

Charmed-hadron fragmentation functions from CERN LEP1 revisited

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In Phys. Rev. D **58**, 014014 (1998) and **71**, 094013 (2005), we determined nonperturbative D^0 , D^+ , D^{*+} , D_s^+ , and Λ_c^+ fragmentation functions, both at leading and next-to-leading order in the $\overline{\text{MS}}$ factorization scheme, by fitting e^+e^- data taken by the OPAL Collaboration at CERN LEP1. The starting points for the evolution in the factorization scale μ were taken to be $\mu_0 = 2m_Q$, where $Q = c, b$. For the reader's convenience, in this paper, we repeat this analysis for $\mu_0 = m_Q$, where the flavor thresholds of modern sets of parton density functions are located.

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I. INTRODUCTION

The OPAL Collaboration presented measurements of the fractional energy spectra of inclusive D^{*+} [1], D^0 , D^+ , D_s^+ , and Λ_c^+ [2] production in Z-boson decays based on their entire LEP1 data sample. Apart from the full cross sections, they also determined the contributions arising from $Z \rightarrow b\bar{b}$ decays. This enabled us, partly in collaboration with Binnewies, to determine lowest-order (LO) and next-to-leading-order (NLO) sets of fragmentation functions (FF's) for these charmed (X_c) hadrons [3,4]. We took the charm-quark FF to be of the form proposed by Peterson *et al.* [5] and thus obtained new values of the ϵ parameter, which are specific for our choice of factorization scheme.

We worked in the QCD-improved parton model implemented in the pure modified minimal-subtraction ($\overline{\text{MS}}$) renormalization and factorization scheme with $n_f = 5$ massless quark flavors (zero-mass variable-flavor-number scheme). This scheme is particularly appropriate if the characteristic energy scale of the considered production process, i.e., the center-of-mass energy \sqrt{s} in the case of e^+e^- annihilation and the transverse momentum p_T of the X_c hadron in other scattering processes, is large compared to the bottom-quark mass m_b . Owing to the factorization theorem [6], the FF's defined in this scheme satisfy two desirable properties: (i) their scaling violations are ruled by the timelike Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [7] evolution equations; and (ii) they are universal. Thus, this formalism is predictive and suitable for global data analyses.

We verified that the values of the branching and average momentum fractions of the various $c, b \rightarrow X_c$ transitions evaluated at LO and NLO using our FF's [3,4] are in reasonable agreement with the corresponding results from OPAL [1,2] and other experiments [8].

We tested the scaling violations of our D^0 , D^+ , D_s^+ , and Λ_c^+ FF's [4] by comparing the fractional energy spectra of these hadrons measured in nonresonant e^+e^- annihilation at $\sqrt{s} = 10.55$ GeV [9], 29 GeV [10], and 34.7 [11] with

our LO and NLO predictions to find reasonable agreement. Since events of X_c -hadron production from X_b -hadron decay were excluded from the data samples at $\sqrt{s} = 10.55$ GeV, we obtained a clean test of our charm-quark FF's.

In Refs. [3,4], the starting points μ_0 for the DGLAP evolution in the factorization scale μ were taken to be $\mu_0 = 2m_Q$, where $Q = c, b$. This choice is phenomenologically motivated by the observation that, in e^+e^- annihilation, which has been providing the most constraining input for the determinations of FF's, these values of μ_0 represent the very production thresholds of the respective flavors. Unfortunately, this choice is inconsistent with the convention underlying modern sets of parton density functions (PDF's) [12], which prefer to place the flavor thresholds at $\mu_0 = m_Q$. For the reader's convenience, in this addendum to Refs. [3,4], we thus repeat the analysis of that papers for the choice $\mu_0 = m_Q$, so as to provide alternative LO and NLO sets of X_c FF's that can be conveniently utilized together with those PDF's. The FF's presented below were already used as input for a NLO study [13] of charmed-meson hadroproduction in $p\bar{p}$ collisions, which yielded agreement within errors with data collected by the CDF Collaboration in run II at the Fermilab Tevatron [14]. We note in passing that, in the case of perturbatively induced FF's, which is quite different from the case of nonperturbative FF's (involving substantial intrinsic components) under consideration here, the choice $\mu_0 = m_Q$ is more natural, since, at NLO, it avoids finite matching conditions at the flavor thresholds [15].

