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Avoidance of future singularities in loop quantum cosmology

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We consider the fate of future singularities in the effective dynamics of loop quantum cosmology. Nonperturbative quantum geometric effects which lead to ρ^2 modification of the Friedmann equation at high energies result in generic resolution of singularities whenever energy density ρ diverges at future singularities of Friedmann dynamics. Such quantum effects lead to the avoidance of a big rip, which is followed by a recollapsing universe stable against perturbations. Resolution of sudden singularity, the case when pressure diverges but energy density approaches a finite value depends on the ratio of the latter to a critical energy density of the order of the Planck value. If the value of this ratio is greater than unity, the universe escapes the sudden future singularity and becomes oscillatory.

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I. INTRODUCTION

Observations suggest that the current universe is dominated by a matter component which leads to an accelerated expansion of the universe—called dark energy (see Refs. [1] for review). This has stimulated a study that our universe may face a future singularity. Such future singularities typically arise if the universe is dominated by matter which violates dominant energy condition and causes a state of superacceleration of the universe before leading it to a singularity. They can occur due to divergence either of the energy density ρ and/or the pressure density p of the matter content. For example if the universe is filled with a phantom dark energy with a constant equation of state w less than -1 [2], this leads to a big rip singularity at which both ρ and p diverge with a finite time [3]. Barrow pointed out a possibility to obtain a sudden future singularity at which ρ is finite but p diverges [4]. Depending on the equation of state of dark energy, future singularities have been categorized in different classes [5].

The existence of future singularities in Friedmann-Robertson-Walker (FRW) cosmology reflects the vulnerability of standard Friedmann dynamics whenever ρ or p become of the order of Planck values. This indicates that limit of validity of general relativity has been reached and inputs from quantum gravity are necessary to probe the dynamics near the singularity. Resolution of singularities using Wheeler-DeWitt quantization has been attempted [6] but has met with little success. One of the primary reasons for its failure has been a lack of a fundamental theory which can guide quantization in the Wheeler-DeWitt framework. The issues of resolution of past [7] and future singularities [8] have been investigated using perturbative corrections in string theoretic models. These analysis indicate that generic resolution of singularities may only be accomplished using nonperturbative corrections. In particular in the absence of an analysis which uses nonperturbative quantum gravitational modifications to model the dynamics of dark energy, the fate of future singularities has remained an open problem.

Loop quantum gravity (LQG) is a leading background independent nonperturbative quantization of gravity [9] which has been very well understood in the cosmological setting in loop quantum cosmology (LQC) [10]. To its success, LQG has dealt with various singularities in the cosmological setting [11–15] and techniques have also been used to resolve singularities in black hole spacetimes [16]. Recent investigations have revealed that nonperturbative loop quantum effects lead to a ρ^2 modification of the Friedmann equation with a negative sign [13,17,18]. The modification becomes important when energy density of the universe becomes of the same order of a critical density ρ_c. The resulting dynamics generically leads to a bounce when our flat expanding universe is evolved backwards [12–15].

Since important insights have been gained on resolution of spacelike singularities in LQC, it offers a natural arena to investigate the fate of future singularities. This is the goal of the present work. Using the effective Friedmann dynamics which has emerged from LQC, we would analyze the way nonperturbative quantum gravitational effects modify the dynamics near future singularities. The plan of this paper is as follows. In the next section we would briefly review the way an effective modified Friedmann dynamics is obtained from the discrete quantum dynamics in LQC. Since Wheeler-DeWitt quantization, which like LQC is a mini-superspace approach, has been unsuccessful in resolving spacelike singularities, we would highlight some differences which emerge with LQC (for details see

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Ref. [13]). In Sec. III we analyze in detail the fate of three types of future singularities—type I (the big rip): scale factor a, energy density ρ and pressure p becoming infinite in finite time, type II (sudden): p becoming infinite with finite ρ in finite time, and type III: ρ and p diverging with finite a in finite time. We will show the LQC can successfully resolve type I and type III singularities for generic choice of initial conditions. Resolution of type II singularities though depends on the amplitude of model parameters. We conclude with a summary of our results in Sec. IV.

