

**Methods of approaching decoherence in the flavor sector due to space-time foam**

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In the first part of this work we discuss possible effects of stochastic space-time foam configurations of quantum gravity on the propagation of “flavored” (Klein-Gordon and Dirac) neutral particles, such as neutral mesons and neutrinos. The formalism is not the usually assumed Lindblad one, but it is based on random averages of quantum fluctuations of space-time metrics over which the propagation of the matter particles is considered. We arrive at expressions for the respective oscillation probabilities between flavors which are quite distinct from the ones pertaining to Lindblad-type decoherence, including in addition to the (expected) Gaussian decay with time, a modification to oscillation behavior, as well as a power-law cutoff of the time-profile of the respective probability. In the second part we consider space-time foam configurations of quantum-fluctuating charged-black holes as a way of generating (parts of) neutrino mass differences, mimicking appropriately the celebrated Mikheyev-Smirnov-Wolfenstein (MSW) effects of neutrinos in stochastically fluctuating random media. We pay particular attention to disentangling genuine quantum-gravity effects from ordinary effects due to the propagation of a neutrino through ordinary matter. Our results are of interest to precision tests of quantum-gravity models using neutrinos as probes.

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**I. INTRODUCTION AND MOTIVATION**

The important feature of classical general relativity, is the fact that space-time is not *simply* a frame of coordinates on which events take place, but is itself a dynamical entity. For conventional quantization this poses a problem, since the space-time coordinates themselves appear “fuzzy.” The “fuzzyness” of space-time is associated with microscopic quantum fluctuations of the metric field, which may be singular. For instance, one may have Planck size ( $10^{-35}$  m) black holes, emerging from the quantum-gravity (QG) “vacuum,” which may give space-time a “foamy,” topologically nontrivial structure.

An important issue arises which concerns the existence of a well-defined scattering matrix in the presence of black holes, especially such microscopic ones (i.e. for strong gravity); the information encoded in matter fields may not be delivered intact to asymptotic observers. In this context we refer the reader to a recent claim by S. Hawking [1] according to which information is not lost in the black-hole case, but is entangled in a holographic way with the portion of space-time outside the horizon. It is claimed that this can be understood formally within a Euclidean space-time path-integral formulation of QG. In this formulation the path-integral over the topologically trivial metrics is unitary, but the path-integral over the topologically nontrivial black-hole metrics, leads to correlation functions that decay to zero for asymptotically long times. Consequently only the contributions over trivial topologies are important asymptotically, and so information is preserved. In simple terms, according to Hawking himself, the information is not lost but may be so mangled that it cannot be easily extracted by an asymptotic observer. He drew the analogy to information encrypted in “a burnt out encyclopedia,” where the information is

radiated away in the environment, but there is no paradox, despite the fact that it is impossibly difficult to recover.

However, there are fundamental issues we consider as unanswered by the above interesting arguments. This makes the situation associated with the issue of unitarity of effective matter theories in foamy space-times unresolved. On the technical side, one issue that causes concern is the Euclidean formulation of QG. According to Hawking this is the only sensible way to perform the path integral over geometries. However, given the uncertainties in analytic continuation, it may be problematic. Additionally, it has been argued [1] that the dynamics of formation and evaporation of (microscopic) black holes is unitary using Maldacena’s holographic conjecture of AdS/CFT correspondence [2] for the case of anti-de Sitter (supersymmetric) space-times. This framework describes the process in a very specific category of foam, and may not be valid generally for theories of QG. However even in this context the rôle of the different topological configurations is actually important, a point recently emphasized by Einhorn [3]. In Maldacena’s treatment of black holes [4], the non-vanishing of the contributions to the correlation functions due to the topologically nontrivial configurations is required by unitarity. Although such contributions vanish in semiclassical approximations, the situation may be different in the full quantum theory, where the rôle of stretched and fuzzy (fluctuating) horizons may be important, as pointed out by Barbon and Rabinovici [5].

The information paradox is acutest [3] in the case of gravitational collapse to a black hole from a pure quantum mechanical state, without a horizon; the subsequent evaporation due to the celebrated Hawking-radiation process, leaves an apparently “thermal” state. It is in this sense that the analogy [1] is made with the encoding of information in the radiation of a burning encyclopedia. However

the mangled form of information in the burnt out encyclopedia, is precisely the result of an interaction of the encyclopedia with a heat bath that burned its pages, thereby leading to an *irreversible* process. The information cannot be retrieved due to entropy production in the process.

