# Ultraviolet regularization in de Sitter space

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The ultraviolet regularization of Yukawa theory in de Sitter space is considered. We rederive the one-loop effective Candelas-Raine potentials, such that they agree with the corresponding Coleman-Weinberg potentials in flat space. Within supersymmetry, this provides a mechanism for the lifting of flat directions during inflation. For the purpose of calculating loop integrals, we employ the dimensional regularization procedure by Onemli and Woodard and show explicitly that the resulting self-energies are also invariant. This implies the absence of anomalous de Sitter breaking terms, which are reported in the literature. Furthermore, transplanckian effects do not necessarily leave an imprint on the spectrum of cosmic perturbations generated during inflation.

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## I. INTRODUCTION

Quantum theory in de Sitter space predicts a scale-invariant primordial spectrum of density fluctuations generated during inflation [1–5]. This effect is in compelling accordance with observation (see e.g. [6]) and it can be calculated by solving the field equations of motion in a de Sitter background at tree level. There is now strong evidence for a small deviation from scale-invariance [6], this can, however, be explained by the fact that in all realistic inflationary models the background is not exactly de Sitter space, leaving the methods for computing density perturbations still valid.

As an example for a loop calculation, one can compute the Coleman-Weinberg [7] effective potential in curved space-times using methods devised by DeWitt [8]. In the setting of a de Sitter background, these effective potentials have been calculated for fermion and scalar loops by Candelas and Raine [9]. Their results recently have been rederived for the case of Yukawa theory [10] and have lead to the proposal of a new relaxation mechanism for the cosmological constant [11]. In Sec. II, we rederive the Candelas-Raine potentials and confirm the original results [9]. As a consistency check, we confirm that the results presented here have the advantage of reducing to the flat-space limit [7] when taking the expansion rate to zero. We point out that the Hubble induced mass terms arising from the effective potential can be of relevance for supersymmetric models of inflation.

Loops in de Sitter space have also been computed to obtain self-energies [12–19], which induce corrections to the field equations of motion. For this type of calculation, a powerful ultraviolet regularization procedure has been introduced by Onemli and Woodard [20]. Employing this technique, it is reported that local de Sitter breaking terms occur, which have been interpreted as anomalous so far [12–14,16,19]. An ultraviolet induced breaking of the de Sitter symmetry also plays a role in context of the transplanckian problem [21–23], where time translation and boost invariance is assumed to be explicitly broken, such that one may suspect these effects to be related. The answer is that the boundary conditions imposed to effectively take account of transplanckian effects decouple from renormalizations in four dimensional de Sitter space [24,25]. Nonetheless, it is an interesting question whether one can explicitly construct an invariant regularization procedure in field theory or whether ultraviolet effects necessarily lead to the breakdown of de Sitter invariance. In Sec. III, it is shown that the procedure by Onemli and Woodard in fact preserves invariance, and as an example, this method is applied to Yukawa theory. The de Sitter breaking terms are shown to be not anomalous but cancel with contributions which have been neglected so far.

## II. PROPAGATORS AND EFFECTIVE POTENTIALS

We discuss Yukawa theory as described by the Lagrangian

$$\mathcal{L} = \sqrt{-g}\Big\{-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_\phi^2\phi^2 + \bar{\psi}\nabla\!\!\!/\,\psi - m\bar{\psi}\psi - \mu^{2-(D/2)}f\phi\bar{\psi}\psi\Big\}, \tag{1}$$

where $\mu$ is a constant of mass dimension, which is introduced to ensure that the Yukawa-coupling $f$ is dimensionless for any space-time dimension $D$, and $\nabla\!\!\!/$ denotes the covariant derivative acting on spinors. We choose conformal coordinates for de Sitter space expanding at the Hubble rate $H$, such that

$$g_{\mu\nu} = a^2\eta_{\mu\nu}, \tag{2}$$

where $a = -1/(H\eta)$ is the scale factor, $\eta \in ]-\infty, 0[$ is the conformal time and

$$\eta_{\mu\nu} = \mathrm{diag}(-1, \underbrace{1, \ldots, 1}_{D-1\ \text{times}}) \tag{3}$$

is the $D$-dimensional Minkowski metric. When expressed in these coordinates, the Lagrangian (1) takes the form [12]

 

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{2}a^{D-2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \tfrac{1}{2}a^D m_\phi^2\phi^2 \\ &\quad + (a^{(D-1)/2}\bar\psi)\mathrm{i}\not\partial(a^{(D-1)/2}\psi) - a^D m\bar\psi\psi \\ &\quad - a^D\mu^{2-(D/2)}f\phi\bar\psi\psi. \end{aligned} \tag{4}$$

Here, we assume $\langle\phi\rangle = 0$ and regard the effective potentials as functions of $m$ and $m_\phi$, respectively. The reason is purely notational, since alternatively, we could set $m = 0$ and take $\mu^{2-(D/2)}f\langle\phi\rangle$ with $\langle\phi\rangle \neq 0$ as the mass term. Likewise, the scalar mass $m_\phi$ could be substituted by introducing a self-interaction for $\phi$ and redefining the vacuum expectation value such that $\langle\phi\rangle \neq 0$.

