Leading log solution for inflationary Yukawa theory

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We generalize Starobinskiĭ's stochastic technique to the theory of a massless, minimally coupled scalar interacting with a massless fermion in a locally de Sitter geometry. The scalar is an ''active'' field that can engender infrared logarithms. The fermion is a passive field that cannot cause infrared logarithms but which can carry them, and which can also induce new interactions between the active fields. The procedure for dealing with passive fields is to integrate them out, then stochastically simplify the resulting effective action following Starobinskiĭ. Because Yukawa theory is quadratic in the fermion this can be done explicitly using the classic solution of Candelas and Raine. We check the resulting stochastic formulation against an explicit two loop computation. We also derive a nonperturbative, leading log result for the stress tensor. Because the scalar effective potential induced by fermions is unbounded below, backreaction from this model might dynamically cancel an arbitrarily large cosmological constant.

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I. INTRODUCTION

Massless, minimally coupled scalars and gravitons are unique in achieving zero mass without classical conformal invariance. This means that inflation rips their virtual quanta out of the vacuum, which greatly strengthens the quantum loop effects they mediate [1]. In the expectation values of familiar operators these enhanced quantum effects typically manifest as *infrared logarithms*. A simple example is provided by the stress tensor of a massless, minimally coupled scalar with a quartic self-interaction,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{\lambda}{4!}\varphi^4\sqrt{-g} + \text{counter terms.} \quad (1)$$

When the expectation value of the stress tensor of this theory is computed in de Sitter background,

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} \quad \text{with} \quad a(t) = e^{Ht}, \quad (2)$$

and renormalized so as to make quantum effects vanish at $t = 0$, the results for the quantum-induced energy density and pressure are [2,3],

$$\rho(t) = \frac{\lambda H^4}{(2\pi)^4}\left\{\frac{1}{8}\ln^2(a)\right\} + O(\lambda^2), \quad (3)$$

$$p(t) = \frac{\lambda H^4}{(2\pi)^4}\left\{-\frac{1}{8}\ln^2(a) - \frac{1}{12}\ln(a)\right\} + O(\lambda^2). \quad (4)$$

Infrared logarithms are the factors of $\ln(a) = Ht$. They arise from the fact that inflationary particle production drives the free scalar field strength away from zero [4–6]

$$\langle\Omega|\varphi^2(x)|\Omega\rangle_0 = \frac{H^2}{4\pi^2}\ln(a) + \text{divergent constant.} \quad (5)$$

This increases the vacuum energy contributed by the quartic potential and the result is evident in (3) and (4).

Infrared logarithms arise in the one particle irreducible (1PI) functions of this theory [7]. They occur as well in massless, minimally coupled scalar quantum electrodynamics (SQED) [8–11] and in massless Yukawa theory [12,13]. The 1PI functions of pure gravity fields on de Sitter background show infrared logarithms [14–16]. A recent all orders analysis of scalar-driven inflation was unable to exclude the possibility that they might even contaminate loop corrections to the power spectra of cosmological perturbations [17]. And infrared logarithms have been discovered in the 1PI functions of gravity + Dirac fields [18,19].

Infrared logarithms are fascinating because they introduce a secular element into the usual, static expansion in the loop counting parameter. No matter how small the coupling constant λ is in (3) and (4), the continued growth of the inflationary scale factor must eventually overwhelm it. When this happens perturbation theory typically breaks down. For example, the general form of the induced energy density (3) is

$$\rho(t) = H^4\sum_{\ell=2}^{\infty}\lambda^{\ell-1}\{C_0^\ell\ln^{2\ell-2}(a) + C_1^\ell\ln^{2\ell-3}(a) + \ldots + C_{2\ell-4}^\ell\ln^2(a)\}. \quad (6)$$

The $\lambda^{\ell-1}C_0^\ell\ln^{2\ell-2}(a)$ terms are the *leading logarithms* at ℓ loop order; the remaining terms are *subdominant logarithms*. Assuming that the numerical coefficients C_k^ℓ are of order one, we see that the leading infrared logarithms all become order one at $\ln(a) \sim 1/\sqrt{\lambda}$. At this time the highest subdominant logarithm terms are still perturbatively small ($\sim\sqrt{\lambda}$), so it seems reasonable to attempt to follow the nonperturbative evolution by resumming the series of leading infrared logarithms,

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$$\rho_{\rm lead} = H^4 \sum_{\ell=2}^{\infty} C_0^{\ell} (\lambda \ln^2(a))^{\ell-1}. \tag{7}$$

This is known as the *leading logarithm approximation*.

Starobinskiĭ has long maintained that his stochastic field equations reproduce the leading logarithm approximation [20]. With Yokoyama he exploited this conjecture to explicitly solve for the nonperturbative, late time limit of any model of the form [21],

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}, \tag{8}$$

assuming only that the potential $V(\varphi)$ is bounded below. When the potential is unbounded below the conjecture still gives the leading infrared logarithms at each order, however, the theory fails to approach a static limit.

Starobinskiĭ's conjecture has recently been proved to all orders [22,23]. The field equations are first rewritten in Yang-Feldman form [24], then the free field mode expansion is truncated at horizon crossing, and the free field mode functions are replaced with their leading long wave length forms. This procedure converts the original quantum field into a commuting random variable, but it preserves the leading infrared logarithms. Although it was not evident at the time, the reason the field can be infrared truncated is that every pair of fields in a simple potential model of the form (8) is capable of inducing an infrared logarithm, and the leading log term derives from requiring them to do so. Because only the infrared part of the field is responsible for infrared logarithms, we can truncate and take the long wave length limit of the mode functions, $u(t, k) \to H/\sqrt{2k^3}$.[1] For example, the infrared logarithm in (5) is

$$\langle \Omega | \varphi^2(x) | \Omega \rangle_0 = \int \frac{d^3k}{(2\pi)^3} \| u(t,k) \|^2 \to \int_H^{Ha} \frac{dk}{k} \frac{H^2}{4\pi^2} = \frac{H^2}{4\pi^2} \ln(a). \tag{9}$$

A field which can generate infrared logarithms is called *active*. Scalar potential models of the form (8) possess only active fields. However, more general theories can possess fields which are not themselves capable of engendering an infrared logarithm. We call these *passive* fields. A example of such a model is SQED,

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} - (\partial_\mu - ieA_\mu)\varphi^*(\partial_\nu + ieA_\nu)\varphi g^{\mu\nu}\sqrt{-g}. \tag{10}$$

In this model the charged scalar is active whereas the photon is passive.

[1] It is also necessary to take the first nonzero term in the long wavelength limit for the retarded Green's function of the Yang-Feldman equation. This turns out to require a higher order term in $u(t, k)$.

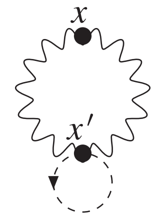

FIG. 1. Two loop contribution to $\langle \Omega | F_{\mu\nu}(x) F_{\rho\sigma}(x) | \Omega \rangle$.

Although passive fields cannot cause infrared logarithms, they can propagate their effects. That is, an expectation value of passive fields can acquire an infrared logarithm from a loop correction involving an active field. For example, the diagram in Fig. 1 gives a contribution to $\langle \Omega | F_{\mu\nu}(x) F_{\rho\sigma}(x) | \Omega \rangle$ which acquires an infrared logarithm through the scalar loop at the bottom.

Passive fields can also induce interactions between active fields. For example, the photon loop in Fig. 2 induces an effective $(\varphi^*\varphi)^2$ interaction in SQED.

SQED—and also gravity fields—feature another complication in which derivatives of active fields can induce interactions between undifferentiated active fields. For example, the 3-point interaction of SQED,

$$ie(\varphi^*\partial_\mu\varphi - \partial_\mu\varphi^*\varphi)A_\nu g^{\mu\nu}\sqrt{-g}, \tag{11}$$

can induce an effective $\varphi^*\varphi$ coupling through the diagram of Fig. 3, this is part of the full 1PI 2-point function which has recently been computed at one loop order [25].

In generalizing Starobinskiĭ's technique to theories which include passive fields, and/or differentiated active fields, it is crucial to realize that *the ultraviolet parts of passive fields and differentiated active fields contribute on an equal footing with the infrared parts* in propagating infrared logarithms and in mediating interactions between undifferentiated active fields. So one cannot infrared truncate the passive fields, or even differentiated active fields. Instead the correct procedure is:

(1) Integrate out the passive fields and renormalize the resulting effective action.
(2) Integrate out the differentiated active fields and renormalize the resulting effective action. Note that this can always be done because the original action is at most quadratic in derivatives.
(3) Infrared truncate and stochastically simplify the effective action of active fields.

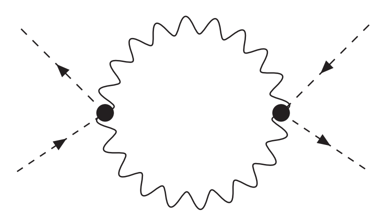

FIG. 2. Effective $(\varphi^*\varphi)^2$ coupling in SQED.

One might suspect that the third step is not possible owing to the nonlocality of the effective action. However, this nonlocality must be mediated by differential operators which, *precisely because they derive from passive fields*, must contain positive powers of the scale factor whose rapid time dependence weights the integral overwhelmingly at its upper limit and totally dominates the logarithms which might derive from the active fields. A typical example is the integral,

$$\int_0^t dt' a'^2 \ln(a') = \int_0^t dt' e^{2Ht'} Ht' = \frac{a'^2}{2H}\Big\{\ln(a') - \frac{1}{2}\Big\}\Big|_0^t \simeq \frac{1}{2H} a^2 \ln(a). \tag{12}$$

For $\ln(a) \gg 1$ it is as though we simply divide the integrand by $2H$ and evaluate it at the upper limit. Hence the hopelessly complicated "effective action" degenerates, in the leading log approximation, to a very tractable "effective potential," and the resulting local theory assumes the form (8) already solved by Starobinskiĭ [20,21].

Yukawa theory is especially simple because it possesses no differentiated active fields, and because it is free of the subtle gauge fixing problems of SQED [25] and gravity fields [18,26]. In Sec. II we review the full apparatus of perturbation theory for massless Yukawa theory on a locally de Sitter background. In Sec. III we integrate out the fermion and renormalize the effective potential. A curious and possibly significant property of Yukawa theory is that its effective potential is unbounded below, a fact that survives in the flat space limit and has even earned a place in standard model parameter estimation. We check the stochastic formalism against an explicit two loop vacuum expectation value in Sec. IV. In Sec. V we employ the stochastic formalism to obtain the leading log approximation for the Yukawa theory stress tensor. Because the effective potential depends upon the inflationary Hubble constant, the induced vacuum energy of the stress tensor does not quite agree with it. In fact the latter is initially positive whereas the former is always negative. However, their asymptotic large field behaviors are identical, so there seems no avoiding the conclusion that gravitational back-reaction in this model must eventually halt inflation, although not in an acceptable fashion. Our conclusions comprise Sec. VI. An appendix presents the less interesting, technical analysis behind the perturbative calculation described in Sec. IV.

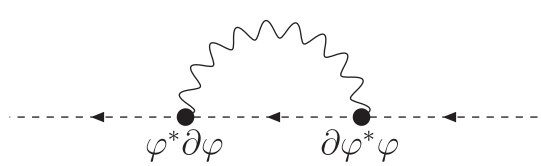

FIG. 3. Effective $\varphi^*\varphi$ coupling in SQED.

