Eternal observers and bubble abundances in the landscape

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We study a class of ''landscape'' models in which all vacua have positive energy density, so that inflation never ends and bubbles of different vacua are endlessly ''recycled.'' In such models, each geodesic observer passes through an infinite sequence of bubbles, visiting all possible kinds of vacua. The bubble abundance p_j can then be defined as the frequency at which bubbles of type j are visited along the worldline of an observer. We compare this definition with the recently proposed general prescription for p_j and show that they give identical results.

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I. INTRODUCTION

Nearly all models of inflation are eternal to the future. Once inflation has started, it continues forever, producing an unlimited number of pocket universes [1–3]. If there is a number of different types of pockets, as in the landscape picture suggested by string theory [4,5], all the possible types are produced in the course of eternal inflation. A natural question is, then, What is the relative abundance p_j of pockets of type j?

This question has proved to be surprisingly difficult to answer. The total number of pockets is divergent, so one needs to introduce some sort of a cutoff. If we cut off the count at a constant-time hypersurface $\Sigma : t = \text{const}$, the resulting abundances are very sensitive to the choice of the time coordinate t [6]. The reason is that the number of pockets in an eternally inflating universe is growing exponentially with time, so at any time a substantial fraction of pockets have just nucleated. Which of these pockets are crossed by the surface Σ depends on how the surface is drawn; hence the gauge dependence of the result.

A new prescription for the calculation of p_j, which does not suffer from the gauge-dependence problem, has been recently suggested in [7]. To simplify the discussion, we shall focus on models where transitions between different vacua occur through bubble nucleation, so the role of pocket universes is played by bubbles. To determine the bubble abundance, one starts with a congruence of geodesics emanating from some (finite) initial spacelike hypersurface Σ_0. As they extend to the future, the geodesics will generally cross a number of bubbles before ending up in one of the terminal bubbles, having negative or zero vacuum energy density, where inflation comes to an end. The geodesics provide a mapping of all bubbles encountered by the congruence back on the initial hypersurface. The proposal is to count only bubbles greater than a certain comoving size ϵ, and then take the limit $\epsilon \to 0$:

$$p_j \propto \lim_{\epsilon\to 0} N_j(\epsilon). \tag{1}$$

Here, $N_j(\epsilon)$ is the number of bubbles of type j with comoving size greater than ϵ. The comoving size of a bubble is defined as the size of its image on Σ_0.

In this prescription, the bubble count is dominated by bubbles formed at very late times and having very small comoving sizes. (The asymptotic number of bubbles is infinite even though the initial hypersurface Σ_0 is assumed to be finite.) The resulting values of p_j are independent of the choice of the initial hypersurface, because of the universal asymptotic behavior of eternal inflation [8].

An alternative prescription for p_j has been suggested by Easther, Lim, and Martin [9]. They randomly select a large number N of worldlines out of a congruence of geodesics and define p_j as being proportional to the number of bubbles of type j intersected by at least one of these worldlines in the limit $N \to \infty$. As the number of worldlines is increased, the average comoving distance ϵ between them (on Σ_0) gets smaller, so most bubbles of comoving size larger than ϵ are counted. In the limit of $N \to \infty$, we have $\epsilon \to 0$, and it can be shown [7] that this definition is equivalent to that of [7] (except in a special case indicated below). We shall not distinguish between the two definitions in what follows.

The prescription of [7,9] for p_j has some very attractive features. Unlike the earlier prescriptions, it is applicable in the most general case and does not depend on any arbitrary choices, such as the choice of gauge or of a spacelike hypersurface. It is also independent of the initial conditions at the onset of inflation. It is not clear, however, how uniquely the new prescription is selected by these requirements. Are there any alternative prescriptions with the same properties?

In this paper we shall analyze an attractive alternative, which suggests itself in models including only recyclable (nonterminal) vacua. In such models each geodesic observer passes through an infinite sequence of bubbles, visiting all possible kinds of vacua. The bubble abundance p_j can then be defined as the frequency at which j-type

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bubbles are visited along the worldline of a given observer:

$$p_j \propto \lim_{\tau\to\infty} N_j(\tau), \tag{2}$$

where $N_j(\tau)$ is the number of times the observer had visited vacuum j by the time τ. This definition is clearly independent of gauge or initial conditions. An added attraction here is that the bubble abundances are defined in terms of observations accessible to a single observer—a property that some string theorists find desirable [5,10].

We note that the proposal of Easther, Lim, and Martin [9] cannot be applied to models with full recycling [11]. The reason is that in this case each geodesic worldline intersects an infinite number of bubbles. We shall therefore focus on the prescription of Ref. [7] in what follows.