The separation between two coordinate points is ($\eta = x^0$)

$$\Delta x^2(x;x') = -(|\eta-\eta'| - \mathrm{i}\epsilon)^2 + \sum_{i=1}^{D}|x^i - x'^i|^2, \tag{5}$$

where the $\mathrm{i}\epsilon$ term is introduced according to the Feynman-pole prescription (see e.g. [17]). This is, however, not the same as the physical geodesic distance $\ell(x;x')$ between two points. The function $\ell(x;x')$ is de Sitter invariant and most conveniently expressed in terms of the also de Sitter invariant function $y(x;x')$ as

$$y = 4\sin^2\left(\frac{1}{2}H\ell\right) = aa'H^2\Delta x^2, \tag{6}$$

where here and in the following we abbreviate $a = a(\eta)$ and $a' = a(\eta')$.

The concept of point-splitting regularization [26–28] is to expand two-point functions in a series

$$\cdots + a_{-1}\frac{1}{\ell^2} + c + a_0\log\ell + a_1\ell^2 + \cdots, \tag{7}$$

where the $a_i$ and $c$ are constants. Each term is manifestly covariant, in particular, one can subtract the terms which are ultraviolet divergent when $\ell \to 0$, without breaking general coordinate invariance. Namely, these divergent contributions are the negative powers of $\ell$ and the $\log\ell$ term. When noting that

$$y = (H\ell)^2 + O([H\ell]^6), \tag{8}$$

we can replace $y \to (H\ell)^2$ in these expansions, as long as the expansion does only range over the coefficients $a_{-1}$, $a_0$, and $c$ and as long as $\ell \lesssim H^{-1}$ is of subhorizon scale. This is the regularization procedure we follow in this section.

Let us first construct the one-loop effective potential for fermions in de Sitter background [9]. The Feynman function is given by

$$\mathrm{i}\,S(x,x') = \frac{\int\mathcal{D}\psi\mathcal{D}\bar\psi\,\psi(x)\bar\psi(x')\mathrm{e}^{\mathrm{i}\int d^Dx\mathcal{L}}}{\int\mathcal{D}\psi\mathcal{D}\bar\psi\,\mathrm{e}^{\mathrm{i}\int d^Dx\mathcal{L}}}, \tag{9}$$

and it satisfies

$$\sqrt{-g}(\mathrm{i}\not\nabla - m)\mathrm{i}S(x,x') = \mathrm{i}\delta^D(x-x'). \tag{10}$$

This equation may be solved in de Sitter background using the ansatz [10,11]

$$\begin{aligned}\mathrm{i}\,S(x;x') &= a(a^{-(D+1/2)}\mathrm{i}\not\partial a^{(D-1/2)} + m) \\ &\quad \times (aa')^{-(1/2)}\sum_{\pm}\mathrm{i}S_\pm(y)\frac{1\pm\gamma^0}{2}, \end{aligned} \tag{11}$$

such that we find, when substituting the above into Eq. (10),

$$\begin{aligned}&\left[(y^2-4y)\frac{\partial^2}{\partial y^2} - D(2-y)\frac{\partial}{\partial y} + \frac{D}{2}\left(\frac{D}{2}-1\right)\right. \\ &\qquad \left. - \mathrm{i}\gamma^0\frac{m}{H} + \frac{m^2}{H^2}\right]\sum_{\pm}S_\pm(y)\frac{1\pm\gamma^0}{2} = 0. \end{aligned} \tag{12}$$

The solution to the latter equation is a hypergeometric function [9–11],

$$\begin{aligned}\mathrm{i}S_\pm &= \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma(\frac{D}{2}-1\mp\mathrm{i}\frac{m}{H})\Gamma(\frac{D}{2}\pm\mathrm{i}\frac{m}{H})}{\Gamma(\frac{D}{2})} \\ &\quad \times {}_2F_1\left(\frac{D}{2}-1\mp\mathrm{i}\frac{m}{H}, \frac{D}{2}\pm\mathrm{i}\frac{m}{H}; \frac{D}{2}; 1-\frac{y}{4}\right). \end{aligned} \tag{13}$$

Note that for the purpose of point-splitting regularization, we could set $D = 4$ from the outset. In the following expansion, however, we keep the leading terms in general space-time dimension, as they are needed for the dimensional regularizations taken out in Sec. III. For the subleading (logarithmic and constant) terms, we readily take $D = 4$ ($\varepsilon = 0$). Having said this, we expand two-point function $S_\pm$ in $y$ and find, when and writing $D = 4 - \varepsilon$,

$$\begin{aligned}\mathrm{i}S_\pm &= \frac{H^{2-\varepsilon}}{4\pi^{2-(\varepsilon/2)}}\Gamma\left(1-\frac{\varepsilon}{2}\right)\frac{1}{y^{1-(\varepsilon/2)}} \\ &\quad + \frac{\mp\mathrm{i}Hm + m^2}{16\pi^2}\left[\log\frac{y}{4} + \psi\left(1\mp\mathrm{i}\frac{m}{H}\right) + \psi\left(2\pm\mathrm{i}\frac{m}{H}\right)\right. \\ &\quad \left. - 1 + 2\gamma_{\mathrm{E}}\right]. \end{aligned} \tag{14}$$