II. EFFECTIVE DYNAMICS IN LOOP QUANTUM COSMOLOGY

In LQG the phase space of classical general relativity is expressed in terms of SU(2) connection A_a^i and densitized triads E_i^a. In loop quantum cosmology (LQC) [10], due to underlying symmetries of the FRW spacetime the phase space structure simplifies and can be cast in terms of canonically conjugate connection $\mathfrak{c}$ and triad $\mathfrak{p}$ which satisfy $\{\mathfrak{c}, \mathfrak{p}\} = \kappa\gamma/3$, where $\kappa = 8\pi G$ (G is gravitational constant) and γ is the dimensionless Barbero-Immirzi parameter (which is set by the black hole thermodynamics in LQG, as $\gamma \approx 0.2375$). On the space of physical solutions of general relativity they are related to scale factor and its time derivative as: $\mathfrak{c} = \gamma \dot{a}$ and $\mathfrak{p} = a^2$.

The elementary variables used for quantization in LQC are the triads and holonomies of connection over edges of loops: $h_i(\mu) = \cos(\mu\mathfrak{c}/2) + 2\sin(\mu\mathfrak{c}/2)\tau_i$, where τ_i are related to Pauli spin matrices as $\tau_i = -i\sigma_i/2$ and μ is related to the length of the edge over which holonomy is evaluated. The algebra generated by holonomies is that of almost periodic functions of $\mathfrak{c}$ with elements of the form: $\exp(i\mu\mathfrak{c}/2)$. On quantization, though holonomies have well-defined quantum operators, there are no quantum operators for $\mathfrak{c}$ in LQC (as in LQG). The kinematical Hilbert space in LQC is $\mathcal{H} = L^2(\mathbb{R}_{\rm Bohr}, d\mu_{\rm Bohr})$ where $\mathbb{R}_{\rm Bohr}$ is the Bohr compactification of the real line and $\mu_{\rm Bohr}$ is the Haar measure on it. Note that the Hilbert space is different from the one in Wheeler-DeWitt quantization: $\mathcal{H}_{\rm WDW} = L^2(\mathbb{R}, d\mu)$. The triad and thus the scale factor operator in LQC have a discrete eigenvalue spectrum and quantum constraint, obtained by expressing the classical constraint in terms of holonomies and positive powers of triad and then quantized, in LQC leads to a discrete quantum difference equation whose *all* solutions are nonsingular—another important distinction from the Wheeler-DeWitt theory.

Physical predictions can be extracted from LQC by construction of a physical Hilbert space. By identifying Dirac observables on this space, information about dynamics can be extracted using ideas of emergent time. On constructing coherent states we can then find out the expectation values of Dirac observables and compare the quantum dynamics with the classical one. It turns out that when a flat expanding universe is evolved backward using loop quantum dynamics, instead of ending in big bang singularity it bounces at Planck scale to a contracting branch [12–14].

The coherent states used to analyze the details of quantum dynamics also play an important role in obtaining an effective Hamiltonian description of dynamics governed by the quantum difference equation. This can be done by using methods of geometric formulation of quantum mechanics [19] where one notes that quantum Hilbert space can be regarded as a quantum phase space with a bundle structure. The classical phase space forms the base of this bundle, whereas fibers consist of states with same expectation values of conjugate variables. Horizontal sections of the bundle are isomorphic to the classical phase space. Using coherent states one can then find horizontal sections which are preserved by quantum evolution which then leads us to an effective Hamiltonian with loop quantum modifications [20,21]:

$$\mathcal{C}_{\rm eff} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2} a \sin^2(\bar{\mu}\mathfrak{c}) + \mathcal{C}_{\rm M}. \tag{1}$$

Here $\bar{\mu}$ is the kinematical length of the edge of a square loop which has the area given by the minimum eigenvalue of the area operator in LQG [18] and $\mathcal{C}_{\rm M}$ corresponds to matter Hamiltonian which in general contains modifications due to regularization of the inverse scale factor [10]. These modifications are negligible for large universes and would not be considered in the present work.

