New $\mathcal{N} = 8$ nonlinear supermultiplet

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We construct a new off-shell $\mathcal{N} = 8$, $d = 1$ nonlinear supermultiplet $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ proceeding from the nonlinear realization of the $\mathcal{N} = 8$, $d = 1$ superconformal group $OSp(4^\star|4)$ in its supercoset $\frac{OSp(4^\star|4)}{SU(2)_{\mathcal{R}}\otimes\{D,K\}\otimes SO(4)}$. The irreducibility constraints for the superfields automatically follow from appropriate covariant conditions on the $osp(4^\star|4)$-valued Cartan superforms. We present the most general sigma-model type action for $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ supermultiplet. The relations between linear and nonlinear $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ supermultiplets and linear $\mathcal{N} = 8$ $(\mathbf{5}, \mathbf{8}, \mathbf{3})$ vector supermultiplet are discussed.

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I. INTRODUCTION

During the last few years it has become clear that in Supersymmetric Quantum Mechanics (SQM) with extended $\mathcal{N} = 4, 8$ supersymmetries the nonlinear supermultiplets play an essential role [1–6]. The main reason for resort to nonlinearities is the presence of too strong restrictions on the bosonic target-space metrics, in the case of theories with linear supermultiplets. Indeed, the general consideration of $d = 1$ sigma models with $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetries reveal the following possible bosonic target geometries: hyper-Kähler with torsion, for $\mathcal{N} = 4$ supersymmetric theories with four physical bosons, and octonionic-Kähler with torsion for $\mathcal{N} = 8$ ones, in the case of eight physical bosonic fields [7]. Moreover, the detailed analysis of the components and superfield actions for $\mathcal{N} = 4, 8$ cases with diverse numbers of physical bosonic degrees of freedom shows that only conformally flat geometries, with the additional restriction on the metrics of bosonic manifolds to be harmonic functions, may arise [8–15]. Being quite general, these results keep open only a unique way to have more complicated bosonic target-space geometries—i.e. to introduce nonlinear supermultiplets.

When dealing with nonlinear supermultiplets one should be able to overcome at least two obstacles:

(i) it is not clear how to find the proper superfield constraints defining the irreducible nonlinear supermultiplets
(ii) the construction of the invariant superfield actions is not evident.

One should mention that the dimensional reduction is not too useful for obtaining the invariant $d = 1$ superfield actions and the irreducible constraints on the superfields. Although any $d = 1$ super Poincaré algebra can be obtained from a higher-dimensional one via dimensional reduction, this is generally not true for $d = 1$ super *conformal* algebras [16,17] and off-shell $d = 1$ multiplets. For instance, no $d = 4$ analog exists for the $\mathcal{N} = 4$, $d = 1$ multiplet with off-shell content $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ [14] or $(\mathbf{3}, \mathbf{4}, \mathbf{1})$ [11,13]. Moreover, there exist off-shell $d = 1$ supermultiplets containing no auxiliary fields at all, something impossible for $d \geq 3$ supersymmetry.

However, a convenient superfield approach to $d = 1$ models which does not resort to dimensional reduction and is self-contained in $d = 1$ exists. It is based on superfield nonlinear realizations of $d = 1$ superconformal groups. It was pioneered in [14] and recently advanced in [1,18,19]. In this approach the physical bosons and fermions, together with the $d = 1$ superspace coordinates, prove to be coset parameters associated with the appropriate generators of the superconformal group. The conditions which identify the fermionic components of the bosonic superfields with the cosets fermionic parameters are just the irreducible constraints singling out the proper supermultiplets.

Using the nonlinear realizations approach, in [1] all known linear off-shell multiplets of $\mathcal{N} = 4$, $d = 1$ Poincaré supersymmetry were recovered and a two novel nonlinear ones were found. Concerning $\mathcal{N} = 8$, $d = 1$ supermultiplets, in [19] a similar analysis has been started along the same line. It has been shown that the $(\mathbf{5}, \mathbf{8}, \mathbf{3})$ and $(\mathbf{3}, \mathbf{8}, \mathbf{5})$ multiplets come out as the Goldstone ones, parameterizing the specific cosets of the supergroup $OSp(4^\star|4)$. Consequently, in [20] a superfield description of all other linear off-shell $\mathcal{N} = 8$, $d = 1$ supermultiplets with $\mathbf{8}$ fermions, in both $\mathcal{N} = 8$ and $\mathcal{N} = 4$ superspaces, was given. Finally, the first $\mathcal{N} = 8$ nonlinear supermultiplet $(\mathbf{2}, \mathbf{8}, \mathbf{6})$ has been constructed in [4]. However, the task of deriving an exhaustive list of off-shell $\mathcal{N} = 8$ supermultiplets and the relevant constrained $\mathcal{N} = 8$, $d = 1$ superfields is much more complicated as compared to the $\mathcal{N} = 4$ case, in view of the existence of many nonequivalent $\mathcal{N} = 8$ superconformal groups ($OSp(4^\star|4)$, $OSp(8|2)$, $F(4)$ and $SU(1, 1|4)$, see e.g. [16]), with a large number of different coset supermanifolds. Moreover, the

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explicit construction of linear [21] and nonlinear [4] $(\mathbf{2}, \mathbf{8}, \mathbf{6})$ supermultiplets demonstrates that the constraints which follow from the nonlinear realization approach should be accompanied by additional, second order in spinor covariant derivatives, constraints, in order to define irreducible $\mathcal{N}=8$ supermultiplets.

The subject of the present paper is the study of the nonlinear $\mathcal{N}=8$ supermultiplet with field content $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ obtained by a nonlinear realization of the $OSp(4^\star|4)$ supergroup on a particular coset supermanifold $\frac{OSp(4^\star|4)}{SU(2)_{\mathcal{R}}\otimes\{D,K\}\otimes SO(4)}$. After reviewing some basic facts on the $\mathcal{N}=8$, $d=1$ superspace and the Lie superalgebra $osp(4^\star|4)$ in Section II we give a $\mathcal{N}=8$ superfield formulation of the nonlinear multiplet $(\mathbf{4}, \mathbf{8}, \mathbf{4})$. In Sec. III we obtain the superfield constraints defining irreducible linear and nonlinear $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ supermultiplets by the reduction procedures from the $(\mathbf{5}, \mathbf{8}, \mathbf{3})$ vector multiplet. Using these results in Sec. IV we construct the most general action for our nonlinear supermultiplet and provide a detailed analysis of its bosonic part. A summary of our results and an outlook are the contents of the concluding Sec. V.

