Probing cosmic acceleration beyond the equation of state: Distinguishing between dark energy and modified gravity models

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If general relativity is the correct theory of physics on large scales, then there is a differential equation that relates the Hubble expansion function, inferred from measurements of angular diameter distance and luminosity distance, to the growth rate of large scale structure. For a dark energy fluid without couplings or an unusual sound speed, deviations from this consistency relationship could be the signature of modified gravity on cosmological scales. We propose a procedure based on this consistency relation in order to distinguish between some dark energy models and modified gravity models. The procedure uses different combinations of cosmological observations and is able to find inconsistencies when present. As an example, we apply the procedure to a universe described by a recently proposed 5-dimensional modified gravity model. We show that this leads to an inconsistency within the dark energy parameter space detectable by future experiments.

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I. INTRODUCTION

Several cosmological observations, e.g. [1–8], have demonstrated that the expansion of the universe has entered a phase of acceleration. The cosmic acceleration and the questions associated with it constitute one of the most important and challenging problems for several fields of physics (astrophysics, gravitation, high energy physics, and fundamental physics), e.g. [9–14]. A dark energy component that represents $2/3$ of the entire energy content of the universe has been proposed to explain cosmic acceleration [15–18]. This dark energy component can lead to repulsive gravity because of its negative equation of state.

Besides dark energy, several models based on modifications to the gravity sector have been proposed recently as alternatives to explain the cosmic acceleration, e.g. [19–21].

These two families of models, dark energy and modified gravity, are fundamentally different, and a question of major importance is to distinguish between the two possibilities using cosmological data. In this paper, we propose a procedure that uses different pairs of cosmological observations in order to address this question. The procedure is able to find inconsistencies in the dark energy parameter space due to an underlying modified gravity model.

Furthermore, a burning question that comes to mind after an equation of state of dark energy has been determined from cosmological observations is as follows: Is this really a dark energy equation of state or just a results obtained because one tried to fit a dark energy model on the top of some modified gravity model? The procedure proposed also addresses this question.

The outline of the paper is as follows. In Sec. II, we introduce the basic idea of the consistency test. Next, in Sec. III, we recall some of the commonly used parametrizations of dark energy and also an example of modified gravity models. In Sec. IV, we discuss constraints on the expansion history while in Sec. V we discuss constraints on the growth rate of large scale structure. Section VI describes our approach. In Sec. VII, we describe our implementation of the procedure using cosmological observations and present our results. We discuss our results and conclude in the last section.

II. GENERAL RELATIVITY CONSISTENCY RELATIONSHIP

Einstein's equations of general relativity relate the curvature of spacetime to the matter and energy content of the universe, thus describing the cosmological dynamics. Cosmic acceleration affects both the expansion history of the universe (given by the Hubble function $H(z) \equiv \frac{1}{a} \times \frac{da}{dt}|_{a=(z+1)^{-1}}$, where a is the scale factor and z is the redshift) and the rate at which clusters of galaxies grow (given by the growth rate of large scale structure, $D = \delta(a)/\delta(a_i)$, where $\delta(a) \equiv \frac{\rho-\bar{\rho}}{\bar{\rho}}$ is the overdensity).

Importantly, the expansion function must be consistent with the growth rate function via Einstein's equations. If the cosmic acceleration is not due to a dark energy component in Einstein's equations then the presence of significant deviations from the consistency relation between the expansion and the growth rate will be a symptom of a breakdown of general relativity at cosmological scales and a hint to possible modified gravity models at very large scales.

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In this paper, the consistency test is implemented using specific combinations of simulated future cosmological observations.

III. DARK ENERGY MODELS VERSUS MODIFIED GRAVITY

A. Dark energy parametrization

We assume that dark energy can be described in terms of a cosmic fluid, with an equation of state $w = P/\rho$ that can vary with redshift. We parametrize the variation of $w(z)$ using:

(i) $w(z) = w_0 + w_1 z$ if $z < 1$ and $w(z) = w_0 + w_1$ otherwise, e.g. [4,22], and also the Taylor expansion

(ii) $w(a) = w_0 + w_a(1 - a) + w_b(1 - a)^2$, e.g. [23,24].

