Spontaneous absorption of an accelerated hydrogen atom near a conducting plane in vacuum

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We study, in the multipolar coupling scheme, a uniformly accelerated multilevel hydrogen atom in interaction with the quantum electromagnetic field near a conducting boundary and separately calculate the contributions of the vacuum fluctuation and radiation reaction to the rate of change of the mean atomic energy. It is found that the perfect balance between the contributions of vacuum fluctuations and radiation reaction that ensures the stability of ground-state atoms is disturbed, making spontaneous transition of ground-state atoms to excited states possible in a vacuum with a conducting boundary. The boundary-induced contribution is effectively a nonthermal correction, which enhances or weakens the nonthermal effect already present in the unbounded case, thus possibly making the effect easier to observe. An interesting feature worth noting is that the nonthermal corrections may vanish for atoms on some particular trajectories.

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I. INTRODUCTION

Understanding the physical origin of radiative properties of atoms, such as spontaneous emission and radiative level shifts, is a very stimulating problem. So far mechanisms such as vacuum fluctuations [1,2] and radiation reaction [3], or a combination of them [4], have been proposed as the possible physical interpretations. The ambiguity arises because of the freedom in the choice of ordering of commuting operators of the atom and field in a Heisenberg picture approach to the problem. As a result, there exists an indetermination in the separation of effects of vacuum fluctuations and radiation reaction such that distinct contributions of vacuum fluctuations and radiation reaction to the spontaneous emission of atoms do not possess an independent physical meaning. Therefore, although quantitative results for spontaneous emission and radiative level shifts are well established, the physical interpretations remained controversial until Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) argued in [5,6] that there exists a symmetric operator ordering of atom and field variables where the distinct contributions of vacuum fluctuations and radiation reaction to the rate of change of an atomic observable are separately Hermitian. If one demands such an ordering, an independent physical meaning can be assigned to each contribution. Using this prescription one can show that for ground-state atoms, the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean excitation energy cancel exactly and this cancellation forbids any transitions from the ground state and thus ensures atom's stability in vacuum. While for any initial excited state, the rate of change of atomic energy acquires equal contributions from vacuum fluctuations and from radiation reaction.

Recently, Audretsch, Müeller, and Holzmann [7–9] have generalized the formalism of DDC [6] to evaluate vacuum fluctuations and radiation reaction contributions to the spontaneous excitation rate and radiative energy shifts of an accelerated two-level atom interacting with a scalar field in a unbounded Minkowski space. In particular, their results show that when an atom is accelerated, then the delicate balance between vacuum fluctuations and radiation reaction is altered since the contribution of vacuum fluctuations to the rate of change of the mean excitation energy is modified while that of the radiation reaction remains the same. Thus transitions to excited states for ground-state atoms become possible even in vacuum. This result not only is consistent with the Unruh effect [10], which is closely related to the Hawking radiation of black holes, but also provides a physically appealing interpretation of it, since the spontaneous excitation of accelerated atoms can be considered as the actual physical process underlying the Unruh effect. Physically, this gives a transparent illustration for why an accelerated detector clicks (See Ref. [11] for a discussion in a different context).

Therefore, one sees that the Unruh effect is intrinsically related to the effects of modified vacuum fluctuations induced by the acceleration of the atom (or detector) in question. On the other hand, however, it is well-known that the presence of boundaries in a flat space-time also modifies the vacuum fluctuations of quantum fields, and it has been demonstrated that this modification (or changes) in vacuum fluctuations can lead to a lot of novel effects, such as the Casimir effect [12], the light-cone fluctuations when gravity is quantized [13–15], and the Brownian (random) motion of test particles in an electromagnetic vacuum [16–19] (Also see [20–22]), just to name a few. Therefore, it remains interesting to see what happens to the radiation properties of accelerated atoms found in Ref. [7] when the vacuum fluctuations are further modified by the presence of boundaries. Recently the effects of modified vacuum fluctuations and radiation reaction due to the presence of a conducting plane boundary upon the spontaneous excitation of both an inertial and a uniformly accelerated atom

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interacting with a quantized real massless scalar field have been discussed [23]. It is found that the modifications induced by the presence of a boundary make the spontaneous radiation rate of an excited inertial atom to oscillate near the boundary and this oscillatory behavior may offer a possible opportunity for experimental tests for geometrical (boundary) effects in flat space-time. While for accelerated atoms, the transitions from ground states to excited states are found to be possible even in vacuum due to changes in the vacuum fluctuations induced by both the presence of the boundary and the acceleration of atoms. Meanwhile the contribution of radiation reaction is now dependent on the acceleration of the atom, in sharp contrast to the unbounded Minkowski space where it has been shown that for accelerated atoms on arbitrary stationary trajectory, the contribution of radiation reaction is generally not altered from its inertial value [9].

However, a two-level atom interacting with a scalar field is more or less a toy model, and a more realistic system would be a multilevel atom, a hydrogen atom, for instance, in interaction with a quantized electromagnetic field. Let us note that such a system was examined in terms of the radiative energy shifts of an accelerated atom [24] using the method of Ref. [8], where nonthermal corrections to the energy shifts were found in addition to the usual thermal ones associated with the temperature $T = a/2\pi$. Recently, the spontaneous excitation rate of an accelerated atom in the same system has been studied [25]. It has been found that both the effects of vacuum fluctuations and radiation reaction on the atom are changed by the acceleration. This is in sharp contrast to the scalar field case where the contribution of radiation reaction is not altered by the acceleration. A dramatic feature is that the contribution of electromagnetic vacuum fluctuations to the spontaneous emission rate contains an extra nonthermal term proportional to a^2, the proper acceleration squared, in contrast to the scalar field case where the effect of acceleration is purely thermal. Therefore the equivalence between uniform acceleration and thermal fields is lost when the scalar field is replaced by the electromagnetic field as has been argued elsewhere in other different contexts [26,27]. However, one may wonder what happens to the spontaneous emission of accelerated multilevel atoms in interaction with quantized electromagnetic fields found in Ref. [25], when the vacuum fluctuations are further modified by the presence of boundaries. This is what we plan to address in the present paper; we will calculate the effects of modified vacuum fluctuations and radiation reaction due to the presence of a conducting plane boundary upon the spontaneous excitation of both an inertial and a uniformly accelerated multilevel atom interacting with a quantized electromagnetic field in the multipolar coupling scheme. It should be pointed out that the multilevel atom in the dipole coupling with electromagnetic fields only serves as a model for discussion and it is still a crude representation of a hydrogen atom in reality.