In our view, if microscopic black holes, or other defects forming space-time foam, exist in the vacuum state of quantum gravity (QG), this state will constitute an “environment” which will be characterized by some *entanglement entropy*, due to its interaction with low-energy matter. This approach has been followed by the authors [6,7] in many phenomenological tests or microscopic models of space-time foam [8], within the framework of noncritical string theory; the latter, in our opinion, is a viable (non-equilibrium) theory of space-time foam [9], based on an identification of time with the Liouville mode. The latter is viewed as a dynamical local renormalization-group scale on the world-sheet of a nonconformal string. The non-conformality of the string is the result of its interaction with backgrounds which are out of equilibrium, such as those provided by twinkling microscopic black holes in the foam. The entropy in this case can be identified with the world-sheet conformal anomaly of a  $\sigma$ -model describing the propagation of a matter string in this fluctuating background [9,10]. Although within critical string theory, arguments have been given that entanglement entropy can characterize the number of microstates of anti-de Sitter black holes [11], we do not find these to be entirely convincing.

It should be stressed at this stage that our approach, and the underlying conceptual background, to (Liouville-string) quantum space-time foam ground state of QG differs from conventional local effective theories of quantum gravity involving loops of virtual particles such as gravitons, discussed, for instance, in [12]. In such local field theory models, dressing of a matter quantum state with virtual gravitons, say, does not lead to decoherence, since the latter does not induce any additional “environmental” interaction that would induce irreversibility and decoherence. If virtual local black holes behaved like virtual photons within a framework of ordinary quantum field theory, as argued by Diósi [13], it is the state dressed (by such virtual particles) in such models that is accessible to experimenters, and thus no decoherence is expected, unless these virtual black holes emit somehow “real” gravitons.

The above picture for QG effects on matter is quite different from our point of view in important ways. Blackholes, microscopic or otherwise, have horizons, which are real, in a sense to be clarified below, within the context of Liouville strings, which is a specific string-inspired framework for which one can discuss such ideas in a quantitative manner. In our picture there are several parallel three-brane worlds, one of which represents the observable Universe, embedded in a higher-dimensional bulk where only gravitational (closed) string states propa-

gate. On the brane world there are only open string states propagating, representing ordinary matter, with their ends attached to the brane hypersurface. Of course, there are also closed string states, either propagating along the longitudinal brane directions, or crossing the brane boundary from the bulk. As discussed in [14] consistent supersymmetric models of  $D$ -particle foam can be constructed, in which the bulk space between, say, two parallel brane worlds is populated by pointlike  $D$ -particle defects which cross brane boundaries. These  $D$ -particle defects can even represent compactified black holes from a four-dimensional view point, with the extra dimensions being wrapped up appropriately in Planckian size compactifications. One may then encounter a situation in which  $D$ -particle pointlike space-time defects from the higher-dimensional bulk space-time cross the three brane (where ordinary-matter resides). Such constructions clearly lead to a radically different picture from virtual excitations in a vacuum.

In Ref. [8] we have discussed the details of dynamical formation of horizons on the brane world (in the context of (Liouville) string theory), as a result of the encounter of brane matter with the crossing  $D$ -particle defect. Schematically, ordinary string matter on the brane creates - through backreaction (recoil) effects due to scattering off  $D$ -particles- sufficient distortion of space-time for dynamical horizons, surrounding the defect, to appear. The appearance of horizons in this way looks - from the point of view of a four-dimensional observer—as a dynamical “flashing on and off of a black hole,” coming from the “vacuum.” The lifetime of such objects is of the order of the Planck time, since this is the time uncertainty for the defect to cross the brane world and interact with stringy matter excitations. Once horizons form there is entropy production [10] and through this irreversibility and decoherence. Consequently such stringy black-hole defects are not equivalent to ordinary virtual particles in flat space-time field theories or to effective local quantum-gravity approaches from the point of view of decoherence (cf. [12]).

The presence of dynamical horizons is a real effect of the ground state of quantum gravity (at least in such Liouville-string approaches to QG), which implies real environmental entanglement of matter systems with (gravitational) degrees of freedom behind the horizons. This leads to the problem of loss of information for particles propagating outside the horizon, and as such can lead to microscopic time-irreversibility à la Wald [15], and thus to  $CPT$  violation and QG-induced decoherence. There is then a consequent nonunitary evolution of particles outside the horizon. Somewhat general arguments (even in flat space-times but with a boundary) have been put forward in the literature [16] to support this point of view. The generic property of nonunitary evolution has then been extracted and modeled by phenomenological Lindblad master equations [17,18] over two decades [7,19] to describe particles evolving in space-time foam.