Given $w(z)$, the dark energy density as a function of redshift can be described using the dimensionless function

$$\mathcal{Q}(z) \equiv \rho_{\rm de}(z)/\rho_{\rm de}(0) = \exp 3\int_0^z \frac{1 + w(z'))dz'}{1 + z'}. \quad (1)$$

We assume a spatially flat universe and do not include massive neutrinos or isocurvature perturbations. For supernova (SN Ia) calculations, we use the parameters

$$p^\alpha = \{\Omega_\Lambda, w_0, w_1, \mathcal{M}\}. \quad (2)$$

For weak gravitational lensing (WL) calculations, we use:

$$p^\alpha = \{\Omega_m h^2, \Omega_\Lambda, w_0, w_1, n_s, \sigma_8^{\rm lin}, z_p, \xi_s, \xi_r\}. \quad (3)$$

For CMB, we use:

$$p^\alpha = \{\Omega_m h^2, \Omega_b h^2, \Omega_\Lambda, w_0, w_1, n_s, \sigma_8^{\rm lin}, \tau\} \quad (4)$$

where $\Omega_{\rm de} \equiv \rho_{\rm de}(0)/\rho_{\rm crit}(0)$ is the dark energy density fraction, and $\rho_{\rm crit}(0) \equiv 3H_0^2/8\pi G$; $\Omega_m h^2$ is the physical matter density; $\Omega_b h^2$ is the physical baryon density; τ is the optical depth to the surface of last scattering; $\sigma_8^{\rm lin}$ is the amplitude of linear fluctuations; n_s is the spectral index of the primordial power spectrum; z_p is the characteristic redshift of source galaxies for weak lensing [25,26]; ξ_s and ξ_r are the lensing absolute and relative calibration parameters as defined in [25].

B. Modified gravity models

In this paper, we are interested in using a good example of modified gravity model but only in order to illustrate the procedure proposed. We are not interested here in evaluating any particular model of modified gravity.

Although several models of modified gravity have been proposed recently as alternatives to dark energy, we chose in this analysis to use the Dvali-Gabadadze-Porrati (DGP) model. The model is inspired by higher dimensional physics and is consistent with current observations, e.g. [27,28]. As mentioned, our interest is to use the DGP model as an example of deviation from general relativity in order to test the proposed procedure. We refer the reader to studied dedicated to the DGP model phenomenology [27–30]

We provide in this subsection a brief introduction to the model, but we refer the reader to [19,31] for a full description. The action for the five-dimensional theory is [19,31]

$$S_{(5)} = \frac{1}{2}M_{(5)}^3 \int d^4x dy \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{2}M_{(4)}^2 \int d^4x \sqrt{-g_{(4)}} R_{(4)} + S_{\rm matter}, \quad (5)$$

where the subscripts 4 and 5 denote quantities on the brane and in the bulk, respectively; $M_{(5)}$ is the five-dimensional reduced Planck mass; $M_{(4)} = 2.4 \times 10^{18}$ GeV is the four dimensional effective reduced Planck mass; R and g are the Ricci scalar and the determinant of the metric, respectively. The first and second terms on the right hand side describe the bulk and the brane, respectively, while $S_{\rm matter}$ is the action for matter confined to the brane. The two different prefactors $M_{(5)}^3/2$ and $M_{(4)}^2/2$ in front of the bulk and brane actions give rise to a characteristic length, scale [32]