The paper is organized as follows, we give, in Sec. II, a review of the general formalism developed in Ref. [7] and generalized in Refs. [24,25] to the case of a multilevel atom interacting with a quantized electromagnetic field in the multipolar coupling scheme, then apply it to the case of an inertial atom in Sec. III and to the case of an accelerated atom in Sec. IV. Finally we will conclude with some discussions in Sec. V.

II. THE GENERAL FORMALISM FOR VACUUM FLUCTUATION AND RADIATION REACTION

We consider a multilevel hydrogen atom in interaction with electromagnetic fields. To study the modifications of the spontaneous emission rate of atoms caused by the presence of a conducting plane boundary in vacuum, we assume that the conducting boundary is located at $z = 0$ in space and consider a pointlike hydrogen atom on a stationary space-time trajectory $x(\tau)$, where τ denotes the proper time on the trajectory. The stationary trajectory guarantees the existence of a series of stationary atomic states $|n\rangle$, with energies ω_n. The Hamiltonian that governs the time evolution of the atom with respect to the proper time τ can then be written as[1]

$$H_A(\tau) = \sum_n \omega_n \sigma_{nn}(\tau), \tag{1}$$

where $\sigma_{nn}(\tau) = |n\rangle\langle n|$. The free Hamiltonian of the quantum electromagnetic field that governs its time evolution with respect to τ is

$$H_F(\tau) = \sum_k \omega_{\vec{k}} a^\dagger_{\vec{k}} a_{\vec{k}} \frac{dt}{d\tau}, \tag{2}$$

where $\vec{k}$ denotes the wave vector and polarization of the field modes. We couple the hydrogen atom and the quantum electromagnetic field in the multipolar coupling scheme [28]

$$H_I(\tau) = -e\mathbf{r}(\tau) \cdot \mathbf{E}(x(\tau)) = -e\sum_{mn} \mathbf{r}_{mn} \cdot \mathbf{E}(x(\tau))\sigma_{mn}(\tau), \tag{3}$$

where e is the electron electric charge, $e\mathbf{r}$ the atomic electric dipole moment, $x(\tau) \leftrightarrow (t(\tau), \mathbf{x}(\tau))$, the space-time coordinates of the hydrogen atom. In the present case the dipole moment must be kept fixed with respect to the proper frame of reference of the atom, otherwise the rotation of the dipole moment will bring in extra time dependence in addition to the intrinsic time evolution [27].

Let us note that, since both $\mathbf{r}(\tau)$ and $\mathbf{E}(x)$ are not world vectors, the interaction Hamiltonian H_I is ambiguous when we deal with the situation of moving atoms. However, a manifestly coordinate invariant generalization of H_I can be given [27]:

[1]Lorentz-Heaviside units with $\hbar = c = 1$ will be used here.

$$H_I(\tau) = -er^\mu(\tau)F_{\mu\nu}(x(\tau))u^\nu(\tau), \tag{4}$$

where $F_{\mu\nu}$ is the field strength, $r^\mu(\tau)$ is a four vector such that its temporal component in the frame of the atom (proper reference frame) vanishes and its spatial components in the same frame are given by $\mathbf{r}(\tau)$, and u^ν is the four velocity of the atom. Since $u^\nu(\tau) = (1, 0, 0, 0)$ in the frame of the atom, this extended interaction Hamiltonian reduces to that given by Eq. (3) in the reference frame of the atom. In what follows, we choose to work in this reference frame.

We can now obtain the Heisenberg equations of motion for the dynamical variables of the hydrogen atom and the electromagnetic field from the Hamiltonian $H = H_A + H_F + H_I$. The solutions of the equations of motion can be split into two parts: a free part, which is present even in the absence of the coupling, and a source part, which is caused by the interaction of the atom and field. We assume that the initial state of the field is the vacuum $|0\rangle$, while the atom is in the state $|b\rangle$. Our aim is to identify and separate the two physical mechanisms that contribute to the rate of change of atomic observables $O(\tau)$: the contribution of vacuum fluctuations and that of radiation reaction. For this purpose, we choose a symmetric ordering between atom and field variables and identify the contribution of the vacuum fluctuations and radiation reaction to the rate of change of $O(\tau)$. Since we are interested in the spontaneous emission and absorption of the atom, we will concentrate on the mean atomic excitation energy $\langle H_A(\tau)\rangle$. The contribution of vacuum fluctuations (vf) and radiation reaction (rr) to the rate of change of $\langle H_A(\tau)\rangle$ can be written as (cf. Refs. [5–7,25])

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm vf} = 2ie^2 \int_{\tau_0}^{\tau} d\tau' C^F_{ij}(x(\tau), x(\tau')) \frac{d}{d\tau} (\chi^A_{ij})_b(\tau, \tau'), \tag{5}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm rr} = 2ie^2 \int_{\tau_0}^{\tau} d\tau' \chi^F_{ij}(x(\tau), x(\tau')) \frac{d}{d\tau} (C^A_{ij})_b(\tau, \tau'), \tag{6}$$

where $|\rangle = |b, 0\rangle$. The statistical functions of the atom, $(C^A_{ij})_b(\tau, \tau')$ and $(\chi^A_{ij})_b(\tau, \tau')$, are defined as

$$(C^A_{ij})_b(\tau, \tau') = \tfrac{1}{2}\langle b|\{r^f_i(\tau), r^f_j(\tau')\}|b\rangle, \tag{7}$$

$$(\chi^A_{ij})_b(\tau, \tau') = \tfrac{1}{2}\langle b|[r^f_i(\tau), r^f_j(\tau')]|b\rangle, \tag{8}$$

and those of the field are

$$C^F_{ij}(x(\tau), x(\tau')) = \tfrac{1}{2}\langle 0|\{E^f_i(x(\tau)), E^f_j(x(\tau'))\}|0\rangle, \tag{9}$$

$$\chi^F_{ij}(x(\tau), x(\tau')) = \tfrac{1}{2}\langle 0|[E^f_i(x(\tau)), E^f_j(x(\tau'))]|0\rangle. \tag{10}$$