Next, we insert this into the fermion propagator (13) and consistently keep only terms which are $\sim y^{-1}$, $\sim\log y$, and constant in $D = 4$, such that we obtain

$$\begin{aligned}\mathrm{i}S(x,x') &= (aa')^{-(3-\varepsilon/2)}\frac{1}{4\pi^{2-(\varepsilon/2)}}\Gamma\left(1-\frac{\varepsilon}{2}\right)\mathrm{i}\not\partial\frac{1}{(\Delta x^2)^{1-(\varepsilon/2)}} + (aa')^{\varepsilon/2}\frac{m}{4\pi^{2-(\varepsilon/2)}}\Gamma\left(1-\frac{\varepsilon}{2}\right)\frac{1}{(\Delta x^2)^{1-(\varepsilon/2)}} \\ &\quad + \frac{1}{16\pi^2}(m^3 - 2\mathrm{i}\gamma^0Hm^2 + H^2m)\left(-1 + 2\gamma_{\mathrm{E}} + \log(aa'H^2\Delta x^2) - 2\log2 + \psi\left(1-\mathrm{i}\frac{m}{H}\right) + \psi\left(1+\mathrm{i}\frac{m}{H}\right)\right). \end{aligned} \tag{15}$$

The one-loop effective potential now can be calculated easily using [8,9]

$$\frac{\partial V_{\rm eff}}{\partial m} = -{\rm tr}\sqrt{-g}{\rm i}S(x,x). \qquad (16)$$

Noting that in the limit $m \gg H$, the Euler functions have the asymptotic property

$$\psi\Big(1-{\rm i}\frac{m}{H}\Big)+\psi\Big(1+{\rm i}\frac{m}{H}\Big) \sim \log\frac{m^2}{H^2}, \qquad (17)$$

we obtain [9,10,29]

$$V^{\psi}_{\rm eff} = -\frac{m^2}{2\pi^2}\frac{1}{\varrho^2}+\frac{1}{16\pi^2}\Big\{-m^4\log(\varrho^2m^2) - 2H^2m^2\log(\varrho^2m^2)+\Big(\frac{3}{2}-2\gamma_{\rm E}+\frac{1}{2}\log 2\Big)m^4 + (4-4\gamma_{\rm E}+\log 2)H^2m^2\Big\}. \qquad (18)$$

Out of the terms in curly brackets, important are only the logarithmic ones, since the analytic contributions are regularization scheme dependent and can always be canceled by adding counterterms to the Lagrangian. In addition, we have introduced a constant physical cutoff scale $\varrho$ with the dimension of a length, which is used to regulate expressions which are divergent as $aa'\Delta x^2$ goes to zero. An important consistency check is to note that in the limit $H \to 0$, the above expression reduces to the celebrated Coleman-Weinberg potential [7]. The same result has been derived by Miao and Woodard [10], who suggest to use $\varrho = H^{-1}$ as a regulator. This is motivated by imposing the renormalization condition that the scalar field remains free and massive in the large $H$ limit. In turn, the scalar mass and couplings then diverge logarithmically when $H \to 0$. Such a behavior may lead to a solution to the cosmological constant problem [11]. The numerical factors in the terms presented here and in [10,29] differ from the earlier results [9]. Note, however, that the latter effective potentials for the fermion and for the scalar loop do not reduce to the Coleman-Weinberg form as $H \to 0$, albeit being well defined in that limit. In particular, all contributions $\sim m^4\log m$ and $\sim m_\phi^4\log m_\phi$ cancel, in disagreement with the flat-space result.[1] Our regularization procedure differs, but we point out that this is not the origin of the disagreement, which should be attributed to a calculational mishap or a typo in Ref. [9]. A result very similar to Eq. (18) is reported by Elizalde and Odintsov [30]. The difference lies within renormalization-scheme-dependent terms, but moreover an overall factor of minus one occurs when comparing the coefficients of the logarithms.

[1]When comparing Eq. (18) of this article with the corresponding Eq. (30) of Ref. [9] (Eq. (23) here is to be compared with Eqn. (21) of Ref. [9] for the scalar case), one should be aware that the integrals over the Euler-$\psi$ functions in Ref. [9] comprise logarithms at leading order in $H/m$.

Since the scalar case is more familiar, we go into less details. The Green function for the scalar field is of the following de Sitter invariant form [17,19,20,31]:

$$ {\rm i}\Delta(x,x') = \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{(4\pi)^{D/2}\Gamma(\frac{D}{2})}H^{D-2} \times {}_2F_1\Big(\frac{D-1}{2}+\nu,\frac{D-1}{2}-\nu,\frac{D}{2},1-\frac{y}{4}\Big), \qquad (19)$$

where

$$\nu = \Big[\Big(\frac{D-1}{2}\Big)^2-\frac{m_\phi^2+\xi R}{H^2}\Big]^{1/2}, \qquad (20)$$

$R = D(D-1)H^2$ denotes the Ricci scalar curvature of de Sitter space, and $\xi$ is the coupling constant of the scalar field to curvature [$\xi = (D-2)/(4D-4)$ corresponds to conformal coupling, $\xi = 0$ to minimal coupling].

