$

In the local subgroup H formed by algebra $\{M_{ij}, D\}$, under the scale transformation $x^m \to x'^m = e^d x^m$, the interval transforms as $ds^2 \to ds'^2 = e^{2d}ds^2$. In terms of metric tensor g_{mn}, because $ds^2 = g_{mn}dx^m dx^n$, then it is understanding that the metric tensor have a weight 2 under the Weyl scale transformation, i.e.

$$g'_{mn} = e^{2d}g_{mn}. \tag{12}$$

Conversely, g^{mn} has scale weight -2. Its total infinitesimal transformation induced by the general coordinate variation $x^m \to x'^m = x^m + dx^m + \varepsilon^m(x)$ is given by

$$g'_{mn} = g_{mn} + 2dg_{mn} - (\partial_m\varepsilon_n + \partial_n\varepsilon_m) \tag{13}$$

where $\varepsilon^m(x) = a^m - i(\bar{\xi}\gamma^m\theta + \bar{\eta}\gamma^m\lambda) - \phi b^m + \varepsilon^{mnr}\alpha_n x_r$.

In order to construct an invariant action, we can obtain dreibein and connection one-forms by using Cartan form $\Omega^{-1}d\Omega$, which is expanded with respect to the G generators:

$$\begin{aligned}
\Omega^{-1}d\Omega &= i(\omega^a p_a + \bar{\omega}_{qi}q_i + \bar{\omega}_{si}s_i + \omega_Z Z + \omega_k^a K_a \\
&\quad + \omega_M^a M_a + \omega_D D)
\end{aligned} \tag{14}$$

Under the transformation $\Omega \to \Omega'$, the Cartan forms transform as

$$\Omega'^{-1}d\Omega' = h(\Omega^{-1}d\Omega)h^{-1} + hdh^{-1}. \tag{15}$$

It is obvious that all the forms transform homogeneously under G except the connection one from ω_M^a which transforms by a shift. These forms are invariant under the global left action of G on G/H. Under the local right action $\Omega \to \Omega' h$ with h given by Eq. (10), the forms ω^a transform as the dreibein on the tangent bundle to G/H, while ω_M^a transforms as a connection to this bundle. The Cartan forms associated with the unbroken spacetime generators P involve the exterior derivative d which is independent of the coordinate system used to parameterize the embedded submanifold and is reparametrization invariant. After choosing the static gauge $\xi^m = x^m$, the dreibein $e_m{}^a$ is obtained by expanding spacetime one-forms ω^a with respect to the coordinate differentials dx^m, i.e. $\omega^a = dx^m e_m{}^a$. The connection one-forms ω_M^a, on the other hand, can be used to construct the covariant derivative of the fields $\nabla\tilde{\phi}(x) = (d + i\omega_M^a\Gamma_a + i\omega_D\Gamma')\tilde{\phi}(x)$, where Γ_a and Γ' are respective representations of the generators M_a and D with respect to the fields $\tilde{\phi}(x)$. These are the building blocks that can be used to construct invariant actions under G. Considering Eqs. (4) and (B4), we have

$$\omega_M^a = (\cosh\sqrt{u^2} - 1)\frac{u_b du_c}{u^2}\varepsilon^{abc}$$

$$\omega^a = (dx^m + id\theta\gamma^0\gamma^m\theta + id\lambda\gamma^0\gamma^m\lambda)\cdot\left(\delta_m{}^a + (\cosh\sqrt{u^2} - 1)\frac{u_m u^a}{u^2}\right) + (d\phi + d\theta\gamma^0\lambda - d\lambda\gamma^0\theta)\frac{\sinh\sqrt{u^2}}{\sqrt{u^2}}u^a$$

$$\omega_D = 0 \tag{16}$$

where $a = 0, 1, 2$. We use a, b, c to represent the tangent spacetime index, and i, j, k to represent $2 + 1$ general coordinates in what follows. Since D is the automorphism generator of the (super)spacetime position group, and from the commutation relations of Eq. (2), it is found that D is not involved in the Cartan forms here, which is in consistent with $\omega_D = 0$. The dreibein

$$e_m{}^a = (\delta_m{}^b + i\partial_m\theta\gamma^0\gamma^b\theta + i\partial_m\lambda\gamma^0\gamma^b\lambda)\cdot\left(\delta_b{}^a + (\cosh\sqrt{u^2} - 1)\frac{u_b u^a}{u^2} + (\tilde{D}_b\phi + \tilde{D}_b\theta\gamma^0\lambda - \theta\gamma^0\tilde{D}_b\lambda)\frac{\sinh\sqrt{u^2}}{\sqrt{u^2}}u^a\right)$$