Let us note that C^A is also called the symmetric correlation function of the atom in the state $|b\rangle$, χ^A its linear susceptibility, while C^F and χ^F are also known as the Hadamard function and Pauli-Jordan or Schwinger function of the field, respectively. The explicit forms of the statistical functions of the atom are given by

$$(C^A_{ij})_b(\tau, \tau') = \frac{1}{2}\sum_d [\langle b|r_i(0)|d\rangle\langle d|r_j(0)|b\rangle e^{i\omega_{bd}(\tau-\tau')} + \langle b|r_j(0)|d\rangle\langle d|r_i(0)|b\rangle e^{-i\omega_{bd}(\tau-\tau')}], \tag{11}$$

$$(\chi^A_{ij})_b(\tau, \tau') = \frac{1}{2}\sum_d [\langle b|r_i(0)|d\rangle\langle d|r_j(0)|b\rangle e^{i\omega_{bd}(\tau-\tau')} - \langle b|r_j(0)|d\rangle\langle d|r_i(0)|b\rangle e^{-i\omega_{bd}(\tau-\tau')}], \tag{12}$$

where $\omega_{bd} = \omega_b - \omega_d$ and the sum extends over a complete set of atomic states.

In order to calculate the statistical functions of the field, let us recall that the two-point function for the photon field may be expressed as

$$D^{\mu\nu}(x, x') = \langle 0|A^\mu(x)A^\nu(x')|0\rangle = D^{\mu\nu}_0(x - x') + D^{\mu\nu}_b(x, x'), \tag{13}$$

where $D^{\mu\nu}_0(x - x')$ is the two-point function in the usual Minkowski vacuum, and $D^{\mu\nu}_b(x, x')$, is the correction induced by the presence of boundary, which can be obtained by the method of images. In the Feynman gauge, at a distance z from the boundary, we have, in the laboratory frame,

$$D^{\mu\nu}_0(x - x') = \frac{\eta^{\mu\nu}}{4\pi^2[(t - t' - i\varepsilon)^2 - (x - x')^2 - (y - y')^2 - (z - z')^2]} \tag{14}$$

and

$$D^{\mu\nu}_b(x, x') = -\frac{\eta^{\mu\nu} + 2n^\mu n^\nu}{4\pi^2[(t - t' - i\varepsilon)^2 - (x - x')^2 - (y - y')^2 - (z + z')^2]}. \tag{15}$$

Here, $\eta^{\mu\nu} = {\rm diag}(1, -1, -1, -1)$ and the unit normal vector $n^\mu = (0, 0, 0, 1)$. Note that the two-point function Eq. (13) is constructed in such way that the tangential components of the electric field two-point function vanish on the conducting plane. The electric field two-point function can be expressed as a sum of the Minkowski vacuum term and a correction term due to the boundary:

$$\langle 0|\mathbf{E}(x)\mathbf{E}(x')|0\rangle = \langle 0|\mathbf{E}(x)\mathbf{E}(x')|0\rangle_0 + \langle 0|\mathbf{E}(x)\mathbf{E}(x')|0\rangle_b. \tag{16}$$

Since the boundary-independent contributions caused by the Minkowski vacuum term have been studied in Ref. [25], in the present paper, we will only calculate the boundary-dependent contributions, and write

$$\langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_b = \frac{1}{4\pi^2}[(\delta_{ij} - 2n_i n_j)\partial_0\partial_0' - \partial_i\partial_j']\frac{1}{(t - t' - i\varepsilon)^2 - (x - x')^2 - (y - y')^2 - (z + z')^2}, \tag{17}$$

where $\varepsilon \to +0$ and ∂' denotes the differentiation with respect to x'. The statistical functions of the field can be calculated using (17).

III. SPONTANEOUS EMISSION FROM A UNIFORMLY MOVING ATOM

In this section, we apply the previously developed formalism to study, in the presence of a conducting plane boundary, the spontaneous emission of an inertial multilevel atom interacting with quantized electromagnetic fields in the multipolar coupling scheme. We consider the atom moving in the x-direction with a constant velocity v at a distance z from the plane, thus its trajectory is given by

$$t(\tau) = \gamma\tau, \qquad x(\tau) = x_0 + v\gamma\tau, \qquad y(\tau) = y_0, \qquad z(\tau) = z, \tag{18}$$

where $\gamma = (1 - v^2)^{-(1/2)}$. From the general form Eq. (17) we can obtain the nonzero electric field two-point functions in the frame of the atom

$$\langle 0|E_x(x(\tau))E_x(x(\tau'))|0\rangle_b = \langle 0|E_y(x(\tau))E_y(x(\tau'))|0\rangle_b = -\frac{u^2 + 4z^2}{\pi^2[(u - i\varepsilon)^2 - 4z^2]^3}, \tag{19}$$

and

$$\langle 0|E_z(x(\tau))E_z(x(\tau'))|0\rangle_b = \frac{1}{\pi^2[(u - i\varepsilon)^2 - 4z^2]^2}. \tag{20}$$

where $u = \tau - \tau'$. Performing calculations using the above result lead to the nonzero Hadamard functions of the field:

$$C^F_{xx}(x(\tau), x(\tau')) = C^F_{yy}(x(\tau), x(\tau')) = -\frac{1}{2\pi^2}\left(\frac{u^2 + 4z^2}{[(u - i\varepsilon)^2 - 4z^2]^3} + \frac{u^2 + 4z^2}{[(u + i\varepsilon)^2 - 4z^2]^3}\right), \tag{21}$$

$$C^F_{zz}(x(\tau), x(\tau')) = \frac{1}{2\pi^2}\left(\frac{1}{[(u - i\varepsilon)^2 - 4z^2]^2} + \frac{1}{[(u + i\varepsilon)^2 - 4z^2]^2}\right), \tag{22}$$

and the Pauli-Jordan, or Schwinger functions:

$$\chi^F_{xx}(x(\tau), x(\tau')) = \chi^F_{yy}(x(\tau), x(\tau')) = -\frac{i}{4\pi z}\frac{u^2 + 4z^2}{6u^2 + 8z^2}(\delta''(u - 2z) - \delta''(u + 2z)), \tag{23}$$

$$\chi^F_{zz}(x(\tau), x(\tau')) = \frac{i}{8\pi z}\frac{1}{u}(\delta'(u + 2z) - \delta'(u - 2z)). \tag{24}$$

Here δ' and δ'' are the first and the second derivative of the delta function, respectively.

