PHYSICAL REVIEW D **74,** 045012 (2006)

Remark on integrating out heavy moduli in flux compactification

Hiroyuki Abe,[1,*] Tetsutaro Higaki,[2,†] and Tatsuo Kobayashi[3,‡]
[1]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[2]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[3]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
(Received 20 June 2006; published 16 August 2006)

We study two steps of moduli stabilization in type IIB flux compactification with gaugino condensations. We consider the condition that one can integrate out heavy moduli first with light moduli remaining. We give appendix, where detail study is carried out for potential minima of the model with a six dimensional compact space with $h_{1,1} = h_{2,1} = 1$, including the model, whose respective moduli with $h_{1,1}, h_{2,1} \neq 1$ are identified.

DOI: 10.1103/PhysRevD.74.045012 PACS numbers: 04.65.+e, 11.25.Mj, 11.25.Wx

I. INTRODUCTION

Moduli stabilization in superstring theory is one of important issues to study. Indeed, several scenarios have been proposed so far. Flux compactifications are studied intensively in these years, because several moduli can be stabilized through flux compactification. For example, the dilaton S and complex structure moduli U^α can be stabilized within the framework of type IIB string theory [1], while Kähler moduli T remain not stabilized. Recently, in Ref. [2] a new scenario was proposed to lead to a de Sitter (or Minkowski) vacuum, where all of moduli are stabilized in type IIB string models, and it is the so-called KKLT scenario. The KKLT scenario consists of three steps. At the first step, it is assumed that the dilaton and complex structure moduli are stabilized through flux compactification. At the second step, we introduce nonperturbative superpotential terms, which depend on the Kähler moduli. That leads to a supersymmetric anti de Sitter (AdS) vacuum. At the third step, the AdS vacuum is uplifted by introducing anti D3 branes, which break supersymmetry (SUSY) explicitly.

Phenomenological aspects like soft SUSY breaking terms have been studied [3]. The KKLT scenario predicts the unique pattern of SUSY breaking terms and they have significant phenomenological implications [4–6].

On the other hand, the flux compactification has been studied in explicit models [7–9]. Moreover, the three steps of moduli stabilization has been studied, in particular, the first two steps. It has been shown that such two or three steps of moduli stabilization may be inconsistent in some models showing instability of assumed vacua [10,11].

Furthermore, in Ref. [12] models with S-T mixing nonperturbative superpotential terms have been discussed with the assumption that S is already stabilized through flux compactification. Such models lead to interesting phenomenological and cosmological aspects. For example, these models have a rich structure of soft SUSY breaking terms compared with the original KKLT scenario. Also, a certain class of these models have moduli potential forms different from the original KKLT, and may avoid the overshooting problem [13] and destabilization due to finite temperature effects [14], from which the original KKLT potential suffers. At any rate, in this new scenario it is the crucial point that one of moduli, say S, in nonperturbative superpotential is already stabilized through the flux compactification.

Thus, it is important to study the validity of the two-step moduli stabilization, in particular, the KKLT type models with moduli-mixing nonperturbative superpotential. That is our purpose of this paper. Here we concentrate to IIB string models, but our discussions on validity of integrating out heavy moduli can be easily extended into generic string theory.

This paper is organized as follows. In Sec. II, we give a brief review on the KKLT scenario and its generalization with moduli-mixing superpotential. In Sec. III, we study validity of two-steps moduli stabilization. Section IV is devoted to conclusion and discussion. In appendix validity of our procedure is studied by examining potential minima explicitly and carefully.

II. REVIEW ON KKLT SCENARIO

Here we give a brief review on the KKLT scenario for moduli stabilization through the flux compactification. In the KKLT scenario, three types of moduli, the dilaton S, Kähler moduli and complex structure moduli U^α are stabilized through two steps. For simplicity, we consider the string model with a single Kähler modulus field T, although it is straightforward to extend our discussions to models with more than 1 Kähler moduli. We use the unit such that $M_{\rm Pl} = 1$, where $M_{\rm Pl}$ is the $4D$ reduced Planck mass.

At the first step, we consider a nontrivial background with nonvanishing flux, which generates a superpotential of S and U^α in type IIB string theory [15],

*Electronic address: abe@yukawa.kyoto-u.ac.jp
†Electronic address: tetsu@tuhep.phys.tohoku.ac.jp
‡Electronic address: kobayash@gauge.scphys.kyoto-u.ac.jp

$$W_{\rm flux}(S, U^\alpha) = \int_{M_6} G_3 \wedge \Omega, \tag{1}$$

where $G_3 = F_3^{RR} - 2\pi i S H_3^{NS}$ and Ω is the holomorphic 3-form. Note that T does not appear in the flux-induced superpotential in type IIB string theory. The Kähler potential is written as

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) - \ln\left(-i\int_{M_6}\Omega\wedge\bar{\Omega}\right). \tag{2}$$

The scalar potential in generic supergravity model is written as

$$V = e^K(D_aW\overline{D_bW}K^{a\bar{b}} - 3|W|^2), \tag{3}$$

where $D_aW = (\partial_aK)W + \partial_aW$. Thus, the above superpotential and the Kähler potential lead to the following scalar potential,

$$V = e^K\left(\sum_{i,j=S,U^\alpha} D_iW\overline{D_jW}K^{i\bar{j}}\right), \tag{4}$$

because of the no-scale form of the Kähler potential of T. We obtain the same result, e.g. in models with three moduli fields T^i. By this potential, the moduli fields, S and U^α, except the Kähler modulus T can be stabilized at the point, $D_SW = D_{U^\alpha}W = 0$.