II. RESULTS

In the following, we concentrate on the most important results of Refs. [3,4] that are affected by the shift in μ_0 . These include the fit parameters N , α , β , and ϵ defining the x distributions of the $Q \rightarrow X_c$ FF's $D_Q(x, \mu^2)$ at $\mu = \mu_0$,

$$D_c(x, \mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2}, \quad (1)$$

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TABLE I. Fit parameters of the charm- and bottom-quark FF's in Eqs. (1) and (2), respectively, for the various X_c hadrons at LO and NLO. The corresponding starting scales are $\mu_0 = m_c = 1.5$ GeV and $\mu_0 = m_b = 5$ GeV, respectively. All other FF's are taken to be zero at $\mu_0 = m_c$.

X_c	Order	Q	N	α	β	ϵ
D^0	LO	c	0.694	0.101
		b	81.7	1.81	4.95	...
	NLO	c	0.781	0.119
		b	100	1.85	5.48	...
D^+	LO	c	0.282	0.104
		b	52.0	2.33	5.10	...
	NLO	c	0.266	0.108
		b	60.8	2.30	5.58	...
D^{*+}	LO	c	0.174	0.0554
		b	69.5	2.77	4.34	...
	NLO	c	0.192	0.0665
		b	20.8	1.89	3.73	...
D_s^+	LO	c	0.0498	0.0322
		b	27.5	1.94	4.28	...
	NLO	c	0.0381	0.0269
		b	27.5	1.88	4.48	...
Λ_c^+	LO	c	0.00677	0.00418
		b	41.2	2.02	5.92	...
	NLO	c	0.00783	0.00550
		b	34.9	1.88	6.08	...

TABLE II. $\chi^2/\text{d.o.f.}$ achieved in the LO and NLO fits to the OPAL [1,2] data on the various X_c hadrons. In each case, $\chi^2/\text{d.o.f.}$ is calculated for the $Z \rightarrow b\bar{b}$ sample (b), the full sample (All), and the combination of both (Average).

X_c	Order	b	All	Average
D^0	LO	1.26	0.916	1.09
	NLO	1.10	0.766	0.936
D^+	LO	0.861	0.658	0.759
	NLO	0.756	0.560	0.658
D^{*+}	LO	1.19	1.12	1.16
	NLO	1.07	1.01	1.04
D_s^+	LO	0.246	0.111	0.178
	NLO	0.290	0.112	0.201
Λ_c^+	LO	1.05	0.117	0.583
	NLO	1.05	0.112	0.579

$$D_b(x, \mu_0^2) = Nx^\alpha(1-x)^\beta, \quad (2)$$

the χ^2 values per degree of freedom ($\chi^2/\text{d.o.f.}$) achieved in the fits, and the branching fractions $B_Q(\mu)$ and average momentum fractions $\langle x \rangle_Q(\mu)$,

$$B_Q(\mu) = \int_{x_{\text{cut}}}^1 dx D_Q(x, \mu^2), \quad (3)$$

$$\langle x \rangle_Q(\mu) = \frac{1}{B_Q(\mu)} \int_{x_{\text{cut}}}^1 dx x D_Q(x, \mu^2), \quad (4)$$

TABLE III. Branching fractions (in %) of $Q \rightarrow X_c$ for $Q = c, b$ and the various X_c hadrons evaluated according to Eq. (3) in LO and NLO at the respective production thresholds $\mu = 2m_Q$ and at the Z-boson resonance $\mu = M_Z$.

X_c	Order	$B_c(2m_c)$	$B_c(M_Z)$	$B_b(2m_b)$	$B_b(M_Z)$
D^0	LO	72.8	67.6	57.5	52.7
	NLO	71.6	65.8	54.3	49.3
D^+	LO	28.9	26.8	19.0	17.7
	NLO	26.4	24.3	18.5	17.1
D^{*+}	LO	29.0	27.2	24.3	23.1
	NLO	27.8	25.9	24.5	22.8
D_s^+	LO	12.3	11.7	23.1	21.2
	NLO	10.6	10.0	22.1	20.2
Λ_c^+	LO	6.17	6.06	15.1	13.7
	NLO	6.12	5.87	14.3	12.8

TABLE IV. Average momentum fractions of $Q \rightarrow X_c$ for $Q = c, b$ and the various X_c hadrons evaluated according to Eq. (4) in LO and NLO at the respective production thresholds $\mu = 2m_Q$ and at the Z-boson resonance $\mu = M_Z$.