$$= A_m{}^b\cdot\left(\delta_b{}^a + (\cosh\sqrt{u^2} - 1)\frac{u_b u^a}{u^2} + (\tilde{D}_b\phi + \tilde{D}_b\theta\gamma^0\lambda - \theta\gamma^0\tilde{D}_b\lambda)\frac{\sinh\sqrt{u^2}}{\sqrt{u^2}}u^a\right) \tag{17}$$

has a tangent space index a, which has the transformation property induced by Eq. (15) in the local tangent space, with $L_b{}^a$ the local H representation with vector indices

$$e'_m{}^a = e_m{}^b L_b{}^a \tag{18}$$

In Eq. (17), $\tilde{D}_b = A_b^{-1m}\partial_m$ is the Akulov-Volkov derivative, defined by $A_m{}^b = \delta_m{}^b + i\partial_m\theta\gamma^0\gamma^b\theta + i\partial_m\lambda\gamma^0\gamma^b\lambda$ [3,4,10]. Imposing the invariant condition $\omega_z = 0$ on the covariant derivative, as a result of the inverse Higgs Mechanism [10,20], the field u_m can be eliminated by the following relation

$$u_b\frac{\tanh\sqrt{u^2}}{\sqrt{u^2}} = -(\tilde{D}_b\phi + \tilde{D}_b\theta\gamma^0\lambda - \theta\gamma^0\tilde{D}_b\lambda) = -\tilde{D}_b\Phi \tag{19}$$

Plugging this into Eq. (17), the dreibein hence has the simple form

$$e_m{}^a = A_m{}^b\cdot\left(\delta_b{}^a + \frac{u_b u^a}{u^2}\left(\frac{1}{\cosh\sqrt{u^2}} - 1\right)\right)$$
$$= A_m{}^b\cdot\left(\delta_b{}^a + \frac{\tilde{D}_b\Phi\tilde{D}^a\Phi}{(D\Phi)^2}\left(\sqrt{1 - (\tilde{D}\Phi)^2} - 1\right)\right). \tag{20}$$

The metric tensor becomes

$$g_{mn} = e_m{}^a e_n{}^b\eta_{ab} = A_m{}^a A_n{}^b\eta_{ab} - \partial_m\Phi\partial_n\Phi \tag{21}$$

Introduce four dynamic variables $X^\mu = (X^a, X^3) = (X^a, \Phi)$, which are defined as following

$$dX^a = dx^m A_m{}^a$$
$$dX^3 = (\partial_m\phi + \partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda)dx^m \tag{22}$$

After integrating from both sides, we have

$$X^0 = x^0 + f^0(\theta, \lambda), \qquad X^1 = x^1 + f^1(\theta, \lambda),$$
$$X^2 = x^2 + f^2(\theta, \lambda), \qquad X^3 = \phi + F(\theta, \lambda), \tag{23}$$

where $f(\theta, \lambda)$, $F(\theta, \lambda)$ are functions of $\theta(x)$, $\lambda(x)$, decided by the integration of Eq. (22). Therefore,in the static gauge $\xi^m = x^m$, by using Eq. (23), the metric tensor in Eq. (21) now becomes

$$g_{mn} = \eta_{\mu\nu}\frac{\partial X^\mu}{\partial\xi^m}\frac{\partial X^\nu}{\partial\xi^n} = \eta_{\mu\nu}\frac{\partial x^\mu}{\partial\xi^m}\frac{\partial x^\nu}{\partial\xi^n} + \text{other terms} \tag{24}$$

where $x^\mu = (x^0, x^1, x^2, \phi)$. Consequently, in contrast with the normal spacetime induced metric $g_{mn} = \eta_{\mu\nu}\frac{\partial x^\mu}{\partial\xi^m}\frac{\partial x^\nu}{\partial\xi^n}$ on p brane world volume, there are modification terms of the metric through the functions $f(\theta, \lambda)$, $F(\theta, \lambda)$ which are contributed from the Nambu-goldstone fields $\theta(x)$, $\lambda(x)$ corresponding to the broken symmetries in the superspace coordinates directions (see Eq. (28) for details).

