

**Nonsingular bouncing universes in loop quantum cosmology**

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Nonperturbative quantum geometric effects in loop quantum cosmology (LQC) predict a  $\rho^2$  modification to the Friedmann equation at high energies. The quadratic term is negative definite and can lead to generic bounces when the matter energy density becomes equal to a critical value of the order of the Planck density. The nonsingular bounce is achieved for arbitrary matter without violation of positive energy conditions. By performing a qualitative analysis we explore the nature of the bounce for inflationary and cyclic model potentials. For the former we show that inflationary trajectories are attractors of the dynamics after the bounce implying that inflation can be harmoniously embedded in LQC. For the latter difficulties associated with singularities in cyclic models can be overcome. We show that nonsingular cyclic models can be constructed with a small variation in the original cyclic model potential by making it slightly positive in the regime where scalar field is negative.

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**I. INTRODUCTION**

One of the cornerstone problems in cosmology is that of the initial conditions of our universe. In standard big bang cosmology the weight of this problem is shifted to quantum gravity which is expected to generate ideal conditions for the genesis of our universe. Generally, it is envisioned that our universe started in a hot expanding phase from the big bang and entered a stage of inflation before becoming radiation and matter dominated. However the idea that our expanding universe was preceded by a contracting phase of a classical universe has had many avatars. This idea, though promising to eliminate some of the fundamental problems related with initial conditions in standard cosmology and providing an alternative to inflation, has remained unsuccessful due to absence of generic nonsingular evolution from the contracting to the expanding branch. For a flat  $k = 0$  cosmological model, which is of interest of the present work, based on classical general relativity (GR) such an evolution is forbidden by powerful singularity theorems, unless one assumes an exotic form of matter which violates positive energy conditions. Though quantum corrections have been introduced to achieve a nonsingular evolution between contracting and expanding phases in special conditions, it has been realized that a generic nonsingular evolution is difficult to achieve unless one incorporates nonperturbative effects of quantum gravity. In absence of any nonperturbative quantum gravitational modifications to dynamics, innovative ideas like pre

big bang string cosmology [1] and ekpyrotic/cyclic models [2,3] have so far had limited viability [4].

Loop quantum gravity (LQG) is a leading nonperturbative background independent approach to quantize gravity [5]. The underlying geometry in LQG is discrete and the continuum spacetime is obtained from quantum geometry in a large eigenvalue limit. The application of LQG techniques to homogeneous spacetimes results in loop quantum cosmology (LQC) [6] which has led to important insights on the resolution of singularities in various situations [7–9]. Using extensive analytical and numerical methods the analysis of the evolution of semiclassical states for the flat  $k = 0$  model with a massless scalar field has shown that a contracting semiclassical universe passes through the quantum regime bouncing to an expanding semiclassical universe [10–12]. Thus within the context of the model considered the idea of the nonsingular bounce is realized in a natural fashion.

The underlying dynamics in LQC is governed by a discrete quantum difference equation in quantum geometry. However, using semiclassical states we can construct an effective Hamiltonian description on a *continuum* spacetime which has been shown to very well approximate the quantum dynamics [11–13]. In particular, we can obtain effective equations for the modified Friedmann dynamics which can be used to investigate the role of nonperturbative quantum corrections. An important feature of the dynamics is that the classical Friedmann equation is modified to include a  $\rho^2$  term which is relevant in the high energy regime. The modified term in the Friedmann equation is negative definite implying a bounce (or more generally a turnaround) when the energy density reaches a critical value on the order of the Planck density in accordance with the results from the quantum evolution.

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An important question which gains immediate attention is the viability of the bounce picture for more general matter sources and compatibility of inflationary and cyclic model ideas with LQC. In this regard various questions arise: Does the bounce occur for various inflationary potentials? Do inflationary solutions form the attractors of the dynamics after the bounce? Since the modifications which lead to a nonsingular bounce are obtained from nonperturbative quantum gravity effects is it possible that these can lead to construction of nonsingular cyclic models using cyclic/bi-cyclic potentials? What insights are obtained to alleviate the problems of cyclic models?

This work seeks to answer these questions using the effective theory in LQC. The plan of this paper is as follows. In the next section we briefly review the effective theory of LQC and classify the differences between quantum bounce generated by loop quantum dynamics and the classical bounce originating from general relativity. In Sec. IV we perform a qualitative analysis using phase space diagrams to study the dynamics for inflationary potentials ( $\phi^2$  and  $\phi^4$ ) and show that inflationary trajectories are attractors of the dynamics after the bounce. In Sec. V we apply the qualitative analysis to negative potentials of the cyclic/bi-cyclic models. We show that the bi-cyclic potential [14] can successfully lead to nonsingular cycles, whereas the original cyclic potential can exhibit cycling of the scale factor but not the scalar field. A truly cyclic model can be easily constructed if the cyclic potential becomes slightly positive in the regime where the scalar field is negative, as the bi-cyclic potential does. The nonsingular cyclic behavior obtained for negative potentials is unavoidable and occurs for generic values of parameters. We conclude the paper with a discussion and open issues in Sec VI.

## II. EFFECTIVE DYNAMICS IN LOOP QUANTUM COSMOLOGY

LQC is a canonical quantization of homogeneous spacetimes based on techniques used in LQG. As in LQG, the classical phase space for the gravitational sector is denoted by two conjugate variables the connection and the triad, which encode curvature and spatial geometry, respectively. Using symmetries of the homogeneous and isotropic spacetime the dynamical part of the connection is determined by a single quantity labeled  $c$  and likewise the triad is determined by a parameter  $p$ . For the flat  $k = 0$  model (which we will only consider in this work) on classical solutions of general relativity the relations between the new variables and the usual metric variables is

$$c = \gamma \dot{a}, \quad p = a^2, \quad (1)$$

where  $\gamma$  is the Immirzi parameter whose value  $\gamma \approx 0.2375$  is typically constrained by black hole entropy considerations [15].

In the Hamiltonian formulation for homogeneous and isotropic spacetimes the dynamical equations of motion can be determined completely by the Hamiltonian constraint. Under quantization, the Hamiltonian constraint gets promoted to an operator and the quantum wave functions are annihilated by that operator. It is thus at the level of the Hamiltonian constraint that modifications due to LQC will appear and from the modified Hamiltonian constraint, the effective Friedmann constraint will be derived. Classically in terms of the connection-triads variables the Hamiltonian constraint is given by

$$\mathcal{H}_{\text{cl}} = -\frac{3\sqrt{p}}{\kappa\gamma^2}c^2 + \mathcal{H}_{\text{M}}, \quad (2)$$

with  $\kappa = 8\pi G$  (where  $G$  is Newton's gravitational constant) and  $\mathcal{H}_{\text{M}}$  being the matter Hamiltonian. The complete equations of motion are derived from Hamilton's equations  $\dot{x} = \{x, \mathcal{H}_{\text{cl}}\}$  for any phase space variable  $x$ , and by enforcing that  $\mathcal{H}_{\text{cl}}$  should vanish. The variables  $c$  and  $p$  are canonically conjugate with Poisson bracket  $\{c, p\} = \gamma\kappa/3$  the use of which gives us relation  $c = \gamma\dot{a}$  from Hamilton's equation for  $\dot{p}$  together with  $p = a^2$ . Substituting these relations into the Hamiltonian constraint (2) and enforcing that the constraint vanishes then gives the classical Friedmann equation.