The modified Friedmann equation can then be obtained by using the Hamilton's equation:

$$\dot{\mathfrak{p}} = \{\mathfrak{p}, \mathcal{C}_{\rm eff}\} = -\frac{\kappa\gamma}{3}\frac{\partial \mathcal{C}_{\rm eff}}{\partial \mathfrak{c}} \tag{2}$$

in the effective Hamiltonian constraint $\mathcal{C}_{\rm eff} \approx 0$:

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{3}$$

with $\rho_c = \sqrt{3}/(16\pi\gamma^3 G^2 \hbar)$ [18] where $\hbar$ is the Planck constant. Along with the conservation law:

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{4}$$

Eq. (3) provides an effective description of Friedmann dynamics which very well approximates the underlying discrete quantum dynamics and confirms with the picture of bounce which occurs when $\rho = \rho_c$ (of the order of Planck density) [13–15]. In the classical limit $\hbar \to 0$ one has $\rho_c \to \infty$, thus classically nonsingular bounce is absent. Further, for $\rho \ll \rho_c$ the modified Friedmann equation reduces to the standard one. Interestingly, ρ^2 modifications also appear in string inspired braneworld scenarios and it turns out that there exist interesting dualities between the two frameworks [17]. However, such modifications in braneworlds usually appear with a positive sign and a bounce is absent (unless one assumes the existence of two timelike dimensions [22]). We will now address the issue of future singularities in the effective dynamics of LQC. Our treatment of matter which leads to such singularities would be phenomenological and at an effective level (as in Ref. [23]).

III. AVOIDANCE OF FUTURE SINGULARITIES

For the analysis of the fate of future singularities in LQC, it is useful to first obtain the rate of change of Hubble parameter from Eqs. (3) and (4):

$$\dot{H} = -\frac{\kappa}{2}(1+w)\rho\left(1 - 2\frac{\rho}{\rho_c}\right), \tag{5}$$

where w is the equation of state: $w = p/\rho$ which in general may not be a constant.

It is convenient to define two variables

$$x \equiv \frac{\kappa\rho}{3H^2}, \qquad y \equiv \frac{\rho}{\rho_c}. \tag{6}$$

Then from Eq. (3) we find

$$y = 1 - 1/x. \tag{7}$$

Since H^2 is positive, the variables y and x are in the ranges $0 < y < 1$ and $x > 1$. From Eqs. (4) and (5) we obtain the differential equation for the variable x:

$$\frac{\mathrm{d}x}{\mathrm{d}N} = -3(1+w)x(x-1), \tag{8}$$

where $N \equiv \ln(a)$.

Since H can change the sign, it will be convenient to solve differential equations in terms of a cosmic time t rather than N. Defining two dimensionless quantities $\tilde{t} \equiv H_c t$ and $\tilde{H} \equiv H/H_c$, where $H_c \equiv \sqrt{\kappa\rho_c/3}$, we find that Eqs. (4) and (5) are written as

$$\frac{\mathrm{d}y}{\mathrm{d}\tilde{t}} = -3(1+w)\tilde{H}y, \tag{9}$$

$$\frac{\mathrm{d}\tilde{H}}{\mathrm{d}\tilde{t}} = -\frac{3}{2}(1+w)y(1-2y), \tag{10}$$

together with the constraint equation

$$\tilde{H}^2 = y(1-y). \tag{11}$$

Combining Eqs. (9) and (10) gives

$$\frac{\mathrm{d}^2y}{\mathrm{d}\tilde{t}^2} = \frac{9}{2}(1+w)^2y^2(3-4y) - 3\tilde{H}y\frac{\mathrm{d}w}{\mathrm{d}\tilde{t}}. \tag{12}$$

We will study several equations of state which, in standard Einstein gravity, give rise to various types of future singularities [5]. Our interest is to clarify the role of loop quantum modifications on the following singularities which are known to exist in standard Einstein gravity:

(i) Type I ("big rip"): For $t \to t_s$, $\rho \to \infty$, $|p| \to \infty$, $H \to \infty$ and $a \to \infty$

(ii) Type II ("sudden"): For $t \to t_s$ $\rho \to \rho_s$, $|p| \to \infty$, $H \to H_s$ and $a \to a_s$

(iii) Type III : For $t \to t_s$, $\rho \to \infty$, $|p| \to \infty$, $H \to \infty$ and $a \to a_s$.

