Anomalies, Hawking radiations, and regularity in rotating black holes

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This is an extended version of our previous letter [S. Iso, H. Umetsu, and F. Wilczek, Phys. Rev. Lett. **96**, 151302 (2006).]. In this paper we consider rotating black holes and show that the flux of Hawking radiation can be determined by anomaly cancellation conditions and regularity requirement at the horizon. By using a dimensional reduction technique, each partial wave of quantum fields in a $d = 4$ rotating black hole background can be interpreted as a $(1 + 1)$-dimensional charged field with a charge proportional to the azimuthal angular momentum m. From this and the analysis [S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **95**, 011303 (2005), S. Iso, H. Umetsu, and F. Wilczek, Phys. Rev. Lett. **96**, 151302 (2006).] on Hawking radiation from charged black holes, we show that the total flux of Hawking radiation from rotating black holes can be universally determined in terms of the values of anomalies at the horizon by demanding gauge invariance and general coordinate covariance at the quantum level. We also clarify our choice of boundary conditions and show that our results are consistent with the effective action approach where regularity at the future horizon and vanishing of ingoing modes at $r = \infty$ are imposed (i.e. Unruh vacuum).

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I. INTRODUCTION

Hawking radiation is the most prominent quantum effect to arise for quantum fields in a background space-time with an event horizon. There are several derivations and all of them take the quantum effect in black hole backgrounds into account in various ways. The original derivation by Hawking [1,2] calculates the Bogoliubov coefficients between the in and out states in a black hole background. A tunneling picture [3,4] is based on pair creations of particles and antiparticles near the horizon and calculates WKB amplitudes for classically forbidden paths. A common property in these derivations is the universality of the radiation: i.e. Hawking radiation is determined universally by the horizon properties (if we neglect the gray body factor induced by the effect of scattering outside the horizon.)

Another approach to the Hawking radiation is to calculate the energy-momentum (EM) tensor in the black hole backgrounds. It has a long history and there are many investigations (see for example [5] and references therein). Here we would like to mention the seminal work by Christensen and Fulling [6]. In this paper the authors determined the form of the EM tensor by using symmetry arguments and the conservation law of the EM tensor together with the trace anomaly. In $d = 2$, such information is sufficient to determine the complete form of the EM tensor and accordingly the Hawking radiation can be correctly reproduced. But in $d = 4$ there remains an indeterminable function and the full form of the EM tensor can not be determined by symmetries only. Since the Hawking radiation is a very universal phenomenon, it should be discussed based on fundamental properties at the horizon.

In a previous paper [7], we have shown that the flux of Hawking radiation from Reissner-Nordström black holes can be determined by requiring gauge and general coordinate covariance at the quantum level. The work was based on [8] but with a slightly different procedure. In the following we take the procedure adopted in [7]. The basic idea is the following. We consider a quantum field in a black hole background. Near the horizon, the field can be effectively described by an infinite collection of $(1 + 1)$-dimensional fields on (t, r) space where r is the radial direction. Furthermore, due to the property of the black hole metric, mass or potential terms for quantum fields in it can be suppressed near the horizon. Therefore we can treat the original higher dimensional theories as a collection of two-dimensional quantum fields. In this two-dimensional reduction, outgoing modes near the horizon behave as right moving modes while ingoing modes as left moving modes. Since the horizon is a null hypersurface, all ingoing modes at the horizon can not *classically* affect physics outside the horizon. Then, if we integrate the other modes to obtain the effective action in the exterior region, it becomes anomalous with respect to gauge or general coordinate symmetries since the effective theory is now chiral at the horizon. The underlying theory is of course invariant under these symmetries and these anomalies must be cancelled by quantum effects of the classically irrelevant ingoing modes. We have shown that the condition for anomaly

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cancellation at the horizon determines the Hawking flux of the charge and energy-momentum. The flux is universally determined only by the value of anomalies at the horizon.

In this paper, we further extend the analysis to Hawking radiations of quantum fields from rotating black holes. In the case of Kerr black hole, the metric is axisymmetric and the azimuthal angular momentum is conserved. Because of this isometry, the effective two-dimensional theory for each partial wave has $U(1)$ gauge symmetry. The effective background gauge potential for this $U(1)$ symmetry is written in terms of the metric while the quantum field in the Kerr background has a charge m of this gauge symmetry, where m is an azimuthal quantum number. The effective theory is now interpreted as a two-dimensional field theory of charged particles in a charged black hole. Hence we can apply the same method for the charged black holes to obtain the Hawking flux from rotating black holes.

Our calculation based on anomaly cancellations reproduces the Hawking fluxes in the so-called Unruh vacuum [9]. This vacuum violates the time reversal symmetry by boundary conditions. Namely regularity at the future horizon is imposed, which fixes the flux of the outgoing modes. On the other hand, for ingoing modes, it is assumed that there is no ingoing flux at $r = \infty$. In deriving the flux in our method, we demand that covariant current at the horizon should vanish. We show that this is nothing but the regularity condition. On the other hand, the boundary condition for ingoing modes is also imposed. We clarify these points in this paper.

The content of the paper is as follows. In Sec. II, we will first show that, near the horizon of Kerr black hole, each partial wave of scalar fields behaves as a charged field in two-dimensional charged black hole. In Sec. III, we relate symmetries in the original $d = 4$ system and the dimensionally reduced $(1 + 1)$-dimensional system and derive modified conservation laws of current and energy-momentum tensor in $d = 2$. By using the dimensional reduction technique, we derive the Hawking flux from Kerr black hole in section IV. Here we demand gauge and general coordinate covariance at the horizon and impose that the covariant currents should vanish at the horizon. We discuss our choice of boundary conditions. In Sec. V we derive the flux of Hawking radiation from Kerr-Newman black holes. Section VI is devoted to conclusions and discussions. In Appendix A we calculate the flux of Hawking radiation from Kerr-Newman black hole by integrating the Planck distribution. In Appendix B we derive the flux of radiation from charged black holes by an effective action approach. Since quantum fields near horizons of black holes are effectively described by $(1 + 1)$-dimensional conformal fields, we can explicitly calculate expectation values of a current or an energy-momentum tensor near the horizon by imposing boundary conditions, such that physical quantities are regular at the future horizon and there is no ingoing flux at $r = \infty$. This is the boundary condition for the Unruh vacuum and the fluxes coincide with those derived in this paper.