The elementary variables used for quantization in LQC are the triads and holonomies of the connection:  $h_i(\mu) = \exp(\mu c \tau_i) = \cos(\mu c/2) + 2\sin(\mu c/2)\tau_i$ . Here  $\tau_i$  is related to Pauli spin matrices as  $\tau_i = -i\sigma_i/2$  and  $\mu$  is the eigenvalue of the triad operator  $\hat{p}$ . The gravitational constraint is expressed in terms of these elementary variables and quantized. On quantization the Hamiltonian constraint leads to a discrete quantum difference equation whose solutions are nonsingular [7,8].

The analysis of the quantum Hamiltonian using semiclassical states belonging to the physical Hilbert space reveals that on backward evolution of our expanding phase of the universe, the universe bounces at a critical density into a contracting branch. By finding expectation values of the Dirac observables we can investigate the differences between quantum and classical dynamics. It turns out that quantum geometric effects become dominant only when the energy density ( $\rho$ ) of the universe becomes of the order of the critical density  $\rho_{\text{crit}}$  [12,16] and classical general relativity is a good approximation to the quantum dynamics when  $\rho \ll \rho_{\text{crit}}$ .

The quantum dynamics of LQC can be well approximated by an effective description. Several proposals have been made to derive this effective theory [13,17–19] and the result is that the dynamics can be described in terms of an effective Hamiltonian constraint which to leading order is [20]

$$\mathcal{H}_{\text{eff}} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2}a\sin^2(\bar{\mu}c) + \mathcal{H}_{\text{M}}, \quad (3)$$

where  $\bar{\mu} = \sqrt{3\sqrt{3}/2|\mu|}$  [12] and  $\mu$  is the eigenvalue of the triad operator  $\hat{p}$  in the quantum theory [8]

$$\hat{p}|\mu\rangle = \frac{8\pi\gamma l_{\text{pl}}^2}{6}\mu|\mu\rangle. \quad (4)$$

Here  $l_{\text{pl}} = \sqrt{\hbar G}$  denotes the Planck length (in units of speed of light equal to unity). In this work we will be mainly interested in the matter Hamiltonians corresponding to a massive scalar field  $\phi$  with momentum  $\Pi_\phi$  and potential  $V(\phi)$

$$\mathcal{H}_M = \frac{1}{2} \frac{\Pi_\phi^2}{p^{3/2}} + p^{3/2}V(\phi). \quad (5)$$

The energy density and pressure for the scalar field are given by the same expressions as those classically, i.e.

$$\rho = \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (6)$$

where we have used Hamilton's equations to get  $\dot{\phi} = \Pi_\phi/p^{3/2}$ . It is then straightforward to find the Klein-Gordon equation using Hamilton's equations for  $\dot{\phi}$  and  $\dot{\Pi}_\phi$ , which turns out to be of the same form as the classical equation and therefore the scalar field satisfies the stress-energy conservation law:

$$\dot{\rho}_\phi + 3\frac{\dot{a}}{a}(\rho_\phi + p_\phi) = 0. \quad (7)$$

The modified Friedmann equation can be found by using Hamilton's equations for  $\dot{p}$

$$\begin{aligned} \dot{p} &= \{p, \mathcal{H}_{\text{eff}}\} = -\frac{\gamma\kappa}{3} \frac{\partial}{\partial c} \mathcal{H}_{\text{eff}} \\ &= \frac{2a}{\gamma\bar{\mu}} \sin(\bar{\mu}c) \cos(\bar{\mu}c), \end{aligned} \quad (8)$$

which on using Eq. (1) implies that the rate of change of the scale factor is given by

$$\dot{a} = \frac{1}{\gamma\bar{\mu}} \sin(\bar{\mu}c) \cos(\bar{\mu}c). \quad (9)$$

Furthermore, the vanishing of the Hamiltonian constraint implies

$$\sin^2(\bar{\mu}c) = \frac{\kappa\gamma^2\bar{\mu}^2}{3a} \mathcal{H}_M. \quad (10)$$

Squaring Eq. (9) and plugging in (10) yields the effective Friedmann equation for the Hubble rate  $H = \dot{a}/a$

$$H^2 = \frac{\kappa}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}}\right), \quad (11)$$

with the critical density given by

$$\rho_{\text{crit}} = \frac{\sqrt{3}}{16\pi^2\gamma^3} \rho_{\text{pl}}. \quad (12)$$

where  $\rho_{\text{pl}} = 1/(\hbar G^2)$  is the Planck density. The nonperturbative quantum geometric effects are manifested in the form of a  $\rho^2$  modification of the Friedmann equation. Since the modified term is negative definite, when  $\rho = \rho_{\text{crit}}$  the Hubble parameter vanishes and the universe experiences a turnaround in the scale factor. For  $\rho \ll \rho_{\text{crit}}$ , the modifications to Friedmann dynamics become negligible, and we recover standard Friedmann dynamics. Note that origin of  $\rho_{\text{crit}}$  is purely quantum, since as  $\rho_{\text{pl}} \propto 1/\hbar$ ,  $\rho_{\text{crit}} \rightarrow \infty$  in the classical limit  $\hbar \rightarrow 0$ .

Interestingly, similar  $\rho^2$  modifications have also been obtained in Randall-Sundrum brane-world scenarios based on string theory but those come with a positive sign in the Friedmann equation and a bounce is not possible. If one assumes the existence of a timelike extra dimension then the sign can become negative as in LQC [21] (for an analysis of dynamics of the brane-world model with two timelike dimensions see Refs. [22,23]). A detailed comparison of the effective dynamics of LQC and brane-world scenarios shows interesting features which include a dual relationship between their scaling solutions [24].

### III. QUANTUM AND CLASSICAL BOUNCES

Having established the equations for the effective Friedmann dynamics we now wish to classify the conditions under which effective dynamics given by the modified Friedmann Eq. (11) implies a turnaround, namely, either a bounce or recollapse. As discussed above when the energy density of the universe reaches the critical value  $\rho_{\text{crit}}$ , the Hubble parameter becomes zero and a bounce/recollapse is expected. Since we will also consider the possibility of the scalar field potential being negative we also discuss the case where the matter energy density vanishes which also leads to a turnaround.

More precisely from Eq. (9) we can determine the conditions under which such a turnaround can occur; i.e., when  $\dot{a} = 0$ . It is clear from that equation that there are two possibilities, namely, when  $\bar{\mu}c = (n + \frac{1}{2})\pi$  and  $\bar{\mu}c = m\pi$  for any integers  $n, m$ . The energy density can be computed for both cases and is equal to  $\rho_{\text{crit}}$  for the former and zero for the latter. We thus label turnarounds occurring due quantum geometric effects when  $\bar{\mu}c = (n + \frac{1}{2})\pi$  as *quantum turnarounds* and those which occur when energy density vanishes for  $\bar{\mu}c = m\pi$  as *classical turnarounds*.