$$r_c = M_{(4)}^2/2M_{(5)}^3. \quad (6)$$

If $M_{(5)}$ is much less than $M_{(4)}$, then the brane terms in the action above will dominate over the bulk terms on scales much smaller than r_c, and gravity will appear four dimensional. For example, nonrelativistic small-scale gravity will obey the Newtonian inverse-square force law. On scales larger than r_c, the full five-dimensional physics will be recovered, and the gravitational force law will revert to its five-dimensional $1/r^3$ form. This is usually discussed in terms of gravity leakage into an extra dimension. Reference [33] shows that tuning $M_{(5)}$ to about 10–100 MeV, implying $r_c \sim H_0^{-1}$, is consistent with cosmological data. They have also been discussed in [34]. Low redshift cosmology in DGP brane worlds was studied in [31,32]. Defining the effective energy density

$$\rho_{r_c} \equiv \frac{3}{(32\pi G r_c^2)}, \quad (7)$$

Friedmann's first equation becomes

$$H_{\rm DGP}^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\sqrt{\rho + \rho_{r_c}} + \sqrt{\rho_{r_c}})^2. \quad (8)$$

For each perfect fluid i on the brane, we may define $\Omega_i \equiv \rho_{i,0}/\rho_{\rm crit,0}$. Following [31], we also define $\Omega_{r_c} \equiv \frac{1}{4} r_c^{-2} H_0^{-2}$. We focus here on a flat universe containing only nonrelativistic matter, such as baryons and cold dark matter, in which $\Omega_{r_c} = (\frac{1-\Omega_m}{2})^2$. In this model, the gravitational "leakage" into the fifth dimension, on large length scales, becomes a substitute for dark energy.

IV. CONSTRAINTS FROM THE EXPANSION HISTORY USING SUPERNOVAE OF TYPE IA

For a spatially flat universe with dark energy, the Hubble parameter $H(z)$ can be expressed in terms of its current value H_0 and the function $\mathcal{Q}(z)$ as

$$H_{\rm De}(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)\mathcal{Q}(z)}. \quad (9)$$

Since probes such as the SN Ia surveys give information about the redshift variation of $H(z)$ but not its current value, we define the dimensionless Hubble parameter for dark energy models as

$$\mathcal{H}_{\rm De}(z) \equiv H(z)/H_0 = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)\mathcal{Q}(z)}. \quad (10)$$

For spatially flat DGP model, the expansion is given by

$$\mathcal{H}_{\rm DGP}(z) = \tfrac{1}{2}(1-\Omega_m) + \sqrt{\tfrac{1}{4}(1-\Omega_m)^2 + \Omega_m(1+z)^3}. \quad (11)$$

To probe the expansion history, we consider calibrated type Ia supernovae as standard candles to measure the luminosity distance as a function of the redshift. Recall that the SN Ia apparent magnitude as a function of redshift is given by

$$m(z) = 5\log_{10}(D_L(z)) + \mathcal{M}. \quad (12)$$

Here, $\mathcal{M}$ depends on H_0 as well as the absolute magnitude of type Ia supernovae. We treat $\mathcal{M}$ as a nuisance parameter. Meanwhile, the dimensionless luminosity distance is

$$D_L(z) = (1+z)\int_0^z dz'/\mathcal{H}(z'). \quad (13)$$

As is well known, $m(z)$ depends on w_0 and w_1 only through a complicated integral relation, which ''smears out'' dark energy information [35]. The result is a degeneracy which allows very different models to have nearly identical $m(z)$. Figure 1 shows $m(z)$ for a few dark energy and DGP cosmologies. The figure shows that some DGP models are very degenerate with certain dark energy models, such as ΛCDM models or SUGRA-inspired models. Evident in the expression for $m(z)$ is another degeneracy, between Ω_m and the equation of state parameters. We break this degeneracy by combining SN Ia data with the CMB power spectrum, Fig. 3. We assume the availability of a sample of 2000 supernovae, evenly distributed in redshift between $z_{\rm min} = 0$ and $z_{\rm max} = 1.7$, with a magnitude
uncertainty per supernova of $\sigma_m = 0.2$. As in [22,36], we also include a peculiar velocity uncertainty $\sigma_v = 500$ km/s, and a systematic uncertainty $\delta m = 0.02$ in bins of $\Delta z = 0.1$.