We cannot, of course, advocate at this stage that this (noncritical, Liouville) string approach, is the only consistent approach to quantum gravity. Consequently we cannot rule out other approaches given in given in [12] or the suggestion that the environment due to quantum-gravity is a sort of thermal heat bath, a point of view advocated in Ref. [20]. It is therefore a challenging experimental issue to seek signatures for such quantum-gravity-induced decoherencelike effects, which could definitely discriminate between several models of quantum gravity.

In general, for phenomenological purposes, the important feature of such situations is the fact that gravitational environments, arising from space-time foam or some other, possibly semiclassical feature of QG, can still be described by nonunitary evolutions of density matrices. Such equations have the form

$$\partial_t \rho = \Lambda_1 \rho + \Lambda_2 \rho, \quad (1.1)$$

where

$$\Lambda_1 \rho = \frac{i}{\hbar} [\rho, H]$$

and  $H$  is the hamiltonian with a stochastic element in a classical metric. Such effects may arise from backreaction of matter within a quantum theory of gravity [21] which decoheres the gravitational state to give a stochastic ensemble description. Furthermore within models of  $D$ -particle foam arguments in favor of a stochastic metric have been given [6]. The Liouvillian term  $\Lambda_2 \rho$  gives rise to a nonunitary evolution. A common approach to  $\Lambda_2 \rho$ , not based on microscopic physics, is to parametrize the Liouvillian in a so-called Lindblad form [17,18]. We note at this point that any nonlinear evolutions that may characterize a full theory of QG (see e.g. a manifestation in Liouville strings [22]), can be ignored to a first approximation appropriate for the accuracy of contemporary experimental probes of QG. Generically space-time foam and the backreaction of matter on the gravitational metric may be modeled as a randomly fluctuating environment; formalisms for open quantum mechanical systems propagating in such random media can thus be applied and lead to concrete experimental predictions. The approach to these questions have to be phenomenological to some degree since QG is not sufficiently developed at a nonperturbative level.

One of the most sensitive probes of such stochastic quantum-gravity phenomena are neutrinos [7,23–27], in particular, high-energy ones [28]. It is the point of this article to present various approaches to gravitationally-induced decoherence of matter and to classify some characteristic experimental predictions that could be falsified in current or near future neutrino experiments.

The neutrino, being almost massless, and weakly interacting, can travel long distances in the Universe essentially undisturbed. Thus the detection of high-energy neutrinos, which are produced at early stages of our Universe, say in

gamma-ray-bursters or other violent phenomena, can carry important information on the Universe's past which would not have reached us otherwise. If space-time has therefore a stochastic foamy structure, the longer the neutrino travels the greater the cumulative quantum-gravity effects become. For instance, due to their known mass differences, the neutrinos exhibit oscillations between their various flavors, and such oscillations appear to attenuate with time in stochastic environments. Although such an attenuation may be too small to be detected in laboratory experiments, it may nevertheless be appreciable in the case of ultra-high-energy neutrinos, which have travelled cosmological distances before reaching the observation point on Earth [7,28]. From such (non) observations of damping effects, one may place important bounds on quantum-gravity effects, information that may prove quite useful in our theoretical quest of understanding space-time.

Moreover, there is another interesting possibility regarding neutrinos. As pointed out recently in [23], the tiny mass differences between neutrino flavors may themselves (in part) be the result of a  $CPT$  violating quantum-gravity background. The phenomenon, if true, would be the generalization of the celebrated Mikheyev-Smirnov-Wolfenstein (MSW) effect [29,30]. The latter arises from effective mass differences between the various neutrino flavors, as a result of different type of interactions of the various flavors with matter within the context of the standard model. The phenomenon has been generalized to randomly fluctuating media [31], which are of relevance to solar and nuclear reactor  $\beta$ -decays neutrinos. This stochastic MSW effect will be more relevant for us, since we consider space-time foam, as a random medium which induces flavor-sensitive mass differences. Moreover the Liouvillian in (I) can be derived and is not an assumption in this framework.