The infinitesimal transformation of dreibein in the local tangent space is

$$\delta e_m{}^a = \delta^L e_m{}^a + de_m{}^a \tag{25}$$

with $\delta^L e_m{}^a$ represents the local Lorentz transformation. The second term is the Weyl scale transformation. Hence, the world volume has the scale transformation property

$$dx^3\,\det e \to dx'^3\,\det e' = e^{3d}dx\,\det e. \tag{26}$$

We introduce the intrinsic metric ρ_{mn} on this $p = 2$ brane manifold. Similarly, it has the Weyl scale transformation property $\rho_{mn} \to e^{2d}\rho_{mn}$ and $dx^3\sqrt{|\rho|} \to dx'^3\sqrt{|\rho'|} = e^{3d}dx^3\sqrt{|\rho|}$. Considering Eq. (24), the action of the Weyl scale invariant $p = 2$ brane is then constructed

$$I = -T\int d^3x\sqrt{|\rho|}\left(\frac{1}{3}\rho^{mn}\eta_{\mu\nu}\partial_m X^\mu\partial_n X^\nu\right)^{3/2} \tag{27}$$

here ρ^{mn} is the inverse of the metric ρ_{mn} and ρ stands for the determinant of ρ_{mn}. The auxiliary intrinsic metric ρ_{mn} can be eliminated by using its equation of motion. By using Eq. (22), the action then has an explicit form

$$I = -T \int d^3x \sqrt{|\rho|} \Big(\frac{1}{3} \rho^{mn} \eta_{\mu\nu} \partial_m X^\mu \partial_n X^\nu \Big)^{3/2}$$
$$= -T \int d^3x \sqrt{\rho} \Big\{ \frac{1}{3} \rho^{mn} (\eta_{mn} + i\partial_n\theta\gamma^0\gamma_m\theta + i\partial_n\lambda\gamma^0\gamma_m\lambda + i\partial_m\theta\gamma^0\gamma_n\theta + i\partial_m\lambda\gamma^0\gamma_n\lambda$$
$$+ (i\partial_m\theta\gamma^0\gamma^a\theta + i\partial_m\lambda\gamma^0\gamma^a\lambda) \cdot (i\partial_n\theta\gamma^0\gamma_a\theta + i\partial_n\lambda\gamma^0\gamma_a\lambda) - (\partial_m\phi + \partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda)$$
$$\cdot (\partial_n\phi + \partial_n\theta\gamma^0\lambda - \theta\gamma^0\partial_n\lambda)) \Big\}^{3/2} \quad (28)$$

where T stands for the brane tension.

III. WEYL SCALE INVARIANT D2 BRANE

As discussed in Sec. II, we have constructed the Weyl scale invariant non-BPS $p = 2$ brane action. In the following we derive its dual form, the non-BPS Weyl scale invariant D2 brane supersymmetric Born-Infeld action. From Eqs. (19) and (20), we have

$$\det e = \det(e_m{}^a)$$
$$= \det\Big\{ A_m{}^b \cdot \Big(\delta_b{}^a + \frac{u_b u^a}{u^2} \Big(\frac{1}{\cosh\sqrt{u^2}} - 1 \Big) \Big) \Big\}$$
$$= \det(A_m{}^b) \cdot \det\Big\{ \delta_b{}^a + \frac{u_b u^a}{u^2} \Big(\frac{1}{\cosh\sqrt{u^2}} - 1 \Big) \Big\}$$
$$= \det A \cdot \frac{1}{\cosh\sqrt{u^2}}$$
$$= \det A \cdot \sqrt{1 - (\tilde{D}_b\phi + \tilde{D}_b\theta\gamma^0\lambda - \theta\gamma^0\tilde{D}_b\lambda)} \quad (29)$$

By using the Nambu-Goto type $p = 2$ brane action $-T\int d^3x \det e$, and considering Eq. (29), it allows us to introduce a gauge field strength vector F^r [10] by variation of this action with respect to the field ϕ. It is defined as

$$F^r = \det A \cdot u^a \cdot A_a^{-1r} \cdot \frac{\sinh\sqrt{u^2}}{\sqrt{u^2}}. \quad (30)$$

Its equation of motion results the relation $\partial_r F^r = 0$, which has explicitly U(1) gauge solution A_n, i.e.

$$F_{mn} = \partial_m A_n - \partial_n A_m \quad (31)$$

and F^r is related to F_{mn} by

$$F^r = \frac{1}{2} \varepsilon^{mnr} F_{mn} = \frac{1}{2} \varepsilon^{mnr} (\partial_m A_n - \partial_n A_m). \quad (32)$$