Next, in the second step, the modulus T is stabilized. That is, in Ref. [2], a nonperturbative effect is assumed to induce the following superpotential,

$$W = w_0 - Ce^{-aT}, \tag{5}$$

where $w_0 = \langle W_{\rm flux}(S, U^\alpha)\rangle_{D_SW=D_{U^\alpha}W=0}$. Such term can be generated by gaugino condensation on D7-brane. Then, the modulus T can be stabilized at $D_TW = 0$. It corresponds to

$$a\,{\rm Re}(T) \approx \ln(C/w_0), \tag{6}$$

when $a\,{\rm Re}T \gg 1$. Its mass is estimated as

$$m_T \approx aw_0. \tag{7}$$

The above vacuum has the negative energy, i.e., $V = -3e^K|W|^2 < 0$ unless $W = 0$ at the above point. To realize a de Sitter (or Minkowski) vacuum, we need another step. To achieve it, the uplifting potential,

$$V_L = \frac{D}{(T+\bar{T})^{n_P}}, \tag{8}$$

is added in the KKLT scenario. This uplifting potential slightly shifts the minimum. This uplifting potential is an explicit SUSY breaking term, and the constant D is fine-tuned such as $V + V_L \approx 0$. That is, the size of D is estimated as $D = \mathcal{O}(|w_0|^2)$, and the SUSY breaking scale is w_0.

In Ref. [12], the above scenario has been extended into models with nonperturbative superpotential, where S and T are mixing,[1]

$$W_{\rm np} = \sum_m C_m e^{-(b_mS + a_mT)}. \tag{9}$$

These superpotential terms can be generated by e.g. gaugino condensations, where corresponding gauge kinetic functions are written as linear combinations of S and T. Such type of moduli-mixing appears in several types of string models, e.g. weakly coupled heterotic string models [17], heterotic M models [18], type IIA intersecting D-brane models and type IIB magnetized D-brane models [19]. Here, the exponent constants a_m and b_m can be negative, but they must satisfy the condition $b_m\langle{\rm Re}S\rangle + a_m\langle{\rm Re}T\rangle > 0$. We assume that S is already stabilized through the first step of the flux compactification, and that it is frozen in the above superpotential. That is, the dynamical mode in the above superpotential is only T. Its mass is estimated in a way similar to the original KKLT scenario. This type of models lead to interesting aspects from the viewpoint of particle phenomenology and cosmology [12].

III. INTEGRATING OUT HEAVY MODULI

Here, we study mainly on the first two steps of moduli stabilization. As above, in the KKLT scenario, stabilization of T and the other moduli is considered separately. That is, in the first step S and U^α are stabilized (integrated out), and in the second step T is stabilized. Such potential analysis is valid physically if the moduli fields S and U^α are much heavier than T with the superpotential,[2]

$$W = W_{\rm flux} + W_{\rm np}.$$

Hence, let us evaluate masses of moduli fields. The masses squared of moduli are obtained by the second derivatives of the scalar potential,

$$\begin{pmatrix} V_{a\bar{b}} & V_{ab} \\ V_{\bar{a}\bar{b}} & V_{\bar{a}b}\end{pmatrix}, \tag{10}$$

where each entry is obtained at $D_aW = 0$ as

$$V_{a\bar{b}}|_{D_aW=0} = (m_0)^2_{a\bar{b}} + (m_1)^2_{a\bar{b}} + (m_2)^2_{a\bar{b}}, \tag{11}$$

$$V_{ab}|_{D_aW=0} = (m_1)^2_{ab} + (m_2)^2_{ab}, \tag{12}$$

with

$$(m_0)^2_{a\bar{b}} = e^KK^{c\bar{d}}W_{ca}\bar{W}_{\bar{b}\bar{d}}, \tag{13}$$

$$(m_1)^2_{a\bar{b}} = e^K\bar{W}K^{c\bar{d}}W_{ac}(K_{\bar{b}\bar{d}} - K_{\bar{b}}K_{\bar{d}}) + {\rm H.c.}, \tag{14}$$

[1]See also Ref. [16].
[2]This point is confirmed in Appendix by examining potential minima explicitly and carefully.

$$(m_2)^2_{a\bar{b}} = e^K|W|^2[K^{c\bar{d}}(K_{ca} - K_aK_c)(K_{\bar{b}\bar{d}} - K_{\bar{b}}K_{\bar{d}}) - 3K_{a\bar{b}}], \tag{15}$$

$$(m_1)^2_{ab} = -e^K\bar{W}W_{ab}, \tag{16}$$

$$(m_2)^2_{ab} = -e^K|W|^2(K_{ab} - K_aK_b). \tag{17}$$

We assume that S, T, $U^\alpha = \mathcal{O}(1)$, and also e^K and its derivatives are of $\mathcal{O}(1)$. The above second derivatives of scalar potential include two types of mass scales. One is the superpotential mass, W_{ab}, and the other is supergravity effect, which is represented by the gravitino mass $m_{3/2} = e^{K/2}|W|$. For example, in $V_{a\bar{b}}|_{D_aW=0}$ we have

$$(m_0)^2_{a\bar{b}} = \mathcal{O}(|W_{ab}|^2), \qquad (m_1)^2_{a\bar{b}} = \mathcal{O}(|W_{ab}|m_{3/2}), \qquad (m_2)^2_{a\bar{b}} = \mathcal{O}(m^2_{3/2}). \tag{18}$$

Note that the third term $(m_2)^2_{a\bar{b}}$ appears somehow universally for all of moduli fields. That implies that if

$$|W_{ab}| \gg |m_{3/2}|, \tag{19}$$

the moduli fields corresponding to large superpotential masses can be integrated out first. Furthermore, when all of moduli masses satisfy the above condition and the determinant of mass matrix is nonvanishing, all of masses squared are positive and the SUSY point, $D_aW = 0$, is stable.[3]

Now, let us apply the above discussion to the flux compactification. In general, the superpotential $W_{\rm flux}(S, U^\alpha)$ induces mass terms of S and U^α, and those mass scales are naturally of $\mathcal{O}(M_{\rm Pl})$. On the other hand, the mass scale of T is of $\mathcal{O}(am_{3/2})$ in the above model. Thus, the procedure that first we integrate out S and U^α with T remaining, is valid when

$$|W_{ab}| \gg am_{3/2}, \tag{20}$$

for $a, b = S, U^\alpha$.