To determine whether or not a bounce or recollapse occurs we need to go further and calculate the second time derivative of the triad (which is proportional to  $\ddot{a}$  at the turnaround), where a negative value indicates a recollapse and a positive value a bounce. Taking the time derivative of Eq. (8) and evaluating when  $\dot{p}$  equals zero gives

$$\ddot{p}|_{\dot{p}=0} = \frac{2\sqrt{p}}{\gamma} \cos(2\bar{\mu}c)\dot{c}. \quad (13)$$

To determine the sign of  $\ddot{p}$  we therefore need to know the time derivative of  $c$  which is determined from Hamilton's equations  $\dot{c} = \{c, \mathcal{H}_{\text{eff}}\}$  leading to

$$\begin{aligned} \dot{c} &= \frac{c}{\gamma^{3/2} \sqrt{2\pi} \sqrt{3} l_{\text{pl}}} \sin(\bar{\mu}c) \cos(\bar{\mu}c) \\ &\quad - \frac{3\sqrt{p}}{4\pi\sqrt{3}\gamma^2 l_{\text{pl}}^2} \sin^2(\bar{\mu}c) + \frac{\gamma\kappa}{3} \frac{\partial \mathcal{H}_{\text{M}}}{\partial p} \\ &= -\frac{3\sqrt{p}}{4\pi\sqrt{3}\gamma^2 l_{\text{pl}}^2} \sin^2(\bar{\mu}c) + \frac{1}{3} \gamma\kappa \frac{\partial \mathcal{H}_{\text{M}}}{\partial p}, \end{aligned} \quad (14)$$

where the first term vanishes for both quantum and classical turnarounds. We now need to consider separately the quantum and classical turnarounds and to specify the form of matter. We first consider the quantum turnarounds.

### A. Quantum turnarounds

The quantum turnarounds as stated above are a direct consequence of the discrete quantum geometry effects of LQC. From the fact that  $\bar{\mu}c = (n + \frac{1}{2})\pi$  we have  $\sin(\bar{\mu}c) = 1$  and  $\cos(\bar{\mu}c) = 0$  which implies the matter energy density satisfies  $\rho = \rho_{\text{crit}}$ . Under these conditions the Eq. (14) for the triad acceleration becomes

$$\ddot{p}|_{\dot{p}=0} = \kappa p \left( \rho_{\text{crit}} - \frac{2}{3} \frac{1}{\sqrt{p}} \frac{\partial \mathcal{H}_{\text{M}}}{\partial p} \right). \quad (15)$$

Let us first consider the simplest case of matter with constant equation of state  $w$  with energy density  $\rho \propto a^{-3(1+w)} = p^{-3/2(1+w)}$  which implies  $\mathcal{H}_{\text{M}} \propto p^{-3w/2}$ . It is then straightforward to calculate  $\ddot{p}$  giving

$$\ddot{p}|_{\dot{p}=0} = \kappa p \rho_{\text{crit}} (1 + w), \quad (16)$$

and thus we can make the identification that for  $w < -1$  a recollapse can occur and for  $w > -1$  a bounce can occur. This can be understood in a simple fashion. Matter with equation of state  $w < -1$  violates the null energy condition and hence has increasing energy density as the universe expands. Thus when the energy density hits the quantum critical value a recollapse occurs and the universe begins contracting. In the classical universe the energy density of such matter can grow to infinite values and even lead to a singularity (for example a big rip singularity encountered for phantom field which has a negative kinetic energy [25]). In the context of LQC the implication is that such a singularity does not occur as the energy density remains bounded and the big rip is avoided [26]. On the other hand matter with  $w > -1$  has increasing energy density as the universe contracts and therefore a quantum bounce will occur when the critical density is reached.

Let us now turn to the case of a massive scalar field. Here we consider general forms of the potential which can be negative valued and phantom models where the kinetic term is negative definite. The general form of the matter

Hamiltonian is

$$\mathcal{H}_{\text{M}} = \pm \frac{1}{2} p^{-3/2} \Pi_{\phi}^2 + p^{3/2} V(\phi) \quad (17)$$

where the  $\pm$  indicates normal and phantom matter, respectively. With this form we can calculate  $\frac{\partial \mathcal{H}_{\text{M}}}{\partial p}$  which gives

$$\frac{\partial \mathcal{H}_{\text{M}}}{\partial p} = \frac{3}{2p} \left( \mp \frac{1}{2} p^{-3/2} \Pi_{\phi}^2 + p^{3/2} V(\phi) \right), \quad (18)$$

and we can simplify this by noting that when  $\rho = \rho_{\text{crit}}$  we have  $\pm \frac{1}{2} p^{-3/2} \Pi_{\phi}^2 = p^{3/2} (-V + \rho_{\text{crit}})$  whence  $\ddot{p}$  simplifies to

$$\ddot{p}|_{\dot{p}=0} = 2\kappa p (\rho_{\text{crit}} - V). \quad (19)$$

The implication of this is that if the scalar field potential is less than the critical density at the turnaround then a bounce occurs, otherwise if the potential is larger a recollapse occurs. In this form we can analyze the turnaround behavior for the different types of scalar field and potential.

We start with a normal scalar field where the kinetic term of the matter energy density is positive definite. Because of this when the total energy density is equal to the critical value at the turnaround the potential *must* be less than  $\rho_{\text{crit}}$ . Thus for a normal scalar field, only a bounce can occur in a quantum turnaround. This can be understood intuitively from the previous paragraph given that an ordinary scalar field can not have  $w < -1$ . Since we have not made any assumptions on the form of the potential even negative potentials will not lead to a quantum recollapse (though as we shall see a negative potential can lead to a classical recollapse).

Considering now phantom models with a negative definite kinetic term we find the opposite situation. First a positive potential is required in general from the effective Friedmann equation to get classical behavior of the scale factor. Furthermore since the kinetic term is negative definite when the matter density equals the critical density, the potential is larger than the critical density. Hence  $\ddot{p} < 0$  and the phantom field behaves like a matter with  $w < -1$  and a quantum recollapse can occur. This implies generically that a phantom field can not have a quantum bounce in LQC.

### B. Classical turnarounds

The classical turnaround, characterized by  $\bar{\mu}c = m\pi$  and  $\rho = 0$ , is not a feature limited to LQC but would occur even in the classical theory. For standard perfect fluids with fixed equation of state this does not occur as the energy density is always nonzero. However, if the scalar field potential is negative or a phantom field is present this condition can be met. At this turnaround  $\cos(\bar{\mu}c) = 1$ ,  $\sin(\bar{\mu}c) = 0$  which gives us

$$\begin{aligned} \ddot{p}|_{\dot{p}=0} &= \frac{2}{3} \kappa p^{1/2} \frac{\partial \mathcal{H}_M}{\partial p} \\ &= \kappa p^{-1/2} \left( \mp \frac{1}{2} p^{-3/2} \Pi_\phi^2 + p^{3/2} V \right), \end{aligned} \quad (20)$$

and we can again use  $\rho = 0$  to relate  $\mp \frac{1}{2} p^{-3/2} \Pi_\phi^2 = p^{3/2} V$  giving

$$\ddot{p}|_{\dot{p}=0} = 2\kappa p V. \quad (21)$$

In the case of a normal scalar field the potential must be negative in order for the energy density to become zero hence  $\dot{p} < 0$  at turnaround and a recollapse is possible. In the case of a phantom model we require the potential must be positive and hence a classical bounce can occur. Furthermore, in the course of dynamics it is possible that both a quantum and classical turnaround can occur. This is precisely what can happen when we consider negative potentials as the universe cycles between classical recollapses and quantum bounces.