With all the statistical functions given, we are ready to calculate the contributions of both the vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy. Since the polarization direction of the atom can be arbitrary, in general, the polarization can have nonzero components in both the direction normal and that which is parallel to the plane. So calculations have to be carried out for all nonzero field statistical functions. Take the xx component for example; it is easy to show that the contribution of the changes in vacuum fluctuations induced by the presence of the boundary is given by

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle^{xx}_{b,\mathrm{vf}} = \frac{e^2}{2\pi^2}\sum_d |\langle b|r_x(0)|d\rangle|^2\omega_{bd}\int_{-\infty}^{\infty} du\left(\frac{u^2 + 4z^2}{[(u - i\varepsilon)^2 - 4z^2]^3} + \frac{u^2 + 4z^2}{[(u + i\varepsilon)^2 - 4z^2]^3}\right)e^{i\omega_{bd}u} \tag{25}$$

and that of radiation reaction by

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle^{xx}_{b,\mathrm{rr}} = \frac{ie^2}{4\pi z}\sum_d |\langle b|r_x(0)|d\rangle|^2\omega_{bd}\int_{-\infty}^{\infty} du\,\frac{u^2 + 4z^2}{6u^2 + 8z^2}(\delta''(u - 2z) - \delta''(u + 2z))e^{i\omega_{bd}u}, \tag{26}$$

where we have extended the range of integration to infinity for sufficiently long times $\tau - \tau_0$. The superscript xx denotes contributions associated with the xx component of the statistical functions and b in the subscript indicates boundary-dependent contribution. The integrals in Eqs. (25) and (26) can be evaluated via the residue theorem to get

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{vf}}^{xx} = \frac{e^2}{32\pi}\left(\sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 f_x(z,\omega_{bd}) - \sum_{\omega_b<\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 f_x(z,\omega_{bd}) \right), \tag{27}$$

and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{rr}}^{xx} = \frac{e^2}{32\pi}\left(\sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 f_x(z,\omega_{bd}) + \sum_{\omega_b<\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 f_x(z,\omega_{bd}) \right). \tag{28}$$

Here we have defined

$$f_x(z,\omega_{bd}) = \frac{2}{z^2\omega_{bd}^2}\cos(2z\omega_{bd}) + \frac{4z^2\omega_{bd}^2 - 1}{z^3\omega_{bd}^3}\sin(2z\omega_{bd}) \tag{29}$$

Adding up the contributions of vacuum fluctuations and radiation reaction, we obtain the rate of change of the atomic excitation energy induced by the presence of the boundary.

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{tot}}^{xx} = \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,vf}^{xx} + \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,rr}^{xx} = \frac{e^2}{16\pi} \sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 f_x(z,\omega_{bd}). \tag{30}$$

Equation (30) only gives the correction to the spontaneous excitation rate caused by the presence of boundary and it is an oscillating function of z, the distance of the atom from the boundary. In order to find the total rate, we need to add the Minkowski vacuum contribution, which can be obtained by setting acceleration, a, to zero in the corresponding result given in Ref. [25], and the above boundary-dependent correction term. The result is

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\mathrm{tot}}^{xx} = -\frac{e^2}{3\pi} \sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_x(0)|d\rangle|^2 \times \left(1 - \frac{3}{16} f_x(z,\omega_{bd})\right). \tag{31}$$

Obviously, with merely a substitution of r_y for r_x and $f_y(z,\omega_{bd}) = f_x(z,\omega_{bd})$ for $f_x(z,\omega_{bd})$, the above result also applies for the yy component contributions, that is,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\mathrm{tot}}^{yy} = -\frac{e^2}{3\pi} \sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_y(0)|d\rangle|^2 \times \left(1 - \frac{3}{16} f_y(z,\omega_{bd})\right). \tag{32}$$

Similarly, one has for the zz component case that

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{vf}}^{zz} = \frac{e^2}{32\pi}\left(\sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 f_z(z,\omega_{bd}) - \sum_{\omega_b<\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 f_z(z,\omega_{bd}) \right) \tag{33}$$

for the contribution of vacuum fluctuations to the rate of change of the atomic excitation energy, and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{rr}}^{zz} = \frac{e^2}{32\pi}\left(\sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 f_z(z,\omega_{bd}) + \sum_{\omega_b<\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 f_z(z,\omega_{bd}) \right) \tag{34}$$

for that of radiation reaction, where function $f_z(z,\omega_{bd})$ is given by

$$f_z(z,\omega_{bd}) = \frac{4}{z^2\omega_{bd}^2}\cos(2z\omega_{bd}) - \frac{2}{z^3\omega_{bd}^3}\sin(2z\omega_{bd}). \tag{35}$$

It then follows that

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{b,\mathrm{tot}}^{zz} = \frac{e^2}{16\pi} \sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 f_z(z,\omega_{bd}), \tag{36}$$

and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\mathrm{tot}}^{zz} = -\frac{e^2}{3\pi} \sum_{\omega_b>\omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2 \times \left(1 - \frac{3}{16} f_z(z,\omega_{bd})\right). \tag{37}$$

After having presented all the results of our calculations, a few comments are now in order. First, although the presence of the conducting boundary modifies both the vacuum fluctuations and radiation reaction [refer, for example, to Eqs. (27) and (28)], the effects of both contributions to the spontaneous excitation rate, however, cancel exactly for an atom in the ground state ($\omega_b < \omega_d$) [refer to Eqs. (30), (32), and (36)]. Therefore, the presence of a plane boundary conspires to modify the vacuum fluctuations and radiation reaction in such a way that the delicate balance between the vacuum fluctuations and radiation reaction found in Ref. [25] in the absence of boundaries remains and this ensures the stability of ground-state inertial atoms in vacuum with a conducting boundary. Second, if the atom is polarized in the parallel direction, then, as the atom is placed closer and closer to the boundary ($z \to 0$), the rate of change of the atomic energy vanishes since $f_x(z, \omega_{bd})$ and $f_y(z, \omega_{bd})$ approach zero. This can be understood as a result of the fact that the tangential components of the electric field vanish on the conducting plane. However, if the polarization of the atom is along the normal direction, then $f(z, \omega_{bd}) \approx 2$ when ($z \to 0$), and one has