II. QUANTUM FIELDS IN KERR BLACK HOLE

The metric of the Kerr black hole is given by

$$\begin{aligned} ds^2 &= \frac{\Delta}{r^2 + a^2\cos^2\theta}(dt - a\sin^2\theta d\varphi)^2 \\ &\quad - \frac{\sin^2\theta}{r^2 + a^2\cos^2\theta}(adt - (r^2 + a^2)d\varphi)^2 \\ &\quad - (r^2 + a^2\cos^2\theta)\left(\frac{dr^2}{\Delta} + d\theta^2\right), \end{aligned} \tag{1}$$

where

$$\Delta = r^2 - 2Mr + a^2 = (r - r_+)(r - r_-), \tag{2}$$

and $r_{+(-)}$ are radii of outer (inner) horizons

$$r_\pm = M \pm \sqrt{M^2 - a^2}. \tag{3}$$

We will consider matter fields in the Kerr black hole background. For a while we will consider a scalar field for simplicity. The action consists of the free part

$$\begin{aligned} S_{\rm free} &= \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &= -\frac{1}{2}\int dtdrd\theta d\varphi \sin\theta\phi\bigg[\bigg(\frac{(r^2 + a^2)^2}{\Delta} \\ &\quad - a^2\sin^2\theta\bigg)\partial_t^2 + 2a\bigg(\frac{r^2 + a^2}{\Delta} - 1\bigg)\partial_t\partial_\varphi - \partial_r\Delta\partial_r \\ &\quad - \frac{1}{\sin^2\theta}\partial_\theta\sin^2\theta\partial_\theta - \frac{1}{\sin\theta^2}\bigg(1 - \frac{a^2\sin^2\theta}{\Delta}\bigg)\partial_\varphi^2\bigg]\phi, \end{aligned} \tag{4}$$

and the other parts $S_{\rm int}$ including a mass term, potential terms and interaction terms. Performing the partial wave decomposition of ϕ in terms of the spherical harmonics, $\phi = \sum_{l,m}\phi_{lm}Y_{l,m}$, the theory is reduced to a two-dimensional effective theory with an infinite collection of fields with quantum numbers (l, m). These two-dimensional fields are interacting with each other. Because of the axial symmetry of the Kerr black hole metric in the φ direction, the azimuthal quantum number, m of Y_{lm}, is conserved.

Upon transforming to the r_* tortoise coordinate, defined by

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta} \equiv f(r)^{-1}, \tag{5}$$

and considering the region near the outer horizon r_+, one finds that the effective two-dimensional action is much simplified. The effective radial potentials for partial waves $(\sim l(l + 1)/r^2)$ or mixing terms between fields with different angular momenta contain a suppression factor $f(r(r_*))$ and vanish exponentially fast near the horizon.

The same applies to a mass term or interaction terms S_{int}. Thus it is straightforward to show that the physics near the horizon can be effectively described by an infinite collection of massless (1 + 1)-dimensional fields with the following action,

$$S = \int dt dr (r^2 + a^2)\phi^*_{lm}\left[\frac{r^2 + a^2}{\Delta}\left(\partial_t + \frac{iam}{r^2 + a^2}\right)^2 - \partial_r \frac{\Delta}{r^2 + a^2}\partial_r\right]\phi_{lm}. \tag{6}$$

From this action we find that ϕ_{lm} can be considered as a (1 + 1)-dimensional complex scalar field in the backgrounds of the dilaton Φ, metric $g_{\mu\nu}$ and $U(1)$ gauge field A_μ,

$$\Phi = r^2 + a^2, \qquad g_{tt} = f(r), \qquad g_{rr} = -\frac{1}{f(r)}, \qquad g_{rt} = 0, \qquad A_t = -\frac{a}{r^2 + a^2}, \qquad A_r = 0. \tag{7}$$

The $U(1)$ charge of the two-dimensional field ϕ_{lm} is m.

III. SYMMETRIES AND CONSERVATION LAWS

The $U(1)$ gauge symmetry in the effective two-dimensional theories originates from the axial isometry of the Kerr black hole. Since the fields are in the background of dilaton and gauge potentials, the conservation law for the energy-momentum tensor is modified.

In this section we will see how the $U(1)$ symmetry arises from the general coordinate invariance in the axial direction and then show that the conservation law for the energy-momentum tensor in $d = 4$ is rewritten as modified conservation laws of the $U(1)$ current and energy-momentum tensor in $d = 2$.

The quantum fields in the $d = 4$ Kerr black hole background is invariant under the general coordinate symmetries. In particular we are interested in the general coordinate transformations in the φ direction ξ^φ which is independent of the angles (θ, φ). They generate the $U(1)$ gauge transformations with the transformation parameter $\xi^\varphi(t, r)$. For such transformations, since the metric transforms as $\delta g^{\mu\nu} = -(\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu)$, we can define a gauge potential as $A^\mu = -g^{\mu\varphi}$ with a transformation $\delta A^\mu = \nabla^\mu \xi^\varphi$. Here μ is restricted to t or r. $g^{\varphi\varphi}$ is interpreted as a dilaton which is invariant under the above transformation. A matter field ϕ_{lm} transforms as a charged field with a charge m; $\delta\phi_{lm} = im\xi^\phi\phi_{lm}$.

The energy-momentum tensor $T^{\mu\nu}$ of matter fields in the Kerr black hole background is conserved in $d = 4$,

$$\nabla_\nu T^{\mu\nu} = 0. \tag{8}$$

Since the Kerr background is stationary and axisymmetric, the expectation value of the energy-momentum tensor in the background depends only on r and θ, i.e. $\langle T^{\mu\nu}\rangle = \langle T^{\mu\nu}(r, \theta)\rangle$. (In the following we omit the bracket for notational simplicity.)