#### IV. QUALITATIVE ANALYSIS OF BOUNCE WITH INFLATIONARY POTENTIALS

The analysis of the previous section has established analytically the turnaround behavior *if* the conditions for a quantum or classical turnaround are met. However the analysis does not indicate whether the turnaround is a feature of the dynamics given generic initial conditions. To go further we will use phase portraits to analyze the space of solutions to the dynamics. To illustrate the techniques and as a simple nontrivial model we first consider an ordinary scalar field with  $\phi^2$  and  $\phi^4$  potentials conventionally used for chaotic inflationary scenarios. In the previous section we showed that only a quantum bounce is possible. We are particularly interested in the question if inflation in the post bounce expanding universe is an attractor in the space of solutions.

It is useful to first obtain  $\dot{H}$  equation using Friedmann Eq. (11) and the conservation law (7):

$$\dot{H} = -\frac{\kappa}{2} (\rho + p_\phi) \left( 1 - \frac{2\rho}{\rho_{\text{crit}}} \right), \quad (22)$$

where  $\rho = \rho_\phi$  (6).

Already from (7) and (11) it is evident that both energy density and Hubble parameter are bounded

$$-H_{\text{max}} \leq H \leq H_{\text{max}} = \sqrt{\frac{\kappa \rho_{\text{crit}}}{12}}, \quad (23)$$

$$0 \leq \rho \leq \rho_{\text{crit}}, \quad (24)$$

$$\rho(H = \pm H_{\text{max}}) = \frac{1}{2} \rho_{\text{crit}}, \quad (25)$$

$$H(\rho = \rho_{\text{crit}}) = H(\rho = 0) = 0. \quad (26)$$

For the scalar field with non-negative potentials, the only nontrivial case is a quantum turnaround,  $H(\rho_{\text{crit}}) = 0$  and it always represents a bounce, since  $\rho + p_\phi = \dot{\phi}^2 \geq 0$  and  $\dot{H} > 0$  in (22). Therefore, given any positive definite potentials all solutions of the cosmological Eqs. (7) and (11) are *nonsingular*. As an example, let us discuss the simplest case of a massless scalar field. The solution of (7) can be written as

$$\dot{\phi} = \sqrt{2\rho_{\text{crit}}} \left( \frac{a_c}{a} \right)^3. \quad (27)$$

Substituting this solution in Eq. (11) gives the analytic solution for Hubble parameter

$$H = \pm \sqrt{2\rho_{\text{crit}}} \frac{a_c^3}{a^6} \sqrt{\frac{\kappa}{6} (a^6 - a_c^6)}. \quad (28)$$

Clearly this solution is devoid of any singularity as soon before and after the bounce the Hubble parameter reaches its maximum value.

When the effective potential of the scalar field does not vanish analytical solutions are difficult to find. To understand the qualitative behavior of solutions it is useful to build phase portraits. The dynamical phase space is four dimensional (consisting of  $c(t)$ ,  $p(t)$ ,  $\phi(t)$ ,  $\dot{\phi}(t)$ ) however the vanishing of the constraint can be used to fix one of them given the other three. Therefore a complete phase portrait would be three-dimensional, however for simplicity we will display two dimensional portraits consisting of  $\phi$  and  $\dot{\phi}$ . Since the Hubble parameter only changes sign at a classical or quantum turnaround each phase portrait drawn will consist of either an expanding or contracting phase. The quantum and classical turnarounds will be represented as boundaries in the phase portrait (which we will represent as solid lines in the figures) and any trajectory that reaches the boundary will continue into the corresponding phase portrait with opposite sign of the Hubble rate thus indicating the turnaround. In this sense the expanding and contracting phase portraits are to be glued together at the boundaries.

In order to obtain equations for corresponding dynamical system we can express the Hubble parameter in terms of  $\phi$  and  $\dot{\phi}$  using the effective Friedmann equation and plug into the conservation law Eq. (7) to get

$$\ddot{\phi} = -\frac{\partial V}{\partial \phi} \mp 3\dot{\phi} \left\{ \frac{\kappa}{3} \left( \frac{\dot{\phi}^2}{2} + V \right) \left[ 1 - \frac{1}{\rho_{\text{crit}}} \left( \frac{\dot{\phi}^2}{2} + V \right) \right] \right\}^{1/2}, \quad (29)$$

where the sign “-” corresponds to expansion while the sign “+” gives contraction. Using this equation we can obtain the phase space portraits [27] for the massive scalar field  $V(\phi) = m^2 \phi^2/2$  with  $m = 0.2$  and self-interacting nonlinear scalar field with  $V(\phi) = \lambda \phi^4/4$  with  $\lambda = 10^{-2}$ . As we have discussed in the previous section, in this case only quantum bounce is possible which occurs when  $\rho = \rho_{\text{crit}}$ , that is

$$\frac{1}{2} \dot{\phi}^2 + V(\phi) = \rho_{\text{crit}}. \quad (30)$$

Regions close to the outer boundary correspond to high energy density with quantum effects dominant whereas the low energy limit occurs near the origin in the  $\phi$ ,  $\dot{\phi}$  plane.

The results for the two potentials in the expanding phase are shown in Fig. 1 and 2 respectively. The phase space trajectories obtained via LQC dynamics are compared with those of GR obtained in Ref. [28] which are given as the dotted lines in the figures. All phase trajectories are di-

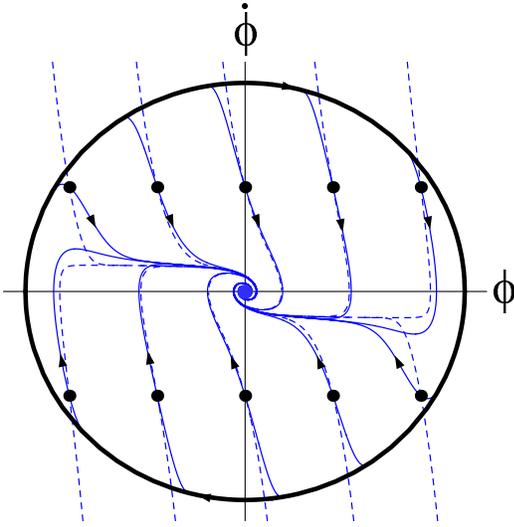


FIG. 1 (color online). Phase portrait for massive scalar field  $\phi^2$  potential. Dashed curves represent GR case and solid curves shown LQC case. Dots correspond to points where initial conditions for numerical solutions were given. The direction of phase trajectories is from the boundary to the origin. Here  $m = 0.2$ .