II. THE $\mathcal{N}=8$ $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ NONLINEAR MULTIPLET

Like the $\mathcal{N}=8$ tensor and vector supermultiplets [19], the new nonlinear $\mathcal{N}=8$ multiplet we are going to consider can be obtained from a proper nonlinear realization of the $\mathcal{N}=8$, $d=1$ superconformal group $OSp(4^\star|4)$ in $\mathcal{N}=8$, $d=1$ superspace. After exposing some basic facts in Subsections II A and II B, in Subsection II C we will give the details of the relevant nonlinear realization procedure giving rise to the nonlinear $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ multiplet.[1] Our basic notations follow those of Ref. [19].

A. The $\mathcal{N}=8$, $d=1$ superspace

The eight real Grassmann coordinates of $\mathcal{N}=8$, $d=1$ superspace $\mathbb{R}^{(1|8)}$ can be arranged into one of three 8-dimensional real irreps of $SO(8)$—the maximal automorphism group of $\mathcal{N}=8$, $d=1$ super Poincaré algebra. However, the constraints defining the irreducible $\mathcal{N}=8$ supermultiplets in general break this $SO(8)$ symmetry. So, it is preferable to split the 8 coordinates into two real quartets

$$\mathbb{R}^{(1|8)}=(t,\theta_{ia},\vartheta_{\alpha A}),\qquad \overline{(\theta_{ia})}=\theta^{ia},\qquad \overline{(\vartheta_{\alpha A})}=\vartheta^{\alpha A},\qquad i,a,\alpha,A=1,2, \tag{2.1}$$

in terms of which only four commuting automorphism $SU(2)$ groups will be explicit. The further symmetry breaking can be understood as the identification of some of these $SU(2)$, whereas additional symmetries, if existing, mix different $SU(2)$ indices. The corresponding covariant derivatives are defined by

$$D^{ia}=\frac{\partial}{\partial\theta_{ia}}+i\theta^{ia}\partial_t,\qquad \nabla^{\alpha A}=\frac{\partial}{\partial\vartheta_{\alpha A}}+i\vartheta^{\alpha A}\partial_t. \tag{2.2}$$

By construction, they obey the algebra[2]:

$$\{D^{ia},D^{jb}\}=2i\epsilon^{ij}\epsilon^{ab}\partial_t,\qquad \{\nabla^{\alpha A},\nabla^{\beta B}\}=2i\epsilon^{\alpha\beta}\epsilon^{AB}\partial_t. \tag{2.3}$$

Thus all our $\mathcal{N}=8$, $d=1$ superfields depend on $(t,\theta_{ia},\vartheta_{\alpha A})$ and the differential constraints on the relevant superfields will be defined directly in terms of the spinor covariant derivatives (2.2).

B. The superalgebra $osp(4^\star|4)$

Let us briefly recall some basic facts about the Lie superalgebra $osp(4^\star|4)$ [17,19]. It contains the following 16 spinor generators:

$$Q_1^{iaA},\qquad Q_2^{i\alpha A},\qquad \overline{(Q^{iaA})}=\epsilon_{ij}\epsilon_{ab}Q^{jbA},\qquad (i,a,\alpha,A=1,2), \tag{2.4}$$

and 16 bosonic generators:

$$T_0^{AB},\qquad T^{ij},\qquad T_1^{ab},\qquad T_2^{\alpha\beta},\qquad U^{a\alpha}. \tag{2.5}$$

Here, the indices A, i, a and α refer to fundamental representations of the mutually commuting $sl(2,\mathbb{R})\sim T_0^{AB}$ and three $su(2)\sim T^{ij}, T_1^{ab}, T_2^{\alpha\beta}$ algebras. The four generators $U^{a\alpha}$ belong to the coset $SO(5)/SO(4)$ with $SO(4)$ generated by T_1^{ab} and $T_2^{\alpha\beta}$.

The commutators of any $SU(2)$-generators with Q have the standard form

$$[T^{ab},Q^c]=-\frac{i}{2}(\epsilon^{ac}Q^b+\epsilon^{bc}Q^a), \tag{2.6}$$

where a, b refer to some particular sort of indices (with other indices of Q being suppressed).

The commutators with the coset $SO(5)/SO(4)$ generators $U^{a\alpha}$ mix the Q_1^{iaA} and $Q_2^{i\alpha A}$ generators

$$[U^{a\alpha},Q_1^{ibA}]=-i\epsilon^{ab}Q_2^{i\alpha A},\qquad [U^{a\alpha},Q_2^{i\beta A}]=-i\epsilon^{\alpha\beta}Q_1^{iaA}. \tag{2.7}$$

Finally, the anticommutators of the fermionic generators read

$$\begin{aligned}
\{Q_1^{iaA},Q_1^{jbB}\}&=-2(\epsilon^{ij}\epsilon^{ab}T_0^{AB}-2\epsilon^{ij}\epsilon^{AB}T_1^{ab}+\epsilon^{ab}\epsilon^{AB}T^{ij}),\\
\{Q_2^{i\alpha A},Q_2^{j\beta B}\}&=-2(\epsilon^{ij}\epsilon^{\alpha\beta}T_0^{AB}-2\epsilon^{ij}\epsilon^{AB}T_2^{\alpha\beta}+\epsilon^{\alpha\beta}\epsilon^{AB}T^{ij}),\\
\{Q_1^{iaA},Q_2^{j\alpha B}\}&=2\epsilon^{ij}\epsilon^{AB}U^{a\alpha}.
\end{aligned} \tag{2.8}$$

For what follows it is convenient to pass to another nota-

[1]We use the notation $(\mathbf{m}, \mathbf{8}, \mathbf{8\text{-}m})$ to identify an off-shell $\mathcal{N}=8$, $d=1$ supermultiplet with $\mathbf{m}$ physical bosons, $\mathbf{8}$ fermions and $\mathbf{8\text{-}m}$ auxiliary bosonic components.