Here we give two illustrating examples. The first example is the model without complex structure moduli. In this model, we obtain

$$W_{\rm flux} = A + SB, \tag{21}$$

where A and B are constants. This superpotential does not include the mass term, i.e. $(W_{\rm flux})_{SS} = 0$. Thus, the dilaton mass is naturally of the gravitino mass, i.e. $m_S = \mathcal{O}(m_{3/2})$. That is, the dilaton is not heavier than the modulus T, and it is not valid to integrate out S first by using $D_SW_{\rm flux} = 0$. Indeed, it has been shown that it is inconsistent to first integrate out S in Ref. [10,11].

The second example is the orientifold model with a single complex structure U [8]. In this model, we obtain

$$W_{\rm flux} = A_0 + A_1U + A_2S + A_3SU, \tag{22}$$

where A_i $(i = 0, 1, 2, 3)$ are constants. This superpotential includes a mass term between S and U, and its natural scale is of $\mathcal{O}(M_{\rm Pl})$. Thus, it is valid to integrate out S and U first with T remaining if $m_T \approx am_{3/2} \ll M_{\rm Pl}$. Note that this mass term has mixing between S and U. That implies that it is not valid to integrate out only U by use of $D_UW_{\rm flux} = 0$, with S remaining. Therefore, we have to integrate out S and U at the same time. If the condition $m_T \approx am_{3/2} \ll M_{\rm Pl}$ is not satisfied and T is heavy, we can not integrate out first S and U. Instead of that, we have to study moduli stabilization for S, U and T at the same time, and the natural order of $m_{3/2}$ is of $M_{\rm Pl}$.[4]

Now, let us consider the condition that we can integrate out S and U, i.e., $m_T \approx am_{3/2} \ll M_{\rm Pl}$. The natural order of $w_0 = \langle W_{\rm flux}(S, U^\alpha)\rangle_{D_SW=D_{U^\alpha}W=0} \approx m_{3/2}$ is of $\mathcal{O}(1)$ in the unit $M_{\rm pl} = 1$. However, the above condition implies that $w_0 = \langle W_{\rm flux}(S, U^\alpha)\rangle_{D_SW=D_{U^\alpha}W=0} \approx m_{3/2} \ll \mathcal{O}(1)$. One way to realize such condition is to fine-tune flux such that $w_0 = \langle W_{\rm flux}(S, U^\alpha)\rangle_{D_SW=D_{U^\alpha}W=0}$ is finite, but suppressed compared with $M_{\rm Pl}$.

Another way is to consider the flux compactification satisfying

$$W_{\rm flux} = (W_{\rm flux})_a = 0, \tag{23}$$

for $a = S, U$. On top of that, we add nonperturbative term, e.g.

$$Ce^{-bS}, \tag{24}$$

which can be induced e.g. by gaugino condensation. The above condition (23) may be rather easily realized compared with the condition $w_0 = \langle W_{\rm flux}(S, U^\alpha)\rangle_{D_SW=D_{U^\alpha}W=0} \approx m_{3/2} \ll \mathcal{O}(1)$ and $w_0 \neq 0$. We give such an example from Ref. [7], which has the following superpotential;

$$W_{\rm flux} = -4(iU^3 + 1) + 2S(U^3 - 3iU^2 - 3U + 2i). \tag{25}$$

The SUSY minimum $D_aW_{\rm flux} = 0$ corresponds to

$$U = -i\omega, \qquad S = -i2\omega, \tag{26}$$

where $\omega = e^{2\pi i/3}$. Indeed, this minimum leads to Eq. (23).

Here we consider the condition leading to Eq. (23) for S and U. We write the flux-induced superpotential

$$W_{\rm flux} = f^{RR}(U) + Sf^{NS}(U), \tag{27}$$

where $f^{RR}(U)$ and $f^{NS}(U)$ are polynomial functions of U. We write values of S and U at the minimum as S_0 and U_0. The above conditions requires

[3]See also Ref. [20].

[4]In this case, the SUSY breaking scale is $M_{\rm Pl}$, even after uplifting to realize de Sitter (or Minkowski) vacuum. That is not good from the phenomenological purpose to realize the low-energy SUSY.

$$f^{RR}(U_0) = f^{NS}(U_0) = 0. \tag{28}$$

Thus, we can write

$$f^{RR}(U) = (U - U_0)^{n_{RR}} \tilde{f}^{RR}(U), \\ f^{NS}(U) = (U - U_0)^{n_{NS}} \tilde{f}^{NS}(U), \tag{29}$$

with positive integers n_{RR} and n_{NS}, where $\tilde{f}^{RR,NS}(U_0) \neq 0$. Furthermore, the above condition $W_U = 0$ at U_0 requires

$$n_{RR}(U - U_0)^{n_{RR}-1} \tilde{f}^{RR}(U_0) + n_{NR} S_0 (U - U_0)^{n_{NS}-1} \tilde{f}^{NS}(U_0) = 0. \tag{30}$$

Obviously, we are interested in the case with $S_0 \neq 0$. Thus, there are three cases: 1) the case with $n_{RR} = n_{NS} = 1$, 2) the case with $n_{RR} = n_{NS} = 2$ and 3) the case where both n_{RR} and n_{NS} are larger than 2, i.e., $n_{RR}, n_{NS} \geq 3$. In the first case, the above condition reduces

$$\tilde{f}^{RR}(U_0) + S_0 \tilde{f}^{NS}(U_0) = 0, \tag{31}$$

that is, S_0 is determined as

$$S_0 = -\frac{\tilde{f}^{RR}(U_0)}{\tilde{f}^{NS}(U_0)}. \tag{32}$$

Furthermore, since the real part of S_0 gives the gauge coupling, the obtained value of S_0 must satisfy $\mathrm{Re}(S_0) > 0$.