Conversely, $F_{mn} = \varepsilon_{mnr} F^r$. In $D = 3$ dimension, there is $D - 2 = 1$ degree of freedom for the U(1) gauge field A_n, which compensates the degree of freedom of field ϕ in Eq. (29). Therefore, by using Eq. (30), we find

$$\cosh\sqrt{u^2} = \sqrt{1 + \sinh^2\sqrt{u^2}} = \sqrt{1 + \frac{F^m A_m{}^b F^n A_n{}^a \eta_{ab}}{\det^2 A}}. \quad (33)$$

Introduce the Akulov-Volkov metric field $\tilde{g}_{mn}$, which is given by

$$\tilde{g}_{mn} = A_m{}^a A_n{}^b \eta_{ab}. \quad (34)$$

It has the explicit form

$$\tilde{g}_{mn} = \eta_{mn} + i\partial_n\theta\gamma^0\gamma_m\theta + i\partial_n\lambda\gamma^0\gamma_m\lambda + i\partial_m\theta\gamma^0\gamma_n\theta$$
$$+ i\partial_m\lambda\gamma^0\gamma_n\lambda + (i\partial_m\theta\gamma^0\gamma^a\theta + i\partial_m\lambda\gamma^0\gamma^a\lambda)$$
$$\cdot (i\partial_n\theta\gamma^0\gamma_a\theta + i\partial_n\lambda\gamma^0\gamma_a\lambda). \quad (35)$$

After explicitly expanding the following determent, it can be shown

$$\det(\tilde{g}_{mn} + F_{mn}) = \det(\tilde{g}_{mn} + \varepsilon_{mnr}F^r)$$
$$= \det\tilde{g} + F^m F^n \tilde{g}_{mn} = \det{}^2 A \cdot \cosh^2\sqrt{u^2} \quad (36)$$

where $\tilde{g} = \det\tilde{g}_{mn}$, and the last equality is a result of Eq. (33). Consider the alternative form of Eq. (29)

$$\det e = \det A \cdot \frac{1}{\cosh\sqrt{u^2}} = \det A \cdot \frac{\cosh^2\sqrt{u^2} - \sinh^2\sqrt{u^2}}{\cosh\sqrt{u^2}}$$
$$= \det A \cdot \cosh\sqrt{u^2} - \det A \cdot \sinh\sqrt{u^2} \tanh\sqrt{u^2}, \quad (37)$$

by using Eq. (19) and (30) and substituting Eq. (36) into (37), the resulting expression is

$$\det e = \sqrt{\det(\tilde{g}_{mn} + F_{mn})} + F^m(\partial_m\phi + \partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda) \quad (38)$$

Introduce an intrinsic tensor field G_{mn}, which has Weyl scale transformation property

$$G_{mn} \to e^{2d} G_{mn}. \quad (39)$$

Hence a spacetime integral of the first part of Eq. (38) has the classically equivalent Weyl invariant form:

$$\int d^3x \Big\{ \tilde{g}^{1/4} G^{1/4} \Big[\frac{1}{3} G^{mn} (\tilde{g}_{mn} - \tilde{g}^{kl} F_{mk} F_{ln}) \Big]^{3/4} \Big\} \quad (40)$$

The equation of motion of the intrinsic tensor field G_{mn}, which can be derived from Eq. (43), is

$$G_{mn} = \Omega(\tilde{g}_{mn} - \tilde{g}^{kl} F_{mk} F_{ln}), \quad (41)$$

where Ω is a constant. The spacetime integral of the second part of Eq. (38) has the form

$$\begin{aligned}\int d^3xF^m(\partial_m\phi + \partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda) &= \int d^3xF^m(\partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda)\\ &= \int d^3x\frac{1}{2}\varepsilon^{mnr}(\partial_mA_n - \partial_nA_m)\cdot(\partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda).\end{aligned} \tag{42}$$

In the first equality we use the relation $\partial_rF^r = 0$ and integrate by parts to drop the field ϕ term. Consider the world volume element d^3x, which is a tensor density with weight -1. Therefore $d^3x\sqrt{\tilde{g}}$ becomes a scalar quantity. Since ε^{mnr} is also a weight -1 tensor density, we can form an ordinary contravariant rank three tensor $\frac{\varepsilon^{mnr}}{\sqrt{\tilde{g}}}$. Under the Weyl scale transformation $\tilde{g}_{mn}\longrightarrow e^{2d}\tilde{g}_{mn}$, Eq. (42) hence keeps invariant.