Here t_s, ρ_s, H_s and a_s are constants. The type I singularity appears for constant w less than -1 [3]. The type II is a sudden future singularity [4] at which ρ and a are finite but p diverges. The type III appears for the model with $p = -\rho - A\rho^\beta$ with $\beta > 1$ [24]. In what follows we shall study each case separately.

A. Type I singularity

Let us consider a constant equation of state, w. In this case Eq. (8) is easily integrated to give

$$x = \frac{1}{1 - Aa^{-3(1+w)}}, \qquad y = Aa^{-3(1+w)}, \tag{13}$$

where A is a positive constant.

When $w > -1$ the solutions in an expanding universe approach the fixed point $(x, y) = (1, 0)$, which corresponds to the standard Einstein gravity. Meanwhile when $w < -1$ one has $x \to \infty$ and $y \to 1$ with a scale factor satisfying $Aa_c^{-3(1+w)} = 1$. In this case ρ approaches a constant value ρ_c as $a \to a_c$. From Eq. (3) we find that the Hubble parameter becomes zero at this point. This equation [or equivalently Eq. (11)] also tells us that the Hubble parameter $\tilde{H}$ varies between its maximum and minimum values given by $\tilde{H}_{\max} = 1/2$ and $\tilde{H}_{\min} = -1/2$ respectively. We also notice from Eq. (10) that $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} < 0$ at the time when y becomes $y = 1$, leading to the decrease of $\tilde{H}$. Equation (9) tell us that y begins to decrease after it has reached its maximum value $y = 1$ corresponding to $\tilde{H} = 0$.

We can now understand the fate of the universe for $w < -1$ in LQC. A qualitative description of the evolution can be obtained by using Eqs. (9)–(11). Let us begin to examine the evolution with a positive initial value of $\tilde{H}$. From Eqs. (9) and (10), we find that both y and $\tilde{H}$ grow until y reaches $y = 1/2$. Such a behavior of ρ and H is generic to a phantom dominated universe with constant w in the standard FRW cosmology which then leads to a big rip. The LQC correction changes this cosmic evolution in a crucial manner. The Hubble rate begins to decrease after it reaches a maximum value at $y = 1/2$, whereas y continues to grow until $\tilde{H}$ drops below zero [see Eqs. (9) and (10)]. As explained above, the variable y starts to decrease with $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} < 0$ after it has reached its maximum value $y = 1$. When y becomes smaller than $1/2$, $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t}$ changes its sign after which $\tilde{H}$ (< 0) increases toward 0. This stage corresponds to a recollapsing universe that asymptotically approaches the fixed point $(x, y) = (1, 0)$. From Eq. (12) together with Eq. (11) we find that the asymptotic behavior is given by $y \propto t^{-2}$ and $H \propto -t^{-1}$. Note that for negative $\tilde{H}$ the fixed point $(x, y) = (1, 0)$ is stable against perturbations as can be checked by linearly perturbing the system

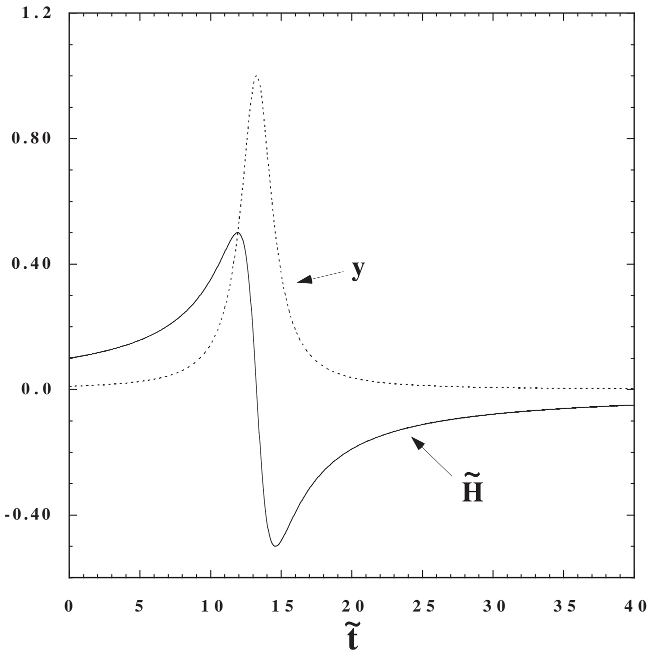

FIG. 1. Evolution of the Hubble parameter and the variable $y = \rho/\rho_c$ for $w = -1.5$ with initial conditions $y_i = 0.01$ and $\tilde{H}_i = [y_i(1-y_i)]^{1/2}$. The big rip singularity is avoided in the presence of loop quantum modifications to the Friedmann dynamics.