On the other hand, in the second case with $n_{RR}, n_{NS} = 2$, the value S_0 is not determined. Actually, we have

$$(W_{\rm flux})_{SU} = 0, \tag{33}$$

at U_0, although we have $(W_{\rm flux})_{UU} \neq 0$ at U_0. That implies that through this type of flux compactification only the U moduli is stabilized, but the dilaton S is not stabilized. In the third case with $n_{RR}, n_{NS} \geq 3$, both the moduli S and U are not stabilized by the flux.

Since in the first case, S has already a larger mass of $\mathcal{O}(M_{\rm Pl})$, the minimum does not shift significantly by adding Ce^{-bS} as well as terms like Eq. (9), and the added term leads to a small gravitino mass $m_{3/2} = e^{K/2}\langle Ce^{-bS}\rangle$, which is needed to stabilize T at the second stage. This possibilities has been pointed out in Ref. [12].

Concerned about stabilizing T at the second step, there is a way not to add the superpotential Ce^{-bS} to $W_{\rm flux}$, but we change T-dependent superpotential $W_{\rm np}$. We consider not a single term e^{-aT}, but more terms like

$$C_1 e^{-a_1 T} - C_2 e^{-a_2 T}, \tag{34}$$

that is, the racetrack model. In this case, the mass of T is obtained

$$m_T \approx a_1 a_2 \left(\frac{|C_1| a_1}{|C_2| a_2}\right)^{a_1/(a_1 - a_2)}. \tag{35}$$

and it can be smaller than $M_{\rm Pl}$.

In the second case with $n_{RR}, n_{NS} = 2$, after the first step of the flux compactification, two moduli S and T remain light. Stabilization of such moduli has been discussed by nonperturbative superpotential, e.g. moduli-mixing racetrack model [21], which leads to a SUSY breaking vacuum with negative vacuum energy before uplifting.

Here we have studied the model with a single U. The above discussion can be extended to models with more than 1 moduli fields U^α.

IV. CONCLUSION AND DISCUSSION

We have studied two steps of moduli stabilization through flux compactification and nonperturbative superpotential. We need mass hierarchy between superpotential masses W_{ab} and the gravitino mass such that the two-step procedure is valid. Such situation would be realized by fine-tuning flux such as $W_{ab} \gg \langle W_{\rm flux}\rangle$, although the natural scale of the gravitino mass through the flux superpotential would be of $\mathcal{O}(M_{\rm Pl})$. If we do not consider such fine-tuning, it would be interesting to use the flux leading to $\langle W_{\rm flux}\rangle = 0$. With this flux, both of U and S are stabilized, or only U is stabilized. Thus, after flux compactification, only T modulus remains light, or two moduli T and S remain light. Remaining moduli can be stabilized at the second step.

ACKNOWLEDGMENTS

H. A., T. H. and T. K. are supported in part by the Grand-in-Aid for Scientific Research No. 182496, No. 171643 and No. 17540251, respectively. T. K. is also supported in part by the Grant-in-Aid for the 21st Century COE "The Center for Diversity and Universality in Physics" from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

APPENDIX A: PERTURBATION OF FLUXED NO-SCALE MINIMUM BY GAUGINO CONDENSATIONS

In this Appendix, we estimate the shift of the potential minimum from the no-scale one (23) caused by the gaugino condensations, and show that the general argument that "when S and U have heavy masses through flux compactification, we can integrate out them at the first step with only T remaining," holds in a concrete and typical situation. We assume an effective theory described by $4D$ $N = 1$ supergravity parameterized by the following Kähler and superpotential[5]:

$$K = -n_S \ln(S + \bar{S}) - n_T \ln(T + \bar{T}) - n_U \ln(U + \bar{U}), \\ W = f(S, U) + g(S, T), \tag{A1}$$

where the superpotential terms $f(S, U) = W_{\rm flux}$ and $g(S, T) = W_{\rm np}$ may originate from the flux and gaugino

[5]Generalization to the case with more than 1 Kähler and complex structure modulus might be straightforward.

condensations with moduli-mixed gauge couplings, respectively, given by

$$f(S,U) = f^{RR}(U) + Sf^{NS}(U),$$
$$g(S,T) = \sum_m C_m e^{-(b_m S + a_m T)}.$$

1. No-scale minimum

First we analyze a SUSY minimum without gaugino condensations, i.e. $W = f(S,U)$. In this case, some combinations of flux may allow the global SUSY minimum realized by conditions $f_S = f_U = f = 0$, which result in $S = -f_U^{RR}(U)/f_U^{NS}(U)$ and $f^{RR}(U) = f^{NS}(U) = 0$ where $f_U^{RR,NS} = \partial_U f^{RR,NS}$. Note that the global SUSY condition $W_a = W = 0$ satisfies the SUSY stationary condition $D_a W = W_a + K_a W = 0$ in the supergravity. We denote S and U satisfying these conditions by S_0 and U_0, i.e.,

$$\langle U\rangle = U_0 \quad \text{such that } f^{RR}(U_0) = f^{NS}(U_0) = 0,$$
$$\langle S\rangle = S_0 = -f_U^{RR}(U_0)/f_U^{NS}(U_0). \tag{A2}$$

Note that Kähler modulus T remains as a flat direction in the case with $n_T = 3$ for which the scalar potential is in the no-scale form (4), and we assume $n_T = 3$ in the following arguments.