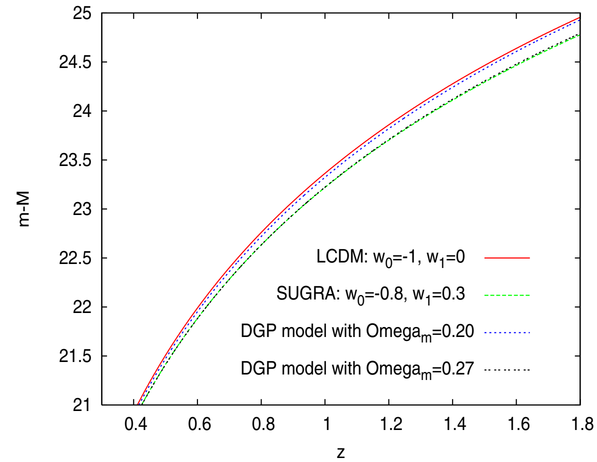

FIG. 1 (color online). Supernova Hubble diagrams for several dark energy and DGP models. Note that the ΛCDM model (red solid line) and the $\Omega_m = 0.20$ DGP model (blue dotted line) have nearly identical Hubble diagrams, but different growth factors as shown in Fig. 2. The same is true of the SUGRA (green dashed line) and $\Omega_m = 0.27$ DGP (black double dotted line) models.

V. CONSTRAINTS FROM THE GROWTH OF LARGE SCALE STRUCTURE AND GRAVITATIONAL LENSING TOMOGRAPHY

For dark energy models, the suppression of the growth of large scale structure is due to an increase in the ratio of dark energy density to matter density. We calculate the growth factor for these models by numerical integration of the differential equation from [37,38],

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1+X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1-w(a)}{1+X(a)}\frac{G}{a^2} = 0, \quad (14)$$

where $' \equiv d/da$, $G = D/a$ is the normalized growth factor,

$$X(a) = \frac{\Omega_m}{(1-\Omega_m)a^3\mathcal{Q}(a)}, \quad (15)$$

and $\mathcal{Q}(a)$ is as given earlier.

For DGP models, the suppression of the growth is due to weakened gravity resulting from leakage into the extra dimension. Following [34,39], we use

$$\ddot{\delta} + 2H_{\rm DGP}\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0 \quad (16)$$

where

$$\beta = 1 - 2r_c H_{\rm DGP}\left(1 + \frac{\dot{H}_{\rm DGP}}{3H_{\rm DGP}^2}\right) \quad (17)$$

Figure 2 shows that, compared to a ΛCDM model with the same matter density, a DGP model has a distinct suppression of the growth factor. Also, Fig. 2 displays how the degenerate models of Fig. 1 show distinct growth factor functions. In the next two sections, we will take advantage

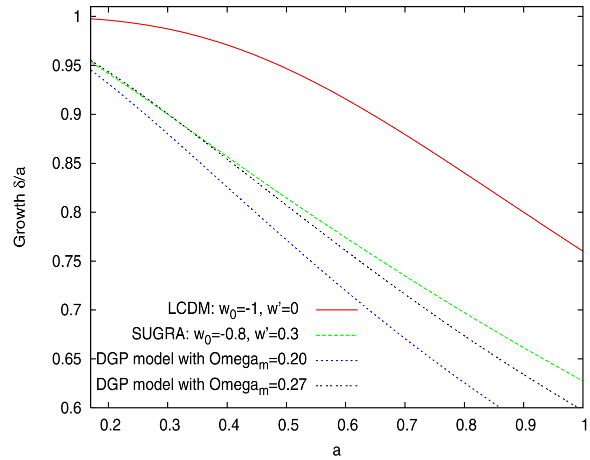

FIG. 2 (color online). Growth factors of linear density perturbations for several dark energy and DGP models. Comparisons among several dark energy and DGP models growth factors of linear density perturbations. Note that the growth factor in the $\Omega_m = 0.27$ DGP model is suppressed with respect to that in the ΛCDM model, which has the same Ω_m.