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{zz} \approx -\frac{2e^2}{3\pi} \sum_{\omega_b > \omega_d} \omega_{bd}^4 |\langle b|r_z(0)|d\rangle|^2. \tag{38}$$

This is two times that of the unbounded case and it can be attributed to the fact that the reflection at the boundary doubles the normal component of the fluctuating electric field. Third, for an atom which polarized in an arbitrary direction, we need to add all contributions together [Eqs. (31), (32), and (37)] and the result is

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} = -\frac{e^2}{3\pi} \sum_{\omega_b > \omega_d} \omega_{bd}^4 |\langle b|\mathbf{r}(0)|d\rangle|^2 - \frac{e^2}{3\pi} \sum_{\omega_b > \omega_d} \frac{3}{16} \omega_{bd}^4 |\langle b|r_i(0)|d\rangle|^2 f_i(z, \omega_{bd}). \tag{39}$$

Clearly the second term involving functions $f_i(z, \omega_{bd})$ give the modifications induced by the presence of the boundary to the rate of change of the mean atomic energy and they are oscillating functions of z with a modulated amplitude. Since f_x and f_y are different from f_z, the polarization of the atom in the direction parallel to the boundary and that in the normal direction are weighted differently in terms of their contributions to the spontaneous emission rate of the atom. Fourth, when the distance of the atom from the boundary approaches infinity ($z \to \infty$), all the oscillating functions approach zero, and we recover the results of the unbounded case. Fifth, let us note that the oscillating behavior of the spontaneous emission rate of the hydrogen atom as a function of z may manifest itself in the intensity of the emission spectrum and therefore might be verified in experiment. Take a typical transition frequency of a hydrogen atom, $\omega_{bd} \sim 10^{15}\ \mathrm{s}^{-1}$, for example, the amplitude of the oscillating functions will show appreciable deviations from 1 when $z \sim \frac{c}{\omega_{bd}} \sim 10^{-5}$ cm, which is orders of magnitude larger than radius of the hydrogen atom. Finally, the readers should be warned that our results are based upon a particular model for the hydrogen atom in which the multipolar coupling between the atom and the electromagnetic fields is assumed.

IV. UNIFORMLY ACCELERATED ATOM

A. Basic results

We now turn to the case in which the atom is uniformly accelerated in a direction parallel to the conducting plane boundary. We assume that the atom is at a distance z from the boundary and is being accelerated in the x-direction with a proper acceleration a. Specifically, the atom's trajectory is described by

$$t(\tau) = \frac{1}{a}\sinh a\tau, \qquad x(\tau) = \frac{1}{a}\cosh a\tau, \qquad y(\tau) = y_0, \qquad z(\tau) = z. \tag{40}$$

Let us introduce a unit vector pointing along the direction of acceleration, $k^\mu = (0, 1, 0, 0)$, then the electric field two-point function for the trajectory (40) can be evaluated from its general form (17) in the frame of the atom with a substitution $u = \tau - \tau'$ as follows

$$\langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle_b = -\frac{a^4}{16\pi^2} \frac{1}{[\sinh^2 \frac{a}{2}(u - i\varepsilon) - a^2z^2]^3} \bigg\{[\delta_{ij} - 2n_in_j + 2az(n_ik_j + k_in_j)]\sinh^2 \frac{au}{2} + a^2z^2\bigg[\delta_{ij}\cosh^2\frac{au}{2} + (\delta_{ij} - 2k_ik_j)\sinh^2\frac{au}{2}\bigg]\bigg\}. \tag{41}$$

From Eq. (41), we obtain the Hadamard functions of the field

$$C_{ij}^F(x(\tau), x(\tau')) = -\frac{a^4}{32\pi^2}\bigg(\frac{1}{[\sinh^2 \frac{a}{2}(u - i\varepsilon) - a^2z^2]^3} + \frac{1}{[\sinh^2 \frac{a}{2}(u + i\varepsilon) - a^2z^2]^3}\bigg) \times \bigg\{[\delta_{ij} - 2n_in_j + 2az(n_ik_j + k_in_j)]\sinh^2\frac{au}{2} + a^2z^2\bigg[\delta_{ij}\cosh^2\frac{au}{2} + (\delta_{ij} - 2k_ik_j)\sinh^2\frac{au}{2}\bigg]\bigg\}, \tag{42}$$

and the Pauli-Jordan or Schwinger functions

$$\chi^F_{ij}(x(\tau), x(\tau')) = -\frac{ia}{16\pi z}\frac{\delta''(\sinh\frac{au}{2} - az) - \delta''(\sinh\frac{au}{2} + az)}{\sinh^2(au) - \cosh(au)\sinh^2\frac{au}{2} + a^2z^2\cosh(au)}\bigg\{[\delta_{ij} - 2n_in_j + 2az(n_ik_j + k_in_j)]\sinh^2\frac{au}{2} + a^2z^2\bigg[\delta_{ij}\cosh^2\frac{au}{2} + (\delta_{ij} - 2k_ik_j)\sinh^2\frac{au}{2}\bigg]\bigg\}. \tag{43}$$

From the above expressions one can see that the only nonzero components of the statistical functions are xx, yy, zz, and xz components. For an accelerating arbitrarily polarized atom, we need to perform calculations for all nonzero statistical functions in order to obtain the boundary-dependent contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy.

Let us now take the xx component, for example, to show how the calculations are to be carried out. It follows from Eqs. (42) and (43) that

$$C^F_{xx}(x(\tau), x(\tau')) = -\frac{a^4}{32\pi^2}\bigg(\frac{\sinh^2\frac{au}{2} + a^2z^2}{[\sinh^2\frac{a}{2}(u - i\varepsilon) - a^2z^2]^3} + \frac{\sinh^2\frac{au}{2} + a^2z^2}{[\sinh^2\frac{a}{2}(u + i\varepsilon) - a^2z^2]^3}\bigg) \tag{44}$$

and

$$\chi^F_{xx}(x(\tau), x(\tau')) = -\frac{ia}{16\pi z}\frac{\sinh^2\frac{au}{2} + a^2z^2}{\sinh^2(au) - \cosh(au)\sinh^2\frac{au}{2} + a^2z^2\cosh(au)}\bigg[\delta''\bigg(\sinh\frac{au}{2} - az\bigg) - \delta''\bigg(\sinh\frac{au}{2} + az\bigg)\bigg]. \tag{45}$$