X_c	Order	$\langle x \rangle_c(2m_c)$	$\langle x \rangle_c(M_Z)$	$\langle x \rangle_b(2m_b)$	$\langle x \rangle_b(M_Z)$
D^0	LO	0.573	0.442	0.318	0.285
	NLO	0.550	0.420	0.304	0.272
D^+	LO	0.571	0.441	0.341	0.302
	NLO	0.557	0.425	0.324	0.287
D^{*+}	LO	0.617	0.472	0.393	0.344
	NLO	0.592	0.448	0.366	0.322
D_s^+	LO	0.654	0.496	0.348	0.310
	NLO	0.653	0.487	0.337	0.299
Λ_c^+	LO	0.765	0.571	0.302	0.272
	NLO	0.738	0.544	0.290	0.261

where $x_{\text{cut}} = 0.1$, at $\mu = 2\mu_0$ and M_Z . In the present analysis, we adopt the up-to-date input information from our 2005 paper [4].

Our new results are presented in Tables I, II, III, and IV. Comparing Tables III and IV with the corresponding tables in Refs. [3,4], we observe that the branching and average momentum fractions are changed very little by the reduction in μ_0 . For a comparison of these observables with experimental data, we refer to Refs. [3,4].

For lack of space, we refrain from presenting here any updated versions of figures included in Refs. [3,4]; they would not exhibit any qualitatively new features. However, as already mentioned in Ref. [13], the reduction in μ_0 has an appreciable effect on the gluon FF's, which are only feebly constrained by e^+e^- data. This effect is visualized for $X_c = D^{*+}$ in Fig. 1, where the $\mu_0 = m_Q$ to $\mu_0 = 2m_Q$ ratios of $D_g(x, \mu^2)$ at $\mu = 5, 10$, and 20 GeV are shown as functions of x . We observe that the reduction in μ_0 leads to a significant enhancement of the gluon FF, especially at low values of x . The results for $X_c = D^0, D^+, D_s^+$, and Λ_c^+ are very similar and, therefore, not shown here.

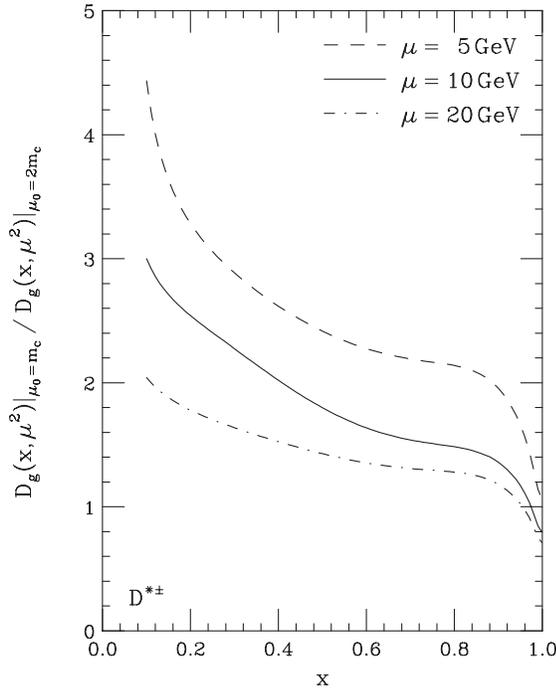


FIG. 1. $\mu_0 = m_Q$ to $\mu_0 = 2m_Q$ ratios of $D_g(x, \mu^2)$ at $\mu = 5$ (dashed line), 10 (solid line), and 20 GeV (dot-dashed line) as functions of x for $X_c = D^{*+}$.

III. CONCLUSIONS

In this addendum to Refs. [3,4], we repeated the fits of nonperturbative D^0 , D^+ , D^{*+} , D_s^+ , and Λ_c^+ FF's, both at

LO and NLO in the $\overline{\text{MS}}$ factorization scheme, to OPAL data from LEP1 [1,2] for the reduced choice $\mu_0 = m_Q$ ($Q = c, b$) of starting point for the DGLAP evolution in the factorization scale μ . These FF's are appropriate for use in connection with modern sets of PDF's [12], which are implemented with the same convention for the heavy-flavor thresholds. A FORTRAN routine that evaluates the values of these FF's as functions of the input variables x and μ may be obtained by electronic mail upon request from the authors.

This reduction in μ_0 is inconsequential for the theoretical interpretation of experimental e^+e^- data because it is compensated by corresponding shifts in the fit parameters N , α , β , and ϵ . However, the gluon FF's, which are only feebly constrained by e^+e^- data, play a significant role in hadroproduction. In fact, detailed analysis [13] revealed that the increase in the gluon FF's due to the extension of the evolution length leads to a rise in cross section and thus improves the agreement with the CDF data of charmed-meson production in run II at the Tevatron [14].

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