The moduli masses $m^2_{a\bar b}$, $m^2_{ab} = \overline{m^2_{\bar a\bar b}}$ are evaluated by computing the second derivatives of the scalar potential,

$$m^2_{a\bar b} = K_{a\bar a}^{-1/2} K_{b\bar b}^{-1/2} V_{a\bar b}, \qquad m^2_{ab} = K_{a\bar a}^{-1/2} K_{b\bar b}^{-1/2} V_{ab}.$$

In the case with $W = f(S,U)$, we find

$$V_{a\bar b}|_{S=S_0,U=U_0} = e^K K^{c\bar d} f_{ca}\bar f_{\bar d\bar b}, \qquad V_{ab}|_{S=S_0,U=U_0} = 0. \tag{A3}$$

The moduli fields S and U typically receive heavy masses if $f_{ab} \neq 0$ $(a = S, U)$, because the parameters in the flux superpotential $f(S,U)$ are expected to be naturally of $\mathcal{O}(1)$ in the unit $M_{\rm Pl} = 1$.

2. Including gaugino condensations

Next we consider the case with $W = f(S,U) + g(S,T)$ and study the perturbation of the previous SUSY vacuum caused by gaugino condensations described by additional superpotential terms in $g(S,T)$. For such purpose, we analyze the shift of the vacuum

$$T = T_0 + \delta T, \qquad S = S_0 + \delta S, \qquad U = U_0 + \delta U, \tag{A4}$$

around the vacuum satisfying $D_S f = D_U f = 0$ and $D_T g = 0$ defined by

$$f|_0 = f_S|_0 = f_U|_0 = 0, \qquad g|_0 = -g_T/K_T|_0 \neq 0, \tag{A5}$$

where $|_0$ stands for $|_{T=T_0,S=S_0,U=U_0}$. The first derivatives of $G = K + \ln|W|^2$ can be expanded as

$$G_A = G_A|_0 + \delta\phi^B G_{AB}|_0 + \mathcal{O}(\delta^2),$$

where the indices $A, B, C, \ldots$ run all the holomorphic and antiholomorphic fields as $A, B, C, \ldots = (S, T, U, \bar S, \bar T, \bar U)$ and $\delta\phi^A$ denotes the deviation of the vacuum value $\delta\phi^A = \delta S, \delta T, \delta U, \delta\bar S, \delta\bar T, \delta\bar U$ with the corresponding index.

At the linear order of $\delta\phi^A$, the solution of the SUSY stationary condition $G_A = 0$ is given by

$$\delta\phi^A = -G^{AB} G_B|_0 + \mathcal{O}(\delta^2), \tag{A6}$$

where $G_{AB}G^{BC} = \delta_A{}^C$. If we assume that all the parameters in $f(S,U)$ and $g(S,T)$ are of order one quantities except for $a_m \sim b_m \gg 1$, we may naturally obtain

$$K|_0, K_A|_0, K_{AB}|_0, \ldots \sim \mathcal{O}(1),$$
$$f_{ab}|_0, f_{abc}|_0, f_{abcd}|_0, \ldots \sim \mathcal{O}(1),$$
$$g|_0 \ll g_{ab}|_0 \ll g_{abc}|_0, \ldots \ll 1,$$

if nonvanishing, by which we can expect the hierarchical structure

$$G_{UU}|_0, G_{SU}|_0, \gg G_{SS}|_0, G_{ST}|_0, G_{TT}|_0, \gg G_{a\bar b}|_0 = K_{a\bar b}|_0, \gg G_{TU}|_0 = 0, \tag{A7}$$

where $G_{ab} = K_{ab} + W_{ab}/W - (W_a/W)(W_b/W)$. From this, we can approximate G_{AB} by the block-diagonal form,

$$G_{AB}|_0 \sim \begin{pmatrix} G_{ab}|_0 & 0 \\ 0 & G_{\bar a\bar b}|_0 \end{pmatrix},$$

and the same for its inverse G^{AB}. This means that Eq. (A6) becomes a holomorphic equation, $\delta\phi^a = -G^{ab}G_b|_0 + \mathcal{O}(\delta^2)$, and with the explicit form of G^{ab} we find

$$\begin{pmatrix} \delta T \\ \delta S \\ \delta U \end{pmatrix} = g \times \left.\begin{pmatrix} -\frac{G_{ST}}{G_{TT}}\frac{1}{f_{SU}}(\frac{f_{UU}}{f_{SU}}G_S - G_U) \\ \frac{1}{f_{SU}}(\frac{f_{UU}}{f_{SU}}G_S - G_U) \\ -\frac{1}{f_{SU}}G_S \end{pmatrix}\right|_0 + \mathcal{O}(g^2), \tag{A8}$$

where

$$G_T|_0 = \mathcal{G}_T|_0 = 0,$$
$$G_S|_0 = \mathcal{G}_S|_0 = (g_S/g + K_S)|_0 \sim \mathcal{O}(b_m) \sim \mathcal{O}(a_m),$$
$$G_U|_0 = \mathcal{G}_U|_0 = K_U|_0 \sim \mathcal{O}(1), \tag{A9}$$

and $\mathcal{G} \equiv K + \ln|g|^2$. Therefore we find $\delta\Phi^a/\Phi_0^a \sim \mathcal{O}(g) \ll 1$ with $\Phi^a = (T, S, U)$ for $\Phi_0^a = \Phi^a|_0 \sim \mathcal{O}(1)$, and this linearized analysis is enough to study the combined effect of the fluxes and the gaugino condensations.