II. MASSLESS YUKAWA THEORY IN DE SITTER

The coupling of gravity fields to particles with half integer spin is usually accomplished by shifting the fundamental gravitational field variable from the metric $g_{\mu\nu}(x)$ to the vierbein $e_{\mu m}(x)$, although there are other approaches [27]. Greek letters stand for coordinate indices, Latin letters denote Lorentz indices, and both sorts take values in the set $\{0, 1, 2, \ldots, (D-1)\}$. One recovers the metric by contracting two vierbeins into the Lorentz metric η^{bc},

$$g_{\mu\nu}(x) = e_{\mu b}(x) e_{\nu c}(x) \eta^{bc}. \tag{13}$$

The coordinate index is raised and lowered with the metric ($e^\mu{}_b = g^{\mu\nu} e_{\nu b}$), while the Lorentz index is raised and lowered with the Lorentz metric ($e_\mu{}^b = \eta^{bc} e_{\mu c}$). We employ the usual metric-compatible and vierbein-compatible connections,

$$g_{\rho\sigma;\mu} = 0 \Rightarrow \Gamma^\rho{}_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma}(g_{\sigma\mu,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}), \tag{14}$$

$$e_{\beta b;\mu} = 0 \Rightarrow A_{\mu cd} = e^\nu{}_c (e_{\nu d,\mu} - \Gamma^\rho{}_{\mu\nu} e_{\rho d}). \tag{15}$$

Fermions also require gamma matrices, γ^b_{ij}. We assume their spinor indices run over $i = 1, \ldots, 4$ in any dimension. The anticommutation relations are

$$\{\gamma^b, \gamma^c\} \equiv (\gamma^b\gamma^c + \gamma^c\gamma^b) = -2\eta^{bc} I. \tag{16}$$

The Dirac Lorentz representation matrices are

$$J^{bc} \equiv \frac{i}{4}[\gamma^b, \gamma^c] = \frac{i}{4}(\gamma^b\gamma^c - \gamma^c\gamma^b). \tag{17}$$

They can be combined with the spin connection (15) to form the Dirac field covariant derivative operator,

$$\mathcal{D}_\mu \equiv \partial_\mu + \frac{i}{2} A_{\mu cd} J^{cd}. \tag{18}$$

In a general vierbein background the bare Lagrangian of massless, minimally coupled, Yukawa theory scalars with massless fermions is

$$\mathcal{L} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\xi_0\varphi^2 R\sqrt{-g} - \frac{1}{4!}\lambda_0\varphi^4\sqrt{-g} + i\bar{\psi} e^\beta{}_b \gamma^b \mathcal{D}_\beta \psi\sqrt{-g} - f_0\varphi\bar{\psi}\psi\sqrt{-g}. \tag{19}$$

The bare fields in this expression are $\varphi(x)$, $\bar{\psi}_i(x)$, and $\psi_i(x)$. The symbols ξ_0, λ_0, and f_0 stand for, respectively, the bare conformal coupling, the bare 4-point coupling constant and the bare Yukawa theory coupling constant. Neither the scalar nor the fermion requires a mass term because we desire the special model with zero renormalized masses, and no mass counterterms are required because mass is multiplicatively renormalized in dimensional regularization.

Renormalization is begun by expressing the bare fields in terms of the renormalized ones,

$$\varphi \equiv \sqrt{Z}\varphi_{\rm r}, \qquad \bar{\psi} \equiv \sqrt{Z_2}\bar{\psi}_{\rm r} \quad \text{and} \quad \psi \equiv \sqrt{Z_2}\psi_{\rm r}. \quad (20)$$

Substituting (20) into the bare Lagrangian (19) gives

$$\mathcal{L} = -\frac{1}{2}Z\partial_\alpha\varphi_{\rm r}\partial_\beta\varphi_{\rm r}g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}Z\xi_0\varphi_{\rm r}^2R\sqrt{-g} - \frac{1}{4!}Z^2\lambda_0\varphi_{\rm r}^4\sqrt{-g} + iZ_2\bar{\psi}_{\rm r}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi_{\rm r}\sqrt{-g} - \sqrt{Z}Z_2f_0\varphi_{\rm r}\bar{\psi}_{\rm r}\psi_{\rm r}\sqrt{-g}. \quad (21)$$

We next enforce the conditions that the renormalized scalar should have neither a conformal coupling nor a 4-point coupling,

$$Z\xi_0 \equiv 0 + \delta\xi, \qquad Z^2\lambda_0 \equiv 0 + \delta\lambda \quad \text{and} \quad \sqrt{Z}Z_2f_0 = f + \delta f. \quad (22)$$

Of course the model *does* require conformal and 4-point counterterms. We also define the field strengths as usual,

$$Z \equiv 1 + \delta Z \quad \text{and} \quad Z_2 \equiv 1 + \delta Z_2. \quad (23)$$

The structure of renormalized perturbation theory is complete when we express the Lagrangian in terms of primitive interactions and counterterms,

$$\mathcal{L} = -\frac{1}{2}\partial_\alpha\varphi_{\rm r}\partial_\beta\varphi_{\rm r}g^{\alpha\beta}\sqrt{-g} + i\bar{\psi}_{\rm r}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi_{\rm r}\sqrt{-g} - f\varphi_{\rm r}\bar{\psi}_{\rm r}\psi_{\rm r}\sqrt{-g} - \frac{\delta Z}{2}\partial_\alpha\varphi_{\rm r}\partial_\beta\varphi_{\rm r}g^{\alpha\beta}\sqrt{-g} + i\delta Z_2\bar{\psi}_{\rm r}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi_{\rm r}\sqrt{-g} - \frac{\delta\xi}{2}\varphi_{\rm r}^2R\sqrt{-g} - \frac{\delta\lambda}{4!}\varphi_{\rm r}^4\sqrt{-g} - \delta f\varphi_{\rm r}\bar{\psi}_{\rm r}\psi_{\rm r}\sqrt{-g}. \quad (24)$$

The preceding analysis has so far been for a general geometry. Of course we are interested in the special case of inflation, for which de Sitter is an excellent paradigm. We work on the open submanifold of D dimensional de Sitter space in the conformal coordinate system for which the invariant element is

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}\cdot d\vec{x}) \quad \text{and} \quad a(\eta) = -\frac{1}{H\eta}. \quad (25)$$

Of course this makes the metric $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$. We also employ the ghost-free, Lorentz symmetric gauge [28] which determines the vierbein,

$$e_{\mu b} = e_{b\mu} \Rightarrow e_{\mu b} = a\eta_{\mu b}. \quad (26)$$

At this stage there is no more point in distinguishing Lorentz indices from coordinate ones. With these conventions the covariant derivative operator takes the simple form,

$$\mathcal{D}_\mu \to \partial_\mu + \tfrac{1}{4}Ha[\gamma^0, \gamma_\mu]. \quad (27)$$

The special case of its contraction into $e^\mu{}_b\gamma^b$ is even simpler,

$$\gamma^b e^\mu{}_b\mathcal{D}_\mu \equiv \not{\mathcal{D}} \to a^{-((D+1)/2)}\not{\partial}a^{((D-1)/2)} \equiv a^{-((D+1)/2)}\gamma^\mu\partial_\mu a^{((D-1)/2)}. \quad (28)$$

The scalar and fermion propagators can be largely expressed in terms of the following function of the invariant length $\ell(x;x')$ between x^μ and x'^μ,

$$y(x;x') \equiv 4\sin^2(\tfrac{1}{2}H\ell(x;x')) = aa'H^2\Delta x^2(x;x'), \quad (29)$$

$$= aa'H^2(\|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'| - i\delta)^2). \quad (30)$$

The most singular term for each case involves the propagator for a massless, conformally coupled scalar,

$$i\Delta_{\rm cf}(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)\Big(\frac{4}{y}\Big)^{(D/2)-1}. \quad (31)$$

It has long been known that there is no de Sitter invariant solution for the propagator of a massless, minimally coupled scalar [29]. If one elects to break de Sitter invariance while preserving homogeneity and isotropy—this is known as the "E(3)" vacuum [30]—the minimal solution is [2,3]

$$i\Delta(x;x') = i\Delta_{\rm cf}(x;x') + \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{\frac{D}{D-4}\frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)}\Big(\frac{4}{y}\Big)^{(D/2)-2} - \pi\cot\Big(\frac{\pi}{2}D\Big) + \ln(aa')\Big\} + \frac{H^{D-2}}{(4\pi)^{D/2}}\sum_{n=1}^\infty\Big\{\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-(D/2)+2}\Big\}. \quad (32)$$

This expression may seem daunting but it is actually simple to use because the infinite sum vanishes in $D = 4$, and the terms of this sum go like higher and higher powers of $y(x;x')$. Hence the infinite sum can only contribute when multiplied by a divergent term, and even then only the first few terms can contribute.

It is useful, in the stochastic analysis to follow, for us to consider the fermion propagator in the presence of an arbitrary, potentially spacetime dependent, scalar field $\varphi(x)$

$$iS[f\varphi](x;x') \equiv \Big\langle x\Big|\frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f\varphi)}\Big|x'\Big\rangle. \quad (33)$$

Of course this can only be evaluated for a handful of field configurations. The case of a constant, $f\varphi(x) = m$, was solved by Candelas and Raine [31],

$$iS[m](x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)\Big(ai\not{\partial}\frac{1}{\sqrt{aa'}} + \sqrt{\frac{a}{a'}}mI\Big)\Big\{\frac{\Gamma(\frac{D}{2}-1+i\frac{m}{H})\Gamma(\frac{D}{2}-i\frac{m}{H})}{\Gamma(\frac{D}{2}-1)\Gamma(\frac{D}{2})}\,{}_2F_1\Big(\frac{D}{2}-1+i\frac{m}{H},\frac{D}{2}-i\frac{m}{H};\frac{D}{2}; 1-\frac{y}{4}\Big)\Big(\frac{I-\gamma^0}{2}\Big) + \frac{\Gamma(\frac{D}{2}-1-i\frac{m}{H})\Gamma(\frac{D}{2}+i\frac{m}{H})}{\Gamma(\frac{D}{2}-1)\Gamma(\frac{D}{2})}\,{}_2F_1\Big(\frac{D}{2}-1-i\frac{m}{H},\frac{D}{2}+i\frac{m}{H};\frac{D}{2};1-\frac{y}{4}\Big)\Big(\frac{I+\gamma^0}{2}\Big)\Big\}. \tag{34}$$

The propagator we use in perturbative calculations is the massless limit of this one,

$$i[S](x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)ai\not{\mathcal{D}}\frac{1}{\sqrt{aa'}}\,{}_2F_1\Big(\frac{D}{2}-1,\frac{D}{2};\frac{D}{2};1-\frac{y}{4}\Big), \tag{35}$$

$$= ai\not{\mathcal{D}}\frac{1}{\sqrt{aa'}}i\Delta_{\rm cf}(x;x'), \tag{36}$$

$$= (aa')^{-((D-1)/2)} \times i\not{\partial} \times \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{D/2}}\frac{1}{\Delta x^{D-2}}. \tag{37}$$

The final expression is just a conformal rescaling of the propagator for a massless fermion in flat space, as it should be in conformal coordinates.