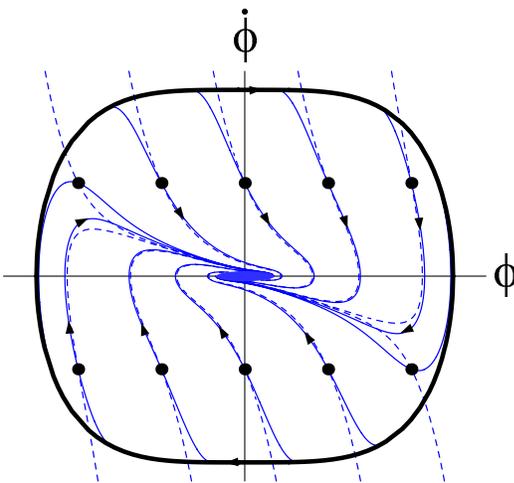


FIG. 2 (color online). Phase portrait for the self-interacting scalar field  $\phi^4$  potential. Trajectories for LQC are shown by solid curves where as dashed curves show GR case. Here  $\lambda = 10^{-2}$ .

rected towards the origin which is the only particular point in the finite region of phase space having the character of a stable node. The scalar field experiences numerous damped oscillation around the origin in agreement with classical GR. The agreement between the two theories is indicative of the fact that  $\rho \ll \rho_{\text{crit}}$  near the origin. In the region near the outer boundary the LQC trajectories deviate radically from the GR case and the existence of the boundary is a fundamental difference between LQC and GR [29] (see analogous discussion for gauge theories of gravity case in [30,31]).

A key feature of these phase portraits is the inflationary separatrix (see details for e.g. in Refs. [14,28,31]) which appears in the figures as the curve in the second and fourth quadrants to which the trajectories are attracted before undergoing oscillatory behavior. It is here that inflationary behavior occurs and it was shown in Ref. [28] that most solutions for  $\phi^2$  and  $\phi^4$  potentials within GR tend to this separatrix in the expansion stage. This led the authors of Ref. [28] to the conclusion that inflation is generic within these models. This separatrix is also present in the LQC case and is qualitatively the same although its position slightly changes with respect to the case of GR especially in the vicinity of the boundary.

However, in LQC the trajectories do not begin at the outer boundary but are preceded by a contracting phase before the bounce. The phase portraits can be obtained for the contracting phase by reflecting the expanding portraits with respect to the vertical axis and inverting the direction of the trajectories. Now all cosmological solutions start in the unstable node at the origin where the universe is contracting and the scalar field is behaving as a driven oscillator (antifrictionlike behavior), reach the outer boundary giving a bounce, return to the inflationary separatrix in the expanding phase and finally approach the stable node at the origin where the scalar field experiences damped oscillations. We show the complete phase portrait of massive scalar field within LQC in Fig. 3 by superimposing the contracting and expanding phases. The dashed curves correspond to the contracting phase and full curves describe the expanding phase. The thick curve gives a concrete example of the closed phase trajectory which starts at the origin, comes to the boundary, experiences a nonsingular bounce and comes back to the origin.

Because of the fact that inflationary separatrix is also present in the case of LQC we come to the conclusion that inflation is as common feature of cosmological solutions within LQC as it is within GR. If one defines a measure of solutions on the boundary (at the bounce) it turns out that only a small fraction of solutions does not contain inflationary phase. Since all solutions are nonsingular, they continue in the past to the contracting phase. The corresponding separatrix has a repulsive nature in contracting phase (i.e. solutions tend to deviate more and more from it in course of cosmological contraction), and the probability

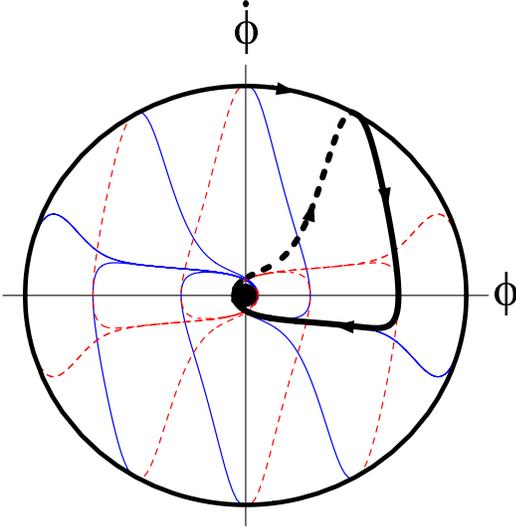


FIG. 3 (color online). Complete phase portrait for massive scalar field within LQC showing both contracting (dashed curve) and expanding phases (full curve). Thick curve shows a concrete example of the phase trajectory.

for solutions to follow this separatrix, given equal measure of initial conditions at the bounce, is small [32]. In fact this separatrix was shown to be exponentially unstable within GR, meaning that solution which is initially close to it deviates exponentially from it in course of time [33]. Therefore, if all initial conditions have equal probability at the boundary, the probability to get noninflating solution is exponentially suppressed. An example of the phase trajectory in Fig. 3 shows that solution can not follow repulsive separatrix on contracting stage but nevertheless has inflationary stage during expansion.

### V. QUALITATIVE ANALYSIS OF BOUNCE WITH NEGATIVE POTENTIALS

Negative potentials have been used in cyclic models to construct alternatives to inflationary scenarios. It is envisioned that seeds for the generation of cosmic structure originated in the contracting phase of the universe which preceded the current expanding phase. A basic difficulty in these scenarios is the singular nature of transition between two phases which has been extensively studied in Ref. [14]. As we noted in Sec. II, a classical bounce is not possible with negative potentials for a normal scalar field, whereas a bounce mediated by loop quantum corrections is possible. We now analyze the nature of this bounce for different negative potentials.

#### A. Massive scalar field

For simplicity we start with massive scalar field with a constant negative potential of form

$$V = \frac{m^2 \phi^2}{2} - V_0, \tag{31}$$

where  $V_0 > 0$ . Without loss of generality for the numerical simulations we use  $V_0 = 0.1$  and  $m = 0.2$  in this subsection. For this potential there exists an inner and an outer boundary in phase portraits. The inner boundary arises due to classical recollapse, a feature shared with classical GR [14]. The outer boundary represents the quantum bounce and is absent classically. The presence of the two boundaries in LQC opens a novel possibility to construct cyclic models where the universe undergoes a series of expansion and contraction phases.