The structure of the article will be the following: we commence our analysis by considering in Sec. II flavor oscillations between two generations of neutrinos, whose dynamics are governed by Klein-Gordon or Dirac Lagrangians in the presence of weakly fluctuating background random gravitational fields. The Klein-Gordon case is an idealization when the effects of neutrino spin are ignored. Moreover it can be of interest in its own right when flavor oscillations of neutral mesons are considered. The case of Dirac particles with two flavors is considered in Sec. III. An effective description in terms of two-level systems is derived and analyzed. We then proceed in Sec. IV to discuss gravitational MSW effects in oscillation phenomena (also for two flavors) for the case when the particles are highly relativistic (a situation applicable to neutrinos). We pay particular attention to disentangling potential genuine quantum-gravity-induced decoherence effects from conventional effects due to the passage of the neutrino probe through ordinary stochastic fluctuating matter. As we shall discuss, the disentanglement is

achieved via the energy  $E$  and oscillation length  $L$  dependence of the relevant probability. In particular, conventional effects attenuate to zero as the parameter  $L/E \rightarrow 0$  [32,33], in contrast to the genuine quantum-gravity decoherence effects which, at least in some models of space-time foam decoherence, exhibit a  $L \cdot E$  dependence. Conclusions and outlook are presented in Sec. V, followed by three appendices that contain some technical details of our formalism.

## II. GRAVITATIONAL DECOHERENCE CALCULATIONS FOR SCALAR PARTICLES

Since the effects of stochastic space-time foam can appear through both  $\Lambda_1 \rho$  and  $\Lambda_2 \rho$  in (I) we shall for clarity isolate their individual signatures. The most satisfactory way of dealing with the effects of such a background is by coupling covariantly the gravitational field to a Klein-Gordon or Dirac Lagrangian. This avoids intuitive arguments which are sometimes presented [34] and correctly incorporates covariance unlike these other approaches.

For the case of scalar particles of mass  $m$ , such as neutral mesons (or in the toy case where the spin of a neutrino of mass  $m$  is ignored), we can describe the motion of the particle in a curved background by means of a Klein-Gordon equation for a field  $\Phi$ . The Klein-Gordon equation

$$g^{\mu\nu} = \begin{pmatrix} -(a_1 + 1)^2 + a_2^2 & -a_3(a_1 + 1) + a_2(a_4 + 1) \\ -a_3(a_1 + 1) + a_2(a_4 + 1) & -a_3^2 + (a_4 + 1)^2 \end{pmatrix}. \quad (2.4)$$

Since the Christoffel symbols  $\Gamma_{\mu\nu}^\alpha = 0$  and  $R = 0$  for static  $a_i$ 's the Klein-Gordon equation is

$$(g^{00} \partial_0^2 + 2g^{01} \partial_0 \partial_1 + g^{11} \partial_1^2) \phi - m^2 \phi = 0. \quad (2.5)$$

For positive energy plane wave solutions

$$\phi(x, t) \sim \varphi(k, w) e^{i(-\omega t + kx)}$$

we have the dispersion relation

$$\omega = \frac{g^{01}}{g^{00}} k + \frac{1}{-g^{00}} \sqrt{(g^{01})^2 k^2 - g^{00}(g^{11} k^2 + M^2)}. \quad (2.6)$$

For an initial  $\alpha$  flavor state with momentum  $k$ , the density matrix  $\rho$  at time  $t$  is

$$\rho(t) = \sum_{j,l,\beta,\gamma} U_{\alpha j} U_{\beta j}^* U_{\alpha l}^* U_{\gamma l} e^{i(\omega_l - \omega_j)t} |f_\beta\rangle \langle f_\gamma|, \quad (2.7)$$

where  $\beta$  is a flavor index and  $j, l (= 1, 2)$  denote indices for mass eigenstates with eigenvalue  $M = m_1$  and  $M = m_2$ . The bras and kets in (2.7) are flavor eigenstates (corresponding to the flavors denoted by the subscripts) and  $U$  is the mixing matrix which can be parametrized by an angle  $\theta$ :

in a gravitational field reads:

$$g^{\alpha\beta} D_\alpha D_\beta \Phi - m^2 \Phi = 0. \quad (2.1)$$

where  $g^{\alpha\beta}$  is the metric tensor and  $D_\alpha$  is a covariant derivative. We will consider the neutrino to be moving in the  $x$ -direction. For simplicity [34] we will examine the situation where the relevant part of the contravariant metric can be regarded as being in 1 + 1 dimension. Moreover if metric fluctuations are caused by  $D$ -particle foam [8] there are further arguments in favor of such a truncated theory. A small stochastic perturbation of the flat metric can be written as