It is useful to recast the Candelas-Raine solution (34) using the transformation formula of hypergeometric functions (see expression 9.131.2 in [32]),

$$iS[m](x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)\Big(ai\not{\mathcal{D}}\frac{1}{\sqrt{aa'}} + \sqrt{\frac{a}{a'}}mI\Big)\Big\{\Big[\frac{\Gamma(1-\frac{D}{2})\Gamma(\frac{D}{2}-1+i\frac{m}{H})\Gamma(\frac{D}{2}-i\frac{m}{H})}{\Gamma(\frac{D}{2}-1)\Gamma(i\frac{m}{H})\Gamma(1-i\frac{m}{H})}\,{}_2F_1\Big(\frac{D}{2}-1+i\frac{m}{H},\frac{D}{2}-i\frac{m}{H};\frac{D}{2};\frac{y}{4}\Big) + \Big(\frac{y}{4}\Big)^{1-(D/2)}{}_2F_1\Big(1-i\frac{m}{H},i\frac{m}{H};2-\frac{D}{2};\frac{y}{4}\Big)\Big]\Big(\frac{I-\gamma^0}{2}\Big) + \Big[\frac{\Gamma(1-\frac{D}{2})\Gamma(\frac{D}{2}-1-i\frac{m}{H})\Gamma(\frac{D}{2}+i\frac{m}{H})}{\Gamma(\frac{D}{2}-1)\Gamma(-i\frac{m}{H})\Gamma(1+i\frac{m}{H})} \times {}_2F_1\Big(\frac{D}{2}-1-i\frac{m}{H},\frac{D}{2}+i\frac{m}{H};\frac{D}{2};\frac{y}{4}\Big) + \Big(\frac{y}{4}\Big)^{1-(D/2)}{}_2F_1\Big(1+i\frac{m}{H},-i\frac{m}{H};2-\frac{D}{2};\frac{y}{4}\Big)\Big]\Big(\frac{I+\gamma^0}{2}\Big)\Big\}. \tag{38}$$

In dimensional regularization all D-dependent powers of $y(x;x')$ vanish at coincidence. Hence we obtain,[2]

$$\lim_{x'\to x} iS[m](x;x') = \frac{mH^{D-2}}{(4\pi)^{D/2}} \times \frac{\Gamma(\frac{D}{2}+i\frac{m}{H})\Gamma(\frac{D}{2}-i\frac{m}{H})\Gamma(1-\frac{D}{2})}{\Gamma(1+i\frac{m}{H})\Gamma(1-i\frac{m}{H})} \times I, \tag{39}$$

$$\lim_{x'\to x} \mathcal{D}_\mu iS[m](x;x') = \frac{mH^{D-2}}{(4\pi)^{D/2}} \times \frac{\Gamma(\frac{D}{2}+i\frac{m}{H})\Gamma(\frac{D}{2}-i\frac{m}{H})\Gamma(1-\frac{D}{2})}{\Gamma(1+i\frac{m}{H})\Gamma(1-i\frac{m}{H})} \times \frac{i}{D}ma\gamma_\mu, \tag{40}$$

$$\lim_{x'\to x} iS[m](x;x')\overleftarrow{\mathcal{D}}'_\mu = \frac{mH^{D-2}}{(4\pi)^{D/2}} \times \frac{\Gamma(\frac{D}{2}+i\frac{m}{H})\Gamma(\frac{D}{2}-i\frac{m}{H})\Gamma(1-\frac{D}{2})}{\Gamma(1+i\frac{m}{H})\Gamma(1-i\frac{m}{H})} \times -\frac{i}{D}ma\gamma_\mu. \tag{41}$$

III. STOCHASTIC EFFECTIVE ACTION

Integrating out the fermions gives rise to a scalar effective action,

$$e^{i\Gamma[\varphi]} = \int[d\bar{\psi}][d\psi]e^{iS[\varphi,\bar{\psi},\psi]} = e^{iS_{\rm s}[\varphi]}\det[\sqrt{-g}(i\not{\mathcal{D}} - f_0\varphi)]. \tag{42}$$

Here $S_{\rm s}[\varphi]$ stands for the action associated with the purely scalar part of the bare Lagrangian,

[2] Note that the back-acting covariant derivative is $\overleftarrow{\mathcal{D}}_\mu = \overleftarrow{\partial}_\mu - \frac{1}{4}Ha[\gamma^0,\gamma_\mu]$.

$$\mathcal{L}_s = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{\xi_0}{2}\varphi^2 R\sqrt{-g} - \frac{\lambda_0}{4!}\varphi^4\sqrt{-g}. \qquad (43)$$

The exact effective scalar field equation is

$$\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} = \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi(x)) - \xi_0\varphi(x)R\sqrt{-g} - \frac{\lambda_0}{6}\varphi^3(x)\sqrt{-g} - \mathrm{Tr}\Big[\frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f_0\varphi)}\frac{\delta}{\delta\varphi(x)} \times \sqrt{-g}(i\not{\mathcal{D}} - f_0\varphi)\Big], \qquad (44)$$

$$= \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi(x)) - \xi_0\varphi(x)R\sqrt{-g} - \frac{\lambda_0}{6}\varphi^3(x)\sqrt{-g} + f_0\,\mathrm{Tr}[i\mathcal{S}[f_0\varphi](x;x)]\sqrt{-g}. \qquad (45)$$

Were we trying to solve the full quantum field theory, Eq. (45) would be a dead end because we lack an explicit expression for the coincidence limit of the fermion propagator in the presence of a general $\varphi(x)$. However, we are focused instead on the leading infrared logarithms, and this fact permits a crucial simplification: *we can evaluate the fermion propagator as if the scalar were constant.* Making use of (39) we infer a local field equation which agrees exactly with the full theory in the leading log approximation,

$$\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} \rightarrow \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) - \xi_0\varphi R\sqrt{-g} - \frac{\lambda_0}{6}\varphi^3\sqrt{-g} + \frac{4f_0^2H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma(\frac{D}{2}+i\frac{f_0\varphi}{H})\Gamma(\frac{D}{2}-i\frac{f_0\varphi}{H})\Gamma(1-\frac{D}{2})}{\Gamma(1+i\frac{f_0\varphi}{H})\Gamma(1-i\frac{f_0\varphi}{H})}\varphi\sqrt{-g}. \qquad (46)$$

Of course the final term in (46) is also related to the effective potential (it is $-V'_{\rm eff}(\varphi)\sqrt{-g}$) and has appeared many times before in this guise [31,33,34].

The factor of $\Gamma(1-\frac{D}{2})$ in Eq. (46) is the only divergence we shall see in the stochastic formalism. It can be removed using the parameters of the scalar potential, ξ_0 and λ_0. In particular, the stochastic formalism does not require either field strength renormalization or renormalization of the Yukawa theory coupling,

$$\delta Z = \delta Z_2 = \delta f = 0 \Rightarrow \xi_0 = \delta\xi \quad \text{and} \quad \lambda_0 = \delta\lambda. \qquad (47)$$

To renormalize (46) we set $D = 4 - \epsilon$ and make use of the following expansions,

$$\Gamma\Big(1-\frac{D}{2}\Big) = -\frac{2}{\epsilon} + \gamma - 1 + O(\epsilon), \qquad (48)$$

$$\frac{\Gamma(\frac{D}{2}+i\frac{f\varphi}{H})\Gamma(\frac{D}{2}-i\frac{f\varphi}{H})}{\Gamma(1+i\frac{f\varphi}{H})\Gamma(1-i\frac{f\varphi}{H})} = 1 + \Big(\frac{f\varphi}{H}\Big)^2 - \epsilon - \Big[1+\Big(\frac{f\varphi}{H}\Big)^2\Big] \times\Big[\psi\Big(1+i\frac{f\varphi}{H}\Big) + \psi\Big(1-i\frac{f\varphi}{H}\Big)\Big]\frac{\epsilon}{2} + O(\epsilon^2). \qquad (49)$$

The symbol "$\psi(z)$" in this last expression of course stands for the psi function rather than the fermi field (see section 8.36 of [32]),

$$\psi(1+z) \equiv \frac{d}{dz}\ln(\Gamma(1+z)) = -\gamma + \sum_{n=2}^{\infty}(-1)^n\zeta(n)z^{n-1}. \qquad (50)$$

Note that (49) is both real and even in φ,

$$\frac{\Gamma(\frac{D}{2}+i\frac{f\varphi}{H})\Gamma(\frac{D}{2}-i\frac{f\varphi}{H})}{\Gamma(1+i\frac{f\varphi}{H})\Gamma(1-i\frac{f\varphi}{H})} = 1 + \Big(\frac{f\varphi}{H}\Big)^2 - (1-\gamma)\epsilon - [\zeta(3)-\gamma]\Big(\frac{f\varphi}{H}\Big)^2\epsilon - \sum_{n=2}^{\infty}(-1)^n[\zeta(2n-1) - \zeta(2n+1)]\Big(\frac{f\varphi}{H}\Big)^{2n}\epsilon + O(\epsilon^2). \qquad (51)$$

From these expansions it is apparent that we can renormalize so as to keep the scalar massless and free to order f^6,

$$\delta\xi = \frac{4f^2H^{D-4}}{(4\pi)^{D/2}}\frac{\Gamma(1-\frac{D}{2})}{D(D-1)} + \frac{f^2}{24\pi^2}(1-\gamma), \qquad (52)$$

$$\delta\lambda = \frac{24f^4H^{D-4}}{(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big) + \frac{3f^4}{\pi^2}[\zeta(3)-\gamma]. \qquad (53)$$

Substituting in (46) and taking the limit $D \rightarrow 4$ gives the following effective equation of motion,

$$\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} \rightarrow \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) + \frac{fH^3}{2\pi^2}\sum_{n=2}^{\infty}(-1)^n[\zeta(2n-1) - \zeta(2n+1)]\Big(\frac{f\varphi}{H}\Big)^{2n+1}\sqrt{-g}, \qquad (54)$$

$$= \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) - \frac{fH^3}{2\pi^2}\zeta(3)\Big(\frac{f\varphi}{H}\Big)^3\sqrt{-g} + \frac{fH^3}{2\pi^2}\Big[1+\Big(\frac{f\varphi}{H}\Big)^2\Big]\Big[\gamma + \frac{1}{2}\psi\Big(1+i\frac{f\varphi}{H}\Big) + \frac{1}{2}\psi\Big(1-i\frac{f\varphi}{H}\Big)\Big]\Big(\frac{f\varphi}{H}\Big)\sqrt{-g}. \qquad (55)$$

It is well to digress at this point to establish an important correspondence limit that bears upon the validity and physical interpretation of our renormalization condition (52). Duffy and Woodard computed the one loop scalar self-mass-squared in this theory and used it to solve the effective scalar field equation [35]. They found (Eqs. (38) and (77) in [35]) that the following choice for the conformal counterterm results in there being no significant late time corrections to the scalar mode functions at one loop order,

$$\delta\xi_{\rm DW} = \frac{f^2 H^{D-4}}{(4\pi)^{D/2}} \frac{(D-2)\Gamma(\frac{D}{2}-2)}{2(D-1)(D-3)} + \frac{f^2}{32\pi^2}, \tag{56}$$

$$= \frac{f^2 H^{D-4}}{(4\pi)^{D/2}}\Big(-\frac{2}{3\epsilon}\Big) - \frac{f^2}{16\pi^2}\Big(\frac{\gamma}{3}+\frac{1}{18}\Big) + O(\epsilon). \tag{57}$$

Up to irrelevant terms of order ϵ, this is precisely the same renormalization (52) as we have used. Therefore our stochastic renormalization conventions agree with the full theory in the regime of significant late time effects, just as they should.