[2]We use the following convention for the skew-symmetric tensor ϵ: $\epsilon_{ij}\epsilon^{jk}=\delta_i^k$, $\epsilon_{12}=\epsilon^{21}\equiv 1$.

tion,

$$P \equiv T_0^{22}, \qquad K \equiv T_0^{11}, \qquad D \equiv -T_0^{12}, \qquad V \equiv T^{22}, \qquad \bar{V} \equiv T^{11}, \qquad V_3 \equiv T^{12}, \qquad Q^{ia} \equiv -Q_1^{ia2}, \qquad \mathcal{Q}^{i\alpha} \equiv -Q_2^{i\alpha 2}, \qquad S^{ia} \equiv Q_1^{ia1}, \qquad \mathcal{S}^{i\alpha} \equiv Q_2^{i\alpha 1}. \tag{2.9}$$

One can check that P and Q^{ia}, $\mathcal{Q}^{i\alpha}$ constitute a $\mathcal{N}=8$, $d=1$ Poincaré superalgebra. The generators D, K and S^{ia}, $\mathcal{S}^{i\alpha}$ stand for the $d=1$ dilatations, special conformal transformations and conformal supersymmetry, respectively.

C. A new supercoset of $OSp(4^\star|4)$

Our goal is to construct the nonlinear supermultiplet with off-shell content (**4**, **8**, **4**). In the nonlinear realization approach the physical bosonic components parameterize some coset of the given supergroup. So the first task is to identify such a four-dimensional bosonic coset in the supergroup $OSp(4^\star|4)$. One of the possible choices is the supercoset $\frac{OSp(4^\star|4)}{U(1)_{\mathcal{R}}\otimes SO(5)}$. For this case the bosonic Goldstone superfields parameterize the coset $D\otimes SU(2)$. The corresponding supermultiplet includes the dilaton and three fields living on the sphere $SU(2)$. It is just the linear (**4**, **8**, **4**) supermultiplet [20].

Another possibility is to consider the supercoset $\frac{OSp(4^\star|4)}{SU(2)_{\mathcal{R}}\otimes\{D,K\}\otimes SO(4)}$. As it can be easily seen, it contains the bosonic coset $SO(5)/SO(4)$; the four physical bosonic fields of the resulting multiplet are nothing but the parameters describing the 4-sphere $S^4 = SO(5)/SO(4)$. Differently from the previously mentioned supercoset, the dilaton associated with the dilatation generator D will not appear. Therefore, despite the fact that the superconformal group $OSp(4^\star|4)$ is perfectly realized on our supercoset there is no possibility to construct superconformally invariant action in our case, because without dilaton there is no possibility to compensate the dilatonic weight of the superspace measure. Nevertheless, the $\mathcal{N}=8$ supersymmetric sigma-model type of the action can be constructed for the supermultiplet in question.

Thus, we are going to realize the superconformal group $OSp(4^\star|4)$ in the coset superspace $\frac{OSp(4^\star|4)}{SU(2)_{\mathcal{R}}\otimes\{D,K\}\otimes SO(4)}$ parameterized as

$$g = e^{itP} e^{\theta_{ia}Q^{ia} + \vartheta_{i\alpha}\mathcal{Q}^{i\alpha}} e^{\psi_{ia}S^{ia} + \xi_{i\alpha}\mathcal{S}^{i\alpha}} e^{iv_{\alpha a}U^{\alpha a}}. \tag{2.10}$$

As usual, in order to find the covariant irreducibility conditions on the coset superfields, we must impose the inverse Higgs constraints [22] on the left-covariant $osp(4^\star|4)$-valued Cartan one-form $\Omega = g^{-1}dg$. Concerning the treated case, the relevant constraints are

$$\omega_U^{\alpha a}| = 0, \tag{2.11}$$

where $|$ denotes the spinor projection. These constraints are manifestly covariant under the left action of the whole supergroup $OSp(4^\star|4)$. Indeed, with respect to the action of this supergroup by a left multiplications on the coset element (2.10), the Cartan forms are rotated by the elements of the stability subgroup $SU(2)_{\mathcal{R}}\otimes\{D,K\}\otimes SO(4)$. Clearly, the constraints (2.11) are invariant under such rotations. Explicitly, the Cartan form $\omega_U^{\alpha a}$ reads

$$\omega_U^{\alpha a} = \frac{2}{2+V^2}\left[idV_{\alpha a} + (\delta_a^b\delta_\alpha^\beta + V_a^\beta V_\alpha^b)\Omega_{\beta b} + V_\alpha^c\omega_{ca} + V_a^\beta\omega_{\beta\alpha}\right], \tag{2.12}$$

where

$$\Omega_{\alpha a} = -2(d\vartheta_{i\alpha}\psi_a^i + d\theta_{ia}\xi_\alpha^i), \qquad \omega_{ab} = -4d\theta_{i(a}\psi_{b)}^i, \qquad \omega_{\alpha\beta} = -4d\vartheta_{i(\alpha}\xi_{\beta)}^i \tag{2.13}$$

and

$$V_{\alpha a} = \frac{\tan\sqrt{\frac{v^2}{2}}}{\sqrt{\frac{v^2}{2}}} v_{\alpha a}, \qquad v^2 = \epsilon^{ab}\epsilon^{\alpha\beta}v_{\alpha a}v_{\beta b}. \tag{2.14}$$

Selecting the $d\theta_{ia}$ and $d\vartheta_{i\alpha}$ projections of the constraints (2.11) we will get

$$iD^{ib}V_{\alpha a} + 2\delta_a^b(\xi_\alpha^i + V_\alpha^c\psi_c^i) + 2V_\alpha^b(\psi_a^i + V_a^\beta\xi_\beta^i) = 0, \tag{2.15}$$

$$i\nabla^{i\beta}V_{\alpha a} + 2V_a^\beta(\xi_\alpha^i + V_\alpha^c\psi_c^i) + 2\delta_\alpha^\beta(\psi_a^i + V_a^\gamma\xi_\gamma^i) = 0. \tag{2.16}$$