Considering Eqs. (40) and (42), the dual form action, i.e. the Weyl scale invariant D2 brane Born-Infeld type action is then constructed

$$\begin{aligned}I = -T\int d^3x\bigg\{&\tilde{g}^{1/4}G^{1/4}\bigg[\frac{1}{3}G^{mn}(\tilde{g}_{mn} - \tilde{g}^{kl}F_{mk}F_{ln})\bigg]^{3/4}\\ &+ \frac{1}{2}\varepsilon^{mnr}(\partial_mA_n - \partial_nA_m)\cdot(\partial_m\theta\gamma^0\lambda - \theta\gamma^0\partial_m\lambda)\bigg\}\end{aligned} \tag{43}$$

where F_{mn} and $\tilde{g}_{mn}$ are given by Eq. (31) and (35) respectively.

IV. LOCALIZED MATTER FIELDS ON THE BRANE

In addition to the massless Nambu-Goldstone fields $\phi(x)$, $\theta(x)$ and $\lambda(x)$ on the brane, there can also be matter field degrees of freedom localized on the brane. The induced localization of the scalar and fermionic degrees of freedom on the submanifold were considered in [10] when the embedded defects spontaneously break the target manifold. In the following model, by using the ingredients of the Cartan one-forms, we present the actions of the matter fields as well as interactions with the Nambu-Goldstone fields. Consider there is different dilatation scale associated with local spacetime points on the brane, i.e.d is a local function of spacetime, a Weyl gauge field $B_m(x)$ is introduced as the compensating field in order to keep the whole action invariant. The action of the Weyl gauge field $B_m(x)$ interacting with the Nambu-Goldstone fields on the brane is also constructed.

For the matter degrees of freedom localized on the $p = 2$ brane, under the unbroken subgroup H, in the tangent space the matter field $M(x)$ transforms as

$$M'(x') = hM(x) \tag{44}$$

in which h is given by Eq. (10). The covariant derivative for the matter field is given through the spin connection and dilatation one-forms:

$$\begin{aligned}\nabla M(x) &= (d + i\omega_M^a\Gamma_a + i\omega_D\Gamma')M(x)\\ &= (d + i\omega_M^a\Gamma_a)M(x)\end{aligned} \tag{45}$$

where Γ_a is the representation of M_m corresponding to field $M(x)$. When Weyl scale parameter d becomes local function of spacetime $d = d(x)$, Eq. (45) will not transform covariantly under H. A new compensating field B_m (Weyl gauge field) is introduced [21–24], therefore in component forms the covariant derivative is written as:

$$\begin{aligned}\nabla_aM(x) &= (e_a^{-1m}\partial_m + i\omega_a{}^b\Gamma_b + B_ad_s)M(x)\\ &= (D_a + i\omega_a{}^b\Gamma_b + B_ad_s)M(x)\end{aligned} \tag{46}$$

where $D_a = e_a^{-1m}\partial_m$, the coefficients $\omega_a{}^b$ is related to spin connection one-from by $\omega_M^b = \omega_a{}^bdx^a$, B_m is related to gauge field B_a in the local tangent space by $B_m = e_m{}^aB_a$, and d_s is the scale dimension(weight) of the matter field $M(x)$. The variation of the matter field under the group H is

$$\begin{aligned}\delta M(x) &= M'(x') - M(x) = \delta_L'M(x) + \delta_D'M(x)\\ &= i\varepsilon^a\cdot(\Gamma_a + L_a)M(x) + id(x)\cdot DM(x)\end{aligned} \tag{47}$$

where $\delta_L'M(x) = i\varepsilon^a\cdot(\Gamma_a + L_a)M(x)$ represents variation under SO(1,2) transformation, the parameter ε^a decided by Eq. (10) is a function of α and b. And L is the angular momentum representation of M_m. The variation of the scale transformation is

$$\begin{aligned}\delta_D'M(x) &= M'(x') - M(x)|_D = id(x)\cdot DM(x)\\ &= d(x\cdot\partial + d_s)M(x)\end{aligned} \tag{48}$$