The contributions of vacuum fluctuations (5) and radiation reaction (6) to the rate of change of the mean atomic energy associated with the above statistical functions can be written as

$$\bigg\langle\frac{dH_A(\tau)}{d\tau}\bigg\rangle^{xx}_{b,\mathrm{vf}} = \frac{e^2a^4}{32\pi^2}\sum_d|\langle b|r_x(0)|d\rangle|^2\omega_{bd}\int_{-\infty}^{\infty}du\bigg(\frac{\sinh^2\frac{au}{2} + a^2z^2}{[\sinh^2\frac{a}{2}(u - i\varepsilon) - a^2z^2]^3} + \frac{\sinh^2\frac{au}{2} + a^2z^2}{[\sinh^2\frac{a}{2}(u + i\varepsilon) - a^2z^2]^3}\bigg)e^{i\omega_{bd}u} \tag{46}$$

and

$$\bigg\langle\frac{dH_A(\tau)}{d\tau}\bigg\rangle^{xx}_{b,\mathrm{rr}} = \frac{iae^2}{16\pi z}\sum_d|\langle b|r_x(0)|d\rangle|^2\omega_{bd}\int_{-\infty}^{\infty}du\frac{\sinh^2\frac{au}{2} + a^2z^2}{\sinh^2(au) - \cosh(au)\sinh^2\frac{au}{2} + a^2z^2\cosh(au)} \times\bigg[\delta''\bigg(\sinh\frac{au}{2} - az\bigg) - \delta''\bigg(\sinh\frac{au}{2} + az\bigg)\bigg]e^{i\omega_{bd}u}. \tag{47}$$

Here we have, as usual, extended the range of integration to infinity for sufficiently long times $\tau - \tau_0$. The integral in Eq. (46) can be evaluated via the residue theorem to get

$$\bigg\langle\frac{dH_A(\tau)}{d\tau}\bigg\rangle^{xx}_{b,\mathrm{vf}} = \frac{e^2}{32\pi}\bigg[\sum_{\omega_b>\omega_d}\omega^4_{bd}|\langle b|r_x(0)|d\rangle|^2f_{xx}(\omega_{bd}, z, a)\bigg(1 + \frac{2}{e^{(2\pi\omega_{bd}/a)} - 1}\bigg) - \sum_{\omega_b<\omega_d}\omega^4_{bd}|\langle b|r_x(0)|d\rangle|^2f_{xx}(\omega_{bd}, z, a)\bigg(1 + \frac{2}{e^{(2\pi|\omega_{bd}|/a)} - 1}\bigg)\bigg], \tag{48}$$

where

$$f_{xx}(\omega_{bd}, z, a) = \frac{2(1 + 4a^2z^2)}{z^2\omega^2_{bd}(1 + a^2z^2)^2}\cos\bigg(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\bigg) + \frac{4z^2\omega^2_{bd} - 1 - 2z^2a^2(1 + 2z^2a^2 - 2z^2\omega^2_{bd})}{z^3\omega^3_{bd}(1 + a^2z^2)^{5/2}} \times\sin\bigg(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\bigg). \tag{49}$$

A comparison of Eq. (48) with that of the unbounded Minkowski space [25] shows that the boundary-dependent contribution is in fact a "nonthermal" correction proportional to the oscillating function $f_{xx}(\omega_{bd}, z, a)$. With the help of the following equations

$$\begin{aligned}\delta\left(\sinh\frac{au}{2}-az\right)&=\frac{2}{a\cdot\cosh[\frac{1}{2}(\frac{au}{2}+\sinh^{-1}(az))]}\delta\left(u-\frac{2}{a}\sinh^{-1}(az)\right)\\ \delta\left(\sinh\frac{au}{2}+az\right)&=\frac{2}{a\cdot\cosh[\frac{1}{2}(\frac{au}{2}-\sinh^{-1}(az))]}\delta\left(u+\frac{2}{a}\sinh^{-1}(az)\right),\end{aligned}\tag{50}$$

we can calculate the contribution of the radiation reaction to the rate of change of the atomic energy to get

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{b,\mathrm{rr}}^{xx}=\frac{e^2}{32\pi}\left(\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2 f_{xx}(\omega_{bd},z,a)+\sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2 f_{xx}(\omega_{bd},z,a)\right).\tag{51}$$

Adding up the two contributions, (48) and (51), we can get the total correction induced by the presence of the boundary

$$\begin{aligned}\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{b,\mathrm{tot}}^{xx}=\frac{e^2}{16\pi}\Bigg[&\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2 f_{xx}(\omega_{bd},z,a)\left(1+\frac{1}{e^{(2\pi\omega_{bd}/a)}-1}\right)\\&-\sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2 f_{xx}(\omega_{bd},z,a)\frac{1}{e^{(2\pi|\omega_{bd}|/a)}-1}\Bigg].\end{aligned}\tag{52}$$

The total rate of change of the atomic energy in the presence of a conducting plane boundary can be obtained by further adding up the Minkowski vacuum contribution given in Ref. [25]

$$\begin{aligned}\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\mathrm{tot}}^{xx}=-\frac{e^2}{3\pi}\Bigg[&\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2\left(1+\frac{a^2}{\omega_{bd}^2}-\frac{3}{16}f_{xx}(\omega_{bd},z,a)\right)\left(1+\frac{1}{e^{(2\pi\omega_{bd}/a)}-1}\right)\\&-\sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_x(0)|d\rangle|^2\left(1+\frac{a^2}{\omega_{bd}^2}-\frac{3}{16}f_{xx}(\omega_{bd},z,a)\right)\frac{1}{e^{(2\pi|\omega_{bd}|/a)}-1}\Bigg].\end{aligned}\tag{53}$$