Equations (2.15) and (2.16) allow one to express the eight fermions ψ_a^i, ξ_α^i in terms of the covariant derivatives of the four bosonic superfields $V_{\alpha a}$, and therefore such equations properly constrain the $V_{\alpha a}$'s. These constraints may be written in two equivalent form

$$(D^{i(a} - V_\beta^{(a}\nabla^{i\beta})V_\alpha^{b)} = 0, \qquad (\nabla^{i(\alpha} - V_b^{(\alpha}D^{ib})V_a^{\beta)} = 0, \tag{2.17}$$

or

$$D^{i(a}X_\alpha^{b)} - X^{\beta(a}\nabla_\alpha^i X_\beta^{b)} = 0, \qquad \nabla^{i(\alpha}X_a^{\beta)} - X^{b(\alpha}D_a^i X_b^{\beta)} = 0, \tag{2.18}$$

where, as usual, the round brackets denote the symmetrization of the enclosed indices, and we introduced

$$X_{a\alpha} \equiv \frac{2}{2-V^2}V_{a\alpha}. \tag{2.19}$$

As it can be seen, such constraints are nonlinear, and therefore the considered $\mathcal{N}=8$, $d=1$ multiplet may be referred to as the (**4**, **8**, **4**) nonlinear supermultiplet. Let us observe that discarding of the nonlinear terms in (2.18) yields the linear (**4**, **8**, **4**) supermultiplet [20].

Besides ensuring the covariance of the constraints (2.18), the coset approach gives the easiest way to find the transformation properties of the coordinates and superfields under the supergroup $OSp(4^\star|4)$.

The $\mathcal{N}=8$, $d=1$ Poincarè supersymmetry is realized in the standard way

$$\delta t = -i(\eta_{ia}\theta^{ia} + \eta_{i\alpha}\vartheta^{i\alpha}), \qquad \delta\theta_{ia} = \eta_{ia}, \qquad \delta\vartheta_{i\alpha} = \eta_{i\alpha} \tag{2.20}$$

and $V_{a\alpha}$ is a scalar with respect to these transformations. For what concerns the transformation properties of the coordinates and superfields $V^{\alpha a}$ under the conformal supersymmetry generated by the left action of the element

$$g_1 = e^{\eta_{ia}S^{ia} + \eta_{i\alpha}S^{i\alpha}}, \tag{2.21}$$

one should note that the coordinates of the superspace are transformed in the same way as in [19]

$$\begin{aligned}
\delta t &= -it(\eta^{ia}\theta_{ia} + \eta^{i\alpha}\vartheta_{i\alpha}) + (\eta^i_a\theta^{ja} + \eta^i_\alpha\vartheta^{j\alpha}) \times (\theta_{ib}\theta^b_j + \vartheta_{i\beta}\vartheta^\beta_j),\\
\delta\theta_{ia} &= t\eta_{ia} - i\eta^j_a\theta_{jb}\theta^b_i + 2i\eta^j_b\theta^b_i\theta_{ja} - i\eta^j_a\vartheta_{j\alpha}\vartheta^\alpha_i + 2i\eta^\alpha_j\vartheta_{i\alpha}\theta^j_a,\\
\delta\vartheta_{i\alpha} &= t\eta_{i\alpha} - i\eta^j_\alpha\vartheta_{j\beta}\vartheta^\beta_i + 2i\eta^j_\beta\vartheta^\beta_i\vartheta_{j\alpha} - i\eta^j_\alpha\theta_{ja}\theta^a_i + 2i\eta^j_a\theta^a_i\vartheta_{j\alpha},
\end{aligned} \tag{2.22}$$

while the superfield $V_{a\alpha}$ transform as

$$\delta V_{\alpha a} = 2i(\delta^\beta_\alpha\delta^b_a + V^b_\alpha V^\beta_a)A_{\beta b} + 2iA_{ab}V^b_\alpha + 2iA_{\alpha\beta}V^\beta_a, \tag{2.23}$$

where

$$A_{\alpha a} = \theta_{ia}\eta^i_\alpha + \vartheta_{i\alpha}\eta^i_a, \qquad A_{ab} = \theta_{ia}\eta^i_b + \theta_{ib}\eta^i_a, \qquad A_{\alpha\beta} = \vartheta_{i\alpha}\eta^i_\beta + \vartheta_{i\beta}\eta^i_\alpha. \tag{2.24}$$

The transformations with respect to other generators of the supergroup $OSp(4^\star|4)$ can be easily found from (2.20) and (2.22) since all bosonic transformations appear in the anticommutators of the conformal and Poincaré supersymmetries. For the reader's convenience we will present here the explicit form of the transformations under the left action of the $SO(5)/SO(4)$ element represented by

$$g_2 = e^{ia_{\alpha a}U^{\alpha a}} \tag{2.25}$$

which read as follows:

$$\delta\theta_{ia} = a_{\alpha a}\vartheta^\alpha_i, \qquad \delta\vartheta_{i\alpha} = a_{\alpha a}\theta^a_i; \tag{2.26}$$

$$\begin{aligned}
\delta V_{a\alpha} &= \left(1 - \frac{V^2}{2}\right)a_{\alpha a} + a_{\beta b}V^{b\beta}V_{a\alpha},\\
\delta X_{a\alpha} &= a_{\alpha a} + 2a_{\beta b}X^{b\beta}X_{a\alpha}.
\end{aligned} \tag{2.27}$$

Thus, the quartet of the $\mathcal{N}=8$ bosonic superfields V_{ia} subjected to the nonlinear constraints (2.17) defines the nonlinear (**4**, **8**, **4**) supermultiplet.