The expanded version of the scalar propagator corresponding to the fermionic case (15) is found to be [17,19]

$$ {\rm i}\Delta(x;x') = \frac{H^{2-\varepsilon}}{4\pi^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)\frac{1}{y^{(D/2)-1}}+\frac{H^2}{16\pi^2}\Big(\frac{m_\phi^2}{H^2}-2\Big) \times\Big[\log\frac{y}{4}+\psi\Big(\frac{3}{2}+\nu\Big)+\psi\Big(\frac{3}{2}-\nu\Big)-1+2\gamma_{\rm E}\Big]. \qquad (21)$$

From this, we now derive the scalar potential using [8,9]

$$\frac{\partial V_{\rm eff}}{\partial m_\phi^2} = \frac{1}{2}\sqrt{-g}{\rm i}\Delta(x,x), \qquad (22)$$

such that we obtain for $m_\phi \gg H$ [29]

$$V^{\phi}_{\rm eff} = \frac{m_\phi^2}{8\pi^2\varrho^2}+\frac{1}{16\pi^2}\Big\{\frac{1}{4}m_\phi^4\log(\varrho^2m_\phi^2)-H^2m_\phi^2\log(\varrho^2m_\phi^2) +\Big(\frac{3}{8}+\frac{1}{2}\gamma_{\rm E}-\frac{1}{8}\log 2\Big)m_\phi^4 +\Big(2-2\gamma_{\rm E}+\frac{1}{2}\log 2\Big)H^2m_\phi^2\Big\}. \qquad (23)$$

Taking $H \to 0$, agreement with the Coleman-Weinberg [7] result is found also for the scalar loop. Just as for the fermionic case, in the final expression reported by Candelas and Raine [9], all logarithmic terms cancel in the flat-space limit $H \to 0$, in disagreement with Coleman and Weinberg [7]. Again, this should not be attributed to the approach or regularization used in [9], but to a minor calculational mistake.

Our results (18) and (23) imply, in particular, for $m = m_\phi$ that

$$4V_{\rm eff}^{\phi} + V_{\rm eff}^{\psi} = -\frac{3}{8\pi^2}H^2m^2\log(\varrho^2m^2) + \frac{1}{16\pi^2}(12 - 12\gamma_{\rm E} + 3\log 2)H^2m^2, \quad (24)$$

and that in Minkowski space ($H = 0$) fermionic and scalar contributions cancel, as they should when assuming the same number of degrees of freedom and the same mass spectrum. De Sitter space with $H \neq 0$ exhibits, however, supersymmetry breaking due to the different curvature coupling of fermions and scalars.

This finding may be of importance for inflationary models when replacing $m$ and $m_\phi$ by $\kappa\langle S\rangle$, where $S$ denotes the slowly rolling inflaton field and $\kappa$ its superpotential coupling to other chiral multiplets. In $F$-term inflation, similar terms are expected from supergravity corrections [32–35] and have consequences for predictions of the spectrum of primordial density fluctuations [36,37]. For a minimal Kähler potential within $F$-term inflation and generally within $D$-term inflation, these kind of corrections originating from supergravity are expected to be absent [38], whereas the de Sitter background induced corrections presented here are still there and should be taken into account. Note that when assuming the renormalization scale to be larger than the mass terms, $\rho^{-1} > m$, and the derived mass-square corrections are always positive.

As a next step, it will be important to derive the one-loop potential for gauge bosons and gauginos in the loop. Using the above results for the scalar and fermion case for a conjecture and taking into account the dimension of the standard model gauge group as well as typical values for the gauge coupling constants suggested by gauge coupling unification, mass corrections which are of order of the Hubble rate might arise also during the postinflationary eras [39]. This intriguing possibility for a universal mechanism of lifting flat directions of the mimimal supersymmetric standard model is subject of ongoing studies.

## III. SELF-ENERGIES

The above expressions for the effective potentials are constructed from the coincidence limits of the fermionic and scalar Green functions, and the occurring ultraviolet divergences are renormalized by subtracting terms which become infinite as we take the short-distance regulator $\varrho$ to zero. This procedure apparently bears some similarity with a cutoff regularization in momentum space. One should be aware, however, of the fact that a large momentum cutoff of a two-point function in momentum space does not result in a short-distance cutoff of its Fourier transform in position space. Therefore, the simple point-splitting method employed in the previous section is not applicable to the case where finite distance effects are of importance.

This can be seen when studying self-energy functions. Examples are the vacuum polarization for scalar electrodynamics [13–18] and the fermion self-energy [12,19] in de Sitter background. The self-energies, which are two-point functions, give rise to corrections to the free field equations, which are in general nonlocal. Momentum space experience is telling us that we obtain from ultraviolet divergent integrals logarithms of functions involving the external momentum $q$ as well as the mass of the particles running in the loop. These contributions are of utmost importance for the predictions of a theory, and therefore a position space technique to separate these terms from the divergent parts is needed. A powerful method to achieve this within dimensional regularization has been suggested by Onemli and Woodard [20].

Some results obtained using this technique seem to introduce as a common feature local operators [12–19], which violate de Sitter invariance, since they are proportional to $\log a(\eta)$ or equivalently $t$, which is the comoving time related to the scale factor as $a(\eta) = {\rm e}^{Ht}$. This observation has been interpreted as a perturbation theory anomaly so far [12–14,16,19]. However, we point out here that these terms are completely canceled by an ostensibly negligible nonlocal contribution. Therefore, with the choice of a de Sitter invariant counterterm, the procedure of ultraviolet regularization does not break de Sitter invariance. We emphasize that for the case of a minimally coupled massless scalar field or gravity, the treatment of ultraviolet divergences suggested here yet leaves behind de Sitter breaking terms originating from the noninvariant propagators.