$$g = O \eta O^T \quad (2.2)$$

with

$$O = \begin{pmatrix} a_1 + 1 & a_2 \\ a_3 & a_4 + 1 \end{pmatrix}, \quad \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.3)$$

and where the static coefficients  $a_i$ 's are Gaussian random variables satisfying  $\langle a_i \rangle = 0$  and  $\langle a_i a_j \rangle = \delta_{ij} \sigma_i$ . This is a simplified model and could be made more complicated, for example, by having a general symmetric covariance matrix for the  $a_i$ 's. Such complications will not affect our qualitative results and magnitudes of estimates. From (2.2):

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \quad (2.8)$$

Now since the  $\omega$ 's are functions of classical random variables (which thus have a positive probability distribution), the averaging of  $\rho(t)$  over these random variables is a positively weighted (generalized) sum over density matrices. Hence the averaged density matrix is also positive and represents a mixed state. The probability of transition from an initial state of flavor 1 to 2 is

$$\text{Prob}(1 \rightarrow 2) = \sum_{j,l} U_{1j} U_{2j}^* U_{1l}^* U_{2l} e^{i(\omega_l - \omega_j)t}, \quad (2.9)$$

where the time dependent part is

$$U_{12} U_{22}^* U_{11}^* U_{21} e^{i(\omega_1 - \omega_2)t} + U_{11} U_{21}^* U_{12}^* U_{22} e^{i(\omega_2 - \omega_1)t}.$$

Since the  $\{a_i\}$  are assumed to be independent Gaussian variables, our covariance matrix  $\Xi$  has the diagonal form

$$\Xi = \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_3} & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_4} \end{pmatrix}, \quad (2.10)$$

with  $\sigma_i > 0$ . The calculation of transition probabilities

requires the evaluation

$$\langle e^{i(\omega_1 - \omega_2)t} \rangle \equiv \int d^4a \exp(-\vec{a} \cdot \vec{\Xi} \cdot \vec{a}) e^{i(\omega_1 - \omega_2)t} \frac{\det \vec{\Xi}}{\pi^2}. \quad (2.11)$$

From (2.6) we obtain

$$\omega_1 - \omega_2 = \frac{1}{-g^{00}} (\sqrt{(g^{01})^2 k^2 - g^{00}(g^{11} k^2 + m_1^2)} - \sqrt{(g^{01})^2 k^2 - g^{00}(g^{11} k^2 + m_2^2)}). \quad (2.12)$$

Now, since fluctuations are small, we can make the expansion

$$\frac{1}{-g^{00}} (\sqrt{(g^{01})^2 k^2 - g^{00}(g^{11} k^2 + m_1^2)} - \sqrt{(g^{01})^2 k^2 - g^{00}(g^{11} k^2 + m_2^2)}) = c(m_l) + \sum_i d_i(m_l) a_i + \sum_{i,j} a_i f_{ij}(m_l) a_j + \mathcal{O}(a^3), \quad (2.13)$$

where the nonzero expansion coefficients are

$$\begin{aligned} c(m_l) &= \sqrt{k^2 + m_l^2}, & d_1(m_l) &= -\sqrt{k^2 + m_l^2}, \\ d_4(m_l) &= \frac{k^2}{\sqrt{k^2 + m_l^2}}, & f_{11}(m_l) &= \sqrt{k^2 + m_l^2}, \\ f_{14}(m_l) &= -\frac{1}{2} \frac{k^2}{\sqrt{k^2 + m_l^2}}, & f_{22}(m_l) &= \frac{m_l^2 + 2k^2}{2\sqrt{k^2 + m_l^2}}, \\ f_{23}(m_l) &= \frac{-k^2}{2\sqrt{k^2 + m_l^2}}, & f_{44}(m_l) &= \frac{1}{2} \frac{k^2 m_l^2}{(k^2 + m_l^2)^{3/2}}. \end{aligned} \quad (2.14)$$

Hence we see that for highly energetic scalar particles the stochastic model of space-time foam leads to a modification of oscillation behavior quite distinct from that of the Lindblad formulation. In particular for the transition probability there is a Gaussian decay with time, a modification of the oscillation period as well an additional power law falloff both decays are invariant under  $t \rightarrow -t$  which is of course related to their origin from  $\Lambda_1$ . From this characteristic time dependence bounds can be obtained for the fluctuation strength of space-time foam. They are compatible with previous estimates and will be discussed later.