III. REDUCTION FROM $\mathcal{N}=8$ VECTOR SUPERMULTIPLET AND $\mathcal{N}=4$ SUPERFIELD FORMULATION

Our construction of the nonlinear (**4**, **8**, **4**) supermultiplet is very similar to the consideration of the (**5**, **8**, **3**) supermultiplet in [19]. The only, though crucial difference, is the absence of the dilaton among the components of our superfields V_{ia}. One may wonder whether it is possible to reconstruct the nonlinear (**4**, **8**, **4**) supermultiplet by a direct reduction from the (**5**, **8**, **3**) one, as in the case of $\mathcal{N}=4$ supermultiplets [23]. Next we demonstrate that such reduction indeed exists. Moreover, there are two different reductions from $\mathcal{N}=8$, $d=1$ vector multiplet (**5**, **8**, **3**) to the supermultiplets (**4**, **8**, **4**)—one reproducing the linear supermultiplets, while second giving rise to the nonlinear one.

A. Two reductions from $\mathcal{N}=8$ vector supermultiplet

In order to properly analyze such reductions, it is convenient to recall some basic facts on the $\mathcal{N}=8$ vector multiplet (see [19] for further elucidation). The $\mathcal{N}=8$ multiplet (**5**, **8**, **3**), already considered in [24], has been obtained in [19] from a nonlinear realization of the same $\mathcal{N}=8$, $d=1$ superconformal group $OSp(4^\star|4)$ in the coset superspace $\frac{OSp(4^\star|4)}{SU(2)_{\mathcal{R}}\otimes SO(4)}$ parameterized as

$$g = e^{itP}e^{\theta_{ia}Q^{ia} + \vartheta_{i\alpha}Q^{i\alpha}}e^{\psi_{ia}S^{ia} + \xi_{i\alpha}S^{i\alpha}}e^{izK}e^{iuD}e^{iv_{\alpha a}U^{\alpha a}}. \tag{3.1}$$

Beside the 4-dimensional bosonic coset $SO(5)/SO(4)$, the physical bosonic field content of the vector multiplet includes the dilaton superfield associated with the generator D. In this case, the invariant constraints read

$$\omega_D = 0, \qquad \omega_U^{\alpha a}| = 0. \tag{3.2}$$

Thus we see that, besides the same constraints on the Cartan forms $SO(5)/SO(4)$, there is an additional one which nullifies the dilaton form ω_D. The constraints (3.2) allow one to express the Goldstone spinor superfields and the boost superfield z in terms of the spinor and t-derivatives of the remaining bosonic Goldstone superfields u, $v_{\alpha a}$. Moreover, they also imply the following irreducibility constraints:

$$D^{ib}\mathcal{V}_{\alpha a} + \delta^b_a\nabla^i_\alpha\mathcal{U} = 0, \qquad \nabla^{i\beta}\mathcal{V}_{\alpha a} + \delta^\beta_\alpha D^i_a\mathcal{U} = 0, \tag{3.3}$$

where

$$\mathcal{V}_{\alpha a} = e^{-u}\frac{2V_{\alpha a}}{2+V^2}, \qquad \mathcal{U} = e^{-u}\left(\frac{2-V^2}{2+V^2}\right), \tag{3.4}$$

with $V_{\alpha a}$ defined in (2.14).

Let us now consider the reductions of the (**5**, **8**, **3**) vector multiplet.

The first reduction procedure is rather trivial. We can start by replacing $D^i_a\mathcal{U}$ and $\nabla^i_\alpha\mathcal{U}$ in (3.2) by arbitrary fermionic superfields Ψ^a_i and Ξ^i_α; Eqs. (3.2) will then define such superfields in terms of covariant spinor derivatives of the Goldstone bosonic superfields $\mathcal{V}_{\alpha a}$, by constraining them as follows:

$$D^{i(a}\mathcal{V}^{b)}_\alpha = 0, \qquad \nabla^{i(\alpha}\mathcal{V}^{\beta)}_a = 0. \tag{3.5}$$

Such constraining conditions are nothing but the ones defining the $\mathcal{N}=8$ linear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet [20]. From the previously performed replacement, it is clear that this reduction procedure corresponds to "removing" the first bosonic component of the superfield $\mathcal{U}$ from the set of physical bosons and replacing it by an auxiliary field.

The second reduction corresponds to the "removal" of the real dilaton superfield u. It is clear from (3.4) that, in order to do this, one has to define the new superfields $X_{\alpha a}$ as follows:

$$X_{\alpha a} \equiv \frac{\mathcal{V}_{\alpha a}}{\mathcal{U}}. \tag{3.6}$$

The rewriting of the constraints (3.2) in terms of $X_{\alpha a}$ gives rise to nothing else than the constraints (2.18).

Summarizing, starting from the multiplet $(\mathbf{5},\mathbf{8},\mathbf{3})$, the dimensional reduction along the first bosonic component of the superfield $\mathcal{U}$ yields the linear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet [20], whereas the removal of the dilaton u yields the previously introduced nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ multiplet. The existence of such a reduction is very useful for the construction of the superfield action (see Sec. IV). Here we will use this reduction, in order to provide a $\mathcal{N}=4$ description of our nonlinear supermultiplet.

B. $\mathcal{N}=4$ superfield formulations

The use of the $\mathcal{N}=8$ superfield formalism is rather convenient when considering the transformation properties, the invariance of the basic constraints, etc. At the same time the $\mathcal{N}=4$ superspace description is preferable for constructing the action. In order to find the $\mathcal{N}=4$ superfields content of our nonlinear supermultiplet we will use its previously established connection with the linear $(\mathbf{5},\mathbf{8},\mathbf{3})$ supermultiplet.