First, the $\mu = \varphi$ component of the conservation law (8) is written as

$$\partial_r(\sqrt{-g}T^r_\varphi) + \partial_\theta(\sqrt{-g}T^\theta_\varphi) = 0. \tag{9}$$

Noting $\sqrt{-g} = (r^2 + a^2\cos^2\theta)\sin\theta$, we define a spacial component of $U(1)$ current $J^r_{(2)}$ for each partial wave mode as follows:

$$J^r_{(2)}(r) \equiv -\int d\Omega_2 (r^2 + a^2\cos^2\theta)T^r_\varphi. \tag{10}$$

Then by integrating the Eq. (9) over the angular coordinates the $U(1)$ current is shown to be conserved,

$$\partial_r J^r_{(2)} = 0. \tag{11}$$

Second, the $\mu = t$ component $\nabla_\nu T^\nu_t = 0$ becomes a modified conservation law of the two-dimensional energy-momentum tensor. It is written as

$$\frac{1}{r^2 + a^2\cos^2\theta}\partial_r[(r^2 + a^2\cos^2\theta)T^r_t] + F_{rt}T^r_\varphi = 0, \tag{12}$$

where $F_{rt} = \partial_r A_t$ with the gauge potential A_t defined in Eq. (7). By defining the energy-momentum tensor $T^r_{t(2)}$ of the two-dimensional effective theory from T^r_t as

$$T^r_{t(2)} = \int d\Omega_2 (r^2 + a^2\cos^2\theta)T^r_t, \tag{13}$$

we find the following conservation law with the $U(1)$ gauge field background A_t,

$$\partial_r T^r_{t(2)} - F_{rt}J^r_{(2)} = 0. \tag{14}$$

This equation is the conservation law for the energy-momentum tensor in an electric field background. (See Eq. (18) in our previous paper [7]).

IV. ANOMALIES AND HAWKING FLUXES

As explained in the introduction, if we neglect classically irrelevant ingoing modes near the horizon, the effective two-dimensional theory becomes chiral near the horizon and the gauge symmetry or the general coordinate covariance becomes anomalous due to the gauge or gravitational anomalies.

The following procedure to obtain the Hawking fluxes from the anomalies is parallel to the analysis for Reissner-Nordström black holes [7].

First we determine the flux of the $U(1)$ current. In the $d = 4$ language, the $U(1)$ flux corresponds to the flux of angular momentum carried by Hawking radiation from rotating black holes. The effective theory outside the horizon r_+ is defined in the region $r \in [r_+, \infty]$. We will divide the region into two. One is a near horizon region where we neglect the ingoing modes since such modes never come out once they fall into black holes. The other region is apart from the horizon. The current is conserved

$$\partial_r J^r_{(o)} = 0, \tag{15}$$

in the latter region. On the contrary, in the near horizon region $r \in [r_+, r_+ + \epsilon]$, since there are only outgoing (right handed) fields, the current obeys an anomalous equation

$$\partial_r J^r_{(2)} = \frac{m^2}{4\pi}\partial_r A_t. \tag{16}$$

The right hand side is a gauge anomaly in a consistent form [10–12]. The current is accordingly a consistent current which can be obtained from the variation of the effective action with respect to the gauge potential. We can solve these equations in each region as

$$J^r_{(o)} = c_o, \tag{17}$$

$$J^r_{(H)} = c_H + \frac{m^2}{4\pi}(A_t(r) - A_t(r_+)), \tag{18}$$

where c_o and c_H are integration constants. c_o is the value of the current at $r = \infty$. c_H is the value of the consistent current of the outgoing modes at the horizon. Current is written as a sum in two regions

$$J^\mu = J^\mu_{(o)}\Theta_+(r) + J^\mu_{(H)}H(r), \tag{19}$$

where $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ and $H(r) = 1 - \Theta_+(r)$ are step functions defined in the region $r \in [r_+, \infty]$. Note that since we have neglected the ingoing modes near the horizon this current is only a part of the total current. The total current including a contribution from the near horizon ingoing modes is given by

$$J^\mu_{\rm total} = J^\mu + K^\mu, \tag{20}$$

where

$$K^\mu = -\frac{m^2}{4\pi}A_t(r)H(r). \tag{21}$$

This cancels the anomalous part in J^μ near the horizon.

We now consider the effective action W where we have neglected the classically irrelevant ingoing modes at the horizon. Hence the variation of the effective action under gauge transformations is given by

$$-\delta W = \int d^2x\sqrt{-g_{(2)}}\lambda\nabla_\mu J^\mu, \tag{22}$$

where λ is a gauge parameter. By integration by parts we have

$$-\delta W = \int d^2x\lambda\left[\delta(r - r_+ - \epsilon)\left(J^r_o - J^r_H + \frac{m^2}{4\pi}A_t\right) + \partial_r\left(\frac{m^2}{4\pi}A_t H\right)\right]. \tag{23}$$

As well as the current (19), this effective action does not contain a contribution from the near horizon ingoing modes. The total effective action must be gauge invariant and the last term should be cancelled by quantum effects of the classically irrelevant ingoing modes. Namely a contribution from the ingoing modes (21) cancels the last term. The coefficient of the delta function should also vanish, which relates the coefficient of the current in two regions;

$$c_o = c_H - \frac{m^2}{4\pi}A_t(r_+). \tag{24}$$

This relation ensures that the total current $J^\mu_{\rm total}$ is conserved in all the regions; $\partial_r J^r_{\rm total} = 0$.

In order to fix the value of the current, we impose that the coefficient of the covariant current at the horizon should vanish. This assumption is based on the following physical requirement. In the near horizon region, we have first neglected ingoing modes. Hence the current there has contributions from only the outgoing modes which depend on $u = t - r_*$. Namely the vanishing condition for the covariant current is nothing but the vanishing condition for the current of the outgoing modes, which is usually imposed to assure regularity of the physical quantities at the future horizon. We will discuss it more in the discussions and in Appendix B. Another condition we have implicitly assumed is the constant value of the ingoing current (21). We could have added an arbitrary constant in (21). The boundary condition for K^μ to vanish at $r = \infty$ corresponds to a condition that there is no ingoing modes at radial infinity.

Since the covariant current $\tilde{J}^r$ is written as $\tilde{J}^r = J^r + \frac{m^2}{4\pi}A_t(r)H(r)$, the condition $\tilde{J}^r(r_+) = 0$ determines the value of the charge flux to be

$$c_o = -\frac{m^2}{2\pi}A_t(r_+) = \frac{m^2 a}{2\pi(r_+^2 + a^2)}. \tag{25}$$

This agrees with the flow of the angular momentum associated with the Hawking thermal (blackbody) radiation. (See Eq. (A3) in the appendix, with $Q = 0$.)