Similarly, with the help of residue theorem and Eq. (50), one can calculate the contributions related to other nonzero components of the statistical functions and the results can be summarized as follows. For a uniformly accelerated arbitrarily polarized atom near a conducting plane, the total boundary-dependent contribution of vacuum fluctuations to the rate of change of the mean atomic energy is given by

$$\begin{aligned}\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{b,\mathrm{vf}}=\frac{e^2}{32\pi}\Bigg[&\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd},z,a)\left(1+\frac{2}{e^{(2\pi\omega_{bd}/a)}-1}\right)\\&-\sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd},z,a)\left(1+\frac{2}{e^{(2\pi|\omega_{bd}|/a)}-1}\right)\Bigg],\end{aligned}\tag{54}$$

while for that of the radiation reaction, the result is

$$\begin{aligned}\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{b,\mathrm{rr}}=-\frac{e^2}{32\pi}\Bigg(&\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd},z,a)\\&-\sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd},z,a)\Bigg),\end{aligned}\tag{55}$$

where summation over repeated indices, i, j, is implied and $f_{xx}(\omega_{bd},z,a)$ is given by Eq. (49) while other nonzero functions by

$$f_{yy}(\omega_{bd},z,a)=\frac{2(1+2a^2z^2)}{z^2\omega_{bd}^2(1+a^2z^2)}\cos\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right)+\frac{4z^2\omega_{bd}^2-1+4a^2z^4\omega_{bd}^2}{z^3\omega_{bd}^3(1+a^2z^2)^{3/2}}\sin\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right),\tag{56}$$

$$f_{zz}(\omega_{bd}, z, a) = \frac{2(2 + a^2z^2 + 2a^4z^4)}{z^2\omega_{bd}^2(1 + a^2z^2)^2}\cos\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right) + \frac{-2 + a^2z^2(-5 + 4z^2\omega_{bd}^2) + 4a^4z^6\omega_{bd}^2}{z^3\omega_{bd}^3(1 + a^2z^2)^{5/2}} \times \sin\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right), \tag{57}$$

and

$$f_{xz}(\omega_{bd}, z, a) = \frac{2a(-1 + 2a^2z^2)}{z\omega_{bd}^2(1 + a^2z^2)^2}\cos\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right) + \frac{4az^2\omega_{bd}^2 + a + 4a^3z^2(1 + z^2\omega_{bd}^2)}{z^2\omega_{bd}^3(1 + a^2z^2)^{5/2}}\sin\left(\frac{2\omega_{bd}\sinh^{-1}(az)}{a}\right). \tag{58}$$

As z, the distance of the atom from the boundary, approaches infinity, all these functions approach zero and the boundary-dependent contributions vanish as expected. We can add the two contributions together to get the total contributions, vacuum fluctuations plus radiation reaction, to the rate of change of the atomic energy induced by the presence of the conducting plane

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{b,\mathrm{tot}} = \frac{e^2}{32\pi}\left[\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd}, z, a)\left(1 + \frac{1}{e^{(2\pi\omega_{bd}/a)} - 1}\right) - \sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|f_{ij}(\omega_{bd}, z, a)\frac{1}{e^{(2\pi|\omega_{bd}|/a)} - 1}\right]. \tag{59}$$

It follows immediately that the total rate of change of the atomic energy with the Minkowski vacuum term [25] included is

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\mathrm{tot}} = -\frac{e^2}{3\pi}\left[\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|\left(\left(1 + \frac{a^2}{\omega_{bd}^2}\right)\delta_{ij} - \frac{3}{16}f_{ij}(\omega_{bd}, z, a)\right)\left(1 + \frac{1}{e^{(2\pi\omega_{bd}/a)} - 1}\right) - \sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_i(0)|d\rangle||\langle d|r_j(0)|b\rangle|\left(\left(1 + \frac{a^2}{\omega_{bd}^2}\right)\delta_{ij} - \frac{3}{16}f_{ij}(\omega_{bd}, z, a)\right)\frac{1}{e^{(2\pi|\omega_{bd}|/a)} - 1}\right]. \tag{60}$$

B. Comments and discussions

A few comments and discussions are now in order for results obtained in the preceding subsection. It is interesting to note that Eq. (60) reveals that for an accelerated atom in the ground state ($\omega_b < \omega_d$), the effects of both contributions do not exactly cancel as in the case of an inertial atom, so the delicate balance between the vacuum fluctuations and radiation reaction no longer exists if the atom is accelerated, although both contributions of the vacuum fluctuations and radiation are altered for accelerated atoms in the presence of the boundary as contrasted with the case without boundaries. There is a positive contribution from the $\omega_b < \omega_d$ term, therefore transitions of ground-state accelerated atoms to excited states are allowed to occur in vacuum with boundaries. The presence of the boundary modulates the transition rate with the functions, $f_{ij}(\omega_{bd}, a, z)$ and makes the rate a function of z, the atom distance from the boundary. It is interesting to note that the boundary-induced contribution is effectively a nonthermal correction, thus depending the atom's distance from the boundary, the nonthermal correction (the term proportional to a^2) which is already present in the unbounded case may get enhanced or weakened by the presence of the boundary. This nonthermal effect which appears even when the boundary is absent may become appreciable for observation when the acceleration is of order necessary to observe the Unruh effect in atomic systems [29]. With the presence of the boundary, the nonthermal effect is expected to be enhanced for atoms on certain trajectories and thus more likely to be observed. For a given atom with a certain polarization, a typical transition frequency and a certain acceleration a, one can find a value of z where the nonthermal correction induced by the presence of the boundary is comparable with that already present without boundaries. For example, if the atom is polarized in the z-direction, then for a typical transition frequency of a hydrogen atom $\omega_{bd} \sim 10^{15}\ \mathrm{s}^{-1}$, and an acceleration $a \sim 10^{25}\ \mathrm{cm/s}^2$, typical acceleration for the Unruh effect to be observable in atomic systems, one can show that this value of z is $z \sim 10^{-5}$ cm.

At the same time, it is interesting to note that, for an accelerated atom which is only polarized in the x or y or z direction, there exists a certain value of z for every pair of a and ω_{bd}, such that $\frac{a^2}{\omega_{bd}^2} - \frac{3}{16}f_{ii}(\omega_{bd}, z, a) = 0$; that is, for atoms accelerated on the trajectory characterized by this value of z, the nonthermal corrections vanish.