Hence, the general filed representation of the scale operator is given by

$$D = -i(x\cdot\partial + d_s). \tag{49}$$

The intrinsic Weyl scale variation of the matter field then can be written as

$$\delta_DM(x) = M'(x) - M(x)|_D = d(x)\cdot d_sM(x). \tag{50}$$

From Eq. (18) and (25) one can find the intrinsic infinitesimal scale variation of dreibein

$$\delta_De_m{}^a = de_m{}^a \tag{51}$$

Therefore the scale dimension for dreibein is 1. Besides, in $2 + 1$ dimensions, the scalar field has weight $d_s(\phi) = -1/2$, and the spinor field is $d_s(\psi) = -1$. Correspondingly, we have

$$\delta_D\phi = -\frac{1}{2}d(x)\phi, \qquad \delta_D\psi = \delta_D\bar{\psi} = -d(x)\psi, \tag{52}$$

with scale transformation of the coordinates in the tangent space

$$\delta_Dx^a = d(x)x^a. \tag{53}$$

Hence, under D operation the derivative of the matter field transforms as

$$\begin{aligned} D_a M(x) \to D'_a M'(x') &= D'_a M'(Dx) \\ &= e^{-d(x)} e_a^{-1m} \partial_m e^{d(x)d_s} M(x) \\ &= e^{-d(x)} e^{d(x)d_s} e_a^{-1m} (d_s d(x)_{,m} M(x) + \partial_m M(x)) \end{aligned} \tag{54}$$

The Weyl gauge covariant derivative then transforms as

$$\begin{aligned} \nabla_a M(x) \to \nabla'_a M'(x') &= (D'_a + i\omega_a{}^b \Gamma_b + B'_a d_s) M'(Dx) \\ &= e^{-d(x)} e^{d(x)d_s} (D_a + d_s d(x)_{,a} + i\omega_a{}^b \Gamma_b + B_a d_s \\ &\quad - d_s d(x)_{,a}) M(x) \\ &= e^{-d(x)} e^{d(x)d_s} \nabla_a M(x) \end{aligned} \tag{55}$$

in which we used $\delta_D \omega_M^b = 0$, therefore $\delta_D \omega_a{}^b = -d(x)\omega_a{}^b$ and the Weyl gauge field B_m has the infinitesimal scale transformation property

$$\delta_D B_m(x) = -d(x)_{,m}. \tag{56}$$

Considering the scalar field localized on the $p = 2$ brane, since $\Gamma_b(\phi) = 0$ and $d_s(\phi) = -1/2$, the Weyl gauge covariant derivative is then constructed

$$\nabla_a \phi(x) = \left(e_a^{-1m}\partial_m - \frac{1}{2} e_a^{-1m} B_m\right)\phi(x). \tag{57}$$

The Lagrangian density of the scalar field is given by

$$\begin{aligned} \ell_\phi &= \eta^{ab} \nabla_a \phi(x) \nabla_b \phi(x) + f\phi^6 \\ &= g^{mn}\left(\partial_m - \frac{1}{2}B_m\right)\phi(x)\left(\partial_n - \frac{1}{2}B_n\right)\phi(x) + f\phi^6 \end{aligned} \tag{58}$$

in which f is the dimensionless coupling constant. The effective action of the scalar matter field on the brane up to the leading term in brane tension expansion is obtained

$$I_\phi = \int d^3x \det e \ell_\phi. \tag{59}$$

For the fermion spinor field, the spinor representation of the operators M_a are $\Gamma_a(\psi) = -\frac{1}{2}\gamma_a$. The covariant derivative of the spinor field is

$$\nabla_a \psi_i(x) = D_a \psi_i - i\frac{1}{2}\omega_a{}^b \gamma_{bij} \psi_j(x) - B_a \psi_i(x) \tag{60}$$

Since $\psi(x)$, $\bar\psi(x)$ interacts with the field B_a in the same manner, the spinor field has no minimal form of coupling to the Weyl gauge field. The Lagrangian of the spinor matter field with Yukawa coupling to the scalar fields is

$$\ell_\psi = \frac{1}{2} i[\bar\psi \gamma^a \nabla_a \psi - \nabla_a \bar\psi \gamma^a \psi] + g\bar\psi\psi\phi^2 \tag{61}$$