Similarly we can determine the flux of the energy-momentum tensor radiated from Kerr black holes. Since there is an effective background gauge potential, the energy-momentum tensor satisfies the modified conservation equation outside the horizon:

$$\partial_r T^r_{t(o)} = F_{rt}J^r_{(o)}. \tag{26}$$

By using $J^r_{(o)} = c_o$ it is solved as

$$T^r_{t(o)} = a_o + c_o A_t(r), \tag{27}$$

where a_o is an integration constant. This is the value of the energy flow at $r = \infty$. In the near horizon region, there are gauge and gravitational anomalies and the conservation equation is modified as

$$\partial_r T^r_t = F_{rt}J^r + A_t\nabla_\mu J^\mu + \partial_r N^r_t, \tag{28}$$

where $N^r_t = (f'^2 + ff'')/192\pi$. (Refer to [7] for the derivation.) The second term comes from the gauge anomaly while the third one is the gravitational anomaly for the consistent energy-momentum tensor [13]. The first and the second terms can be combined in terms of the covariant current $\tilde{J}^r_{(H)}$ as $F_{rt}\tilde{J}^r_{(H)}$. By substituting $\tilde{J}^r_{(H)} = c_o + \frac{m^2}{2\pi}A_t(r)$ into this equation, $T^r_{t(H)}$ can be solved as

$$T^r_{t(H)} = a_H + \int_{r_+}^r dr \partial_r\left(c_o A_t + \frac{m^2}{4\pi}A_t^2 + N^r_t\right). \tag{29}$$

The energy-momentum tensor combines contributions from these two regions, $T^\mu_\nu = T^\mu_{\nu(o)}\Theta_+ + T^\mu_{\nu(H)}H$. This does not contain a contribution from the ingoing modes near the horizon. The total energy-momentum tensor is a sum of T^μ_ν and U^μ_ν, where

$$U^r_t = -\left(\frac{m^2}{4\pi}A_t^2(r) + N^r_t(r)\right)H, \tag{30}$$

is a contribution from the ingoing modes. The freedom to add a constant value is fixed by a requirement that it should vanish at $r = \infty$. This condition corresponds to a condition that there is no ingoing energy flow at $r = \infty$.

Under the following diffeomorphism transformation with a transformation parameter ξ^t, the effective action (without the near horizon ingoing modes) changes as

$$\begin{aligned} -\delta W &= \int d^2x\sqrt{-g_{(2)}}\xi^t\nabla_\mu T^\mu_t \\ &= \int d^2x\xi^t\bigg[c_o\partial_r A_t(r) + \partial_r\left(\frac{m^2}{4\pi}A_t^2 + N^r_t\right) \\ &\quad + \left(T^r_{t(o)} - T^r_{t(H)} + \frac{m^2}{4\pi}A_t^2 + N^r_t\right)\delta(r - r_+ - \epsilon)\bigg]. \end{aligned} \tag{31}$$

The first term is the classical effect of the background electric field for constant current flow. The second term should be cancelled by the quantum effect of the ingoing modes (30). The coefficient of the last term should vanish in order to restore the diffeomorphism covariance at the horizon. This relates the coefficients:

$$a_o = a_H + \frac{m^2}{4\pi}A_t^2(r_+) - N^r_t(r_+). \tag{32}$$

In order to determine a_o, we impose a vanishing condition for the covariant energy-momentum tensor at the horizon. This condition corresponds to the regularity condition for the energy-momentum tensor at the future horizon. Since the covariant energy-momentum tensor is related to the consistent one by [12,14]

$$\tilde{T}^r_t = T^r_t + \frac{1}{192\pi}(ff'' - 2(f')^2), \tag{33}$$

the condition reads

$$a_H = \kappa^2/24\pi = 2N^r_t(r_+), \tag{34}$$

where

$$\kappa = 2\pi/\beta = \frac{1}{2}\partial_r f|_{r=r_+} = \frac{r_+ - r_-}{2(r_+^2 + a^2)} \tag{35}$$

is the surface gravity of the black hole. The total flux of the energy-momentum tensor is given by

$$a_o = \frac{m^2a^2}{4\pi(r_+^2 + a^2)^2} + N^r_t(r_+) = \frac{m^2\Omega^2}{4\pi} + \frac{\pi}{12\beta^2}, \tag{36}$$

where Ω is an angular velocity at the horizon,

$$\Omega = \frac{a}{r_+^2 + a^2}. \tag{37}$$

This value of the flux is the same as the Hawking flux from Kerr black holes in Eq. (A4) with $Q = 0$.

V. QUANTUM FIELDS IN KERR-NEWMAN BLACK HOLE

In this section we generalize our analysis to rotating charged (Kerr-Newman) black holes and obtain Hawking fluxes. The analysis in the previous section can be straightforwardly applied to this case.

The metric of the Kerr-Newman black hole is given by replacing Δ in (1) with

$$\Delta = r^2 - 2Mr + a^2 + Q^2 = (r - r_+)(r - r_-), \tag{38}$$

where Q is the electric charge of the black hole and $r_{+(-)}$ are radii of outer (inner) horizons

$$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}. \tag{39}$$

The background gauge field is given by

$$A = -\frac{Qr}{r^2 + a^2\cos^2\theta}(dt - a\sin^2\theta d\varphi). \tag{40}$$

Let us consider a complex scalar field in this background. As well as the case of the Kerr black hole background, each partial wave mode of fields can be described near the outer horizon by the following effective $(1+1)$-dimensional theory in the $(r-t)$ section

$$\begin{aligned} S = -\int dtdr(r^2 + a^2)\phi^*_{lm}\bigg[\frac{r^2 + a^2}{\Delta}\left(\partial_t + \frac{ieQr}{r^2 + a^2}\right. \\ \left. + \frac{iam}{r^2 + a^2}\right)^2 - \partial_r\frac{\Delta}{r^2 + a^2}\partial_r\bigg]\phi_{lm}, \end{aligned} \tag{41}$$

where e is the electric charge of ϕ. The dilaton background and the metric have the same forms as the ones of the Kerr geometry (7). $U(1)$ gauge field background is now given by

$$\mathcal{A}_t = -\frac{eQr}{r^2 + a^2} - \frac{ma}{r^2 + a^2}. \tag{42}$$

The first term is originated from the electric field of the

Kerr-Newman black hole while the second one is the induced gauge potential from the metric which is associated with the axisymmetry of the Kerr-Newman background.