Let us now note that as the acceleration, a, approaches zero, one has

$$f_{xx}(\omega_{bd}, z, a) = f_x(z, \omega_{bd}) + \left(\frac{13 - 4z^2\omega_{bd}^2}{3\omega_{bd}^2}\cos(2z\omega_{bd}) + \frac{3 - 32z^2\omega_{bd}^2}{6z\omega_{bd}^3}\sin(2z\omega_{bd})\right)a^2 + O[a]^4, \tag{61}$$

$$f_{yy}(\omega_{bd}, z, a) = f_y(z, \omega_{bd}) + \left(\frac{7 - 4z^2\omega_{bd}^2}{3\omega_{bd}^2}\cos(2z\omega_{bd}) + \frac{9 - 32z^2\omega_{bd}^2}{6z\omega_{bd}^3}\sin(2z\omega_{bd})\right)a^2 + O[a]^4, \tag{62}$$

$$f_{zz}(\omega_{bd}, z, a) = f_z(z, \omega_{bd}) + \left(\frac{16z}{3\omega_{bd}}\sin(2z\omega_{bd}) - \frac{16}{3\omega_{bd}^2}\cos(2z\omega_{bd})\right)a^2 + O[a]^4, \tag{63}$$

and

$$f_{xz}(\omega_{bd}, z, a) = \left(\frac{4z^2\omega_{bd}^2 - 1}{z^2\omega_{bd}^3}\sin(2z\omega_{bd}) - \frac{2}{z\omega_{bd}^2}\cos(2z\omega_{bd})\right)a + O[a]^3. \tag{64}$$

This shows that the rate of change of the mean atomic energy will be that for an inertial atom found in the preceding section plus an acceleration-dependent correction, and if the acceleration equals zero, we recover the result of Sec. III.

We now examine what happens as the atom is placed closer and closer to the boundary ($z \to 0$). In this case, one finds for any finite acceleration a that

$$1 + \frac{a^2}{\omega_{bd}^2} - \frac{3}{16}f_{xx}(\omega_{bd}, z, a) = \left(4a^2 + \frac{16a^4}{5\omega_{bd}^2} + \frac{4\omega_{bd}^2}{5}\right)z^2 + O[z]^4, \tag{65}$$

$$1 + \frac{a^2}{\omega_{bd}^2} - f_{yy}(\omega_{bd}, z, a) = \left(2a^2 + \frac{6a^4}{5\omega_{bd}^2} + \frac{4\omega_{bd}^2}{5}\right)z^2 + O[z]^4, \tag{66}$$

$$1 + \frac{a^2}{\omega_{bd}^2} - f_{zz}(\omega_{bd}, z, a) = 2\left(1 + \frac{a^2}{\omega_{bd}^2}\right) - \left(2a^2 + \frac{18a^4}{5\omega_{bd}^2} + \frac{2\omega_{bd}^2}{5}\right)z^2 + O[z]^4, \tag{67}$$

and

$$f_{xz}(\omega_{bd}, z, a) = \left(\frac{32a}{3} + \frac{32a^3}{\omega_{bd}^3}\right)z + O[z]^3. \tag{68}$$

Therefore, if the atom is polarized in a direction parallel to the conducting plane, then the spontaneous excitation rate of the atom diminishes to zero quadratically in z [refer to Eqs. (65) and (66)] as the boundary is approached ($z \to 0$). Recall the result in the preceding section, we see that the fact that the total excitation rate vanishes on the boundary is independent of whether the atom is accelerated or in uniform motion and this can be understood as a result of the fact that the tangential components of the electric field vanish on the conducting plane. However, two parallel directions, the x-direction (along the acceleration) and the y-direction (perpendicular to the acceleration), are now not equivalent as in the inertial case, since f_{xx} and f_{yy} are not equal. On the other hand, if the atom's polarization is perpendicular to the conducting plane, then as z, the distance of the atom from the boundary, approaches zero, we obtain

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot}^{zz} = -\frac{2e^2}{3\pi}\left[\sum_{\omega_b>\omega_d}\omega_{bd}^4|\langle b|r_z(0)|d\rangle|^2\left(\frac{a^2}{\omega_{bd}^2}+1\right)\left(1+\frac{1}{e^{(2\pi\omega_{bd}/a)}-1}\right) - \sum_{\omega_b<\omega_d}\omega_{bd}^4|\langle b|r_z(0)|d\rangle|^2\left(\frac{a^2}{\omega_{bd}^2}+1\right)\frac{1}{e^{(2\pi|\omega_{bd}|/a)}-1}\right], \tag{69}$$

which is just two times the corresponding result in an unbounded Minkowski space [25]. This enhancement can be attributed to the fact that the reflection at the boundary doubles the normal component of the fluctuating electric field. The above analysis tells us that even if the atom is isotropically polarized, each of three equal polarizations will be weighted differently in terms of its contribution to the rate of change of the mean atomic energy.

Finally, another interesting feature to be noted is that if the polarization of the atom is in the $x - z$ plane, the rate of change of the atomic energy gets an extra contribution associated with f_{xz} as compared with the inertial case.

This extra contribution vanishes when a goes to zero and we recover the result of the inertial case as expected. Also, as z, the distance of the atom to the boundary, approaches zero or infinity, the contribution diminishes to zero too.

V. CONCLUSIONS

In conclusion, assuming a multipolar coupling between a multilevel atom and a quantum electromagnetic field, we have studied the spontaneous emission and absorption of both an inertial and a uniformly accelerated atom near a conducting plane in vacuum and separately calculated the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy.

In the case of an inertial atom, our results show that both the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy are modified by the presence of the boundary, but the balance between them remains for ground-state atoms and this ensures the atom's stability in its ground state. The spontaneous emission rate of the atom in this case is an oscillating function of the atom's distance from the boundary and this oscillating behavior may offer a possibility for experimental test.

If the atom moves with constant proper acceleration, the perfect balance between the contributions of vacuum fluctuations and radiation reaction that ensures the stability of ground-state atoms is disturbed, making spontaneous transition of ground-state atoms to excited states possible in a vacuum with a conducting boundary. The presence of the boundary modulates the spontaneous absorption rate with functions dependent on the acceleration and the atom's distance from the boundary. The boundary-induced contribution is effectively a nonthermal correction, which enhances or weakens the nonthermal effect already present in the unbounded case, thus possibly making the effect easier to observe. The appearance of nonthermal correction terms suggest that the effect of electromagnetic vacuum fluctuations is not totally equivalent to that of a thermal field, as is the case for a scalar field in the unbounded space [7]. However, it is interesting to note that for atoms on some particular trajectories, the nonthermal correction induced by the presence of the boundary and that already present in the unbounded case may cancel. The calculations performed in this paper also tell us that each of three polarizations of the atom is weighted differently in terms of its contribution to the rate of change of the mean atomic energy even if the atom is isotropically polarized, as a result of the anisotropy of the configuration due to the presence of the boundary and the atom's acceleration.

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