In order to formulate the nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet in terms of $\mathcal{N}=4$ superfields, it is convenient to recall the $\mathcal{N}=4$ splitting of the $\mathcal{N}=8$ vector multiplet [20]. For our purposes, we just need to define all superfields in the $\mathcal{N}=4$, $d=1$ superspace $\mathbb{R}^{(1|4)}$ which is parameterized by the coordinates $\{t, \theta_{ia}\}$. The constraints (3.2) imply that the spinor derivatives of all involved superfields with respect to $\vartheta_{i\alpha}$ are expressed in terms of the spinor derivatives with respect to θ_{ia}. Consequently, the essential $\mathcal{N}=4$ superfield components in the ϑ-expansion of the physical Goldstone bosonic superfields $\mathcal{V}_{\alpha a}$ and $\mathcal{U}$ of the vector multiplet are only the first ones

$$\hat{\mathcal{V}}_{\alpha a} \equiv \mathcal{V}_{\alpha a}|_{\vartheta=0}, \qquad \hat{\mathcal{U}} \equiv \mathcal{U}|_{\vartheta=0}. \tag{3.7}$$

These five bosonic $\mathcal{N}=4$ superfields, expressing the whole off-shell component content of the $(\mathbf{5},\mathbf{8},\mathbf{3})$ vector multiplet, are subjected by (3.2) to the following irreducibility constraints in $\mathbb{R}^{(1|4)}$ [20]:

$$D^{i(a}\hat{\mathcal{V}}^{b)\alpha} = 0, \qquad D^{i(a}D^{b)}_i\hat{\mathcal{U}} = 0. \tag{3.8}$$

Thus, by adopting such a $\mathcal{N}=4$ superspace perspective, the $\mathcal{N}=8$ vector supermultiplet may be considered as the sum of the $\mathcal{N}=4$, $d=1$ hypermultiplet $\hat{\mathcal{V}}_{\alpha a}$ (with $(\mathbf{4},\mathbf{4},\mathbf{0})$ off-shell component content) and the $\mathcal{N}=4$ "old" tensor multiplet $\hat{\mathcal{U}}$ (with $(\mathbf{1},\mathbf{4},\mathbf{3})$ content).

Beside the explicit $\mathcal{N}=4$ Poincaré supersymmetry directly yielded by the considered $\mathcal{N}=4$ superfield formalism, one should also take into account the additional, implicit $\mathcal{N}=4$ supersymmetry (completing the explicit one to $\mathcal{N}=8$). It is easy to check that the transformation properties of the above defined $\mathcal{N}=4$ superfields read

$$\delta^*\hat{\mathcal{V}}_{\alpha a} = \eta_{i\alpha}D^i_a\hat{\mathcal{U}}, \qquad \delta^*\hat{\mathcal{U}} = \frac{1}{2}\eta_{i\alpha}D^{ia}\hat{\mathcal{V}}^\alpha_a. \tag{3.9}$$

After recalling such facts about the $(\mathbf{5},\mathbf{8},\mathbf{3})$ vector multiplet and considering the definition (3.6), it is now rather easy to get the formulations of the new nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet in terms of $\mathcal{N}=4$ superfields. Indeed, one just needs to introduce the new $\mathcal{N}=4$ superfields

$$\mathcal{L}_{\alpha a} \equiv \frac{\hat{\mathcal{V}}_{\alpha a}}{\hat{\mathcal{U}}}, \qquad \mathcal{W}^{ia} \equiv \frac{D^{ia}\hat{\mathcal{U}}}{\hat{\mathcal{U}}}. \tag{3.10}$$

By rewriting the basic $\mathcal{N}=4$ constraints (3.8) in terms of such $\mathcal{N}=4$ superfields, one obtains

$$D^{i(a}\mathcal{L}^{b)\alpha} + \mathcal{L}^{\alpha(a}\mathcal{W}^{b)i} = 0, \tag{3.11}$$

$$D^{i(a}\mathcal{W}^{b)i} + \mathcal{W}^{i(a}\mathcal{W}^{jb)} = 0. \tag{3.12}$$

It is then immediate to recognize that the constraints (3.11) describe a nonlinear version of the $(\mathbf{4},\mathbf{4},\mathbf{0})$ multiplet, while the constraints (3.12) define a nonlinear version of the $(\mathbf{0},\mathbf{4},\mathbf{4})$ supermultiplet. The transformations of $\mathcal{L}_{\alpha a}$ and $\mathcal{W}^{ia}$ under the implicit $\mathcal{N}=4$ supersymmetry may be easily found by recalling their definition (3.10) and using Eq. (3.9)

$$\begin{aligned}\delta^*\mathcal{L}_{\alpha a} &= \eta_{i\alpha}\mathcal{W}^i_a - \frac{1}{2}\eta_{j\beta}\mathcal{L}_{\alpha a}(D^{jc}\mathcal{L}^\beta_c + \mathcal{L}^\beta_c\mathcal{W}^{jc});\\ \delta^*\mathcal{W}^{ia} &= -\frac{1}{2}\eta_{j\alpha}D^{ia}(D^{jb}\mathcal{L}^\alpha_b + \mathcal{L}^\alpha_b\mathcal{W}^{jb}).\end{aligned} \tag{3.13}$$

Thus we see that our nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet is constructed from two $\mathcal{N}=4$ nonlinear supermultiplets, both of which were never considered before. It also becomes clear what is the role of the dilaton in the "linearization" of our supermultiplet. Indeed, representing the fermionic superfield $\mathcal{W}^{ia}$ as in (3.10) one may easily "linearize" both constraints (3.11) and (3.12), while keeping the fermionic superfield $\mathcal{W}^{ia}$ independent there is no way to have a linear supermultiplet.

IV. ANALYSIS OF THE BOSONIC SECTOR OF THE ACTION

As usual, for constructing the most general superfield action for nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet one should start from the general Ansatz for the $\mathcal{N}=4$ superfield Lagrangian and impose its invariance with respect to implicit $\mathcal{N}=4$ supersymmetry (3.13). This is not so easy because among $\mathcal{N}=4$ superfields spanning the $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet there are bosonic $\mathcal{L}_{\alpha a}$ and fermionic $\mathcal{W}^{ia}$ superfields.