Up to some different renormalization conventions, our expression for the scalar effective potential agrees with Eq. (30) of Candelas and Raine [31],

$$V(\varphi) = -\frac{H^4}{4\pi^2}\sum_{n=2}^{\infty}\frac{(-1)^n}{n+1}[\zeta(2n-1) - \zeta(2n+1)]\Big(\frac{f\varphi}{H}\Big)^{2n+2}, \tag{58}$$

$$= -\frac{H^4}{8\pi^2}\Big\{2\gamma\Big(\frac{f\varphi}{H}\Big)^2 - [\zeta(3)-\gamma]\Big(\frac{f\varphi}{H}\Big)^4 + 2\int_0^{f\varphi/H} dx(x+x^3)[\psi(1+ix)+\psi(1-ix)]\Big\}. \tag{59}$$

We have now reduced the theory to a completely finite, scalar model of the form already solved by Starobinskiĭ [20,21]. An interesting and possibly significant result of applying his technique is that *this model fails to approach a static limit at late times*. This is obvious once one recognizes that the potential $V(\varphi)$ is unbounded from below.

One might expect that $V(\varphi)$ is negative because a nonzero scalar (of either sign) drives the fermion mass positive [12,13], which must lower the vacuum energy. The absence of a lower bound is most easily proved by making use of the asymptotic expansion Stirling's formula implies for the psi function,

$$\psi(z) = \ln(z) - \frac{1}{2z} - \frac{1}{12z^2} + O\Big(\frac{1}{z^4}\Big), \tag{60}$$

to evaluate the strong field limit of (59),

$$V(\varphi) = -\frac{H^4}{8\pi^2}\Big\{\Big(\frac{f\varphi}{H}\Big)^4 \ln\Big(\frac{f|\varphi|}{H}\Big) - \Big[\zeta(3)+\frac{1}{4}-\gamma\Big]\Big(\frac{f\varphi}{H}\Big)^4 + 2\Big(\frac{f\varphi}{H}\Big)^2\ln\Big(\frac{f|\varphi|}{H}\Big) - \Big(\frac{5}{6}-2\gamma\Big)\Big(\frac{f\varphi}{H}\Big)^2 + O\Big(\ln\Big[\frac{f|\varphi|}{H}\Big]\Big)\Big\}. \tag{61}$$

Of course the large φ regime is also the small H regime, at which point we make contact with Eq. (6.10) in the classic paper by Coleman and Weinberg [36]. The negative potential has long been recognized to render pure Yukawa theory unstable in flat space [37]. A constraint on the Higgs mass can be derived in the standard model from the need to avoid this instability for the large Yukawa theory coupling of the top quark [38].

Although we have just seen that the instability is present in flat space, inflation does have a role to play. A flat space scalar would simply roll down an unbounded, negative potential. However, the Hubble friction of expansion retards the scalar's downward progress. The potential's curvature is slight, for small f, so the scalar's evolution is for a very long time driven by the pressure of inflationary particle production. Only when the scalar's magnitude approaches the nonperturbatively large scale of $\varphi \sim H/f$ does the unbounded potential begin to dominate the scalar's evolution. This is distinct from the point at which the potential comes to dominate cosmology. That occurs for $\varphi \sim H/f(GH^2)^{1/4}$, which is when the potential becomes comparable in magnitude to the bare vacuum energy of $3H^2/8\pi G$.

IV. REALITY CHECK

In this section we will test the stochastic formalism by comparing its prediction with an explicit two loop evaluation of the coincident vertex function (see Fig. 4),

$$\langle\Omega|T[\varphi_{\rm r}(x)\bar{\psi}_{\rm r}(x)\psi_{\rm r}(x)]|\Omega\rangle. \tag{62}$$

Because there is no field strength renormalization in the stochastic formalism, we can ignore the distinction between renormalized and unrenormalized fields in working out the stochastic prediction for (62). Integrating the fermions out exactly gives the trace of the field-dependent fermion propagator, which we can again evaluate for constant field configurations in the leading logarithm approximation,

$$e^{-i\Gamma[\varphi]}\int[d\bar{\psi}][d\psi]e^{iS[\varphi,\bar{\psi},\psi]} \times \varphi(x)\bar{\psi}(x)\psi(x) = \varphi(x)\,{\rm Tr}[-i\mathcal{S}[f\varphi](x;x)], \tag{63}$$

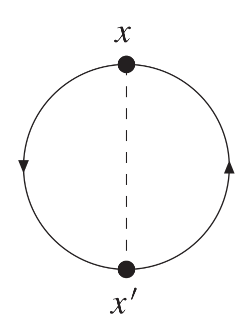

FIG. 4. Lowest order contribution to $\langle\Omega|T[\varphi_{\rm r}(x)\bar{\psi}_{\rm r}(x)\psi_{\rm r}(x)]|\Omega\rangle$.

$$\to -4f\varphi^2(x)\frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\Bigg\|\frac{\Gamma(\frac{D}{2}+i\frac{f\varphi}{H})}{\Gamma(1+i\frac{f\varphi}{H})}\Bigg\|^2, \quad (64)$$

$$= -\frac{4fH^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\Gamma^2\Big(\frac{D}{2}\Big)\varphi^2(x) + O(f^3). \quad (65)$$

Hence our prediction for the order f infrared logarithm is divergent,

$$\langle\Omega|T[\varphi_r(x)\bar{\psi}_r(x)\psi_r(x)]|\Omega\rangle \to +\frac{fH^4}{8\pi^4}\frac{\ln(a)}{\epsilon} + \text{finite} + O(f^3). \quad (66)$$

Note that the divergence arises from integrating out coincident fermion fields. It has nothing to do with the stochastic formalism *per se*, except for being the correct leading log result for the expectation value we have chosen to compute.

That was easy. A measure of the power of the stochastic formalism is that we could just as simply have obtained the leading log result at order f^3 or higher. We turn now to the much more difficult task of perturbatively computing the full order f result for comparison.[3] In models for which the "in" and "out" vacua differ, the in-out matrix elements computed with the usual Feynman rules are not true expectation values. To obtain an expectation value such as (62) one must employ the Schwinger-Keldysh formalism [39–45]. For a recent review of the position-space formalism see [46]. Here we simply summarize the modified Feynman rules:

(1) Each line has a polarity which can be either "+" or "−".
(2) Vertices, including counterterms, are either all + or all −.
(3) A + vertex is the familiar one of the in-out formalism, whereas a − vertex is its complex conjugate.
(4) External lines from time-ordered operators are +, whereas external lines from anti-time-ordered operators are −.
(5) Propagators can be ++, +−, −+, or −−. For our theory these are all obtained from the Feynman propagators (32) and (37) by replacing the conformal coordinate interval $\Delta x^2(x;x')$ with the appropriately polarized interval,

$$\Delta x^2_{++}(x;x') \equiv \|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'|-i\delta)^2, \quad (67)$$

$$\Delta x^2_{+-}(x;x') \equiv \|\vec{x}-\vec{x}'\|^2 - (\eta-\eta'+i\delta)^2, \quad (68)$$

$$\Delta x^2_{-+}(x;x') \equiv \|\vec{x}-\vec{x}'\|^2 - (\eta-\eta'-i\delta)^2, \quad (69)$$

$$\Delta x^2_{--}(x;x') \equiv \|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'|+i\delta)^2. \quad (70)$$

Because the operators in (62) are all time-ordered, the associated external lines have + polarity. At lowest order they can connect to either a + or a − vertex. Hence the Schwinger-Keldysh result for (62) is

$$\langle\Omega|T[\varphi_r(x)\bar{\psi}_r(x)\psi_r(x)]|\Omega\rangle = +if\int d^Dx' a'^D\left\{\begin{matrix} i\Delta_{++}(x;x')i[{}_iS_j]_{++}(x;x')i[{}_jS_i]_{++}(x';x) \\ -i\Delta_{+-}(x;x')i[{}_iS_j]_{+-}(x;x')i[{}_jS_i]_{-+}(x';x)\end{matrix}\right\} + O(f^3). \quad (71)$$

We will ignore the distinction between ++ and +− until it becomes significant. In view of (37) the spinor trace gives

$$i[{}_iS_j](x;x')i[{}_jS_i](x';x) = \frac{\Gamma^2(\frac{D}{2})}{4\pi^D}\frac{\gamma^\mu_{ij}\gamma^\nu_{ji}\Delta x_\mu\Delta x_\nu}{(aa')^{D-1}\Delta x^{2D}} = -\frac{\Gamma^2(\frac{D}{2})}{\pi^D}\frac{(aa')^{1-D}}{\Delta x^{2D-2}}. \quad (72)$$

And the lowest order contribution to the vacuum expectation value (VEV) takes the form

$$\langle\Omega|\varphi_r(x)\bar{\psi}_r(x)\psi_r(x)|\Omega\rangle = -if\frac{\Gamma^2(\frac{D}{2})}{\pi^D a^{D-1}}\times\int d^Dx' a'\Big\{\frac{i\Delta_{++}(x;x')}{\Delta x^{2D-2}_{++}} - \frac{i\Delta_{+-}(x;x')}{\Delta x^{2D-2}_{+-}}\Big\} + O(f^3). \quad (73)$$

Here is where the analysis becomes tedious. We have accordingly moved the technical details to the appendix, and will here quote only the divergent parts of the result. The superficially most singular contribution to (73) derives from the first line of the scalar propagator (32). We call it A,

$$A(x) \equiv -if\frac{\Gamma^2(\frac{D}{2})\Gamma(\frac{D}{2}-1)}{4\pi^{(3/2)D}}a^{2-(3D/2)}\times\int d^Dx'(a')^{2-(D/2)}\Big\{\frac{1}{\Delta x^{3D-4}_{++}} - \frac{1}{\Delta x^{3D-4}_{+-}}\Big\}, \quad (74)$$

$$= \text{finite}. \quad (75)$$

The next most singular contributions come from the second line of (32),

$$B_1(x) \equiv -ifH^{D-2}\frac{\Gamma(\frac{D}{2})\Gamma(D-1)}{2^D\pi^{(3/2)D}a^{D-1}}\frac{D}{D-4}\frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)}\times\Big(\frac{4}{H^2a}\Big)^{(D/2)-2}\int d^Dx'(a')^{3-(D/2)}\times\Big\{\frac{1}{\Delta x^{3D-6}_{++}} - \frac{1}{\Delta x^{3D-6}_{+-}}\Big\}, \quad (76)$$

[3]This computation was done in collaboration with P. M. Ho.

$$= \frac{fH^4\mu^{-2\epsilon}}{8\pi^{4-\epsilon}} \frac{(1-\frac{\epsilon^2}{16})(1-\frac{\epsilon^2}{4})\Gamma^2(2-\frac{\epsilon}{2})}{(1-\frac{3}{4}\epsilon)(1-\epsilon)(1-\frac{3}{2}\epsilon)} \left\{-\frac{1}{2\epsilon^2} - \frac{\ln(a)}{\epsilon}\right\} + \text{finite}, \tag{77}$$

$$B_2(x) \equiv -ifH^{D-2} \frac{\Gamma(\frac{D}{2})\Gamma(D-1)}{2^D\pi^{(3/2)D}a^{D-1}} \times -\pi\cot\left(\frac{\pi}{2}D\right) \times \int d^Dx'a' \left\{\frac{1}{\Delta x_{++}^{2D-2}} - \frac{1}{\Delta x_{+-}^{2D-2}}\right\}, \tag{78}$$

$$= \frac{fH^{4-\epsilon}\mu^{-\epsilon}}{8\pi^{4-\epsilon}} \frac{\pi\epsilon}{2}\cot\left(\frac{\pi\epsilon}{2}\right)\Gamma(1-\epsilon)\left\{\frac{1}{\epsilon^2} + \frac{\ln(a)}{\epsilon}\right\} + \text{finite}, \tag{79}$$

$$B_3(x) \equiv -ifH^{D-2} \frac{\Gamma(\frac{D}{2})\Gamma(D-1)}{2^D\pi^{(3/2)D}a^{D-1}} \int d^Dx'a'\ln(aa') \times \left\{\frac{1}{\Delta x_{++}^{2D-2}} - \frac{1}{\Delta x_{+-}^{2D-2}}\right\}, \tag{80}$$

$$= \frac{fH^{4-\epsilon}\mu^{-\epsilon}}{8\pi^{4-\epsilon}}\Gamma(1-\epsilon)\left\{\frac{3}{4\epsilon} + \frac{\ln(a)}{\epsilon}\right\} + \text{finite}. \tag{81}$$