Let us introduce local counterterms by adding

$$\delta\mathcal{L} = \delta Z_2(a^{(D-1)/2}\bar\psi){\rm i}\not\partial(a^{(D-1)/2}\psi) - a^D\delta m\bar\psi\psi - \tfrac{1}{2}a^{D-2}\delta Z_3\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \tfrac{1}{2}a^D\delta m_\phi^2\phi^2 \quad (25)$$

to the Lagrangian (4). Note that we do not discuss vertex renormalization here. These counterterms are de Sitter invariant, provided $\delta Z_2$, $\delta m$, $\delta Z_3$, and $\delta m_\phi$ are constant. From $\mathcal{L} + \delta\mathcal{L}$, we can straightforwardly derive Feynman rules [12,19] leading to the following self-energy function:

$$\begin{aligned}
-{\rm i}\Sigma(x;x') &= (-{\rm i}f\mu^{\varepsilon/2}a^{4-\varepsilon}){\rm i}S(x;x')(-{\rm i}f\mu^{\varepsilon/2}a^{4-\varepsilon}){\rm i}\Delta(x;x') + {\rm i}\delta Z_2(aa')^{(3-\varepsilon/2)}{\rm i}\not\partial\delta^{4-\varepsilon}(x-x') \\
&\quad + {\rm i}(aa')^{2-(\varepsilon/2)}\delta m\delta^{4-\varepsilon}(x-x') \\
&= -\frac{f^2\mu^\varepsilon(aa')^{3/2}}{32\pi^{4-\varepsilon}}\frac{\Gamma(2-\frac{\varepsilon}{2})\Gamma(1-\frac{\varepsilon}{2})}{1-\frac{\varepsilon}{2}}{\rm i}\not\partial\frac{1}{(\Delta x^2)^{2-\varepsilon}} - \frac{f^2\mu^\varepsilon(aa')^2}{16\pi^{4-\varepsilon}}\Gamma^2\Big(1-\frac{\varepsilon}{2}\Big)\frac{m}{(\Delta x^2)^{2-\varepsilon}} \\
&\quad + {\rm i}\delta Z_2(aa')^{(3-\varepsilon/2)}{\rm i}\not\partial\delta^{4-\varepsilon}(x-x') - {\rm i}(aa')^{2-(\varepsilon/2)}\delta m\delta^{4-\varepsilon}(x-x'). \quad (26)
\end{aligned}$$

In this expression, we have only kept terms which lead to ultraviolet divergences and those which are used to renormalize them. One may interpret it as the self-energy for a fermion of mass $m$ coupled to a conformally coupled scalar ($m_\phi = 0$ and $\xi = \frac{1}{6}$). Important finite contributions arising for the minimally coupled case, $\xi = 0$, are therefore not included here, but they are extensively discussed in Refs. [12,19], where a new mechanism for fermion mass generation is suggested. As these contributions due to minimal coupling are also in the focus of [13–18], the main conclusions drawn in these previous articles remain unaltered when applying the modifications we suggest in the following.

We now need to regulate terms which go as $1/\Delta x^4$ when $\varepsilon \to 0$, since they lead to logarithmic divergences when integrated. Following Onemli and Woodard [20], we make use of the $D$-dimensional representation of the Dirac $\delta$ function

$$\partial^2 \frac{1}{\Delta x_{++}^{D-2}} = \mathrm{i}(D-2)\Omega_D \delta^D(\Delta x), \tag{27}$$

where

$$\Omega_D = \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})} \tag{28}$$

is the area of the $D$-dimensional unit sphere. We then manipulate the logarithmic divergences as

$$\begin{aligned}\left(\frac{1}{\Delta x^2}\right)^{2-\varepsilon} &= -\frac{1}{2\varepsilon(1-\varepsilon)}\partial^2 \frac{1}{\Delta x^{2-2\varepsilon}} = -\frac{\tilde{\partial}^2}{2\varepsilon(1-\varepsilon)}\left[\frac{1}{\Delta x^{2-2\varepsilon}} - \frac{\varrho^\varepsilon}{\Delta x^{2-\varepsilon}}\right] - \varrho^\varepsilon \frac{1}{2\varepsilon(1-\varepsilon)}\partial^2 \frac{1}{\Delta x^{2-\varepsilon}} \\ &\approx -\tilde{\partial}^2 \frac{\log\frac{\Delta x^2}{\varrho^2}}{4\Delta x^2} - \mathrm{i}\frac{1}{\varepsilon}\left(1+\frac{\varepsilon}{2}\right)\varrho^\varepsilon \frac{2\pi^{2-(\varepsilon/2)}}{\Gamma(2-\frac{\varepsilon}{2})}\delta^{4-\varepsilon}(\Delta x).\end{aligned} \tag{29}$$

In the second step, we have added and subtracted the same contribution. For dimensional reasons, similar to the point-splitting approach, we have to introduce a regulator $\varrho$, which has the dimension of a length and may be a function of the space-time points $x_\mu$ and $x'_\mu$. The notation $\tilde{\partial}_\mu$ implies that this is a derivative which does not act on the implicit dependence of $\varrho$ on $x_\mu$,

$$\tilde{\partial}_\mu \varrho(x; x') = 0, \tag{30}$$

and has been introduced to keep notation compact.