The effective action of the spinor field on the brane has the form

$$I_\psi = \int d^3x \det e \ell_\psi \tag{62}$$

The field strength which describes the Weyl gauge field B_m has the normal form

$$F_{mn} = \partial_m B_n - \partial_n B_m \tag{63}$$

Introducing new dynamics variables $F_{ab} = e_a^{-1m} e_b^{-1n} F_{mn}$, their infinitesimal Weyl transformation properties are

$$\delta_D F_{ab} = -2d(x) F_{ab}, \tag{64}$$

on dimension and Weyl scale invariant ground, the effective action of the Weyl gauge field can be constructed

$$I_B = \int d^3x \det e \ell_B = \int d^3x \det e \ell_B(e_a^{-1m}, F_{mn}) \tag{65}$$

where the lagrangian has Weyl dimension -3 and is a function of e_a^{-1m} and Weyl gauge field strength F_{mn}. Considering Eqs. (59), (62), and (65), the full effective action for the matter fields localized on the brane is then given by

$$\begin{aligned} I_{\rm matter} &= I_\phi + I_\psi + I_B = \int d^3x \det e \ell_{\rm matter} \\ &= \int d^3x \det e \ell_\phi + \int d^3x \det e \ell_\psi + \int d^3x \det e \ell_B \end{aligned} \tag{66}$$

In summary, in this letter we have constructed Weyl scale invariant version of the $p = 2$ brane action, which is a result of spontaneous breaking of the target $N = 1$, $D = 4$ superspace with G symmetry to the $W(1, 2)$ symmetry on the embedded the $2 + 1$ world volume. Its low energy fluctuations in directions associated with the broken symmetry generators are described by the dynamics of the Nambu-Goldstone fields. There, unlike the BPS state of the D brane which carries conserved charges or the partially broken supersymmetry on the brane whose central charge saturates the lower bound of the state [25], it is the case of non-BPS state. By this approach of nonlinear realization, one can also find its application to branes of M theory with a large automorphism group of superalgebra [26]. In addition, as described above, the brane, as a defect in spacetime that breaks certain symmetries, may cause the localization of matter fields as well as the gauge fields on it, which is a fact of physical necessity and required to be present in the effective world volume field theories. Additional discussions can also be found in [27,28] and some brane world scenarios as well [29].

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APPENDIX A: DIMENSIONAL REDUCTION OF $N = 1, D = 4$ SUSY TO $N = 2, D = 3$ SUSY

In $N = 2$, $D = 3$ Supersymmetry theory, there are two two-component Majorana spinorial generators Q^1_α, Q^2_β satisfying

$$\{Q^1_\alpha, \bar{Q}^1_\beta\} = (\gamma \cdot P)_{\alpha\beta}, \qquad \{Q^2_\alpha, \bar{Q}^2_\beta\} = (\gamma \cdot P)_{\alpha\beta}, \\ \{Q^1, \bar{Q}^2\} = iZ_0, \tag{A1}$$

where $\alpha, \beta = 1, 2$ are component indices, and $(\gamma^0, \gamma^1, \gamma^2) = (\sigma^2, i\sigma^1, i\sigma^3)$, $\bar{Q}^{1(2)} = Q^{1(2)\dagger}C = Q^{1(2)\dagger}\gamma^0$, with Majorana condition $Q^{1(2)} = -Q^{1(2)*}$, and Z_0 is the central charge. If we introduce a Dirac spinor $Q' = \frac{1}{\sqrt{2}}(Q^1 + iQ^2)$, then we have

$$\{Q', \bar{Q}'\} = \gamma \cdot P + Z_0. \tag{A2}$$

In $N = 1$, $D = 4$ Supersymmetry theory, from Eq. (1), we have

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu. \tag{A3}$$

Imposing rotation operations $e^{-iM^{23}\pi/2}$ and $e^{iM^{13}\pi/2}$ consecutively on Eq. (A3), the four momentum vector has the corresponding transformation

$$P_0 \to P_0 \qquad P_1 \to P_2 \qquad P_2 \to -P_3 \qquad P_3 \to -P_1 \tag{A4}$$

and the spinor becomes

$$Q \to W \tag{A5}$$

with

$$W = \begin{pmatrix} Q_1 e^{-i\pi/4}\cos\frac{\pi}{4} + Q_2 e^{i\pi/4}\sin\frac{\pi}{4} \\ Q_2 e^{i\pi/4}\cos\frac{\pi}{4} - Q_1 e^{-i\pi/4}\sin\frac{\pi}{4} \end{pmatrix} \tag{A6}$$

Equation (A3) then has the form

$$\{W, W^\dagger\} = 2\sigma^0 P_0 + 2\sigma^1 P_2 + 2\sigma^2(-P_3) + 2\sigma^3(-P_1) \tag{A7}$$