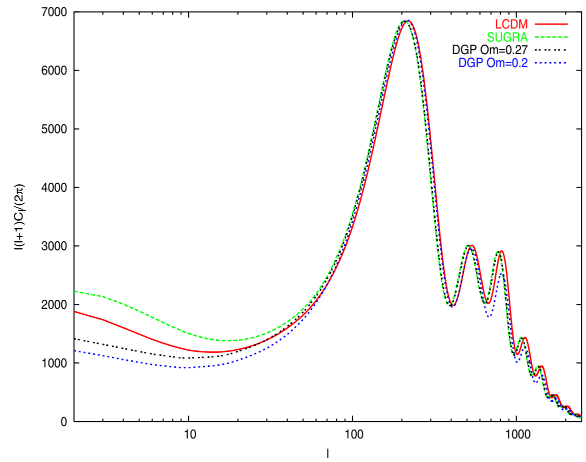

FIG. 3 (color online). CMB power spectra for several dark energy and DGP models: ΛCDM model is in red solid line; SUGRA model is in green dashed line; $\Omega_m = 0.27$ DGP model is in black double dotted line; $\Omega_m = 0.20$ DGP model is in blue dotted line; Following [27], a modified version of CMBFAST [52] was used to generate the DGP plots.

of this difference in order to detect possible signatures of DGP models.

We consider weak gravitational lensing (cosmic shear) tomography as a probe of the growth factor of large scale structure. Indeed, cosmic shear observations capture the effect of the cosmic acceleration on both the expansion and the growth factor. We follow the formalism and conventions as used in [25,36] and we use the convergence power spectrum as our statistic [40–42],

$$P_l^\kappa = \frac{9}{4} H_0^4 \Omega_m^2 \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(\frac{l}{\sin_K(\chi)}, \chi\right) d\chi, \tag{18}$$

where P_{3D} is the $3D$ nonlinear power spectrum of the matter density fluctuation, δ; $a(\chi)$ is the scale factor; and $\sin_K(\chi) = K^{-1/2} \sin(K^{1/2}\chi)$ is the comoving angular diameter distance to χ (for the spatially flat universe used in this analysis, this reduces to χ). The weighting function $g(\chi)$ is the source-averaged distance ratio given by

$$g(\chi) = \int_\chi^{\chi_H} n(\chi') \frac{\sin_K(\chi' - \chi)}{\sin_K(\chi')} d\chi' \tag{19}$$

where $n(\chi(z))$ is the source redshift distribution (we use the distribution [43] $n(z) = \frac{z^2}{2z_0^3} e^{-z/z_0}$, which peaks at $z_p = 2z_0$). We integrate numerically the growth factor function given by Eqs. (14) or (16) above, and we use the linear-to-nonlinear mapping procedure HALOFIT [44]. We used the HALOFIT for the DGP models as well because we are only interested into using them as an example of deviation from general relativity in order to test our procedure. Dedicated studies to DGP phenomenology should consider using a better approximation for these models. As mentioned above, the expansion history is contained in the window function, while the growth factor is contained in the $3D$ nonlinear matter power spectrum. Figure 4 shows convergence power spectra corresponding to dark energy models and modified gravity (DGP) models.