In this case, there are two $U(1)$ gauge symmetries and correspondingly two gauge currents. One is the original gauge symmetry while the other is the induced gauge symmetry associated with the isometry along the φ direction. The gauge potential (42) is a sum of these two fields,

$$\mathcal{A}_t = eA_t^{(1)} + mA_t^{(2)}. \quad (43)$$

The $U(1)$ current j^r associated with the original gauge symmetry is defined from the electric current J^r in the four-dimensional space-time as

$$j^r = \int d\Omega_2 (r^2 + a^2 \cos^2\theta) J^r. \quad (44)$$

Since the background is time-independent, the current in the Kerr-Newman background satisfies

$$\partial_r j^r = 0. \quad (45)$$

In the region $r \in [r_+, r_+ + \epsilon]$, this equation is modified by the gauge anomaly,

$$\partial_r j^r = \frac{e}{4\pi} \partial_r \mathcal{A}_t. \quad (46)$$

Following the procedure in the case of Kerr black hole, we can obtain the flux of the electric charge as

$$-\frac{e}{2\pi} \mathcal{A}_t(r_+) = \frac{e}{2\pi}\left(\frac{eQr_+}{r_+^2 + a^2} + \frac{ma}{r_+^2 + a^2}\right). \quad (47)$$

This reproduces the flux of the electric current derived from the Hawking radiation in (A2).

Next the current $J^r_{(2)}$ associated with the axial symmetry can be defined from the (r, ϕ)-component of the four-dimensional energy-momentum tensor T^r_φ as Eq. (10). The anomalous equation near the horizon is

$$\partial_r J^r_{(2)} = \frac{m}{4\pi} \partial_r \mathcal{A}_t. \quad (48)$$

Hence the flux of the angular momentum is obtained as

$$-\frac{m}{2\pi} \mathcal{A}_t(r_+) = \frac{m}{2\pi}\left(\frac{eQr_+}{r_+^2 + a^2} + \frac{ma}{r_+^2 + a^2}\right), \quad (49)$$

which is equal to (A3). It should be noted that j^r and $J^r_{(2)}$ are not independent for a fixed azimuthal angular momentum m. Actually as is clear from the gauge potential (43) their expectation value in the Kerr-Newman background are related as $\frac{1}{e} j^r = \frac{1}{m} J^r_{(2)} (\equiv \mathcal{J}^r)$.

Finally the anomalous equation for the energy-momentum tensor in the region $r \in [r_+, r_+ + \epsilon]$ is given by

$$\partial_r T^r_t = \mathcal{F}_{rt} \mathcal{J}^r + \mathcal{A}_t \partial_r \mathcal{J}^r + \partial_r N^r_t, \quad (50)$$

where $\mathcal{F}_{rt} = \partial_r \mathcal{A}_t$. $\mathcal{J}^\mu$ is defined above and satisfies $\partial_r \mathcal{J}^r = \frac{1}{4\pi} \partial_r \mathcal{A}_t$. Applying the same method as in the previous section, the flux of the energy-momentum is determined as

$$\frac{1}{4\pi} \mathcal{A}_t^2(r_+) + N^r_t(r_+) = \frac{1}{4\pi}\left(\frac{eQr_+}{r_+^2 + a^2} + \frac{ma}{r_+^2 + a^2}\right)^2 + \frac{\pi}{12\beta^2}, \quad (51)$$

where β is the Hawking temperature of the Kerr-Newman black hole,

$$\frac{2\pi}{\beta} = \frac{r_+ - r_-}{2(r_+^2 + a^2)}. \quad (52)$$

This is the flux of energy expected from the Hawking radiation (A4).

VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we extended our previous analysis of Hawking radiation from charged black holes based on gauge and gravitational anomalies to the cases of rotating black holes, i.e. Kerr and Kerr-Newman black holes. In the case of Hawking radiations from Kerr black hole, though there is no gauge symmetry in the original four-dimensional setting, the technique for a Reissner-Nordström black hole can be utilized since the effective two-dimensional theory near the horizon can be described by charged matter fields in an electric field. This is because the axial direction of the four-dimensional general coordinate transformations can be interpreted as $U(1)$ gauge symmetry for each partial mode. The charge of the field is given by the azimuthal quantum number. By this identification, we have reproduced the correct Hawking flux from Kerr black holes by demanding gauge and diffeomorphism symmetry. This analysis was straightforwardly extended to the Hawking radiations of charged particles from a Kerr-Newman black hole.

The derivation is based only on the anomaly equation for gauge current and energy-momentum tensor in effective two-dimensional field theories near horizons and the result is universal. Namely it does not either depend on the detailed dynamics of fields apart from the horizon or the spin of the radiated particles. Of course, when these radiated particles travel to the infinity, they experience potentials or interactions and the spectrum is modified. Our treatment considered only the near horizon effect and neglected such the scattering effect outside the horizon (i.e. gray body factor).

Our derivation is partial since we have not been able to derive the frequency-dependent spectrum of the Hawking radiation. For this purpose, we may need to develop frequency-dependent formulation of anomalies or renormalization group type analysis near the horizon. This is left for future investigation.