3. Moduli mass

Here we estimate the moduli masses and mixing at the vacuum (A4) determined by Eq. (A8). Up to the second order of $\delta\phi^i$ we can expand the moduli masses as[6]

$$m_{AB}^2 = K_{AA}^{-1/2}K_{BB}^{-1/2}V_{AB} = m_{AB}^2|_0 + \delta\phi^C\partial_C m_{AB}^2|_0 + \frac{1}{2}\delta\phi^C\delta\phi^D\partial_C\partial_D m_{AB}^2|_0 + \mathcal{O}(\delta^3)$$
$$= Z_{AB}^{(2)}V_{AB}|_0 + \delta\phi^C Z_{AB}^{(1)}V_{ABC}|_0 + \frac{1}{2}\delta\phi^C\delta\phi^D Z_{AB}^{(0)}V_{ABCD}|_0 + \mathcal{O}(\delta^3),$$

where

$$Z_{AB}^{(2)} = Z_{AB}^{(1)} - \frac{1}{2}Z_{AB}^{(0)}\Big\{K_{AA}^{-1}K_{AA,CD} + K_{BB}^{-1}K_{BB,CD} - \frac{3}{2}(K_{AA}^{-2}K_{AA,C}K_{AA,D} + K_{BB}^{-2}K_{BB,C}K_{BB,D}) - \frac{1}{2}(K_{AA}^{-1}K_{BB}^{-1}K_{AA,C}K_{BB,D} + K_{AA}^{-1}K_{BB}^{-1}K_{AA,D}K_{BB,C})\Big\}\delta\phi^C\delta\phi^D, \tag{A10}$$

$$Z_{AB}^{(1)} = Z_{AB}^{(0)}\Big\{1 - \frac{1}{2}(K_{AA}^{-1}K_{AA,C} + K_{BB}^{-1}K_{BB,C})\delta\phi^C\Big\}, \qquad Z_{AB}^{(0)} = K_{AA}^{-1/2}K_{BB}^{-1/2}. \tag{A11}$$

All the derivatives of scalar potential V_A, V_{AB}, ... can be written in terms of $G = K + \ln|W|^2$ and its derivatives, G_A, G_{AB}, ... At the point (A5), these can be written as

$$G|_0 = \mathcal{G}|_0, \qquad G_A|_0 = \mathcal{G}_A|_0, \qquad G_{AB} = \mathcal{G}_{AB}|_0 + g^{-1}f_{AB}|_0, \ldots,$$

where $\mathcal{G} = K + \ln|g|^2$. By calculating V_{AB}, V_{ABC}, V_{ABCD} and taking Eq. (A7) into account, we find the leading contributions to each component in moduli mass matrices squared as

$$e^{-K}m_{a\bar{b}}^2 \sim \left.\begin{pmatrix} (\mu_{T\bar{T}} + \delta\mu_{T\bar{T}})|g|^2 & (\mu_{T\bar{S}} + \delta\mu_{T\bar{S}})|g|^2 & \delta\mu_{T\bar{U}}g \\ (\overline{\mu_{T\bar{S}}} + \overline{\delta\mu_{T\bar{S}}})|g|^2 & K_{S\bar{S}}^{-1}K_{U\bar{U}}^{-1}|f_{SU}|^2 & K_{S\bar{S}}^{-1/2}K_{U\bar{U}}^{-3/2}f_{SU}\bar{f}_{\bar{U}\bar{U}} \\ \overline{\delta\mu_{T\bar{U}}}\,\bar{g} & K_{S\bar{S}}^{-1/2}K_{U\bar{U}}^{-3/2}\bar{f}_{\bar{S}\bar{U}}f_{UU} & K_{U\bar{U}}^{-1}(K_{S\bar{S}}^{-1}|f_{SU}|^2 + K_{U\bar{U}}^{-1}|f_{UU}|^2) \end{pmatrix}\right|_0, \tag{A12}$$

$$e^{-K}m_{ab}^2 \sim \left.\begin{pmatrix} (\mu_{TT} + \delta\mu_{TT})|g|^2 & (\mu_{TS} + \delta\mu_{TS})|g|^2 & \delta\mu_{TU}|g|^2 \\ (\mu_{TS} + \delta\mu_{TS})|g|^2 & (\mu_{SS} + \delta\mu_{SS})|g|^2 & \delta\mu_{SU}\bar{g} \\ \delta\mu_{TU}|g|^2 & \delta\mu_{SU}\bar{g} & \delta\mu_{UU}\bar{g} \end{pmatrix}\right|_0, \tag{A13}$$

where the row and column correspond to $a, b = (T, S, U)$.

The coefficients μ_{AB} represent the contributions from $\mathcal{G}$,

$$\mu_{AB} = K_{AA}^{-1/2}K_{BB}^{-1/2}e^{-\mathcal{G}}\mathcal{V}_{AB},$$
$$\mathcal{V} = e^{\mathcal{G}}(\mathcal{G}^{a\bar{b}}\mathcal{G}_a\mathcal{G}_{\bar{b}} - 3),$$

and given by, e.g.,

$$\mu_{T\bar{T}} = K_{T\bar{T}}^{-1}(K_{T\bar{T}}^{-1}|\mathcal{G}_{TT}|^2 + K_{S\bar{S}}^{-1}|\mathcal{G}_{ST}|^2) - 2 + K_{S\bar{S}}^{-1}|\mathcal{G}_S|^2 \sim \mathcal{O}(a_m^4),$$
$$\mu_{TT} = (K_{T\bar{T}}K_{S\bar{S}})^{-1}(|\mathcal{G}_S|^2\mathcal{G}_{TT} + \bar{\mathcal{G}}_{\bar{S}}\mathcal{G}_{TTS}) - \mathcal{G}_{TT} \sim \mathcal{O}(a_m^4),$$