The separation of source galaxies into tomographic bins improves significantly the constraints on cosmological parameters, and particularly those of dark energy [45,46]. The constraints obtained from different bins are complementary, and add up to reduce the final uncertainties on the parameters. Based on recently proposed surveys (e.g. [47]), we use a reference survey with a sky coverage of 10%, a source density $\bar{n} = 100\ \mathrm{gal/arcmin}^2$, a rms intrinsic ellipticity $\langle\gamma_{\rm int}^2\rangle^{1/2} \approx 0.25$, and a median redshift $z_{\rm med} = 1.5$. We assume a reasonable photometric redshift uncertainty

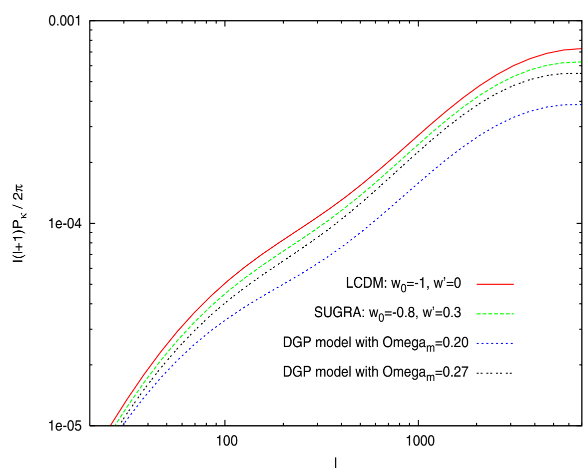

FIG. 4 (color online). Lensing convergence power spectra for several dark energy and DGP models: ΛCDM model is in red solid line; SUGRA model is in green dashed line; $\Omega_m = 0.27$ DGP model is in black double dotted line; $\Omega_m = 0.20$ DGP model is in blue dotted line;

of $\sigma(z) = 0.05$, and we split the sources into 10 tomographic bins over the range $0.0 < z < 3.0$ with intervals of $\Delta z = 0.3$. We have used $\ell_{\max} = 3000$, since the assumption of a Gaussian shear field, underlying the Fisher formalism and the HALOFIT approximation, may not be valid for larger ℓs. For the minimum ℓ, we take the fundamental mode approximation, i.e., we consider only lensing modes for which at least one wavelength can fit inside the survey area.

VI. THE FUNDAMENTALS OF THE CONSISTENCY TEST: CONSTRAINTS FROM THE EXPANSION VERSUS CONSTRAINTS FROM THE GROWTH OF LSS

The cosmic acceleration affects cosmology in two ways: (i) It affects the expansion history of the universe and (ii) It suppresses the growth rate of large scale structure in the universe. The idea that we explore is that, for dark energy models, these two effects must be consistent one with another because they are related by general relativity. The presence of significant inconsistencies between the expansion history and the growth rate would be the signature of some modified gravity models at cosmological scales. We propose a procedure that detects such inconsistencies when they are present.

The approach we explore is as follows. We assume that the true cosmology is described by a DGP model, and then ask what contradictions arise when the data are instead analyzed based on the assumption of a dark energy model (as mentioned earlier, we are not particularly interested in the DGP cosmology here but we want to use it as an example to illustrate the procedure.)

Because we will generate the data using the DGP model, the consistency relation from general relativity between the expansion history and the growth factor of large scale structure will be broken. The dark energy equation of state $w_{\rm exp}(z)$ which best fits measurements of the expansion will not be consistent with the equation of state $w_{\rm growth}(z)$ which best fits measurements the growth.

The methods and steps we use are as follows.

(i) We simulate the data for the expansion and the growth using a fiducial DGP cosmology (for example, supernova and weak lensing data);

(ii) We use a χ^2 minimization method in order to find the best fit dark energy model to measurements of the expansion (the χ^2 minimization method was discussed in detail in [22] and was shown to give similar results to those from Monte Carlo Markov Chain method although the χ^2 minimization can have a better handle on degeneracies [22]);

(iii) we use the χ^2 minimization in order to find the best fit dark energy model to measurements of the growth of structure,

(iv) Next, we use a standard Fisher matrix approach to calculate the confidence regions (or χ^2 contours) around the two best fit dark energy models (this standard procedure is discussed in detail in Refs. [48]);

(v) We look for significant differences between the two effective dark energy parameter spaces. These differences will signal an inconsistency between the expansion and the growth factor. The source of the inconsistency is from point (i) where the data was generated using a DGP model, i.e. from our hypothesis that the true cosmology is that of a modified gravity DGP model.