(9) and (10). In fact two eigenvalues of a matrix for perturbations [25] are 0 and $-3(1+w)\tilde{H}$, where the latter is negative for $\tilde{H} < 0$.

We have numerically solved Eqs. (9) and (10) for $w < -1$ with initial conditions $\tilde{H} > 0$. In Fig. 1 we plot an example for the evolution of $\tilde{H}$ and y when $w = -1.5$. Our numerical results clearly confirm the qualitative behavior of the evolution presented above.

Thus we have shown the big rip singularity is beautifully avoided in the framework of LQC. The solutions finally approach a contracting universe in standard Einstein gravity ($H \to 0$ and $\rho \to 0$ as $t \to \infty$). We note that when $w > -1$ bouncing solutions can be obtained if $H < 0$ initially [12–15].

B. Type II singularity

In standard Einstein gravity the type II singularity appears when the pressure density p diverges as ρ approaches some constant value ρ_0. For example, this is realized when p is given by [5]

$$p = -\rho - \frac{B}{(\rho_0 - \rho)^\gamma}, \tag{14}$$

where B, ρ_0, and γ are positive constants. This singularity appears at a finite time as ρ approaches ρ_0.

Let us consider the cosmological dynamics in the presence of the loop correction. The equation of state is now dependent on ρ, i.e., $w = -1 - B/\rho(\rho_0 - \rho)^\gamma$. Substituting this expression for Eq. (8) by using the relation (7), we get

$$\frac{\mathrm{d}x}{\mathrm{d}N} = \tilde{B}\frac{x^2}{(r - 1 + 1/x)^\gamma}, \tag{15}$$

where $\tilde{B} \equiv 3B/\rho_c^{1+\gamma}$ and $r \equiv \rho_0/\rho_c$. Integrating this equation gives

$$\left(r - 1 + \frac{1}{x}\right)^{\gamma+1} = \left(r - 1 + \frac{1}{x_i}\right)^{\gamma+1} - \tilde{B}(\gamma+1)N, \tag{16}$$

where we chose the initial condition $x = x_i$ at $N = 0$. This shows that x gets larger with the increase of N, in which case $y = \rho/\rho_c$ grows from Eq. (7). The solutions approach $x \to \infty$ and $y \to 1$ provided that ρ does not pass the singularity at $\rho = \rho_0$ before reaching $\rho = \rho_c$.

When $\rho_0 > \rho_c$ the system reaches $\rho = \rho_c$ with a finite time N_c satisfying $\tilde{B}(\gamma+1)N_c = (r - 1 + 1/x_i)^{\gamma+1} - (r-1)^{\gamma+1}$. The Hubble parameter vanishes at this point, since $H^2 = \kappa\rho/3x$ from Eq. (6). From Eqs. (9) and (10) the differential equations for y and H are given by

$$\frac{\mathrm{d}y}{\mathrm{d}\tilde{t}} = \frac{\tilde{B}}{(r-y)^\gamma}\tilde{H}, \qquad \frac{\mathrm{d}\tilde{H}}{\mathrm{d}\tilde{t}} = \frac{\tilde{B}(1-2y)}{2(r-y)^\gamma}. \tag{17}$$

When $y = 1$ one has $\mathrm{d}y/\mathrm{d}\tilde{t} = 0$ and $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} < 0$. Then the Hubble parameter becomes negative, which is accompanied by the decrease of y. From Eq. (17) we find $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} > 0$ for $y < 1/2$, during which $\tilde{H}$ increases. In the type I case y and $\tilde{H}$ asymptotically approach zero with time-dependence $y \propto t^{-2}$ and $H \propto -t^{-1}$. The type II case is different because of a time-dependent equation of state. In fact when $\tilde{H} = 0$ and $y = 0$ we find $\mathrm{d}y/\mathrm{d}\tilde{t} = 0$ and $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} > 0$. This behavior is clearly seen in Fig. 2. Both $\tilde{H}$ and y increase after the system passes the point $\tilde{H} = 0$ and $y = 0$, which is followed by the maximum value of $\tilde{H}$ at $y = 1/2$. After that the evolution of the universe mimics the previous one, namely $\tilde{H}$ oscillates between $-1/2$ and $1/2$ together with the oscillation of y between 0 and 1. Hence the universe repeats the cycle of expansion and contraction without reaching any singularities (see Fig. 2).