The starting point for the dimensional reduction procedures outlined in Subsect. III A is the most general sigma-model type action for the $(\mathbf{5},\mathbf{8},\mathbf{3})$ supermultiplet written in the terms of the $\mathcal{N}=4$ superfields defined in (3.7) [19]

$$S=\kappa\int dtd^4\theta\mathcal{L}(\hat{\mathcal{V}}_{a\alpha},\hat{\mathcal{U}}), \tag{4.1}$$

with the additional constraint that the Lagrangian $\mathcal{L}$ be a harmonic function

$$\frac{\partial^2\mathcal{L}}{\partial\hat{\mathcal{V}}^{a\alpha}\partial\hat{\mathcal{V}}_{a\alpha}}+2\frac{\partial^2\mathcal{L}}{\partial\hat{\mathcal{U}}^2}=0. \tag{4.2}$$

Performing the θ-integration in (4.1) and disregarding all fermionic terms, one obtains the bosonic action

$$S_B=-6\kappa\int dtg(v_{a\alpha},u)\left[\dot{u}^2+2\dot{v}^{a\alpha}\dot{v}_{a\alpha}-\frac{1}{8}C^{ij}C_{ij}\right], \tag{4.3}$$

where

$$v_{a\alpha}\equiv\hat{\mathcal{V}}_{a\alpha}|_{\theta=0},\qquad u\equiv\hat{\mathcal{U}}|_{\theta=0},\qquad C^{ij}\equiv D^{(ia}D_a^{j)}\hat{\mathcal{U}}|_{\theta=0} \tag{4.4}$$

and the metric $g(v_{a\alpha},u)$ of the 5-dim. physical bosonic manifold is defined as

$$g(v_{a\alpha},u)\equiv\frac{\partial^2\mathcal{L}}{\partial\hat{\mathcal{V}}^{a\alpha}\partial\hat{\mathcal{V}}_{a\alpha}}\Bigg|_{\theta=0} \tag{4.5}$$

and obeys the constraints

$$\frac{\partial^2 g}{\partial v^{a\alpha}\partial v_{a\alpha}}+2\frac{\partial^2 g}{\partial u^2}=0. \tag{4.6}$$

One may wonder whether we can learn something from all this for the cases of $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplets, keeping in mind the existence of the reductions from $(\mathbf{5},\mathbf{8},\mathbf{3})$ to $(\mathbf{4},\mathbf{8},\mathbf{4})$. Now we are going to demonstrate that starting from (4.3) we are able to construct the most general sigma-model actions for $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplets together with a particular potential term, in full analogy with $\mathcal{N}=4$ supersymmetric cases [23]. For the sake of simplicity, we will consider only bosonic sectors. The fermionic terms can be easily restored, if needed.

A. Reduction to the linear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet

Such a reduction corresponds to constraining the metric $g(v_{a\alpha},u)$ to be independent on u

$$g(v_{a\alpha},u)=g_1(v_{a\alpha}). \tag{4.7}$$

This functional restriction, when inserted in Eq. (4.2), allows one to write the Lagrangian density $\mathcal{L}(\hat{\mathcal{V}}_{a\alpha},\hat{\mathcal{U}})$ in (4.1) as

$$\mathcal{L}(\hat{\mathcal{V}}_{a\alpha},\hat{\mathcal{U}})=f_1(\hat{\mathcal{V}})+f_2(\hat{\mathcal{V}})\hat{\mathcal{U}}+f_3(\hat{\mathcal{V}})\hat{\mathcal{U}}^2, \tag{4.8}$$

with the additional constraints

$$\frac{\partial^2 f_1}{\partial\hat{\mathcal{V}}^{a\alpha}\partial\hat{\mathcal{V}}_{a\alpha}}\Bigg|_{\theta=0}=g_1,\qquad \frac{\partial^2 f_2}{\partial\hat{\mathcal{V}}^{a\alpha}\partial\hat{\mathcal{V}}_{a\alpha}}\Bigg|_{\theta=0}=0,$$
$$f_3|_{\theta=0}=-\frac{1}{4}g_1,\qquad \frac{\partial^2 g_1}{\partial v^{a\alpha}\partial v_{a\alpha}}=0. \tag{4.9}$$

Thus, in order to perform the reduction to the linear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet, the metric $g_1(v_{a\alpha})$ must obey the 4-dimensional Laplace equation.

Next, we follow the same procedure exploited in the $\mathcal{N}=4$ case in [23]. We replace $\dot{u}$ by a new auxiliary field B in the action (4.3) and add the simplest Fayet-Iliopoulos (FI) term (linear in B)

$$S_1=-6\kappa\int dtg_1(v_{a\alpha})\left[B^2+2\dot{v}^{a\alpha}\dot{v}_{a\alpha}-\frac{1}{8}C^{ij}C_{ij}\right]-6\kappa\int dtmB. \tag{4.10}$$

Eliminating the auxiliary fields in (4.10) by their equations of motion, one obtains the following action for physical bosonic components:

$$S_1=-12\kappa\int dt\left[g_1\dot{v}^{a\alpha}\dot{v}_{a\alpha}-\frac{m^2}{8g_1}\right]. \tag{4.11}$$

The action (4.11) corresponds to the general action for the (4,8,4) linear supermultiplet [25] with the specific potential term.

B. Reduction to the nonlinear $(\mathbf{4},\mathbf{8},\mathbf{4})$ supermultiplet

In order to perform the reduction to the nonlinear supermultiplet, it is convenient to introduce the new variables

$$l^{a\alpha}\equiv\frac{v^{a\alpha}}{u},\qquad y\equiv\frac{\dot{u}}{u},\qquad \hat{C}^{ij}\equiv\frac{C^{ij}}{u}. \tag{4.12}$$

By substituting such definitions in the action (4.3), one gets

$$S_2 = -6\kappa \int dt g(v_{a\alpha}, u) u^2 \Big[(1+2l^2) y^2 + 2\dot{l}^{a\alpha}\dot{l}_{a\alpha} + 4y l^{a\alpha}\dot{l}_{a\alpha} - \frac{1}{8}\hat{C}^{ij}\hat{C}_{ij} \Big]. \tag{4.13}$$

It is easy to conclude that the action (4.13) will correspond to the nonlinear (**4**, **8**, **4**) supermultiplet iff

$$g(v_{a\alpha}, u)u^2 = g_2(l^{a\alpha}). \tag{4.14}$$

Equation (4.2) implies the metric $g_2(l)$ to satisfy the following differential equation:

$$\frac{\partial^2}{\partial l^{a\alpha}\partial l_{a\alpha}} g_2(l) + 2 l^{a\alpha} l^{b\beta} \frac{\partial^2}{\partial l^{a\alpha}\partial l^{b\beta}} g_2(l) + 12 l^{a\alpha}\frac{\partial}{\partial l^{a\alpha}} g_2(l) + 12 g_2(l) = 0. \tag{4.15}$$