In the following sections we use the formalism developed in [7] to compare the bubble abundances measured by an "eternal observer" with those obtained using the prescription of [7]. We find that the two methods give identical results.

II. BUBBLE ABUNDANCES ACCORDING TO [7]

In this section we closely follow the analysis given in [7], specializing it to the case of fully recyclable vacua.

The fraction of comoving volume $f_j(t)$ occupied by vacuum of type j at time t is given by the evolution equation [12]

$$\frac{df_i(t)}{dt} = \sum_{j=1}^{n} M_{ij} f_j, \tag{3}$$

where

$$M_{ij} = \kappa_{ij} - \delta_{ij} \sum_{r=1}^{n} \kappa_{ri}, \tag{4}$$

and κ_{ij} is the probability per unit time for an observer who is currently in vacuum j to find herself in vacuum i. f_i are assumed to be normalized as

$$\sum_{i=1}^{n} f_i = 1. \tag{5}$$

The magnitude of κ_{ij} depends on the choice of the time variable t [12]. The most convenient choice for our purposes is to use the logarithm of the scale factor as the time variable; this is the so-called scale-factor time

$$a(t) \equiv e^t. \tag{6}$$

With this choice [13]

$$\kappa_{ij} = (4\pi/3) H_j^{-4} \Gamma_{ij}, \tag{7}$$

where

$$\Gamma_{ij} = A_{ij} e^{-I_{ij} - S_j}, \tag{8}$$

I_{ij} is the tunneling instanton action,

$$S_j = \frac{\pi}{H_j^2} \tag{9}$$

is the Gibbons-Hawking entropy of jth vacuum, and H_j is the corresponding expansion rate. The instanton action and the prefactor A_{ij} are symmetric with respect to the interchange of i and j [14]. Hence, we can write

$$\kappa_{ij} = \lambda_{ij} H_j^{-4} e^{-S_j} \tag{10}$$

with

$$\lambda_{ij} = \lambda_{ji}. \tag{11}$$

Assuming that all vacua are recyclable and that the matrix M_{ij} is irreducible (each vacuum is accessible from every other one), it can be shown [7,12] that Eq. (3) has a unique stationary solution with $df_j/dt = 0$ and

$$\sum_{j=1}^{n} M_{ij} f_j = 0. \tag{12}$$

In fact, the solution can be found explicitly:

$$f_j \propto H_j^4 e^{S_j}. \tag{13}$$

This can be easily verified by substituting (13) in (4) and (12) and making use of (10) and (11).

f_j has the meaning of the fraction of time spent by a geodesic observer in bubbles of type j. As one might have expected, Eq. (13) shows that it is proportional to the statistical weight of the corresponding vacuum, $\exp(S_j)$.

We shall now use the prescription of Ref. [7] to determine the bubble abundance. The increase in the number of j-type bubbles due to jumps from other vacuum states in an infinitesimal time interval dt can be expressed as

$$dN_j(t) = \sum_{i=1}^{n} \frac{\kappa_{ji} f_i}{\frac{4\pi}{3} R_i(t)^3} dt. \tag{14}$$

Here, $R_i(t)$ is the comoving radius of the bubbles nucleating in vacuum i, which is set by the comoving horizon size at the time t of bubble nucleation,

$$R_i(t) = H_i^{-1} a^{-1}(t) = H_i^{-1} e^{-t}, \tag{15}$$

where $a(t)$ is the scale factor and we have used the definition of scale-factor time in (6).

Bubbles of comoving size greater than ϵ are created at $t < -\ln(\epsilon H_i)$. Integrating Eq. (14) up to this time, we obtain

$$N_j = \frac{\epsilon^{-3}}{4\pi} \sum_{i=1}^{n} \kappa_{ji} f_i. \tag{16}$$

The prescription of [7] is that $p_j \propto N_j$, and thus

$$p_j \propto \sum_{i=1}^{n} \kappa_{ji} f_i \propto \sum_{i=1}^{n} \lambda_{ji}, \tag{17}$$

where we have used Eqs. (10) and (13).

III. ETERNAL OBSERVERS

We consider a large ensemble of eternal observers. They evolve independently of one another, yet statistically all of them are equivalent. The worldline of each observer can be parametrized by discrete jumps to different vacuum states, so the time variable τ takes values in natural numbers, $\tau = 1, 2, 3 \ldots$, and is incremented by one whenever the observer jumps to a different vacuum state.