and similarly $\mu_{T\bar{S}}$, μ_{SS}, $\mu_{TS} \sim \mathcal{O}(a_m^4)$. On the other hand, the terms with coefficients $\delta\mu_{AB}$ come from the remaining contributions (g-f mixed terms in V_{AB}, V_{ABC}, V_{ABCD}), and are given by, e.g.,

$$\delta\mu_{T\bar{T}} = -K_{S\bar{S}}^{-1}|\mathcal{G}_S|^2 \sim \mathcal{O}(a_m^2),$$
$$\delta\mu_{T\bar{U}} = (K_{T\bar{T}}K_{U\bar{U}})^{-1/2}K_{S\bar{S}}^{-1}\mathcal{G}_{ST}\bar{f}_{\bar{S}\bar{U}} \sim \mathcal{O}(a_m^2),$$
$$\delta\mu_{TT} = -(K_{T\bar{T}}K_{S\bar{S}})^{-1}(|\mathcal{G}_S|^2\mathcal{G}_{TT} + \bar{\mathcal{G}}_{\bar{S}}\mathcal{G}_{TTS}) \sim \mathcal{O}(a_m^4),$$

and similarly $\delta\mu_{TS}$, $\delta\mu_{TU}$, $\delta\mu_{SS}$, $\delta\mu_{SU}$, $\delta\mu_{UU} \le \mathcal{O}(a_m^4)$.

First, from the lower right 2×2 submatrix in Eq. (A12), $m_{a\bar{b}}^2$ with $a, b = (S, U)$, we find that the moduli S and U generically receive $\mathcal{O}(M_{\rm Pl})$ of heavy masses. Second, since $\mathcal{O}(a_m^4)$ of contributions in μ_{TT} and $\delta\mu_{TT}$ cancel each other, $\mu_{TT} + \delta\mu_{TT} = -\mathcal{G}_{TT} \sim \mathcal{O}(a_m^2)$ while

[6]The coefficient $K_{AA}^{-1/2}K_{BB}^{-1/2}$ in m_{AB}^2 originates from the normalization of kinetic terms. Note that here we are assuming the Kähler potential without moduli mixing (A1).

$\mu_{T\bar{T}} + \delta\mu_{T\bar{T}} \sim \mathcal{O}(a_m^4)$, we find $m_{T\bar{T}}^2 \gg m_{TT}^2$ and the imaginary direction, ImT is not destabilized.

So far, to estimate μ_{TT}, $\mu_{T\bar{T}}$ and their deviations, we have considered the case with $g(S, T)$, where two or more a_m are nonvanishing. When only a single a_m is nonvanishing and $g(S, T)$ includes other S-dependent terms as well as constant term, the above estimation would change because of e.g. $\mathcal{G}_{TT} = \mathcal{O}(a_m)$. However, the imaginary direction, ImT is stable still.

The third point is that, the T-U mixing $m_{T\bar{U}}^2 \sim \mathcal{O}(a_m^2|g|)$ can affect the lightest eigenvalue of $m_{a\bar{b}}^2$ that is $\mathcal{O}(a_m^4|g|^2)$. If we normalize $m_{a\bar{b}}^2$ and define the 3×3 matrix[7]

$$\mathcal{M} \equiv e^{-K} K_{S\bar{S}} K_{U\bar{U}} |f_{SU}|^{-2} m_{a\bar{b}}^2 = \begin{pmatrix} Z|g|^2 & 0 & Yg \\ 0 & 1 & X \\ \bar{Y}\bar{g} & \bar{X} & 1+|X|^2 \end{pmatrix},$$

where

$$X = (K_{S\bar{S}}/K_{U\bar{U}})^{1/2}(f_{\bar{U}\bar{U}}/f_{\bar{S}\bar{U}}),$$
$$Y = (K_{U\bar{U}}/K_{T\bar{T}})^{1/2}(\mathcal{G}_{ST}/f_{SU}),$$
$$Z = K_{S\bar{S}} K_{U\bar{U}} |f_{SU}|^{-2}\{K_{T\bar{T}}^{-1}(K_{T\bar{T}}^{-1}|\mathcal{G}_{TT}|^2 + K_{S\bar{S}}^{-1}|\mathcal{G}_{ST}|^2) - 2\},$$

the eigenvalue equation for $\mathcal{M}$ is given by

$$-\lambda^3 + (2 + |X|^2 + Z|g|^2)\lambda^2 - (1 + \{Z(2+|X|^2) - |Y|^2\}|g|^2)\lambda + (Z - |Y|^2)|g|^2 = 0,$$

where λ is the eigenvalue of $\mathcal{M}$. For the lightest eigenvalue $\lambda \sim \mathcal{O}(|g|^2)$, this equation is approximated as

$$-\lambda + (Z - |Y|^2)|g|^2 + \mathcal{O}(|g|^4) = 0,$$

[7] Here we assume $f_{SU} \neq 0$, and omit $m_{T\bar{S}}^2$, $m_{S\bar{T}}^2 \sim |g|^2$ which does not affect the following order estimation when $f_{SU} \neq 0$.

and we find the mass squared of the lightest mode $\mathcal{T}$ as

$$m_{\mathcal{T}\bar{\mathcal{T}}}^2 \equiv e^K (K_{S\bar{S}} K_{U\bar{U}})^{-1} |f_{SU}|^2 \lambda = e^K (K_{T\bar{T}}^{-2} |\mathcal{G}_{TT}|^2 - 2)|g|^2|_0.$$

This is actually the same as the mass squared $m_{\mathcal{T}\bar{\mathcal{T}}}^2$ calculated from the generalized Kähler potential

$$G(\mathcal{T}) = K(S_0, \mathcal{T}, U_0) + \ln|g(S_0, \mathcal{T})|^2, \tag{A14}$$

at the SUSY point $G_{\mathcal{T}}(\mathcal{T}) = 0$, which supports the fact that the low-energy effective theory of the light mode $\mathcal{T}$ is described by $G(\mathcal{T})$. In addition, large eigenvalues of $\mathcal{M}$ are obtained as

$$\lambda = \frac{1}{2}(2 + |X|^2 \pm |X|\sqrt{4 + |X|^2}) + O(|g|^2). \tag{A15}$$

These coincide with masses squared, which are obtained from (A3) and are positive.