Numerical solutions for the time evolution given a set of initial conditions are shown in Fig. 4 and the phase portrait is shown in Fig. 5. Both the Hubble parameter and energy density are bounded subject to the constraints given in (23) and (24). As can be seen from Fig. 4, the scalar field starts from positive values and rolls down the potential giving rise to a period of inflation with the Hubble parameter positive and almost constant. This continues until the scalar field enters the region of negative potential and the energy density reaches zero at which point the classical recollapse occurs and  $H = 0$ . In classical GR the field would continue its motion towards negative  $\phi$  with ever increasing  $\dot{\phi}$  (because of the antifriction term in the Klein-Gordon equation) and finally ending in a big crunch sin-

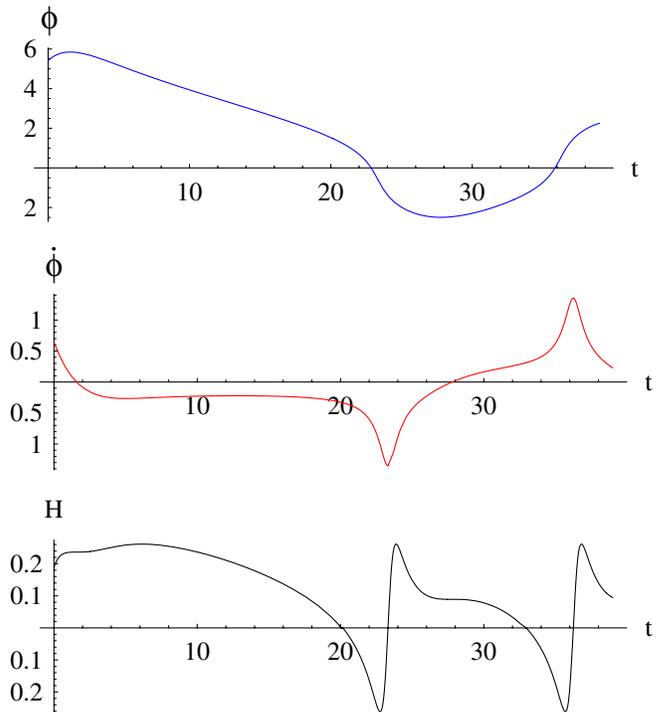


FIG. 4 (color online). Behavior of  $\phi$ ,  $\dot{\phi}$  and  $H$  is shown for the massive scalar field potential (31) in LQC. The field starts rolling down the negative potential from positive value of  $\phi$  and turns around due to loop quantum effects reaching the value where it started from. The scale factor also bounces as is evident from the turnaround in the sign of Hubble parameter. Initial conditions are  $\dot{\phi} = 0.640$ ,  $\phi = 5.422$ ; parameters are  $V_0 = 0.1$ ,  $m = 0.2$ .

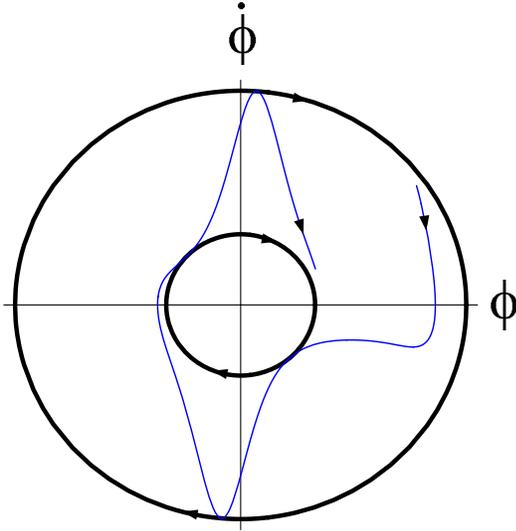


FIG. 5 (color online). Phase trajectory of the solution shown in Fig. 4. The inner boundary corresponds to classical bounce ( $\rho = 0$ ) and the outer boundary to quantum bounce ( $\rho = \rho_{\text{crit}}$ ). The trajectory formally corresponds to infinite asymmetric oscillations, although is shown for a finite time interval.

gularity. However, in LQC when  $\dot{\phi}$  reaches a value such that the energy density of the field becomes comparable to  $\rho_{\text{crit}}$ , the trajectory deviates from the classical one and the magnitude of the Hubble parameter starts decreasing and quickly becomes zero at  $\rho = \rho_{\text{crit}}$ . The universe bounces and immediately afterward enters a phase of superinflation, where  $\dot{H} > 0$  (which has been shown to be a generic feature in LQC for  $\rho > \rho_{\text{crit}}/2$  [24]). Since the universe expands quickly after the bounce,  $\dot{\phi}$  decreases and  $\rho$  becomes smaller than  $\rho_{\text{crit}}/2$ . The phase of superinflation ends and Hubble rate starts decreasing. The field is now negative valued and climbs up the potential only to turn-around, undergo slow-roll inflation and repeat the process in the opposite direction. The cycling continues indefinitely which is evident from the phase portrait in Fig. 5.

As can be seen from Fig. 4, the oscillations of the dynamical variables are asymmetric. This is due to the fact that during expansion the inflationary separatrix (which is still present, like in the cases discussed in Sec. IV) has an attractive nature, while during contraction it is repulsive one. Therefore the expanding universe spends more time in the inflationary phase as opposed to the time spent in the contracting phase and the universe experiences overall net growth in the scale factor. The oscillations of the scalar field together with a monotonic Hubble parameter, which are typical for usual chaotic inflationary potentials, occur only if the inner boundary is close enough to the origin (corresponding to a small value of  $V_0$ ) allowing for the scalar field to oscillate before being reflected by the inner boundary.

As is evident from the phase portrait in Fig. 5 given the parameters chosen these oscillations do not occur. The

presence of the two boundaries leads to the only one global direction of phase trajectories, the clockwise direction seen from Fig. 5.

## B. Cyclic potential

Let us now consider the cyclic potential in the form [3]

$$V = V_0(1 - e^{-\sigma\phi}) \exp(-e^{-\omega\phi}). \quad (32)$$

In the cyclic model, the value  $V_0$  is chosen so as to give the correct magnitude of the current cosmic acceleration and the other parameters are typically constrained to give the correct amplitude for density perturbations. For the purposes of qualitative analysis and without any loss of generality, we choose the following parameters:  $\sigma = 0.3\sqrt{8\pi}$ ,  $\omega = 0.09\sqrt{8\pi}$ , and  $V_0 = 0.1$ . The minimum of the potential is displaced from the origin and the inner boundary is situated to the left of the origin. The phase diagrams for expansion and contraction stages with this potential are represented in Fig. 6.

Here, as in the previous case, both a classical recollapse and quantum bounce can occur. Again, there is a separatrix in the phase space located to the right from the inner boundary. As in the previous cases during contraction phase trajectory is directed towards the outer boundary and during expansion it is directed towards the inner boundary.

The evolution of the Hubble parameter in LQC with the cyclic model potential can be understood from Fig. 7. As in the previous case we investigate the dynamics when the field starts from positive  $\phi$  where the potential is positive. The field rolls down towards negative values of  $\phi$  and upon entering the region of negative potential its kinetic energy cancels the potential energy leading to  $H = 0$  and the classical recollapse. The universe starts contracting, heading towards a big crunch but experiences a quantum bounce when  $\rho = \rho_{\text{crit}}$ . After the bounce, the universe briefly enters the phase of superinflation before the Hubble parameter starts decreasing. However, unlike the previous case the potential is still negative valued and the field can not turn around and continues to  $\phi = -\infty$ . Another round of collapse and bounce occurs, but this behavior terminates and the universe approaches the state  $H \rightarrow 0$  with  $\dot{\phi} \rightarrow 0$  and  $V \rightarrow 0$  at  $t \rightarrow \infty$ . Thus in the case of cyclic model potential though the scale factor bounces and universe escapes the big crunch singularity, the dynamics does not lead to cycles. For such cycles to exist the scalar field must turn around and return to the region of positive  $\phi$ . To achieve this the potential must become positive at some point for negative  $\phi$  as was the case for the quadratic potential with negative constant. Since the cyclic model potential is negative for all values of  $\phi < 0$  and approaches zero asymptotically, the possibility of such a turnaround in  $\phi$  does not exist.