Only the $n = 1$ term from the third line makes a nonzero contribution in the limit of $D = 4$,

$$C_1(x) \equiv -\frac{ifH^{D-2}\Gamma^2(\frac{D}{2})}{2^D\pi^{(3/2)D}a^{D-1}} \frac{\Gamma(\frac{D}{2}+2)}{D-6}\left(\frac{H^2a}{4}\right)^{3-(D/2)} \times \int d^Dx'(a')^{4-(D/2)}\left\{\frac{1}{\Delta x_{++}^{3D-8}} - \frac{1}{\Delta x_{+-}^{3D-8}}\right\}, \tag{82}$$

$$= \frac{fH^4\mu^{-2\epsilon}}{8\pi^{4-\epsilon}} \frac{(1-\frac{\epsilon}{6})(1-\frac{\epsilon}{4})(1-\frac{\epsilon}{2})\Gamma^2(2-\frac{\epsilon}{2})}{(1-\frac{3}{2}\epsilon)(1+\epsilon)}\left\{\frac{3}{8\epsilon}\right\} + \text{finite}, \tag{83}$$

$$C_2(x) \equiv -\frac{ifH^{D-2}\Gamma^2(\frac{D}{2})}{2^D\pi^{(3/2)D}a^{D-1}} \frac{\Gamma(D)}{\Gamma(\frac{D}{2}+1)} \frac{H^2a}{4} \times \int d^Dx'a'^2\left\{\frac{1}{\Delta x_{++}^{2D-4}} - \frac{1}{\Delta x_{+-}^{2D-4}}\right\}, \tag{84}$$

$$= \frac{fH^{4-\epsilon}\mu^{-\epsilon}}{2^{3-\epsilon}\pi^{4-\epsilon}} \frac{(1-\frac{\epsilon}{3})(1-\frac{\epsilon}{2})^2\Gamma(2-\epsilon)}{(1-\frac{\epsilon}{4})(1-\epsilon)}\left\{-\frac{3}{4\epsilon}\right\} + \text{finite}. \tag{85}$$

The $n = 2$ term goes like Δx^4, which is enough to make denominator integrable in $D = 4$. We can therefore take $D = 4$ for these terms, at which point one sees that they vanish. The same argument applies to all terms with $n \geq 2$.

The sum of (75), (77), (79), (81), (83), and (85) gives

$$(\text{divergent constant}) + \frac{fH^4}{8\pi^4}\frac{\ln(a)}{\epsilon} + \text{finite}, \tag{86}$$

in perfect agreement with the stochastic prediction (66). It is worth noting that the only the B-terms contribute divergent infrared logarithms, and those from B_1 and B_2 cancel. So the result seems to derive entirely from B_3, which itself originated from the explicit factor of $\ln(aa')$ on the second line of the scalar propagator (32).

Although it is not really necessary for our purpose of checking the stochastic formalism, we remark that the reason the coincident vertex diverges is that ordinary renormalization does not generally suffice for composite operators such as (62). To renormalize local composite operators one must allow them to mix with operators of the same or lower dimension. There are three local, dimension four operators that can mix with $\varphi_r(x)\bar{\psi}_r(x)\psi_r(x)$,

$$R\varphi_r^2, \qquad \partial_\mu\varphi_r\partial_\nu\varphi_r g^{\mu\nu} \quad \text{and} \quad \bar{\psi}_r e^\mu{}_b\gamma^b\mathcal{D}_\mu\psi_r. \tag{87}$$

The expectation value of the last term vanishes in dimensional regularization. The expectation value's of the first two are

$$\langle\Omega|T[R\varphi_r^2(x)]|\Omega\rangle = \frac{H^{4-\epsilon}}{2^{3-\epsilon}\pi^{2-(\epsilon/2)}}\frac{\Gamma(5-\epsilon)}{\Gamma(2-\frac{\epsilon}{2})} \times \left\{\frac{\pi\epsilon}{2}\cot\left(\frac{\pi\epsilon}{2}\right) + \ln(a)\right\}, \tag{88}$$

$$\langle\Omega|T^*[\partial_\mu\varphi_r(x)\partial_\nu\varphi_r(x)g^{\mu\nu}]|\Omega\rangle = \frac{H^{4-\epsilon}}{2^{5-\epsilon}\pi^{2-(\epsilon/2)}}\frac{(4-\epsilon)\Gamma(3-\epsilon)}{\Gamma(2-\frac{\epsilon}{2})}. \tag{89}$$

We can choose the coefficient of $R\varphi_r^2(x)$ to completely cancel the second term of (86). Then we can choose the coefficient of $\partial_\mu\varphi_r(x)\partial_\nu\varphi_r(x)g^{\mu\nu}$ to cancel whatever constant terms remain.

V. STOCHASTIC STRESS TENSOR

To understand how this model sources gravitational back-reaction we must study the Yukawa theory stress tensor,

$$T_{\mu\nu} \equiv -\frac{1}{\sqrt{-g}}e_{(\mu}{}^b\frac{\delta S}{\delta e^{\nu)b}}, \tag{90}$$

$$= -\frac{i}{2}[\bar{\psi}e_{(\mu b}\gamma^b\mathcal{D}_{\nu)}\psi - \bar{\psi}\overleftarrow{\mathcal{D}}_{(\mu}e_{\nu)b}\gamma^b\psi] + \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi + \delta\xi\left[\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\varphi^2 + g_{\mu\nu}(\varphi^2)^{;\rho}{}_\rho - (\varphi^2)_{;\mu\nu}\right] - \frac{\delta\lambda}{4!}\varphi^4 g_{\mu\nu}. \tag{91}$$

Integrating out the fermions converts the fermionic terms to a purely scalar expression that we can evaluate for constant field configurations using (40) and (41),

$$e^{-i\Gamma[\varphi]}\int[d\bar{\psi}][d\psi]e^{iS[\varphi,\bar{\psi},\psi]}\times\frac{i}{2a}[\bar{\psi}\gamma_{(\mu}\mathcal{D}_{\nu)}\psi-\bar{\psi}\overleftarrow{\mathcal{D}}_{(\mu}\gamma_{\nu)}\psi]$$
$$=-\frac{ia}{2}\lim_{x'\to x}\mathrm{Tr}[-\gamma_{(\mu}\mathcal{D}_{\nu}iS[f\varphi](x;x')+iS[f\varphi](x;x')\overleftarrow{\mathcal{D}}'_{(\mu}\gamma_{\nu)}], \tag{92}$$

$$\to\frac{4H^D}{(4\pi)^{D/2}}\frac{\Gamma(1-\frac{D}{2})}{D}\left\|\frac{\Gamma(\frac{D}{2}+i\frac{f\varphi}{H})}{\Gamma(1+i\frac{f\varphi}{H})}\right\|^2\left(\frac{f\varphi}{H}\right)^2 g_{\mu\nu}, \tag{93}$$

$$=\frac{H^D}{(4\pi)^{D/2}}\left\{\frac{4}{D}\Gamma\left(1-\frac{D}{2}\right)\left[\left(\frac{f\varphi}{H}\right)^2+\left(\frac{f\varphi}{H}\right)^4\right]+2\left(\frac{f\varphi}{H}\right)^2+\left[\left(\frac{f\varphi}{H}\right)^2+\left(\frac{f\varphi}{H}\right)^4\right]\left[\psi\left(1+i\frac{f\varphi}{H}\right)+\psi\left(1-i\frac{f\varphi}{H}\right)\right]+O(\epsilon)\right\}g_{\mu\nu}. \tag{94}$$

Because the differentiated fields in $T_{\mu\nu}$ cannot contribute leading order logarithms we see that the stress tensor takes the form

$$T_{\mu\nu}\to -V_{\rm s}(\varphi)g_{\mu\nu}, \tag{95}$$

where the potential is

$$V_{\rm s}(\varphi)=\frac{H^4}{8\pi^2}\left\{\left[\frac{1}{2}-\gamma\right]\left(\frac{f\varphi}{H}\right)^2+\left[\frac{1}{4}-\gamma+\zeta(3)\right]\left(\frac{f\varphi}{H}\right)^4-\frac{1}{2}\left[\left(\frac{f\varphi}{H}\right)^2+\left(\frac{f\varphi}{H}\right)^4\right]\left[\psi\left(1+i\frac{f\varphi}{H}\right)+\psi\left(1-i\frac{f\varphi}{H}\right)\right]\right\}, \tag{96}$$

$$=\frac{H^4}{8\pi^2}\left\{\frac{1}{2}\left(\frac{f\varphi}{H}\right)^2+\frac{1}{4}\left(\frac{f\varphi}{H}\right)^4-\sum_{n=2}^{\infty}(-1)^n[\zeta(2n-1)-\zeta(2n+1)]\left(\frac{f\varphi}{H}\right)^{2n+2}\right\}. \tag{97}$$

This is not the same potential $V(\varphi)$ we found in section III! Unlike that potential, $V_{\rm s}(\varphi)$ has positive curvature at $\varphi=0$. However, the leading asymptotic behavior for large φ is the same

$$V_{\rm s}(\varphi)=-\frac{H^4}{8\pi^2}\left\{\left(\frac{f\varphi}{H}\right)^4\ln\left(\frac{f|\varphi|}{H}\right)-\left[\zeta(3)+\frac{1}{4}-\gamma\right]\left(\frac{f\varphi}{H}\right)^4+\left(\frac{f\varphi}{H}\right)^2\ln\left(\frac{f|\varphi|}{H}\right)-\left(\frac{5}{12}-\gamma\right)\left(\frac{f\varphi}{H}\right)^2+O\left(\ln\left[\frac{f|\varphi|}{H}\right]\right)\right\}. \tag{98}$$

The evolution of φ is controlled by $V(\varphi)$. So the scalar rolls away from $\varphi=0$, even though this initially means *moving up* the potential $V_{\rm s}(\varphi)$.