Since $\varrho$ corresponds to a *comoving* length, we propose to employ a de Sitter invariant constant *physical*-length regulator, which is given by

$$\varrho(x; x') = \sqrt{\frac{1}{aa'\mu^2}}, \tag{31}$$

where $\mu$ is a constant regularization scale. The choice made in Ref. [20] is

$$\varrho = \mu^{-1}, \tag{32}$$

which corresponds to a physical length shrinking with the scale factor. Of course, there is nothing wrong with this choice, because the manipulation (29) just amounts to adding and subtracting the same contribution. Indeed, tracing the calculation by Onemli and Woodard further [20], both terms of the last expression in (29) are fully taken into account. However, in Refs. [12–19], the first term is eventually neglected. This contribution however, with the choice of the regulator (32), is not de Sitter invariant. Therefore, a de Sitter breaking remainder in form of a local term occurs, which is misinterpreted as anomalous [12–14,16,19].

Applying the above procedure to the self-energy (26), we find

$$\begin{aligned}\Sigma(x;x') = &-\frac{f^2(aa')^{3/2}}{2^7\pi^4}\tilde{\not\partial}\tilde{\partial}^2\frac{\log\frac{\Delta x^2}{\varrho^2}}{\Delta x^2} + \mathrm{i}\frac{f^2(aa')^2}{2^6\pi^4}m\tilde{\partial}^2\frac{\log\frac{\Delta x^2}{\varrho^2}}{\Delta x^2} - \frac{f^2(aa')^{3/2}\mu^\varepsilon\varrho^\varepsilon}{2^4\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\left(1-\frac{\varepsilon}{2}\right)\mathrm{i}\not\partial\delta^{4-\varepsilon}(x-x') \\ &-\frac{f^2(aa')^2\mu^\varepsilon\varrho^\varepsilon}{2^3\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\left(1-\frac{\varepsilon}{2}\right)m\delta^{4-\varepsilon}(x-x') - \delta Z_2(aa')^{(3-\varepsilon/2)}\mathrm{i}\not\partial\delta^{4-\varepsilon}(x-x') \\ &-(aa')^{2-(\varepsilon/2)}\delta m\delta^{4-\varepsilon}(x-x').\end{aligned} \tag{33}$$

As a consistency check, we may convince ourselves that also for the momentum space result in flat space, the infinite contributions $\sim 1/\varepsilon$ to the self-energy satisfy

$$\varepsilon \frac{\partial \Sigma}{\partial \not{d}} = \frac{1}{2} \varepsilon \frac{\partial \Sigma}{\partial m} + O(\varepsilon). \tag{34}$$

In order to render $\Sigma(x;x')$ finite, we choose the counterterms

$$\begin{aligned} -\delta Z_2 &= \frac{1}{2^4 \pi^{2-(\varepsilon/2)}} \frac{1}{\varepsilon} \frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}} \Gamma\left(1-\frac{\varepsilon}{2}\right), \\ \delta m &= \frac{m}{2^3 \pi^{2-(\varepsilon/2)}} \frac{1}{\varepsilon} \frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}} \Gamma\left(1-\frac{\varepsilon}{2}\right). \end{aligned} \tag{35}$$

With the regulator (31), we obtain

$$\begin{aligned} \Sigma(x;x') = &-\frac{f^2(aa')^{3/2}}{2^7 \pi^4} \tilde{\not{\partial}} \tilde{\partial}^2 \frac{\log(aa'\Delta x^2)}{\Delta x^2} \\ &+ \mathrm{i} \frac{f^2(aa')^2}{2^6 \pi^4} m \tilde{\partial}^2 \frac{\log(aa'\Delta x^2)}{\Delta x^2}. \end{aligned} \tag{36}$$

$\Sigma(x;x')$ is de Sitter invariant if it scales as $(aa')^4$, as can be seen explicitly when employing it to modify the conformally rescaled Dirac equation [12,19]. Noting that $\Delta x^2$ scales as $(aa')^{-1}$ ($y = aa'\Delta x^2$ is de Sitter invariant) and that furthermore the arguments of the logarithms are scale-invariant, this property can indeed be verified for each of the two above contributions to the self-energy $\Sigma(x;x')$.

In contrast, when choosing the regulator (32), we find

$$\begin{aligned} \Sigma(x;x') = &-\frac{f^2(aa')^{3/2}}{2^7 \pi^4} \tilde{\not{\partial}} \tilde{\partial}^2 \frac{\log \Delta x^2}{\Delta x^2} \\ &+ \mathrm{i} \frac{f^2(aa')^2}{2^6 \pi^4} m \tilde{\partial}^2 \frac{\log \Delta x^2}{\Delta x^2} - \frac{f^2(aa')^{3/2}}{2^5 \pi^2} \\ &\times \log(aa') \mathrm{i} \not{\partial} \delta^4(x-x') - \frac{f^2(aa')^2}{2^6 \pi^2} \\ &\times \log(aa') m \delta^4(x-x'). \end{aligned} \tag{37}$$

None of the individual contributions are de Sitter invariant here, although the sum is equal to the manifestly invariant expression (36). In the work on Yukawa theory [12,19], the first two terms of (37) are not discussed further and it is incorrectly claimed that the third and fourth term break de Sitter invariance anomalously. The same applies to the corresponding terms being identified as anomalous in the work on scalar quantum electrodynamics [13,14,16]. We emphasize, however, that the putatively anomalous terms are not in the main focus of the papers [12–14,16,19] but are rather treated as a side effect. What we point out here is that the ultraviolet regularization can be taken out without introducing additional de Sitter breaking contributions.