Finally we would like to comment on our choice of boundary conditions and the regularity of the physical quantities at the future horizon. As is well known, Hawking radiation is derived by assuming regularity of the energy-momentum tensor at the future horizon and an assumption that there is no ingoing current at the past horizon [9]. This boundary condition corresponds to the Unruh vacuum, and fluxes for other vacua correspond to other choices of boundary conditions. In the case of Reissner-Nordström black hole or rotating black holes, each partial mode of four-dimensional fields is effectively described by a massless free two-dimensional conformal field in an electric and gravitational background. Hence we can calculate the effective action exactly and the currents or energy-momentum tensor are also exactly obtained up to boundary conditions of Green functions. (See Appendix B.) If we impose a regularity at the future horizon and absence of ingoing fluxes at $r = \infty$, we can obtain the fluxes for Unruh vacuum. (In the Schwarzschild case, see a review [15].) We have chosen the boundary condition that the radial component of the covariant current should vanish at the horizon. This corresponds to the above regularity condition in the following sense. Near the horizon, we have first neglected the quantum effect of ingoing modes. Hence the current in the near horizon region should be considered as the outgoing current. In the (u, v) coordinates where $u = t - r_*$ and $v = t + r_*$, the vanishing condition for the covariant current corresponds to the condition $J_u \to 0$ at the future horizon. This is the regularity condition at the future horizon. On the other hand, the boundary condition for ingoing modes at infinity is implicitly assumed. We have derived the Hawking flux by using anomalies and conservation laws for currents or energy-momentum tensor. But there is a freedom to add an extra constant ingoing flux in the whole region because such an addition does not break the conservation laws. (Such a constant cannot be added to the outgoing flux because this addition violates the regularity at the future horizon.) We have taken into account the quantum effect of ingoing modes through the anomalous contribution (WZ term). This corresponds to the boundary condition that the ingoing modes should vanish at infinity.

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APPENDIX A: BLACKBODY RADIATION

In the body of the paper, we have treated a scalar field in rotating black holes. The same trick to reduce the system to effective $d = 2$ theory can be applied to fermions with slightly more complication. In this appendix we calculate the flux of Hawking radiations in the case of fermions in order to avoid the problem of superradiance. The Hawking distribution is given by the Planck distribution with chemical potentials for an azimuthal angular momentum m and an electric charge e of the fields radiated from the black hole. For fermions the distribution for the Kerr-Newman black hole is given by

$$N_{e,m}(\omega) = \frac{1}{e^{\beta(\omega - e\Phi - m\Omega)} + 1}, \tag{A1}$$

where $\Phi = Qr_+/(r_+^2 + a^2)$, and Ω was defined in Eq. (37). The inverse temperature β is defined in (52). From this distribution, we can calculate fluxes of the electric current j, angular momentum $J_{(2)}$ and energy-momentum tensor, defined, respectively, as F_Q, F_a, and F_M;

$$\begin{aligned} F_Q &= e\int_0^\infty \frac{d\omega}{2\pi}(N_{e,m}(\omega) - N_{-e,-m}(\omega)) \\ &= \frac{e}{2\pi}(e\Phi + m\Omega), \end{aligned} \tag{A2}$$

$$\begin{aligned} F_a &= m\int_0^\infty \frac{d\omega}{2\pi}(N_{e,m}(\omega) - N_{-e,-m}(\omega)) \\ &= \frac{m}{2\pi}(e\Phi + m\Omega), \end{aligned} \tag{A3}$$

$$\begin{aligned} F_M &= \int_0^\infty \frac{d\omega}{2\pi}\omega(N_{e,m}(\omega) + N_{-e,-m}(\omega)) \\ &= \frac{1}{4\pi}(e\Phi + m\Omega)^2 + \frac{\pi}{12\beta^2}. \end{aligned} \tag{A4}$$

Here we added contributions from a particle with a quantum number (e, m) and its antiparticle with $(-e, -m)$ in order to compare our results.

APPENDIX B: EFFECTIVE ACTION AND HAWKING RADIATION

It is shown that each partial wave mode in black hole backgrounds can be described near the outer horizon by a two-dimensional effective theory. Since mass, potential, and interaction terms can be neglected near the horizon, the effective $d = 2$ theories are free conformal theories. Thus we can evaluate fluxes of the current or energy by calculating the effective action directly. Here we will calculate such fluxes in the Reissner-Nordström case because Kerr or Kerr-Newman cases are reduced to the same calculation, as explained in the body of the paper.

In the Schwarzschild case, many works have been done to derive Hawking flux from effective actions in black hole background [15,16]. Since the effective $d = 2$ theories contain dilaton background, it seems necessary to include the effect of dilaton in such investigations and there have been many discussions on it. However, as we have briefly commented in our previous paper [7], the effect of the dilaton does not change the property of the (r, t)-component of the energy-momentum tensor and accordingly the Hawking flux is independent of the dilaton background. It only affects the other nonuniversal components like T^t_t. In this sense, Hawking radiation is universal. Once we impose the boundary conditions, the value of the flux is determined only by the value of anomalies at the horizon. Therefore, in the following, we calculate fluxes of current and energy in the Reissner-Nordström black hole for free massless scalar fields without dilaton backgrounds.

The gauge potential and the metric of the Reissner-Nordström black hole is given by

$$A = -\frac{Q}{r}dt, \tag{B1}$$

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{B2}$$

where $f(r)$ is

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r - r_+)(r - r_-)}{r^2}. \tag{B3}$$

$r_\pm = M \pm \sqrt{M^2 - Q^2}$ are the radii of the outer and inner horizons. We also use the other coordinate system (u, v),

$$v = t + r_*, \qquad u = t - r_*, \qquad \left(dr_* = \frac{1}{f}dr\right), \tag{B4}$$

and the Kruskal coordinates

$$U = -e^{-\kappa_+ u}, \qquad V = e^{\kappa_+ v}, \tag{B5}$$

where κ_+ is the surface gravity on the outer horizon, $\kappa_+ = \frac{r_+ - r_-}{2r_+^2}$.