The basic idea explored here was also discussed in Refs. [34,49] but here we propose to implement the idea using pairs of cosmological data sets. In the next section, we will show how a procedure using observations from supernovae, weak lensing tomography, and the CMB allows one to achieve an observational implementation of the consistency test.

Note that our approach is totally different from that of [50,51]. Their work defined 16 new parameters (additional to the cosmological parameters) describing the distance and growth factor as functions of redshift. A weak lensing survey and CMB power spectra were then used to constrain all 16 parameters simultaneously in order to search for inconsistencies. Our work requires no new parameters and we explore inconsistencies between constraints obtained from different pairs of cosmological probes as we illustrate in the next section.

VII. A PROCEDURE TO DETECT SIGNATURE OF MODIFIED GRAVITY USING [SN IA + CMB] VERSUS [WEAK LENSING + CMB] OBSERVATIONS

We provide here an implementation of the consistency test using simulated cosmological observations. This observational test allows one to distinguish between dark energy models and modified gravity DGP models. We assume the true cosmology to be a DGP model and we analyze two combinations of simulated data sets using dark energy models. We use the methods and steps indicated in the previous section. The procedure we use is as follows.

(i) We use a fiducial DGP model (with $\Omega_m = 0.27$) and generate supernova magnitudes, weak lensing convergence power spectrum, and CMB power spectrum.

(ii) Then, we determine the dark energy model that is the best fit to the supernova magnitudes and to the CMB spectrum generated using the fiducial DGP model.

(iii) Next, we determine the dark energy model that is the best fit to the fiducial weak lensing spectrum and to the CMB spectrum.

(iv) Then, we compare the allowed regions in the $\{\Omega_{\rm de}, w_0, w_1\}$ parameter space between the two

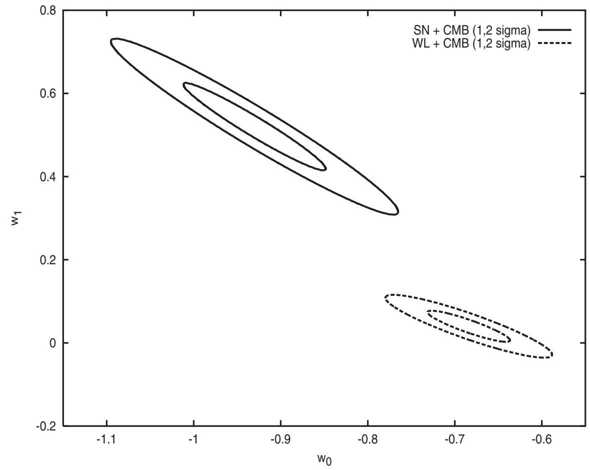

FIG. 5. Equations of state found using two different combinations of data sets. Solid contours are for fits to SN Ia and CMB data, while dashed contours are for fits to weak lensing and CMB data. The significant difference (inconsistency) between the equations of state found using these two combinations is a signature of the DGP model and should be detectable by future experiments described in Secs. IV and V. The inconsistency is an observational detection of the underlying modified gravity DGP model (assumed here to generate the data).

data combinations in order to look for inconsistencies.

The supernova combination contains information on the effect of the acceleration on the expansion history, while the weak lensing combination contains information on the growth factor in addition to the expansion. This translates into a difference between the two allowed regions in the dark energy parameter space.

Our results in Figs. 5–7 show that the two allowed regions in the dark energy parameter space are significantly different. This signals the expected inconsistency between the expansion history and the growth rate of large scale structure.