It would of course be desirable to obtain a manifestly covariant expression for $\Sigma(x;x')$ in general backgrounds, as may be constructed employing DeWitt's technique for expanding two-point functions in terms of the geodesic distance [8]. Future work will show whether this leads to a fairly simple and manageable result.

An immediate consequence of the correct application of the ultraviolet regularization procedure is of course the vanishing of de Sitter breaking terms of ultraviolet origin found for Yukawa theory in Refs. [12,19], which have been misinterpreted as anomalous.[2] However, Onemli's and Woodard's regularization procedure is directly applicable to all calculations involving logarithmic divergences. For example, in the original work [20], Onemli and Woodard study $\phi^4$ theory. Note, however, that they use a de Sitter breaking scalar propagator, as necessary for a massless scalar field [40], which reads in $D = 4$ space-time dimensions [20]

$$\mathrm{i}\Delta(x;x') = \frac{H^2}{4\pi^2}\left\{\frac{1}{y(x;x')} - \frac{1}{2}\log y(x;x') + \frac{1}{2}\log(aa')\right\}. \tag{38}$$

The last term obviously breaks de Sitter invariance, but is of an utterly different, namely, infrared, origin. In loop calculations for renormalizable theories such as Yukawa or $\phi^4$, it does not lead to ultraviolet divergences, but yet gives rise to important finite de Sitter breaking effects. Note, however, that the focus of the present paper is not on the discussion of the peculiarities of the massless minimally coupled scalar field but on the regularization of logarithmic ultraviolet divergences resulting from the leading term $\propto 1/y$ of the scalar propagator, which is universal for scalar fields of different mass and curvature couplings.

A detailed and excellent discussion of these different—ultraviolet and infrared—de Sitter breaking contributions for the example of scalar electrodynamics can be found in Ref. [14], where, however, the conformal anomaly of gauge theory is inappropriately made responsible for the ultraviolet induced breakdown of de Sitter invariance. We point out that employing the invariant regularization as the procedure proposed here, ultraviolet induced de Sitter breaking also can be casted off from scalar electrodynamics [13–18].

Let us finally treat the wave function renormalization of the scalar field, since this is the case which is important for the generation of density perturbations during inflation. In order to do so, it turns out that we need to keep track of the dependence of $\mathrm{i}S_{\pm}(x;x')$ on $\varepsilon \neq 0$ up to logarithmic order [cf. Eq. (14)],

[2] Explicitly, the second term in Eq. (14) of Ref. [19] can be combined with the first term in a de Sitter invariant way. The same statement applies for the second term of Eq. (24) of Ref. [12], where Yukawa interactions of a minimally coupled massless scalar field are investigated. Note that in the latter case, the additional de Sitter breaking contribution originating from the scalar propagator persists even when taking account of the absence of an anomaly.

$$\mathrm{i}S_{\pm}(x;x') = \frac{H^{2-\varepsilon}}{4\pi^{2-(\varepsilon/2)}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big)\frac{1}{y^{1-(\varepsilon/2)}} + \frac{H^{-\varepsilon}}{16\pi^{2-(\varepsilon/2)}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big)(\mp \mathrm{i}Hm + m^2)\Big\{\frac{2}{\varepsilon}(y^{\varepsilon/2}-1) - 2\log 2 + \psi\Big(1\mp \mathrm{i}\frac{m}{H}\Big) + \psi\Big(2\pm \mathrm{i}\frac{m}{H}\Big) - 1 + 2\gamma_{\mathrm{E}}\Big\}. \tag{39}$$

Then, the divergent contributions to the scalar self-energy $\mathrm{i}\Pi(x;x')$ can be regularized as [cf. the fermionic case, Eq. (33)]

$$\Pi(x;x') = \mathrm{i}\frac{f^2aa'}{2^7\pi^4}\tilde{\partial}^4\frac{\log\frac{\Delta x^2}{\varrho^2}}{\Delta x^2} + \mathrm{i}\frac{f^2(aa')^2}{2^5\pi^4}m^2\tilde{\partial}^2\frac{\log\frac{\Delta x^2}{\varrho^2}}{\Delta x^2} - \frac{f^2aa'\mu^\varepsilon\varrho^\varepsilon}{2^4\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big)\partial^2\delta^{4-\varepsilon}(x-x') - \frac{f^2(aa')^2\mu^\varepsilon\varrho^\varepsilon}{2^2\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big)m^2\delta^{4-\varepsilon}(x-x') \tag{40}$$

$$+ \delta Z_3(aa')^{(2-\varepsilon)/2}\partial^2\delta^{4-\varepsilon}(x-x') + (aa')^{(4-\varepsilon)/2}\delta m_\phi^2\delta^{4-\varepsilon}(x-x'), \tag{41}$$

which is renormalized by the counterterms

$$\delta Z_3 = \frac{1}{2^4\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big), \qquad \delta m_\phi^2 = \frac{m_\phi^2}{2^2\pi^{2-(\varepsilon/2)}}\frac{1}{\varepsilon}\frac{1+\frac{\varepsilon}{2}}{1-\frac{\varepsilon}{2}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big). \tag{42}$$

Again, a de Sitter invariant expression is obtained when using the regulator (31) and consistency with the flat-space result derived in momentum space

$$\varepsilon\frac{\partial\Pi}{\partial q^2} = \frac{1}{4}\varepsilon\frac{\partial\Pi}{\partial m^2} + O(\varepsilon) \tag{43}$$

is found.