When $\rho_0 < \rho_c$, independently of $\rho > \rho_0$ or $\rho < \rho_0$, the solutions reach the sudden future singularity at $\rho = \rho_0$ in a finite time.

C. Type III singularity

The type III singularity appears for the model

$$p = -\rho - C\rho^\beta, \qquad \beta > 1, \tag{18}$$

where C is a positive constant. Integrating Eq. (4) for the equation of state (18), we find that the scale factor is given by

$$a = a_0 \exp\left(\frac{\rho^{1-\beta}}{3C(1-\beta)}\right), \tag{19}$$

where a_0 is a constant. In Einstein gravity one has $\rho \to \infty$

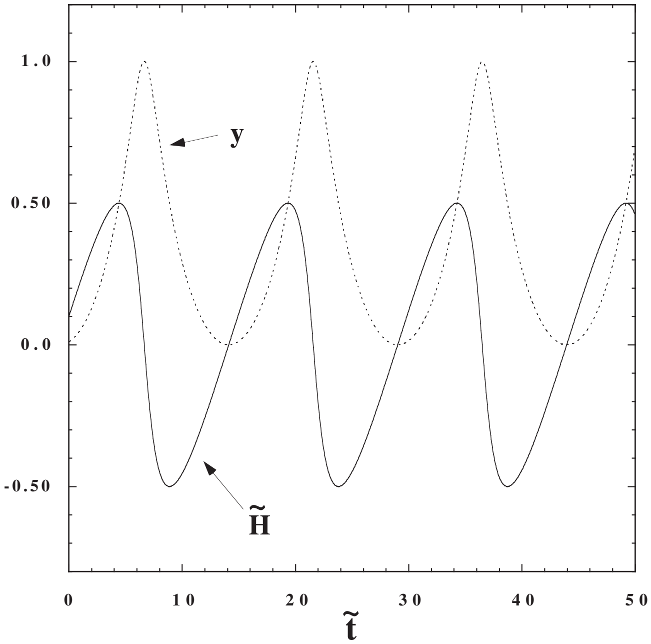

FIG. 2. Evolution of the Hubble parameter and the variable $y = \rho/\rho_c$ for the model (14) with $\tilde{B} = 1$, $r = \rho_0/\rho_c = 2$ and $\gamma = 2$. We choose initial conditions $y_i = 0.01$ and $\tilde{H}_i = [y_i(1 - y_i)]^{1/2}$. The Hubble parameter $\tilde{H}$ oscillates between $-1/2$ and $1/2$ without reaching the singularity at $\rho = \rho_0$.

and $|p| \to \infty$ in a finite time, but a is finite when $\beta > 1$. Hence this is different from the big rip singularity at which scale factor diverges.

In LQC Eq. (8) gives

$$\frac{\mathrm{d}x}{\mathrm{d}N} = \tilde{C}x^2\left(1 - \frac{1}{x}\right)^\beta, \tag{20}$$

where $\tilde{C} \equiv 3C\rho_c^{\beta-1}$. This is integrated as

$$\left(\frac{1}{1 - 1/x}\right)^{\beta-1} = \left(\frac{1}{1 - 1/x_i}\right)^{\beta-1} - \tilde{C}(\beta - 1)N. \tag{21}$$

Then we get $x \to \infty$, $y \to 1$ and $H \to 0$ as $N \to N_c$, where N_c is given by $\tilde{C}(\beta - 1)N_c = (1/(1 - 1/x_i))^{\beta-1} - 1$. The differential equations (9) and (10) are

$$\frac{\mathrm{d}y}{\mathrm{d}\tilde{t}} = \tilde{C}\tilde{H}y^\beta, \qquad \frac{\mathrm{d}\tilde{H}}{\mathrm{d}\tilde{t}} = \frac{\tilde{C}}{2}y^\beta(1 - 2y). \tag{22}$$