What $V_{\rm s}(\varphi)$ gives is the expectation value of the operator which is the source of gravitational back-reaction. The two potentials disagree because the scalar effective potential $V(\varphi)$ involves the square of the Hubble parameter, which is really $H^2\to R/12$ for a general metric.[4] That has consequences for the way $V(\varphi)$ sources gravity fields. One can see this from the familiar case of a conformal coupling term in the Lagrangian, $-\frac{1}{2}\varphi^2R\sqrt{-g}$. In 4-dimensional de Sitter the Ricci scalar is $R=12H^2$, so the contribution to the scalar effective potential is

$$\Delta\mathcal{L}=-\tfrac{1}{2}\varphi^2R\sqrt{-g}\Rightarrow\Delta V(\varphi)=6H^2\varphi^2. \tag{99}$$

But the stress tensor takes account of the way the Ricci scalar depends upon the metric for a general geometry,

$$\Delta\mathcal{L}=-\tfrac{1}{2}\varphi^2R\sqrt{-g}\Rightarrow\Delta T_{\mu\nu}=(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R)\varphi^2+g_{\mu\nu}(\varphi^2)^{;\rho}{}_{\rho}-(\varphi^2)_{;\mu\nu}. \tag{100}$$

Differentiated scalars cannot contribute leading order infrared logarithms, and the de Sitter Einstein tensor is $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-3H^2g_{\mu\nu}$, so the induced potential in the stress tensor is

$$\Delta\mathcal{L}=-\tfrac{1}{2}\varphi^2R\sqrt{-g}\Rightarrow\Delta V_{\rm s}(\varphi)=3H^2\varphi^2. \tag{101}$$

VI. DISCUSSION

Any theory which includes either massless minimally coupled scalars or gravitons will show infrared logarithms in the expectations values of certain operators. These enhance loop effects by powers of $\ln[a(t)]$, where $a(t)\sim e^{Ht}$ is the inflationary scale factor. Of course loop effects are still down by powers of the (presumed small) loop counting parameter, but continued evolution must eventually bring about a situation in which the factors of $\ln[a(t)]$ overcome the small loop counting parameter and cause the breakdown of perturbation theory. Starobinskiĭ has long advocated gaining control over this nonperturbative regime by studying the series comprised of the leading infrared logarithm at each loop order [20]. In fact he has completely solved for the leading log limit of a massless, minimally coupled scalar with arbitrary potential [21].

In scalar potential models every field is "active." That is, it can potentially contribute to an infrared logarithm. Because the leading log result requires that *all* fields at a given order contribute infrared logarithms, one can perform an infrared truncation on the fields. This is why Starobinskiĭ's formalism ends up being so wonderfully simple. More general theories also possess "passive" fields which cannot themselves contribute to an infrared logarithm. However, these passive fields can propagate infrared logarithms obtained from interaction with active

[4]This nonlinear dependence upon the Ricci scalar does not imply the usual kinetic instability associated with higher derivative gravity fields [47].

fields. They can also mediate interactions between active fields. Differentiated active fields play much the same role.

In propagating infrared logarithms, and mediating interactions between active fields, the ultraviolet parts of the passive fields (and differentiated actives) contribute on an equal footing with the infrared. It is therefore invalid to infrared truncate either passive fields or differentiated active fields which appear in interactions. The correct procedure instead is to formally integrate out the passive fields, and the differentiated active fields, both from the action and from whatever operator is being studied. The expression which results is generally not local, but it contains only active fields. Because the nonlocality is confined to inverse differential operators of passive fields, which *cannot* cause infrared logarithms, the associated Green's functions are always dominated by positive powers of the scale factor whose explosive growth weights the result completely at the upper limit in the leading logarithm approximation. Hence the nonlocal effective action degenerates to a completely local and computable effective potential. At this point one has a local potential model of the form Starobinskiĭ has already solved *in toto* [20,21].

Massless Yukawa theory is a wonderfully simple testing ground for these ideas because it contains a passive field—the fermion—without any differentiated active fields.[5] It also lacks the subtle gauge fixing problems of SQED [25] and gravity fields [18,26]. In this paper we have exploited the classic solution of Candelas and Raine [31] to derive the Yukawa theory stochastic effective potential $V(\varphi)$ (59). We have checked the technique with an explicit two loop computation of the coincident vertex function. The result is in perfect agreement with the stochastic prediction.

We have also obtained a leading log result for the stress tensor as a function of the scalar. Although this stress tensor takes the form $-g_{\mu\nu}V_s(\varphi)$, our result for $V_s(\varphi)$ (96) it is not quite the same as the effective potential $V(\varphi)$ (59) which governs the scalar's evolution. The reason for this is that both potentials depend upon the dimensionless quantity, $(f\varphi/H)^2$, and the factors of $H^2 = R/12$ in this exert a nontrivial influence upon the way in which this model sources gravity fields. So one determines the scalar's evolution using $V(\varphi)$, and one finds its impact upon gravity fields from $V_s(\varphi)$. The two potentials differ, but they are each correct.

A curious and potentially significant feature of both potentials is that they are unbounded below. The physics behind this seems to be very solid: inflationary particle production drives the scalar away from zero, which induces a fermion mass. That increases the magnitude of the fermion 0-point energy, which makes for a negative effective potential because fermion vacuum energy is negative. We note that scalars seem always to induce growing mass [11–13,51], so we expect that the effective potential of SQED will be positive for large fields. By comparison, gravitons seem to induce a growing field strength renormalization [19]. It is intriguing to speculate on what that might mean for back-reaction in theories of gravity fields plus matter.

Because the scalar effective potential is unbounded below, this model should decay forever. However, Hubble friction will make the evolution dominated by inflationary particle production until the scalar reaches nonperturbatively large values. Although the initial effect is to raise the gravitating energy density, the large field results for $V(\varphi)$ and $V_s(\varphi)$ agree. It seems inevitable that pure Yukawa theory must be unstable against slow decay to a phase of deflation which culminates in a big rip [52]. In the standard model of flat space this same tendency is controlled by the positive effective potential from the gauge bosons. One naturally wonders what the result might be for inflation, and whether or not this might be parleyed into a model in which inflation might be gotten to end *without* endless decay.

[5]Yukawa theory is so much simpler than SQED that this paper was complete well before a very similar analysis of SQED which was begun at approximately the same time [48–50].

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APPENDIX: INTEGRALS FROM SEC. IV

In Sec. IV we reduced the perturbative expression for the two loop contribution to (62) to a sum of dimensionally regulated integrals (74), (76), (78), (80), (82), and (84). The next step is to partially integrate the inverse powers of Δx^2 until they become integrable in $D = 4$. There is no distinction between $++$ and $+-$ terms at this stage. The identities we need are

$$\frac{1}{\Delta x^{3D-8}} = \frac{\partial^2}{(3D-10)2(D-4)}\left(\frac{1}{\Delta x^{3D-10}}\right), \tag{A1}$$

$$\frac{1}{\Delta x^{3D-6}} = \frac{\partial^4}{(3D-8)(3D-10)4(D-3)(D-4)} \times\left(\frac{1}{\Delta x^{3D-10}}\right), \tag{A2}$$

$$\frac{1}{\Delta x^{3D-4}} = \frac{\partial^6}{(3D-6)(3D-8)(3D-10)8(D-2)(D-3)(D-4)}\left(\frac{1}{\Delta x^{3D-10}}\right), \tag{A3}$$

$$\frac{1}{\Delta x^{2D-4}} = \frac{\partial^2}{(2D-6)(D-4)}\left(\frac{1}{\Delta x^{2D-6}}\right), \tag{A4}$$

$$\frac{1}{\Delta x^{2D-2}} = \frac{\partial^4}{(2D-4)(2D-6)(D-2)(D-4)}\left(\frac{1}{\Delta x^{2D-6}}\right). \tag{A5}$$

Because we are integrating over x'^{μ}, derivatives with respect to x^{μ} can be taken outside the integral, leaving an integrand which is integrable in $D = 4$ dimensions. The limit $D = 4$ could be taken at this point except for the factors of $1/(D-4)$ which were picked up from the last partial integration. To segregate the divergence on a local term we add zero in the form,

$$\partial^2\left(\frac{1}{\Delta x^{D-2}_{++}}\right) - \frac{i4\pi^{D/2}}{\Gamma(\frac{D}{2}-1)}\delta^D(x-x') = 0 = \partial^2\left(\frac{1}{\Delta x^{D-2}_{+-}}\right). \tag{A6}$$

Once this has been added we can take the limit $D = 4$ in the nonlocal term.

We will work this out for the $+-$ term

$$\frac{\partial^2}{2D-8}\left\{\frac{1}{\Delta x^{3D-10}_{+-}}\right\} = \frac{\partial^2}{2D-8}\left\{\frac{1}{\Delta x^{3D-10}_{+-}} - \frac{\mu^{2D-8}}{\Delta x^{D-2}_{+-}}\right\}, \tag{A7}$$

$$= \frac{\partial^2}{2D-8}\left\{\frac{\mu^{3(D-4)}}{\Delta x^2_{+-}}[(\mu^2\Delta x^2_{+-})^{-(3/2)(D-4)} - (\mu^2\Delta x^2_{+-})^{-(1/2)(D-4)}]\right\}, \tag{A8}$$

$$= \mu^{3(D-4)}\partial^2 \times\left\{\frac{-\frac{1}{2}\ln(\mu^2\Delta x^2_{+-}) + \frac{(D-4)}{2}\ln^2(\mu^2\Delta x^2_{+-}) + O((D-4)^2)}{\Delta x^2_{+-}}\right\}. \tag{A9}$$

The other result $+-$ we need is

$$\frac{\partial^2}{D-4}\left\{\frac{1}{\Delta x^{2D-6}_{+-}}\right\} = \frac{\partial^2}{D-4}\left\{\frac{1}{\Delta x^{2D-6}_{+-}} - \frac{\mu^{D-4}}{\Delta x^{D-2}_{+-}}\right\}, \tag{A10}$$

$$= \mu^{2(D-4)}\partial^2\left\{\frac{-\frac{1}{2}\ln(\mu^2\Delta x^2_{+-}) + \frac{3(D-4)}{8}\ln^2(\mu^2\Delta x^2_{+-}) + O((D-4)^2)}{\Delta x^2_{+-}}\right\}. \tag{A11}$$

It is only on account of the explicit factors of $1/(D-4)$ in $B_1(x)$ and $\cot(\frac{\pi}{2}D)$ in $B_2(x)$ that we must keep the order $(D-4)$ terms in relations (A9) and (A11). The analogous $++$ relations are

$$\frac{\partial^2}{2D-8}\left\{\frac{1}{\Delta x^{3D-10}_{++}}\right\} = \frac{\mu^{2D-8}}{2D-8}\frac{i4\pi^{D/2}}{\Gamma(\frac{D}{2}-1)}\delta^D(x-x') + \mu^{3(D-4)}\partial^2\left\{\frac{-\frac{1}{2}\ln(\mu^2\Delta x^2_{++}) + \frac{(D-4)}{2}\ln^2(\mu^2\Delta x^2_{++}) + O((D-4)^2)}{\Delta x^2_{++}}\right\}. \tag{A12}$$

$$\frac{\partial^2}{D-4}\left\{\frac{1}{\Delta x^{2D-6}_{++}}\right\} = \frac{\mu^{D-4}}{D-4}\frac{i4\pi^{D/2}}{\Gamma(\frac{D}{2}-1)}\delta^D(x-x') + \mu^{2(D-4)}\partial^2\left\{\frac{-\frac{1}{2}\ln(\mu^2\Delta x^2_{++}) + \frac{3(D-4)}{8}\ln^2(\mu^2\Delta x^2_{++}) + O((D-4)^2)}{\Delta x^2_{++}}\right\}. \tag{A13}$$