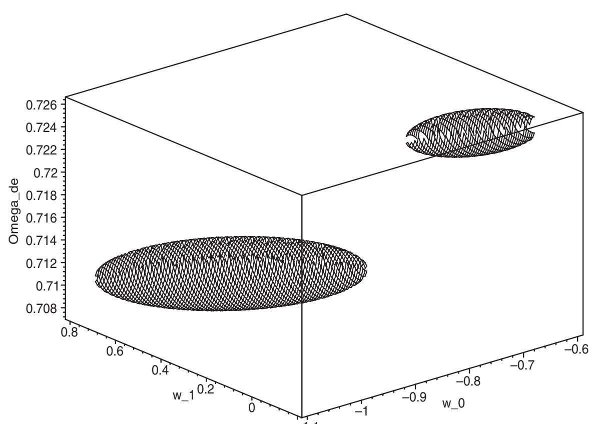

FIG. 6. Dark energy parameter spaces (Ω_{de}, w_0, w_1) found using two different combinations of data sets. The ellipsoid to the left is for fit to SN Ia and CMB data, while ellipsoid to the right is for fits to weak lensing and CMB data. The significant difference (inconsistency) between the parameter spaces found using these two combinations is a signature of the DGP model and should be detectable by future experiments described in Secs. IV and V. The inconsistency is an observational detection of the underlying modified gravity DGP model (assumed here to generate the data).

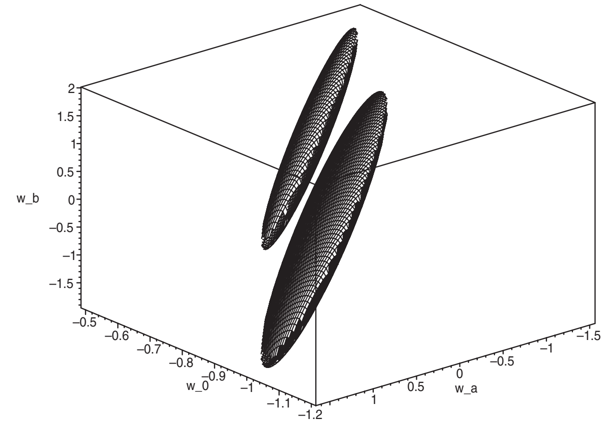

FIG. 7. Three parameter equations of state (w_0, w_a, w_b) (from the Taylor expansion in Sec. III A) found using two different combinations of data sets. The ellipsoid to the right is for fit to SN Ia and CMB data, while the ellipsoid to the left is for fits to weak lensing and CMB data. The inconsistency between the parameter spaces found using these two combinations is a result of the assumed underlying DGP model and should be detectable by future experiments described in Secs. IV and V.

VIII. DISCUSSION

The source of the inconsistency in Figs. 5–7 is that we generated the data using a fiducial DGP model but then we used dark energy models to fit the data. The inconsistency is a consequence of the fact that the fiducial cosmological model is DGP, and therefore it constitutes an observational signature of the underlying modified gravity model. Figure 7 shows that the inconsistency between the two equations of state persists even when we consider a third term in the Taylor expansion of the equation of state (Sec. III A). This indicates that the test is robust to the functional form used for the equation of state.

However, we stress that we did not consider in this study dark energy models with couplings or with an unusual sound speed. The study of the effect of the sound speed of dark energy on the procedure and also the inclusion of other models of dark energy and modified gravity will be considered in subsequent work.

Also, another point worth exploring in future work is the effect of systematic uncertainties. If such a procedure is performed using real data and an inconsistency is found, then one has to develop methods to test whether the inconsistency is due to physics or to systematics in either of the data sets. Further work will be necessary in order to make this type of test robust and generic.

In summary, the comparison of measurements of expansion history and measurements of the growth rate of structure tests the behavior of gravity on large scales. We proposed a procedure that uses different pairs of cosmological data sets in order to explore this comparison and to distinguish between some models of dark energy and modified gravity as the cause of the cosmic acceleration. Being able to distinguish between the two possibilities is an important step in the quest to understand cosmic acceleration.

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