We finally comment that all the analyses and results in this appendix would be applied to the perturbation of (fine-tuned) AdS minimum

$$f|_0 = -f_S/K_S|_0 = -f_U/K_U|_0 \equiv w_0 \sim \mathcal{O}(g|_0),$$
$$W|_0 = w_0 + g|_0 = -g_T/K_T|_0,$$

instead of the no-scale minimum (A5). This will be done by replacing $g|_0 \to w_0 + g|_0$ everywhere, $g(S_0, \mathcal{T}) \to w_0 + g(S_0, \mathcal{T})$ in Eq. (A14) and forgetting Eq. (A9), at least as long as the following condition holds

$$f_a|_0 = -K_a|_0 w_0 \sim \mathcal{O}(g|_0) \ll g_b|_0 \sim \mathcal{O}(a_m g|_0),$$

for $a = (S, U)$ and $b = (S, T)$. However, as shown in Sec. III, a (fine-tuned) nonvanishing value of $w_0 \sim \mathcal{O}(g|_0)$ is not necessary in order to stabilize $\mathcal{T}$ through $G(\mathcal{T})$. To do that, we can assume, e.g., the existence of (24) in $g(S, T)$ which generates a constant superpotential term $Ce^{-bS_0} \sim \mathcal{O}(g|_0)$ naturally and effectively in $G(\mathcal{T})$.

[1] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002).

[2] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).

[3] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, Nucl. Phys. **B718**, 113 (2005).

[4] K. Choi, K. S. Jeong, and K. i. Okumura, J. High Energy Phys. 09 (2005) 039; M. Endo, M. Yamaguchi, and K. Yoshioka, Phys. Rev. D **72**, 015004 (2005); A. Falkowski, O. Lebedev, and Y. Mambrini, J. High Energy Phys. 11 (2005) 034.

[5] K. Choi, K. S. Jeong, T. Kobayashi, and K. i. Okumura, Phys. Lett. B **633**, 355 (2006); R. Kitano and Y. Nomura, Phys. Lett. B **631**, 58 (2005); O. Lebedev, H. P. Nilles, and M. Ratz, hep-ph/0511320; A. Pierce and J. Thaler, hep-ph/0604192.

[6] K. Choi and K. S. Jeong, hep-th/0605108; J. P. Conlon and F. Quevedo, J. High Energy Phys. 06 (2006) 029.

[7] S. Kachru, M. B. Schulz, and S. Trivedi, J. High Energy Phys. 10 (2003) 007.

[8] D. Lust, S. Reffert, W. Schulgin, and S. Stieberger, hep-th/0506090.

[9] G. Curio and V. Spillner, hep-th/0606047.

[10] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, J. High Energy Phys. 11 (2004) 076.

[11] S. P. de Alwis, Phys. Lett. B **626**, 223 (2005).

[12] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D **73**, 046005 (2006).

[13] R. Brustein and P. J. Steinhardt, Phys. Lett. B **302**, 196 (1993).
[14] W. Buchmuller, K. Hamaguchi, O. Lebedev, and M. Ratz, Nucl. Phys. **B699**, 292 (2004); J. Cosmol. Astropart. Phys. 01 (2005) 004.
[15] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000); **B608**, 477(E) (2001).
[16] M. Becker, G. Curio, and A. Krause, Nucl. Phys. **B693**, 223 (2004); G. Curio, A. Krause, and D. Lust, Fortschr. Phys. **54**, 225 (2006).
[17] K. Choi and J. E. Kim, Phys. Lett. B **165**, 71 (1985); L. E. Ibanez and H. P. Nilles, Phys. Lett. B **169**, 354 (1986); J. P. Derendinger, L. E. Ibanez, and H. P. Nilles, Nucl. Phys. **B267**, 365 (1986); L. J. Dixon, V. Kaplunovsky, and J. Louis, Nucl. Phys. **B355**, 649 (1991).
[18] T. Banks and M. Dine, Nucl. Phys. **B479**, 173 (1996); K. Choi, Phys. Rev. D **56**, 6588 (1997); H. P. Nilles and S. Stieberger, Nucl. Phys. **B499**, 3 (1997); E. I. Buchbinder and B. A. Ovrut, Phys. Rev. D **69**, 086010 (2004); A. Lukas, B. A. Ovrut, and D. Waldram, Nucl. Phys. **B532**, 43 (1998); Phys. Rev. D **57**, 7529 (1998).
[19] D. Cremades, L. E. Ibanez, and F. Marchesano, J. High Energy Phys. 07 (2002) 009; D. Lust, P. Mayr, R. Richter, and S. Stieberger, Nucl. Phys. **B696**, 205 (2004).
[20] J. J. Blanco-Pillado, R. Kallosh, and A. Linde, J. High Energy Phys. 05 (2006) 053.
[21] H. Abe, T. Higaki, and T. Kobayashi, Nucl. Phys. **B742**, 187 (2006).