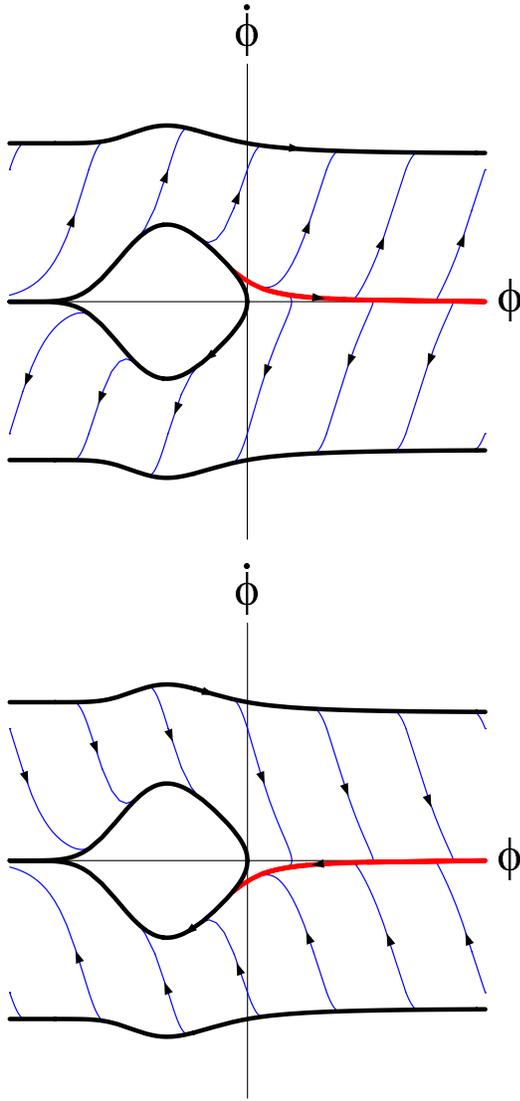


FIG. 6 (color online). Phase diagrams for contraction (above) and expansion (below) stages with the cyclic potential (32). Repulsive (attractive) inflationary separatrix is to the right from the inner boundary. Here  $\sigma = 0.3\sqrt{8\pi}$ ,  $\omega = 0.09\sqrt{8\pi}$ , and  $V_0 = 0.1$ . Outer boundary corresponds to  $\rho = \rho_{\text{crit}}$  and inner boundary to  $\rho = 0$ .

The structure of the phase space shown at Fig. 6 suggests that all solutions begin with  $\phi = -\infty$  and  $\dot{\phi} > 0$  and end at  $\phi = -\infty$  with  $\dot{\phi} < 0$ . The boundaries define the overall direction for the phase space flow which is clockwise, like in the case of massive negative potential, but the outer boundary is not closed, so the scalar field escapes to negative infinity. This is illustrated at Fig. 8 with a particular solution. We have set initial conditions in a way to have positive  $\phi$  at the beginning with the purpose to illustrate the dynamics with cyclic potential within LQC. However the evolution can be traced continuously to the past with the field having large negative values.

The problem of nonsingular bounces for cyclic potential in LQC has been investigated earlier [34], but by consid-

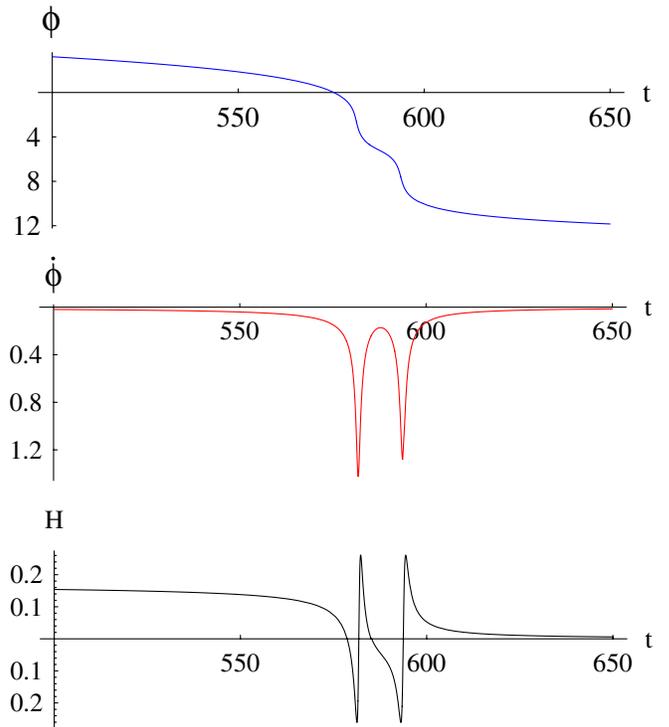


FIG. 7 (color online). Behavior of  $\phi$ ,  $\dot{\phi}$  and  $H$  with the cyclic model potential (32). Initial conditions for this solution are:  $\dot{\phi} = 16/3$ ,  $\phi = 1.066$ . Parameters are:  $\sigma = 0.3\sqrt{8\pi}$ ,  $\omega = 0.09\sqrt{8\pi}$ , and  $V_0 = 0.1$ .

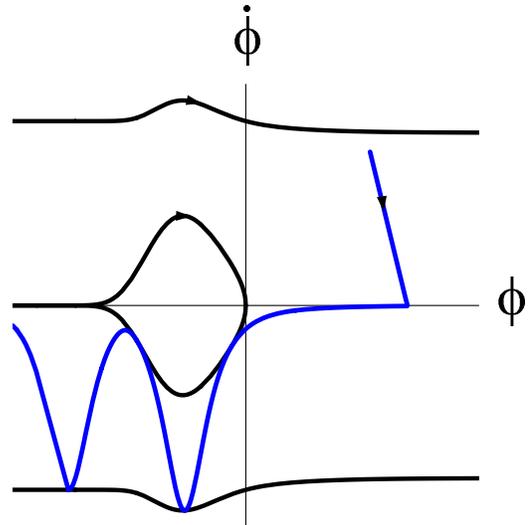


FIG. 8 (color online). Phase portrait of solution shown at Fig. 7, for wider time interval. Here again we draw only finite time evolution of the solution. Inner boundary corresponds to  $\rho = 0$  and outer to  $\rho = \rho_{\text{crit}}$ .

ering modifications to only matter part of the Hamiltonian. The analysis of Ref. [34] was able to show the existence of nonsingular bounce in scale factor for cyclic model for some choices of parameters but not in general. A limitation

of that analysis was the problem of energy density becoming super-Planckian as the field rolls down the cyclic potential [35]. As we understand from the present analysis, incorporation of modifications to the gravitational part in the effective Hamiltonian automatically solve this problem, as the energy density is bounded above by a critical value.

### C. Bi-cyclic potential

The bi-cyclic potential was discussed in Refs. [14,36] as a better choice over the cyclic potential in the attempt to construct cosmologically viable cyclic models. In fact there exists the crucial difference between the cyclic and inflationary models. The principle difficulty of the former is the cosmic singularity, but in a much more drastic way than appears in standard inflation. In contrast to inflation, within cyclic models perturbations are created at the contraction stage and cannot be traced through singularity. Instead, in our case, given regular background dynamics perturbations can in principle be evolved through the regular bounce once these are incorporated in LQC. At present we are interested in comparing its phase portraits with the cyclic potential.