When the condition (4.14) is fulfilled, one can introduce the simplest Fayet-Iliopoulos term (linear in y)

$$-6\kappa \int dt m y \tag{4.16}$$

and eliminate the auxiliary field y by its equation of motion, obtaining the following bosonic action:

$$S_2 = -12\kappa \int dt \Big\{ g_2 \Big[\dot{l}^{a\alpha}\dot{l}_{a\alpha} - 2\frac{(l^{a\alpha}\dot{l}_{a\alpha})^2}{1+2l^2} \Big] - \frac{m^2}{8}\frac{1}{g_2(1+2l^2)} \Big\} \tag{4.17}$$

Moreover, by defining the new fields

$$z^{a\alpha} \equiv \frac{\sqrt{2}}{1+\sqrt{1+2l^2}} l^{a\alpha}, \tag{4.18}$$

one can rewrite the action (4.17) in the following nice form:

$$S_2 = -24\kappa \int dt \Big[\frac{g_2}{(1-z^2)^2}\dot{z}^{a\alpha}\dot{z}_{a\alpha} - \frac{m^2}{16}\frac{(1-z^2)^2}{g_2(1+z^2)^2} \Big] \tag{4.19}$$

It is interesting to notice that, by performing the change of variable (4.18), the differential Eq. (4.15) can be rewritten in the remarkably simple form

$$\frac{\partial^2}{\partial z^{a\alpha}\partial z_{a\alpha}} \Big[\frac{1+z^2}{(1-z^2)^2} g_2(z) \Big] = 0, \tag{4.20}$$

which is nothing but the 4-dim. Laplace equation

$$\frac{\partial^2 \mathcal{G}(z)}{\partial z^{a\alpha}\partial z_{a\alpha}} = 0 \tag{4.21}$$

for the redefined metric function

$$\mathcal{G}(z) \equiv \frac{1+z^2}{(1-z^2)^2} g_2(z). \tag{4.22}$$

By inserting the redefinition (4.22) in the action (4.19), one finally gets

$$S_2 = -24\kappa \int dt \Big[\frac{\mathcal{G}}{1+z^2}\dot{z}^{a\alpha}\dot{z}_{a\alpha} - \frac{m^2}{16}\frac{1}{\mathcal{G}(1+z^2)} \Big]. \tag{4.23}$$

Thus, we see that the net effect of using the nonlinear (**4**, **8**, **4**) supermultiplet is the deformation of the metric and potential term in the bosonic sector (together with the deformation of the fermionic terms).

Finally, it is interesting to note that the particular solution of (4.21)

$$\mathcal{G} = \frac{1+z^2}{z^2} \tag{4.24}$$

gives rise to the action

$$S_2 = -24\kappa \int dt \Big[\frac{1}{z^2}\dot{z}^{a\alpha}\dot{z}_{a\alpha} - \frac{m^2}{16}\frac{z^2}{(1+z^2)^2} \Big]. \tag{4.25}$$

The metric $\frac{1}{z^2}$ is the solution of the four-dimensional Laplace equation and therefore the sigma-model part of the action (4.25) coincides with the action (4.11) for the linear (**4**, **8**, **4**) supermultiplet with $g_1 = \frac{1}{z^2}$. Nevertheless, the potential term in (4.25) is completely different.

V. CONCLUSIONS

In this paper we constructed a new nonlinear off-shell $\mathcal{N}=8$ supermultiplet with (**4**, **8**, **4**) components content. We showed that this multiplet can be described in a $\mathcal{N}=8$ superfield form as properly constrained Goldstone superfields associated with suitable cosets of the nonlinearly realized $\mathcal{N}=8$, $d=1$ superconformal group $OSp(4^\star|4)$. The $\mathcal{N}=8$ superfield irreducibility conditions were derived as a subset of covariant constraints on the Cartan super one-forms. The superconformal transformation properties of these $\mathcal{N}=8$, $d=1$ Goldstone superfields were explicitly given, alongside with the transformation of the coordinates of $\mathcal{N}=8$, $d=1$ superspace. Although the whole superconformal group $OSp(4^\star|4)$ has a perfect realization on the nonlinear (**4**, **8**, **4**) supermultiplet the most general action is invariant only under $\mathcal{N}=8$ Poincaré supersymmetry.

Apart from the $\mathcal{N}=8$ superfield description, we presented also $\mathcal{N}=4$ superfield formulations of this multiplet. We also established the relations of this new nonlinear supermultiplet with the linear (**5**, **8**, **3**) one. More concretely, there exist reductions from (**5**, **8**, **3**) to (**4**, **8**, **4**) linear and nonlinear supermultiplets. Moreover, these reductions being applied to the action give rise to the most general sigma-model type action for (**4**, **8**, **4**) supermultiplets with some sort of potential terms.

The present considerations provide another proof of the statement that the $\mathcal{N} = 4, 8$ supermultiplets which do not contain the dilaton among their components fields are all nonlinear. In this respect, it seems interesting to analyze the nonlinear supermultiplets related with the other $\mathcal{N} = 8$, $d = 1$ superconformal groups $OSp(8|2)$, $F(4)$ and $SU(1,1|4)$ [16,17]. The corresponding $\mathcal{R}$-symmetries groups are $SO(8)$, $SO(7)$ and $SO(6)$. Therefore one might expect to, respectively, obtain (**7**, **8**, **1**), (**6**, **8**, **2**) and (**5**, **8**, **3**) nonlinear supermultiplets.

In this paper, when constructing the superfield actions, we preferred to deal with $\mathcal{N} = 4$, $d = 1$ superfields. Thus, only half of the supersymmetries were manifest. Of course, it would be nice to have a description with all $\mathcal{N} = 8$ supersymmetries manifest. This can be achieved only in harmonic superspace [26].

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