Of course, we would like to reproduce now the contribution to $V_{\mathrm{eff}}^{\psi}$ from $\Pi(x;x')$. At first glance, it is disturbing that there appears to be no divergences in $\Pi(x;x')$ which are proportional to $m^2H^2$, while these terms occur in $V_{\mathrm{eff}}^{\psi}$, Eq. (18). However, when calculating the potential from $\Pi(x;x')$ and keeping only the divergences $\sim 1/\varepsilon$, we observe when setting $m = f\phi$

$$\begin{aligned} V_\Pi^\psi &= \frac{1}{2}\frac{1}{a^4}\int d^4x' f\phi(x)\phi(x;x')f\phi(x') + O(\varepsilon^0) \\ &= -\frac{1}{\varepsilon}\frac{1}{a^4}\int d^4x'\Big\{\frac{1}{8\pi^2}m^4(aa')^2\delta^4(x-x') + \frac{1}{32\pi^2}m^2aa'\partial^2\delta^4(x-x')\Big\} + O(\varepsilon^0) \\ &= -\frac{1}{\varepsilon}\Big(\frac{1}{8\pi^2}m^4 + \frac{1}{16\pi^2}m^2H^2\Big) + O(\varepsilon^0). \end{aligned} \tag{44}$$

Hence, we have recovered a divergence $\sim m^2H^2$. Note, however, that a full reconstruction of the effective potential at leading order will also involve tadpole diagrams. Furthermore, we completely miss the terms $\propto \log m$. We suspect that the solution to this problem lies in the treatment of the terms in the first line of Eq. (40) and equivalently of Eq. (33), which correspond to expressions $\propto \log(F(m,q))$, which are familiar from momentum space and where $F(m,q)$ is some function.

## IV. CONCLUSIONS

In this paper, we rederive the Candelas-Raine effective potentials [9] for fermions and for scalars. The expressions we find are benign as $H \to 0$ and moreover they reduce to the Coleman-Weinberg form in that limit. The different curvature coupling of fermions and scalars gives rise to curvature-induced supersymmetry breaking, evident from the potential sum (24). This may be of relevance for inflationary model building.

We also point out that the ultraviolet regularization procedure by Onemli and Woodard [20] does not lead to an anomalous breakdown of de Sitter invariance. Yet, we have to resolve the question how the remaining nonanalytic terms $\propto \log\Delta x^2$ have to be evaluated. Techniques to approach this problem have been developed and successfully applied to compute the effect of self-energy corrections on the equations of motion of various quantum fields [12–19]. We emphasize that position space techniques are not only useful for computations of effects within curved spacetimes, but for any kind of backgrounds, for example, inhomogeneous electric fields [41,42].

The possibility of an invariant renormalization implies that within field theory, a transplanckian problem does not necessarily exist, because all processes can be imposed to be fully de Sitter invariant by the choice of an invariant renormalization scheme. This is a consequence of the manifest coordinate invariance of the underlying Lagrangian (1) and that this invariance does not appear to be broken by perturbation theory anomalies. Since there is no observational evidence for a breaking of Lorentz symmetry in flat space, where one therefore routinely uses covariant regularization procedures, one may consider also in curved space-times covariant regularization as a natural choice. Note, however, that within string theory, albeit also being formulated in a manifestly covariant way, a breakdown of de Sitter invariance leading to signatures in the primordial perturbation spectrum is expected [23,43,44]. A discussion of the renormalization of these boundary effects and some aspects of renormalization of self-interacting scalar theories, which may be compared to the results presented here, is provided in Refs. [24,25].

We emphasize that the ultraviolet regularization in de Sitter space is not directly related to the breakdown of de Sitter symmetry for massless minimally coupled scalar fields [40,45]. It is a tree level effect and is due to the fact that it is not possible to construct a de Sitter invariant Green function which takes the Hadamard form in this case. It corresponds to an infrared divergence, which becomes manifest when taking $m_\phi \to 0$ and $\xi \to 0$ in Eqs. (19) and (20). Because of its infrared origin, this type of de Sitter breaking for massless minimally coupled fields is not related to the transplanckian problem.

Concerning the prospects of the work presented here, a goal of future efforts should be to obtain a dictionary translating from position space expressions to their familiar momentum space counterparts, such that new effects in curved space-time may reliably be identified. Furthermore, a covariant generalization of the de Sitter space results to more general backgrounds is desirable. While being interesting on their own behalf, quantum loop effects in curved space-time may give relevant input to inflationary model building or even directly lead to observational consequences, such that routine techniques for their calculation are of great importance.


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