The use of these partial integration identities results in each of the terms (74), (76), (78), (80), (82), and (84) having a finite, nonlocal part and a potentially divergent, local part. For $A(x)$ these are

$$A_N(x) = \frac{ifa^{-4}}{2^{10}3\pi^6}\partial^6\int d^4x'\left\{\frac{\ln(\mu^2\Delta x^2_{++})}{\Delta x^2_{++}} - \frac{\ln(\mu^2\Delta x^2_{+-})}{\Delta x^2_{+-}}\right\}, \tag{A14}$$

$$A_L(x) = \frac{f\Gamma^2(\frac{D}{2})}{\pi^D}\frac{\mu^{2D-8}a^{2-(3D/2)}\partial^4 a^{2-(D/2)}}{(3D-6)(3D-8)(3D-10)(2D-4)(2D-6)(2D-8)}. \tag{A15}$$

Before giving the nonlocal and local terms for the $B_i(x)$ and $C_i(x)$ we will evaluate the two above. The local term is simple because the derivatives in the numerator are so easy,

$$a^{2-(3D/2)}\partial^4 a^{2-(D/2)} = \Big(2-\frac{D}{2}\Big)\Big(3-\frac{D}{2}\Big)\Big(4-\frac{D}{2}\Big) \times \Big(5-\frac{D}{2}\Big)H^4 a^{8-2D}. \tag{A16}$$

The factor of $(2-\frac{D}{2})$ means that $A_L(x)$ is actually finite and we can set $D=4$,

$$A_L(x) \longrightarrow -\frac{fH^4}{2^8\pi^4}. \tag{A17}$$

To evaluate any of the nonlocal terms it is best to extract two more derivatives,

$$A_N(x) = \frac{ifa^{-4}}{2^{13}3\pi^6}\partial^8 \int d^4x' \times \left\{ \begin{array}{c} \ln^2(\mu^2\Delta x^2_{++}) - 2\ln(\mu^2\Delta x^2_{++}) \\ -\ln^2(\mu^2\Delta x^2_{+-}) + 2\ln(\mu^2\Delta x^2_{+-}) \end{array}\right\}. \tag{A18}$$

The point of doing this is that we can now exploit the exact cancellation between $++$ and $+-$ terms outside the past light-cone. (Note that we do not want to do this before the last derivative is extracted because the limits of integration must be constant for us to extract derivatives.) We define the temporal and spatial intervals in the obvious ways,

$$\Delta\eta \equiv \eta - \eta', \qquad \Delta r \equiv \|\vec{x} - \vec{x}'\|. \tag{A19}$$

It is then straightforward to show

$$\ln(\mu^2\Delta x^2_{++}) - \ln(\mu^2\Delta x^2_{+-}) = 2\pi i\theta(\Delta\eta - \Delta r), \tag{A20}$$

$$\ln^2(\mu^2\Delta x^2_{++}) - \ln^2(\mu^2\Delta x^2_{+-}) = 4\pi i\theta(\Delta\eta - \Delta r)\ln[\mu^2(\Delta\eta^2 - \Delta r^2)]. \tag{A21}$$

These relations bring the nonlocal term to the form

$$A_N(x) = \frac{-fa^{-4}}{2^{11}3\pi^5}\partial^8\int_{\eta_I}^{\eta} d\eta' \int d^3x'\theta(\Delta\eta - \Delta r) \times \{\ln[\mu^2(\Delta\eta^2 - \Delta r^2)] - 1\}, \tag{A22}$$

where the initial time is $\eta_I \equiv -1/H$. Note that $A_N(x)$ is now manifestly real.

The next step is to make the change of variables $\vec{r} = \vec{x}' - \vec{x}$ and perform the angular integrals

$$A_N(x) = \frac{-fa^{-4}}{2^9 3\pi^4}\partial_0^8\int_{\eta_I}^{\eta} d\eta' \int_0^{\Delta\eta} dr r^2\{\ln[\mu^2(\Delta\eta^2 - r^2)] - 1\}. \tag{A23}$$

We then make the change of variables $r = \Delta\eta z$ and perform the integration over z,

$$A_N(x) = \frac{-fa^{-4}}{2^9 3\pi^4}\partial_0^8\int_{\eta_I}^{\eta} d\eta'\Delta\eta^3\Big\{\frac{2}{3}\ln(2\mu\Delta\eta) - \frac{11}{9}\Big\}. \tag{A24}$$

Owing to the factor of $\Delta\eta^3$, three of the external derivatives can be brought inside the integral,

$$A_N(x) = \frac{-fa^{-4}}{2^9 3\pi^4}\partial_0^5\int_{\eta_I}^{\eta} d\eta' 4\ln(2\mu\Delta\eta). \tag{A25}$$

At this stage one performs the integral over η' and acts the derivatives,

$$A_N(x) = -\frac{fa^{-4}}{2^7 3\pi^4}\partial_0^5\{\Delta\eta_I[\ln(2\mu\Delta\eta_I) - 1]\} = \frac{fH^4}{2^6\pi^4}\frac{1}{(a-1)^4}. \tag{A26}$$

For the B-terms it is best to convert from D to $\epsilon = 4 - D$. All three of the B terms (76), (78), and (80), contain an overall factor of

$$-ifH^{D-2}\frac{\Gamma(\frac{D}{2})\Gamma(D-1)}{2^D\pi^{(3/2)D}a^{D-1}} = -ifH^{2-\epsilon}\frac{\Gamma(2-\frac{\epsilon}{2})\Gamma(3-\epsilon)}{2^{4-\epsilon}\pi^{6-(3/2)\epsilon}a^{3-\epsilon}}. \tag{A27}$$

The integrand of the B_1 term (76) is this overall factor times

$$\begin{aligned}\frac{D}{D-4}\frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)}\Big(\frac{1}{4}H^2aa'\Big)^{2-(D/2)}a'\Big\{\frac{1}{\Delta x^{3D-6}_{++}} - \frac{1}{\Delta x^{3D-6}_{+-}}\Big\} &= \Big(\frac{4-\epsilon}{-\epsilon}\Big)\frac{\Gamma^2(2-\frac{\epsilon}{2})}{\Gamma(3-\epsilon)}\frac{(\frac{1}{4}H^2aa')^{(1/2)\epsilon}a'\partial^2}{(4-3\epsilon)(2-3\epsilon)(2-2\epsilon)(-2\epsilon)} \\ &\quad\times\Big\{\mu^{-3\epsilon}\partial^2\Big[\frac{\epsilon\ln(\mu^2\Delta x^2_{++}) + \epsilon^2\ln^2(\mu^2\Delta x^2_{++}) + \ldots}{\Delta x^2_{++}}\Big] \\ &\quad - (++\to+-) + \frac{i4\pi^{2-(1/2)\epsilon}\mu^{-2\epsilon}}{\Gamma(1-\frac{\epsilon}{2})}\delta^D(x-x')\Big\}.\end{aligned} \tag{A28}$$

The associated local term is,

$$B_{1L}(x) = \frac{fH^2\mu^{-2\epsilon}}{32\pi^{4-\epsilon}}\frac{(1-\frac{\epsilon}{4})(1-\frac{\epsilon}{2})\Gamma^2(2-\frac{\epsilon}{2})}{(1-\frac{3}{4}\epsilon)(1-\epsilon)(1-\frac{3}{2}\epsilon)\epsilon^2} \times a^{-3+(3/2)\epsilon}\partial^2 a^{1+(1/2)\epsilon}, \tag{A29}$$

$$= -\frac{fH^4\mu^{-2\epsilon}}{16\pi^{4-\epsilon}}\frac{(1-\frac{\epsilon^2}{16})(1-\frac{\epsilon^2}{4})\Gamma^2(2-\frac{\epsilon}{2})}{(1-\frac{3}{4}\epsilon)(1-\epsilon)(1-\frac{3}{2}\epsilon)\epsilon^2}a^{2\epsilon}. \tag{A30}$$

The integrand of the B_2 term (78) is the factor (A27) times

$$-\pi\cot\Big(\frac{\pi}{2}D\Big)a'\Big\{\frac{1}{\Delta x_{++}^{2D-2}}-\frac{1}{\Delta x_{+-}^{2D-2}}\Big\}$$
$$=\pi\cot\Big(\frac{\pi\epsilon}{2}\Big)\frac{a'\partial^2}{(4-2\epsilon)(2-2\epsilon)(2-\epsilon)(-\epsilon)}\times\Big\{\mu^{-2\epsilon}\partial^2\Big[\frac{\frac{\epsilon}{2}\ln(\mu^2\Delta x_{++}^2)+\frac{3}{8}\epsilon^2\ln^2(\mu^2\Delta x_{++}^2)+\dots}{\Delta x_{++}^2}\Big]-(++\to+-)+\frac{i4\pi^{2-(1/2)\epsilon}\mu^{-\epsilon}}{\Gamma(1-\frac{\epsilon}{2})}\delta^D(x-x')\Big\}. \tag{A31}$$

The appropriate local term for B_2 is

$$B_{2L}(x)=\frac{-fH^{2-\epsilon}\mu^{-\epsilon}}{2^{6-\epsilon}\pi^{4-\epsilon}}\frac{\pi\cot(\frac{\pi\epsilon}{2})\Gamma(3-\epsilon)}{(1-\frac{\epsilon}{2})(1-\epsilon)\epsilon}a^{-3+\epsilon}\partial^2 a, \tag{A32}$$

$$=\frac{fH^{4-\epsilon}\mu^{-\epsilon}}{8\pi^{4-\epsilon}}\frac{\pi\epsilon}{2}\cot\Big(\frac{\pi\epsilon}{2}\Big)\Gamma(1-\epsilon)\frac{a^\epsilon}{\epsilon^2}. \tag{A33}$$

The integrand of the B_3 term (80) is the factor (A27) times

$$a'\ln(aa')\Big\{\frac{1}{\Delta x_{++}^{2D-2}}-\frac{1}{\Delta x_{+-}^{2D-2}}\Big\}=\frac{a'\ln(aa')\partial^2}{(4-2\epsilon)(2-2\epsilon)(2-\epsilon)(-\epsilon)}\times\Big\{\mu^{-2\epsilon}\partial^2\Big[\frac{\frac{\epsilon}{2}\ln(\mu^2\Delta x_{++}^2)+\dots}{\Delta x_{++}^2}\Big]-(++\to+-)+\frac{i4\pi^{2-(1/2)\epsilon}\mu^{-\epsilon}}{\Gamma(1-\frac{\epsilon}{2})}\delta^D(x-x')\Big\}. \tag{A34}$$

The appropriate local term for B_3 is

$$B_{3L}(x)=\frac{-fH^{2-\epsilon}\mu^{-\epsilon}}{2^{6-\epsilon}\pi^{4-\epsilon}}\frac{\Gamma(3-\epsilon)a^{-3+\epsilon}}{(1-\frac{\epsilon}{2})(1-\epsilon)\epsilon}\{\ln(a)\partial^2 a+\partial^2(a\ln(a))\}, \tag{A35}$$

$$=\frac{fH^{4-\epsilon}\mu^{-\epsilon}}{8\pi^{4-\epsilon}}\frac{\Gamma(1-\epsilon)}{\epsilon}\Big\{\ln(a)+\frac{3}{4}\Big\}a^\epsilon. \tag{A36}$$

For the nonlocal terms it is useful to extract a factor to go with (A27) making a total multiplicative factor of

$$-ifH^{2-\epsilon}\frac{\Gamma(2-\frac{\epsilon}{2})\Gamma(3-\epsilon)}{2^{4-\epsilon}\pi^{6-(3/2)\epsilon}a^{3-\epsilon}}\times\frac{\mu^{-2\epsilon}a'}{16(1-\frac{\epsilon}{2})^2(1-\epsilon)}=-ifH^{2-\epsilon}\mu^{-2\epsilon}\frac{\Gamma(1-\frac{\epsilon}{2})\Gamma(1-\epsilon)}{2^{7-\epsilon}\pi^{6-(3/2)\epsilon}a^{3-\epsilon}}a'. \tag{A37}$$

It is also useful to note the expansions for the Gamma function

$$\Gamma(1-\epsilon)=1+\gamma\epsilon+\Big[\frac{\gamma^2}{2}+\frac{\pi^2}{12}\Big]\epsilon^2+\dots\Rightarrow\frac{\Gamma^2(1-\frac{\epsilon}{2})}{\Gamma(1-\epsilon)}=1-\frac{\pi^2}{24}\epsilon^2+\dots, \tag{A38}$$

and the cotangent

$$\frac{\pi}{2}\cot\Big(\frac{\pi\epsilon}{2}\Big)=\frac{1}{\epsilon}\Big\{1-\frac{\pi^2}{12}\epsilon^2+O(\epsilon^4)\Big\}. \tag{A39}$$

We can expand the nonlocal integrands without regard to $\pm$ variations.