Each partial wave of charged matter fields in the Reissner-Nordström black hole background is effectively described by a charged field in a $d = 2$ charged black hole with a metric

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} \tag{B6}$$

and the gauge potential (B1). The two-dimensional curvature obtained from this metric is given by

$$R_{tt} = \frac{ff''}{2}, \qquad R_{rr} = -\frac{f''}{2f'}, \qquad R_{tr} = 0, \qquad R = f''. \tag{B7}$$

Effective action Γ of a conformal field with a central charge $c = 1$ in this gravitational and electric field background consists of the following two parts; the gravitational part $\Gamma_{\rm grav}$ and gauge field part $\Gamma_{U(1)}$. The gravitational part (Polyakov action) is given by

$$\Gamma_{\rm grav} = \frac{1}{96\pi}\int d^2x d^2y \sqrt{-g}R(x)\frac{1}{\triangle_g}(x, y)\sqrt{-g}R(y), \tag{B8}$$

while the $U(1)$ gauge field part is

$$\Gamma_{U(1)} = \frac{e^2}{2\pi}\int d^2x d^2y \epsilon^{\mu\nu}\partial_\mu A_\nu(x)\frac{1}{\triangle_g}(x, y)\epsilon^{\rho\sigma}\partial_\rho A_\sigma(y). \tag{B9}$$

R is the two-dimensional scalar curvature (B7) and $\triangle_g$ is the Laplacian in this background. From these effective actions, we can obtain the energy-momentum tensor $T_{\mu\nu}$ and $U(1)$ current J^μ (see [17] for a chiral case),

$$T_{\mu\nu} = T^{\rm grav}_{\mu\nu} + T^{U(1)}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\Gamma}{\delta g^{\mu\nu}}, \tag{B10}$$

$$T^{\rm grav}_{\mu\nu} = \frac{1}{48\pi}\Big(2g_{\mu\nu}R - 2\nabla_\mu\nabla_\nu S + \nabla_\mu S\nabla_\nu S - \frac{1}{2}g_{\mu\nu}\nabla^\rho S\nabla_\rho S\Big), \tag{B11}$$

$$T^{U(1)}_{\mu\nu} = \frac{e^2}{\pi}\Big(\nabla_\mu B\nabla_\nu B - \frac{1}{2}g_{\mu\nu}\nabla^\rho B\nabla_\rho B\Big), \tag{B12}$$

$$J^\mu = \frac{1}{\sqrt{-g}}\frac{\delta\Gamma}{\delta A_\mu} = \frac{e^2}{\pi}\frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}\partial_\nu B, \tag{B13}$$

where

$$S(x) = \int d^2y \frac{1}{\triangle_g}(x, y)\sqrt{-g}R(y), \tag{B14}$$

$$B(x) = \int d^2y \frac{1}{\triangle_g}(x, y)\epsilon^{\mu\nu}\partial_\mu A_\nu(y). \tag{B15}$$

Hence B is a solution of the equation,

$$\triangle_g B = \epsilon^{\mu\nu}\partial_\mu A_\nu = -\partial_r A_t(r). \tag{B16}$$

This equation is solved as

$$B = B_0 + b(u) + \tilde{b}(v), \tag{B17}$$

$$\partial_r B_0 = \frac{1}{f}(A_t(r) + c), \tag{B18}$$

where $b(u)$ and $\tilde{b}(v)$ are solutions of the homogeneous equation $\triangle_g B = \frac{4}{f}\partial_u\partial_v B = 0$, and c is an integration constant. Thus the electromagnetic current (B13) becomes

$$J^t = \frac{e^2}{\pi}\partial_r B = \frac{e^2}{\pi f}(A_t(r) + c) + \frac{e^2}{\pi}\partial_r(b(u) + \tilde{b}(v)), \tag{B19}$$

$$J^r = -\frac{e^2}{\pi}\partial_t B = -\frac{e^2}{\pi}\partial_t(b(u) + \tilde{b}(v)). \tag{B20}$$

In the (u, v) coordinates they are given by

$$J_u = \frac{f}{2}\left(J^t - \frac{1}{f}J^r\right) = \frac{e^2}{2\pi}(A_t(r) + c) - \frac{e^2}{\pi}\partial_u b(u), \tag{B21}$$

$$J_v = \frac{f}{2}\left(J^t + \frac{1}{f}J^r\right) = \frac{e^2}{2\pi}(A_t(r) + c) + \frac{e^2}{\pi}\partial_v \tilde{b}(v). \tag{B22}$$

In order to determine the homogeneous parts, we impose the following boundary conditions. First we require that free falling observers see a finite (not infinite) amount of the charged current at the outer horizon and accordingly the current in the Kruskal coordinate U is required to be finite at the future horizon. Since $J_U = -(1/\kappa_+ U)J_u$ and $U \to \sqrt{r - r_+}$ for $r \to r_+$, J_u must vanish on the horizon

$$J_u \overset{r\to r_+}{\longrightarrow} \frac{e^2}{2\pi}(A_t(r_+) + c) - \frac{e^2}{\pi}\partial_u b(u)|_{r=r_+} = 0. \tag{B23}$$

This determines the homogeneous part $b(u)$ as $\partial_u b(u) = \frac{1}{2}(A_t(r_+) + c)$. Second we impose that there is no ingoing current at $r = \infty$ and require

$$J_v \overset{r\to\infty}{\longrightarrow} \frac{e^2}{2\pi}c + \frac{e^2}{\pi}\partial_v \tilde{b}(v)|_{r\to\infty} = 0. \tag{B24}$$

This determines the other homogeneous part $\tilde{b}(v)$ as $\partial_v \tilde{b}(v) = -c/2$. By these boundary conditions the $U(1)$ current is completely determined as

$$J_u = \frac{e^2}{2\pi}(A_t(r) - A_t(r_+)), \tag{B25}$$

$$J_v = \frac{e^2}{2\pi}A_t(r). \tag{B26}$$

In the (t, r)-coordinate, the $U(1)$ current is given by

$$J^r = J_u - J_v = -\frac{e^2}{2\pi}A_t(r_+) = \frac{e^2 Q}{2\pi r_+}, \tag{B27}$$

$$J^t = \frac{1}{f}(J_u + J_v) = \frac{e^2}{\pi f}\left(A_t(r) - \frac{1}{2}A_t(r_+)\right). \tag{B28}$$

This is the expectation value of the current for the Unruh vacuum in the $d = 2$ Reissner-Nordström black hole.

The energy-momentum tensor can be similarly obtained. $S(x)$ satisfies the equation

$$\triangle_g S = R = f'' \tag{B29}$$

and it can be solved to be a sum of an inhomogeneous and homogeneous parts.