Let $x_j(\tau)$ be the fraction of observers in vacuum j at "time" τ. $x_j(\tau)$ is normalized as

$$\sum_{j=1}^{n} x_j = 1 \tag{18}$$

and satisfies the evolution equation

$$x_i(\tau + 1) = \sum_{j=1}^{n} T_{ij} x_j(\tau), \tag{19}$$

where the transition matrix is given by

$$T_{ij} = \frac{\kappa_{ij}}{\kappa_j} \tag{20}$$

and

$$\kappa_j = \sum_{r=1}^{n} \kappa_{rj}. \tag{21}$$

The diagonal elements of the transition matrix are exactly zero,

$$T_{ii} = \kappa_{ii} = 0, \tag{22}$$

since we require each observer to jump to some other vacuum at every time step.

In the case of complete recycling that we are considering here, one expects that the evolution equation (19) has a stationary solution satisfying

$$\sum_{j=1}^{n} (T_{ij} - \delta_{ij}) x_j = 0. \tag{23}$$

And indeed, rewriting Eq. (23) as

$$\sum_{j=1}^{n} M_{ij}(x_j/\kappa_j) = 0, \tag{24}$$

and comparing with Eq. (12), we see that the stationary solution of (24) is

$$x_j = \kappa_j f_j. \tag{25}$$

Here, f_j is the solution of (12), which is given by (13).

Suppose now that we have an ensemble of observers described by the stationary distribution (25). Since the sequences of vacua visited by all observers are statistically equivalent, it is not difficult to see that the distribution of vacua along the observer's worldlines is given by $p_j \propto x_j$, or

$$p_j \propto \sum_{i=1}^{n} \kappa_{ij} f_j. \tag{26}$$

Using Eq. (12) with M_{ij} from (4), we have

$$0 = \sum_{i=1}^{n} M_{ji} f_i = \sum_{i=1}^{n} \kappa_{ji} f_i - \sum_{i=1}^{n} \kappa_{ij} f_j, \tag{27}$$

or

$$\sum_{i=1}^{n} \kappa_{ji} f_i = \sum_{i=1}^{n} \kappa_{ij} f_j. \tag{28}$$

Therefore, Eq. (26) can be also rewritten as

$$p_j \propto \sum_{i=1}^{n} \kappa_{ji} f_i, \tag{29}$$

which is identical to (17).

IV. DISCUSSION

In this paper we considered a special but relatively wide class of models in which all vacua have positive energy density and are therefore inflationary. Transitions between different vacua occur through bubble nucleation, and each geodesic worldline encounters an infinite sequence of bubbles. The bubble abundance can then be defined as the frequency at which bubbles of a given type are encountered in this sequence. We have shown that this natural definition is equivalent (in this class of models) to the prescription of Ref. [7] (which has greater generality).

We wish to emphasize the difference between the stationary distribution f_j [Eq. (13)] and the bubble abundance p_j, which has been the focus of our attention here. The difference is very striking in the case when there are only two vacua. Then Eq. (17) gives

$$p_1/p_2 = \lambda_{12}/\lambda_{21} = 1, \tag{30}$$

while Eq. (13) gives

$$f_1/f_2 = (H_1/H_2)^4 e^{S_1 - S_2}. \tag{31}$$

The stationary solution f_j strongly favors the lower-energy vacuum, which has a higher entropy S_j, while the distribution p_j seems to indicate that the two vacua are equally abundant. The prescription of [7] was recently criticized in [10] for failing to give probabilities proportional to the exponential of the entropy.

We note, however, that the distributions f_j and p_j have very different meanings. f_j is proportional to the average time a geodesic observer spends in vacuum j before transiting to another vacuum. If, as a result of quantum fluctuations, the horizon region accessible to the observer scans all of its quantum states, spending roughly equal time in each of them, then one expects $f_j \propto \exp(S_j)$. This is indeed the case, up to a prefactor. On the other hand, p_j is the frequency at which a given vacuum $j = 1, 2$ appears in the vacuum sequence along a geodesic worldline. In the case of only two vacua, the sequence is $1, 2, 1, 2, 1, \ldots$, and it is clear that both vacua occur with the same frequency.

The prescription of [7] for the bubble abundance is just a proposal. It was not derived from first principles, and its validity would be put into question by any alternative proposal satisfying the necessary invariance and common sense requirements. We therefore find it reassuring that this prescription turned out to be equivalent to that of [9] and to the ''eternal observer'' proposal in their respective ranges of validity.

The bubble abundance is necessary for the calculation of probabilities of various measurements in the landscape. The full expression for the probability includes the volume expansion factor inside the bubbles and the density of observers, in addition to p_j. For a detailed discussion of these factors, see [7,15].

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