The integrand for the nonlocal part of $B_1(x)$ is (A37) times

$$\frac{(1-\frac{\epsilon}{4})(1-\frac{\epsilon}{2})^3}{(1-\frac{3}{4}\epsilon)(1-\epsilon)(1-\frac{3}{2}\epsilon)}\frac{\Gamma^2(1-\frac{\epsilon}{2})}{\Gamma(1-\epsilon)}\Big(\frac{H^2aa'}{4\mu^2}\Big)^{(\epsilon/2)}\frac{\partial^4}{\epsilon}\times\Big\{\frac{\ln(\mu^2\Delta x^2)+\epsilon\ln^2(\mu^2\Delta x^2)}{\Delta x^2}\Big\}=\frac{\partial^4}{\epsilon}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]+\frac{3}{2}\partial^4\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]+\partial^4\Big[\frac{\ln^2(\mu^2\Delta x^2)}{\Delta x^2}\Big]+\frac{1}{2}\ln\Big(\frac{H^2aa'}{4\mu^2}\Big)\partial^4\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]+O(\epsilon). \tag{A40}$$

The integrand for the nonlocal part of $B_2(x)$ is (A37) times

$$-\frac{\pi}{2}\cot\Big(\frac{\pi\epsilon}{2}\Big)\partial^4\Big\{\frac{\ln(\mu^2\Delta x^2)+\frac{3}{4}\epsilon\ln^2(\mu^2\Delta x^2)}{\Delta x^2}\Big\}=-\frac{\partial^4}{\epsilon}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]-\frac{3}{4}\partial^4\Big[\frac{\ln^2(\mu^2\Delta x^2)}{\Delta x^2}\Big]+O(\epsilon). \tag{A41}$$

And the integrand for the nonlocal part of $B_3(x)$ is (A37) times

$$-\frac{1}{2}\ln(aa')\partial^4\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]+O(\epsilon). \tag{A42}$$

Combining (A40)–(A42), multiplying by (A37), including the integral, and taking $\epsilon=0$, gives the following result for the nonlocal part of the B term,

$$B_N(x) = -\frac{ifH^2}{2^7\pi^6}a^{-3}\partial^4\int d^4x'a'\left\{\frac{[\frac{3}{2}+\ln(\frac{H}{2\mu})]\ln(\mu^2\Delta x_{++}^2)+\frac{1}{4}\ln^2(\mu^2\Delta x_{++}^2)}{\Delta x_{++}^2} - \frac{[\frac{3}{2}+\ln(\frac{H}{2\mu})]\ln(\mu^2\Delta x_{+-}^2)+\frac{1}{4}\ln^2(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2}\right\}. \quad \text{(A43)}$$

The reduction of $B_N(x)$ proceeds very much like that of $A_N(x)$. We begin by extracting another d'Alembertian, then combine $++$ and $+-$ parts to make causality and reality manifest, and perform the angular integrations,

$$B_N(x) = -\frac{ifH^2}{2^7\pi^6}a^{-3}\partial^6\int d^4x'a'\left\{\begin{array}{c}\frac{1}{48}\ln^3(\mu^2\Delta x_{++}^2)+\frac{1}{8}\ln\left(\frac{eH}{2\mu}\right)\ln^2(\mu^2\Delta x_{++}^2)-\frac{1}{4}\ln\left(\frac{eH}{2\mu}\right)\ln(\mu^2\Delta x_{++}^2)\\ -\frac{1}{48}\ln^3(\mu^2\Delta x_{+-}^2)-\frac{1}{8}\ln\left(\frac{eH}{2\mu}\right)\ln^2(\mu^2\Delta x_{+-}^2)+\frac{1}{4}\ln\left(\frac{eH}{2\mu}\right)\ln(\mu^2\Delta x_{+-}^2)\end{array}\right\}, \quad \text{(A44)}$$

$$= -\frac{fH^2}{2^6\pi^4}a^{-3}\partial_0^6\int_{\eta_I}^{\eta}d\eta'a'\int_0^{\Delta\eta}drr^2\left\{\frac{1}{4}\ln^2[\mu^2(\Delta\eta^2-r^2)]-\frac{\pi^2}{12}+\ln\left(\frac{eH}{2\mu}\right)\ln[\mu^2(\Delta\eta^2-r^2)]-\ln\left(\frac{eH}{2\mu}\right)\right\}. \quad \text{(A45)}$$

The next step is to make the change of variables $r = \Delta\eta z$ and perform the integration over z. For this purpose it is useful to note the integrals

$$\int_0^1 dzz^2\ln\left(\frac{1-z^2}{4}\right) = -\frac{8}{9} \quad \text{and} \quad \int_0^1 dzz^2\ln^2\left(\frac{1-z^2}{4}\right) = \frac{104}{27}-\frac{\pi^2}{9}. \quad \text{(A46)}$$

Applying these identities gives,

$$B_N(x) = -\frac{fH^2}{2^6\pi^4}a^{-3}\partial_0^6\int_{\eta_I}^{\eta}d\eta'a'\Delta\eta^3\left\{\frac{1}{3}\ln^2(2\mu\Delta\eta) - \frac{8}{9}\ln(2\mu\Delta\eta)+\frac{26}{27}-\frac{\pi^2}{18} + \ln\left(\frac{eH}{2\mu}\right)\left[\frac{2}{3}\ln(2\mu\Delta\eta)-\frac{11}{9}\right]\right\}. \quad \text{(A47)}$$

The next step is to bring three of the derivatives inside

$$B_N(x) = -\frac{fH^2}{2^6\pi^4}a^{-3}\partial_0^5\int_{\eta_I}^{\eta}d\eta'a'\Delta\eta^2\left\{\ln^2(2\mu\Delta\eta) - 2\ln(2\mu\Delta\eta)+2-\frac{\pi^2}{6} + \ln\left(\frac{eH}{2\mu}\right)[2\ln(2\mu\Delta\eta)-3]\right\}, \quad \text{(A48)}$$

$$= -\frac{fH^2}{2^6\pi^4}a^{-3}\partial_0^4\int_{\eta_I}^{\eta}d\eta'a'\Delta\eta\left\{2\ln^2(2\mu\Delta\eta) - 2\ln(2\mu\Delta\eta)+2-\frac{\pi^2}{3} + \ln\left(\frac{eH}{2\mu}\right)[4\ln(2\mu\Delta\eta)-4]\right\}, \quad \text{(A49)}$$

$$= -\frac{fH^2}{2^6\pi^4a^3}\partial_0^3\int_{\eta_I}^{\eta}d\eta'a'\left\{2\ln^2(2\mu\Delta\eta) + 4\ln\left(\frac{e^{(3/2)}H}{2\mu}\right)\ln(2\mu\Delta\eta)-\frac{\pi^2}{3}\right\}. \quad \text{(A50)}$$

Before performing the integral it is best to rearrange the integrand

$$\ln^2(2\mu\Delta\eta)+2\ln\left(\frac{e^{(3/2)}H}{2\mu}\right)\ln(2\mu\Delta\eta)-\frac{\pi^2}{6} = \ln^2(H\Delta\eta)+3\ln(H\Delta\eta)+K, \quad \text{(A51)}$$

where the constant K is

$$K \equiv -\ln^2\left(\frac{2\mu}{H}\right)+3\ln\left(\frac{2\mu}{H}\right)-\frac{\pi^2}{6}. \quad \text{(A52)}$$

The integral gives a complicated result,

$$B_N(x) = \frac{fH}{2^5\pi^4}a^{-3}\partial_0^3\left\{-\frac{1}{3}\ln^3(a)-\frac{\pi^2}{3}\ln(a)+2\zeta(3) - 2\sum_{n=1}^{\infty}\frac{a^{-n}}{n^3}-\ln(a)\ln^2\left(1-\frac{1}{a}\right)+2\ln\left(1-\frac{1}{a}\right) \times\sum_{n=1}^{\infty}\frac{(1-\frac{1}{a})^n}{n^2}-2\sum_{n=1}^{\infty}\frac{(1-\frac{1}{a})^n}{n^2}+\frac{3}{2}\ln^2(a) + \frac{\pi^2}{2}-3\sum_{n=1}^{\infty}\frac{a^{-n}}{n^2}-K\ln(a)\right\}. \quad \text{(A53)}$$

In acting the derivatives there is no point to keeping any but the logarithmically enhanced terms

$$B_N(x) = \frac{fH}{2^5\pi^4}a^{-3}\partial_0^3\left\{-\frac{1}{3}\ln^3(a)+\frac{3}{2}\ln^2(a)+O(\ln(a))\right\}, \quad \text{(A54)}$$

$$= -\frac{fH^4}{2^4\pi^4}\{\ln^2(a)+O(1)\}. \quad \text{(A55)}$$

By comparison the C terms are straightforward. The two nonlocal terms cancel

$$C_{1N}(x) = -\frac{i3fH^4}{2^8\pi^6 a^2}\partial^2 \int d^4x' a'^2 \left\{\frac{\ln(\mu^2\Delta x^2_{++})}{\Delta x^2_{++}} - \frac{\ln(\mu^2\Delta x^2_{+-})}{\Delta x^2_{+-}}\right\}, \tag{A56}$$

$$C_{2N}(x) = +\frac{i3fH^4}{2^8\pi^6 a^2}\partial^2 \int d^4x' a'^2 \left\{\frac{\ln(\mu^2\Delta x^2_{++})}{\Delta x^2_{++}} - \frac{\ln(\mu^2\Delta x^2_{+-})}{\Delta x^2_{+-}}\right\}. \tag{A57}$$

The corresponding local terms are,

$$C_{1L}(x) = +\frac{fH^4\mu^{-2\epsilon}}{2^7\pi^{4-\epsilon}}\frac{(1-\frac{\epsilon}{2})\Gamma(2-\frac{\epsilon}{2})\Gamma(4-\frac{\epsilon}{2})}{(1-\frac{3}{2}\epsilon)(1+\epsilon)}\frac{a^{2\epsilon}}{\epsilon}, \tag{A58}$$

$$C_{2L}(x) = -\frac{fH^{4-\epsilon}\mu^{-\epsilon}}{2^{6-\epsilon}\pi^{4-\epsilon}}\frac{(1-\frac{\epsilon}{2})\Gamma(4-\epsilon)}{(1-\frac{\epsilon}{4})(1-\epsilon)}\frac{a^{\epsilon}}{\epsilon}. \tag{A59}$$

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