The form of the bi-cyclic potential we will discuss is

$$V = V_0[1 - A \cosh^{-1}(\phi)], \quad (33)$$

where  $A > 0$  and  $V_0 > 0$ . For qualitative analysis we choose  $A = 10$  and  $V_0 = 0.03$ . The phase diagram for the expansion stage is represented in Fig. 9. As in the previous cases there exist inner and outer boundaries corresponding to classical recollapses and quantum bounces.

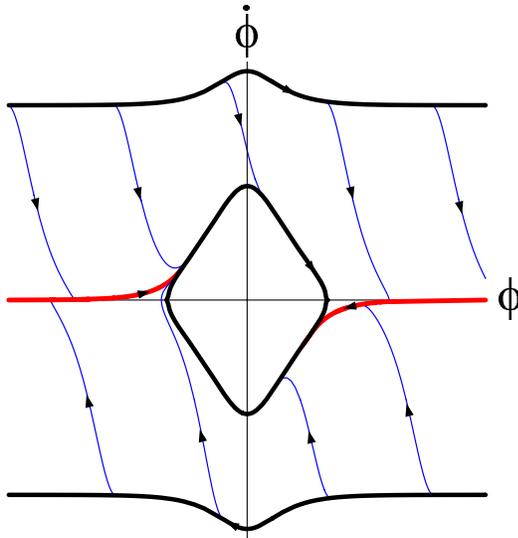


FIG. 9 (color online). Phase diagram for expansion stage with the bi-cyclic potential (33). Inner boundary refers to  $\rho = 0$  and outer to  $\rho = \rho_{crit}$ . Here  $A = 10$  and  $V_0 = 0.03$ .

As can be seen from the phase portrait the outer boundary becomes a horizontal line far from the origin given by the condition  $\dot{\phi}_b = \pm\sqrt{2\rho_{crit}}$  since for large values of  $\phi$  the potential is small. The inner boundary is a closed curve surrounding the origin crossing the  $\phi$  axis at points where the potential (33) vanishes, namely, when  $\phi = \pm \text{arccosh}(A)$ .

The phase portraits for the contracting branch can be obtained by reflecting the phase diagram in Fig. 9 with respect to the vertical axis and inverting the directions of phase trajectories. Example of phase trajectory for bi-cyclic potential is represented at Fig. 10.

A difference between the bi-cyclic model and the previous models is that there is a closed inner boundary but an open outer boundary. However as in the case of massive scalar field with negative constant there is a preferred clockwise direction for phase trajectories. In contrast to the cyclic model, the behavior cycles both in terms of the Hubble expansion and the scalar field. Given initial conditions between the boundaries, the solution is attracted to the inflationary separatrix until reaching the inner boundary. After recollapsing the separatrix is repulsive and the solution is driven to the outer boundary leading to a quantum bounce and the process starts anew. Thus, with the bi-cyclic potential there is an infinite number of inflationary stages within a given solution. The time variation of  $\phi$ ,  $\dot{\phi}$  and  $H$  is qualitatively similar to that for the massive scalar field shown in Fig. 4 with a bounce in both the scalar field and the scale factor. The bi-cyclic model thus leads to a nonsingular cyclic model in LQC. Again the crucial distinction between the cyclic and bi-cyclic potential is the fact that the potential is positive for both large negative and positive values of  $\phi$ . This cycling in the bi-cyclic model is responsible for the better viability of the latter.

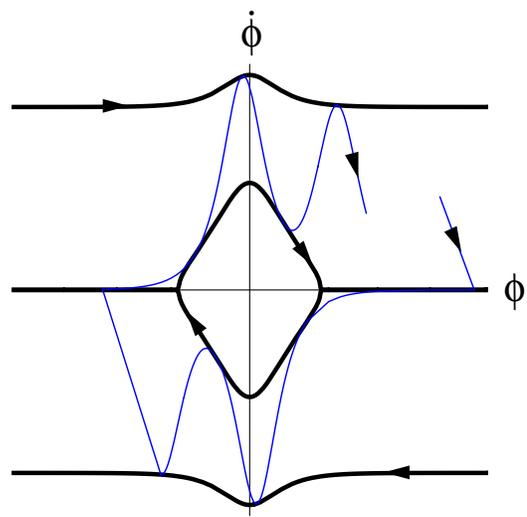


FIG. 10 (color online). Phase portrait with the bi-cyclic potential (33) for a finite time interval. Outer boundary corresponds to  $\rho = \rho_{crit}$  and inner boundary refers to  $\rho = 0$ .

## VI. DISCUSSION AND CONCLUSIONS

The nonperturbative quantum geometric effects lead to a  $\rho^2$  modification with a negative sign in the Friedmann equation in LQC. Since the correction term is negative definite it can lead to a quantum bounce in the high energy regime when loop quantum modifications are dominant. We have investigated the qualitative details of this bounce for inflationary and negative potentials. The existence of the outer boundary given by (30) in the phase portraits guarantees nonsingular behavior of all the solutions with a scalar field in LQC with any kind of potential. For negative potentials the inner boundary also appears corresponding to the classical recollapse. The presence of the two boundaries for negative potentials leads to the possibility of solutions having cyclic behavior. We have also briefly reported on the nature of quantum turnarounds for a phantom field.

The massless scalar field gives a good example of one feature of cosmological solutions within LQC, namely, the nonsingular bounce. In effect, at least solutions with  $\phi^2$  and  $\phi^4$  potentials can be well approximated near the bounce by the corresponding exact solutions for a massless scalar field, because most of them approach the outer boundary of phase space with kinetic energy much larger than the potential energy, thus the potential can be neglected. For these potentials we showed that inflationary trajectories are attractors of dynamics after the bounce. In the same way a massive scalar field with negative potential represents a good example of cycling between expansion and contraction accompanied with inflation, exhibiting the main features of the more complicated potentials. Our analysis shows that there is little qualitative difference between the massive scalar field with negative constant

and bi-cyclic potentials, although their functional form is very different. In both cases cyclic solutions can be found with an infinite number of inflationary stages.

Though the problem of the big crunch can be overcome with potential of the cyclic model, a limitation remains in the lack of a turnaround of  $\phi$  from the negative side of the potential. In the cyclic model this instant is envisioned as the collision of two branes. If the effective potential between branes can become positive prior to collision (arising from higher order perturbative string effects/or nonperturbative corrections), then a turnaround of the field can occur and a viable cyclic model can be constructed. On the other hand, the bi-cyclic potential provides the possibility for the field to turn around and may be used for further development of cyclic models.

The qualitative analysis reported in this work has shown that both inflationary paradigm and the cyclic/bi-cyclic scenarios may be incorporated in LQC. Our analysis has not touched on the aspects which can quantify the viable parameter space of these models with respect to observational data. Further, we have not explored the way LQC may shed insights on the problem of evolution of perturbations through the bounce. Work on including perturbations in the quantum theory and deriving effective equations is in progress, which will eventually address the viability of these ideas with respect to observations.

## ACKNOWLEDGMENTS

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