Hence one has $\mathrm{d}y/\mathrm{d}\tilde{t} = 0$ and $\mathrm{d}\tilde{H}/\mathrm{d}\tilde{t} < 0$ for $y = 1$ and $\tilde{H} = 0$, which is followed by the decrease of y and $\tilde{H}$ (< 0). The evolution of the system is similar to what we discussed in the type I case. After the Hubble rate reaches a minimum at $y = 1/2$, $\tilde{H}$ and y asymptotically approach $\tilde{H} = y = 0$. When $y \ll 1$, in fact, we have $\mathrm{d}^2y/\mathrm{d}\tilde{t}^2 \approx (3\tilde{C}^2/2)y^{2\beta}$ from Eq. (12), which gives $y \propto t^{-2/(2\beta-1)}$ and $\tilde{H} \propto -t^{-1/(2\beta-1)}$. Hence the final attractor is a contracting universe with $\rho \to 0$, $p \to 0$ and $H \to -0$ as $t \to \infty$.

IV. CONCLUSIONS

In this paper we have studied the avoidance of future singularities using the effective dynamics of loop quantum cosmology. Nonperturbative quantum effects give rise to a ρ^2 correction whose effect depends upon the ratio ρ/ρ_c, where ρ_c is of order of Planck density. Typically this type of correction is thought to be important only in early universe whose energy density is close to ρ_c, but it can be also important in the future universe if (phantom) dark energy is present as observations suggest. Note that the modifications we studied are different from those given which emerge from the regularization of inverse scale factor operator in LQC and can be important below a critical scale factor a_* (see for e.g., Ref. [26]). These corrections are negligible for $a \gg a_*$ and are not considered in the present work which deals with late time expansion dynamics in LQC.

There are several types of future singularities which appear in standard Einstein gravity. In the type I case where ρ, p, and a diverge in a finite time and in the type III case where ρ and p are infinite but a is finite in a finite time, we find that the loop quantum modifications generically remove these singularities. The universe transits from an expanding branch to a contracting branch after the energy density approaches critical value ρ_c. After the Hubble parameter reaches a negative minimum when $\rho/\rho_c = 1/2$, it increases toward a stable fixed point $H = 0$ in an infinite time (see Fig. 1). The fate of the universe thus dramatically changes on considering loop quantum modifications in the standard Friedmann dynamics.

In the type II case where p diverges but ρ (ρ_0) and a are finite in a finite time, sudden singularity is not removed when ρ_0 is smaller than ρ_c. When $\rho_0 > \rho_c$, however, the Hubble parameter H exhibits an oscillation around $H = 0$ (see Fig. 2). This corresponds to an oscillating universe without any singularities.

We have thus shown that in most cases the future singularities are avoided because of the presence of loop quantum corrections. Our analysis of the resolution of singularities clearly reflects the important role played by nonperturbative quantum gravity modifications in order to fully understand the dynamics of universe around the Planck energy.

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[1] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); V. Sahni, Lect. Notes Phys. **653**, 141 (2004); S. M. Carroll, Living Rev. Relativity **4**, 1 (2001); T. Padmanabhan, Phys. Rep. **380**, 235 (2003); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); E. J. Copeland, M. Sami, and S. Tsujikawa, hep-th/0603057.

[2] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002).

[3] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).

[4] J. D. Barrow, Classical Quantum Gravity **21**, L79 (2004).

[5] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005).

[6] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); A. Vilenkin, Phys. Rev. D **30**, 509 (1984).

[7] I. Antoniadis, J. Rizos, and K. Tamvakis, Nucl. Phys. **B415**, 497 (1994); M. Gasperini, M. Maggiore, and G. Veneziano, Nucl. Phys. **B494**, 315 (1997); R. Brustein and R. Madden, Phys. Rev. D **57**, 712 (1998); P. Kanti, J. Rizos, and K. Tamvakis, Phys. Rev. D **59**, 083512 (1999); S. Foffa, M. Maggiore, and R. Sturani, Nucl. Phys. **B552**, 395 (1999); C. Cartier, E. J. Copeland, and R. Madden, J. High Energy Phys. 01 (2000) 035; S. Tsujikawa, R. Brandenberger, and F. Finelli, Phys. Rev. D **66**, 083513 (2002); J. C. Fabris, R. G. Furtado, N. Pinto-Neto, and P. Peter, Phys. Rev. D **67**, 124003 (2003); S. Tsujikawa, Classical Quantum Gravity **20**, 1991 (2003).