Hence the energy-momentum tensor is written as

$$T_{uu} = \frac{1}{192\pi}(-f'^2 + 2ff'') + \frac{e^2}{4\pi}(A_t(r) - A_t(r_+))^2 + t(u), \tag{B30}$$

$$T_{vv} = \frac{1}{192\pi}(-f'^2 + 2ff'') + \frac{e^2}{4\pi}A_t^2(r) + \tilde{t}(v), \tag{B31}$$

$$T_{uv} = \frac{1}{96\pi}ff'', \tag{B32}$$

where $t(u)(\tilde{t}(v))$ is an arbitrary function of $u(v)$ that are determined by boundary conditions. Similarly to the $U(1)$ current we impose the following boundary conditions,

$$T_{uu} \overset{r\to r_+}{\longrightarrow} -\frac{1}{192\pi}f'^2(r_+) + t(u)|_{r=r_+} = 0, \tag{B33}$$

$$T_{vv} \overset{r\to\infty}{\longrightarrow} \tilde{t}(v)|_{r\to\infty} = 0. \tag{B34}$$

Then components of the energy-momentum tensor become

$$\begin{aligned} T_t^t &= \frac{1}{f}(T_{uu} + T_{vv} + 2T_{uv}) \\ &= \frac{1}{96\pi f}\left[-(f')^2 + 4ff'' + \frac{1}{2}(f'(r_+))^2\right] \\ &\quad + \frac{e^2}{2\pi f}\left[A_t^2 - A_t(r_+)A_t + \frac{1}{2}A_t^2(r_+)\right], \end{aligned} \tag{B35}$$

$$\begin{aligned} T_r^r &= -\frac{1}{f}(T_{uu} + T_{vv} - 2T_{uv}) \\ &= \frac{f}{96\pi}\left[(f')^2 - \frac{1}{2}f'(r_+)^2\right] \\ &\quad - \frac{e^2}{2\pi f}\left[A_t^2 - A_t(r_+)A_t + \frac{1}{2}A_t^2(r_+)\right], \end{aligned} \tag{B36}$$

$$\begin{aligned} T_t^r &= T_{uu} - T_{vv} \\ &= -\frac{e^2}{2\pi}A_t(r_+)A_t(r) + \frac{1}{192\pi}f'^2(r_+) + \frac{e^2}{4\pi}A_t^2(r_+). \end{aligned} \tag{B37}$$

Therefore the flux of the energy T_t^r is obtained as

$$T_t^r \overset{r\to\infty}{\longrightarrow} \frac{1}{192\pi}f'^2(r_+) + \frac{e^2}{4\pi}A_t^2(r_+). \tag{B38}$$

[1] S. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[2] S. Hawking, Nature (London) **248**, 30 (1974).
[3] M. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).
[4] E. C. Vagenas, Phys. Lett. B **503**, 399 (2001); Mod. Phys. Lett. A **17**, 609 (2002); Phys. Lett. B **533**, 302 (2002); A. J. M. Medved, Classical Quantum Gravity **19**, 589 (2002); Phys. Rev. D **66**, 124009 (2002); E. C. Vagenas, Phys. Lett. B **559**, 65 (2003); M. Angheben, M. Nadalini, L. Vanzo, and S. Zerbini, J. High Energy Phys. 05 (2005) 014; A. J. M. Medved and E. C. Vagenas, Mod. Phys. Lett. A **20**, 2449 (2005); **20**, 1723 (2005); J. Y. Zhang and Z. Zhao, J. High Energy Phys. 10 (2005) 055; Q. Q. Jiang and S. Q. Wu, Phys. Lett. B **635**, 151 (2006); M. Nadalini, L. Vanzo, and S. Zerbini, hep-th/0511250; J. Zhang and Z. Zhao, Phys. Lett. B **638**, 110 (2006); Q. Q. Jiang, S. Q. Wu, and X. Cai, Phys. Rev. D **73**, 064003 (2006); S. Q. Wu and Q. Q. Jiang, hep-th/0603082.
[5] V. P. Frolov and I. D. Novikov, *Black Hole Physics: Basic Concepts and New Developments* (Kluwer Academic, Dordrecht, Netherlands, 1998).
[6] S. Christensen and S. Fulling, Phys. Rev. D **15**, 2088 (1977).
[7] S. Iso, H. Umetsu, and F. Wilczek, Phys. Rev. Lett. **96**, 151302 (2006).
[8] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **95**, 011303 (2005).
[9] W. Unruh, Phys. Rev. D **14**, 870 (1976).
[10] R. Bertlmann, *Anomalies In Quantum Field Theory* (Oxford Science Publications, Oxford, 2000).
[11] K. Fujikawa and H. Suzuki, *Path Integrals and Quantum Anomalies* (Oxford Science Publications, Oxford, 2004).
[12] W. A. Bardeen and B. Zumino, Nucl. Phys. **B244**, 421 (1984).
[13] L. Alvarez-Gaume and E. Witten, Nucl. Phys. **B234**, 269 (1984).
[14] R. Bertlmann and E. Kohlprath, Ann. Phys. (N.Y.) **288**, 137 (2001).
[15] A. Wipf, in *Black Holes: Theory and Observation*, edited by F. W. Hehl, C. Kiefer, and R. Metzler (Springer, Berlin, 1998).
[16] V. Mukhanov, A. Wipf, and A. Zelnikov, Phys. Lett. B **332**, 283 (1994); W. Kummer, H. Liebl, and D. V. Vassilevich, Mod. Phys. Lett. A **12**, 2683 (1997); S. Nojiri and S. D. Odintsov, Phys. Rev. D **57**, 2363 (1998); R. Balbinot and A. Fabbri, Phys. Rev. D **59**, 044031 (1999); M. Buric and V. Radovanovic, Classical Quantum Gravity **16**, 3937 (1999); W. Kummer and D. V. Vassilevich, Annalen Phys. **8**, 801 (1999); S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **16**, 1015 (2001); R. Balbinot, A. Fabbri, V. P. Frolov, P. Nicolini, P. Sutton, and A. Zelnikov, Phys. Rev. D **63**, 084029 (2001); R. Balbinot and A. Fabbri, Classical Quantum Gravity **20**, 5439 (2003); S. N. Solodukhin, Phys. Rev. D **74**, 024015 (2006).
[17] H. Leutwyler, Phys. Lett. B **153**, 65 (1985); **155**, 469(E) (1985).
[18] K. Murata and J. Soda, following Article, Phys. Rev. D **74**, 044018 (2006).