Right multiplication of σ^2 from both sides, it has the form

$$\{W, W^\dagger\sigma^2\} = 2\sigma^2 P_0 + 2i\sigma^3 P_2 + 2(-P_3) + 2i\sigma^1 P_1 \\ = 2\gamma^0 P_0 + 2\gamma^1 P_1 + 2\gamma^2 P_2 + 2(-P_3). \tag{A8}$$

Thus

$$\{W, \bar{W}\} = 2\gamma \cdot P + 2(-P_3), \tag{A9}$$

where $\bar{W} = W^\dagger\gamma^0$. Compare (A9) with (A2), we may identify W with $\sqrt{2}Q'$ and Z_0 with $-P_3$. Using redefined operators

$$Q^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} is_1 \\ is_2 \end{pmatrix}, \qquad Q^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -iq_1 \\ -iq_2 \end{pmatrix}, \tag{A10}$$

from Eqs. (A1), (A2), (A6), and (A10), we have

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}Q_1 e^{i\pi/4} - \frac{1}{2}Q_2 e^{i\pi/4} + \frac{1}{2}\bar{Q}_1 e^{-i\pi/4} - \frac{1}{2}\bar{Q}_2 e^{-i\pi/4} \\ \frac{1}{2}Q_1 e^{-i\pi/4} + \frac{1}{2}Q_2 e^{-i\pi/4} + \frac{1}{2}\bar{Q}_1 e^{i\pi/4} + \frac{1}{2}\bar{Q}_2 e^{i\pi/4} \end{pmatrix}, \\ \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}Q_1 e^{-i\pi/4} - \frac{1}{2}Q_2 e^{-i\pi/4} - \frac{1}{2}\bar{Q}_1 e^{i\pi/4} + \frac{1}{2}\bar{Q}_2 e^{i\pi/4} \\ \frac{1}{2i}Q_1 e^{-i\pi/4} + \frac{1}{2i}Q_2 e^{-i\pi/4} - \frac{1}{2i}\bar{Q}_1 e^{i\pi/4} - \frac{1}{2i}\bar{Q}_2 e^{i\pi/4} \end{pmatrix}. \tag{A11}$$

Hence, the extended centrally charged $N = 2$, $D = 3$ supersymmetry algebra is

$$\{q_i, q_j\} = 2(\gamma^m C)_{ij}P_m, \qquad \{s_i, s_j\} = 2(\gamma^m C)_{ij}P_m, \\ \{q_i, s_j\} = -2iC_{ij}P_3, \qquad [K^m, q_j] = \frac{1}{2}\gamma^m_{ij}s_j, \\ [K^m, s_i] = -\frac{1}{2}\gamma^m_{ij}q_j, \qquad [M^{mn}, q_i] = -\frac{1}{2}\gamma^{mn}_{ij}q_j, \\ [M^{mn}, s_i] = -\frac{1}{2}\gamma^{mn}_{ij}s_j. \tag{A12}$$

APPENDIX B: USEFUL FORMULAS

In derivation of Eqs. (9) and (10), we consider the Baker-Hausdorff formula:

$$\exp(a)\exp(b) = \exp\left(a + b + \frac{1}{2}[a, b] + \frac{1}{12}[a, [a, b]] + \frac{1}{12}[b, [b, a]] + \ldots\right). \tag{B1}$$

For infinitesimal operator a, up to the first order of a, we have

$$\exp(a)\exp(b) = \exp\left(a + b + \frac{1}{2}[a, b] + \frac{1}{12}[b, [b, a]] + \ldots + O(a^2)\right) \\ = \exp(a - ad_{b/2}(a) + ad_{b/2} \cdot \coth(ad_{b/2})(a) + O(a^2)); \tag{B2}$$

and likewise

$$\exp(b)\exp(a) = \exp\left(a + b + \frac{1}{2}[b, a] + \frac{1}{12}[b,[b, a]] + \ldots\right) = \exp(b + ad_{b/2}(a) + ad_{b/2} \cdot \coth(ad_{b/2})(a) + O(a^2)), \tag{B3}$$

in which $ad_{b/2}(a)$ is the adjoint operation with $ad_{b/2}(a) = [\frac{b}{2}, a]$. In derivation of Eq. (16),consider the differentiation formula for exponent:

$$\exp(-b)d\exp(b) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!}(ad_b)^k db \tag{B4}$$

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