[8] M. Sami, A. Toporensky, P. V. Tretyakov, and S. Tsujikawa, Phys. Lett. B **619**, 193 (2005); S. Nojiri, S. D. Odintsov, and M. Sasaki, Phys. Rev. D **71**, 123509 (2005); G. Calcagni, S. Tsujikawa, and M. Sami, Classical Quantum Gravity **22**, 3977 (2005); P. Tretyakov, A. Toporensky, Y. Shtanov, and V. Sahni, Classical Quantum Gravity **23**, 3259 (2006); I. P. Neupane, hep-th/0602097; S. Nojiri, S. D. Odintsov, and M. Sami, Phys. Rev. D **74**, 046004 (2006); S. Nojiri and S. D. Odintsov, hep-th/0601213.

[9] T. Thiemann, Lect. Notes Phys. **631**, 41 (2003); A. Ashtekar and J. Lewandowski, Classical Quantum Gravity **21**, R53 (2004); C. Rovelli, *Quantum Gravity*, Cambridge Monographs on Mathematical Physics (Cambridge Univ. Press, Cambridge, England, 2004).

[10] M. Bojowald, Living Rev. Relativity **8**, 11 (2005).

[11] M. Bojowald, Phys. Rev. Lett. **86**, 5227 (2001); A. Ashtekar, M. Bojowald, and J. Lewandowski, Adv. Theor. Math. Phys. **7**, 233 (2003); M. Bojowald, G. Date, and K. Vandersloot, Classical Quantum Gravity **21**, 1253 (2004); P. Singh and A. Toporensky, Phys. Rev. D **69**, 104008 (2004); G. Date, Phys. Rev. D **71**, 127502 (2005).

[12] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. Lett. **96**, 141301 (2006).

[13] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. D **73**, 124038 (2006).

[14] A. Ashtekar, T. Pawlowski, and P. Singh, gr-qc/0607039.

[15] P. Singh, K. Vandersloot, and G. V. Vereshchagin, gr-qc/0606032.

[16] M. Bojowald, Phys. Rev. Lett. **95**, 061301 (2005); M. Bojowald, R. Goswami, R. Maartens, and P. Singh, Phys. Rev. Lett. **95**, 091302 (2005); A. Ashtekar and M. Bojowald, Classical Quantum Gravity **23**, 391 (2006); R. Goswami, P. S. Joshi, and P. Singh, Phys. Rev. Lett. **96**, 031302 (2006).

[17] P. Singh, Phys. Rev. D **73**, 063508 (2006).

[18] A. Ashtekar, gr-qc/0605011.

[19] A. Ashtekar and T. Schilling, gr-qc/9706069.

[20] J. Willis, Ph.D. Thesis, Penn State, 2004; A. Ashtekar, M. Bojowald, and J. Willis, "*Corrections to Friedmann Equations Induced by Quantum Geometry*," IGPG report, 2004.

[21] P. Singh and K. Vandersloot, Phys. Rev. D **72**, 084004 (2005).

[22] Y. Shtanov and V. Sahni, Phys. Lett. B **557**, 1 (2003).

[23] P. Singh, Classical Quantum Gravity **22**, 4203 (2005).

[24] H. Stefancic, Phys. Rev. D **71**, 084024 (2005).

[25] E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998); B. Gumjudpai, T. Naskar, M. Sami, and S. Tsujikawa, J. Cosmol. Astropart. Phys. 06 (2005) 007.

[26] M. Bojowald, Phys. Rev. Lett. **89**, 261301 (2002); S. Tsujikawa, P. Singh, and R. Maartens, Classical Quantum Gravity **21**, 5767 (2004); J. E. Lidsey, D. J. Mulryne, N. J. Nunes, and R. Tavakol, Phys. Rev. D **70**, 063521 (2004).