### III. DECOHERENCE OF DIRAC PARTICLES

Although scalar flavor oscillation is the relevant case for neutral mesons, for the important case of neutrino oscil-

lations and space-time foam it can only be a rudimentary approximation. The spinorial structure should be incorporated into the description. The usual discrete level descriptions of oscillation phenomena cannot suggest the natural way to incorporate the background and this leads to consideration of the Dirac equation in the presence of a stochastic gravitational background. For definiteness we will take neutrinos to be described by two flavors and by massive Dirac spinors  $\Psi$ ; also a term is introduced which incorporates in mean field the role of a medium that leads to the MSW effect. The neutrinos will interact via the weak interactions with electrons produced via evaporation of microscopic black holes. Any rigorous discussion of such a process would involve a full theory of QG which is not available currently. In the next section some semiclassical arguments from black-hole physics are summarized which

and  $f_{ij}$  is symmetric. In this approximation we find that

$$\begin{aligned} \langle e^{i(\omega_1 - \omega_2)t} \rangle &= \left( \frac{\det \vec{\Xi}}{\det \mathbf{B}} \right)^{1/2} \exp\left(\frac{\chi_1}{\chi_2}\right) \exp(i\tilde{b}t) \\ &= \frac{4\tilde{d}^2}{(P_1 P_2)^{1/2}} \exp\left(\frac{\chi_1}{\chi_2}\right) \exp(i\tilde{b}t), \end{aligned} \quad (2.15)$$

where

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sigma_1} - i\tilde{b}t & 0 & 0 & -\frac{i\tilde{b}}{2\tilde{d}} k^2 t \\ 0 & \frac{1}{\sigma_2} - \frac{i\tilde{b}}{2\tilde{d}} (\tilde{d} - k^2) & \frac{-ik^2 \tilde{b}t}{2\tilde{d}} & 0 \\ 0 & \frac{-ik^2 \tilde{b}t}{2\tilde{d}} & \frac{1}{\sigma_3} & 0 \\ \frac{-i\tilde{b}}{2\tilde{d}} k^2 t & 0 & 0 & \frac{1}{\sigma_4} - \frac{1}{2} ik^2 \tilde{c}t \end{pmatrix},$$

$$\begin{aligned} \chi_1 &= -4(\tilde{d}^2 \sigma_1 + \sigma_4 k^4) \tilde{b}^2 t^2 + 2i\tilde{d}^2 \tilde{b}^2 \tilde{c} k^2 \sigma_1 \sigma_4 t^3, \\ \chi_2 &= 4\tilde{d}^2 - 2i\tilde{d}^2 (k^2 \tilde{c} \sigma_4 + 2\tilde{b} \sigma_1) t + \tilde{b} k^2 (\tilde{b} k^2 - 2\tilde{d}^2 \tilde{c}) \sigma_1 \sigma_4, \\ P_1 &= 4\tilde{d}^2 + 2i\tilde{d} \tilde{b} (k^2 - \tilde{d}) \sigma_2 t + \tilde{b}^2 k^4 \sigma_2 \sigma_3 t^2, \\ P_2 &= 4\tilde{d}^2 - 2i\tilde{d}^2 (k^2 \tilde{c} \sigma_4 + 2\tilde{b} \sigma_1) t + \mathcal{O}(\sigma^2) \end{aligned}$$

with

$$\begin{aligned} \tilde{b} &= \sqrt{k^2 + m_1^2} - \sqrt{k^2 + m_2^2}, \\ \tilde{c} &= m_1^2 (k^2 + m_1^2)^{-3/2} - m_2^2 (k^2 + m_2^2)^{-3/2}, \\ \tilde{d} &= \sqrt{k^2 + m_1^2} \sqrt{k^2 + m_2^2}. \end{aligned} \quad (2.16)$$

It is particularly illuminating to consider the limit  $k \gg m_1, m_2$  for which  $\tilde{d} = k^2$ ,  $\tilde{b} = \frac{(\Delta m)^2}{2k}$ , where  $(\Delta m)^2 = m_1^2 - m_2^2$ , and  $\tilde{c} = \frac{(\Delta m)^2}{k^3}$ . We then have

$$\begin{aligned} P_1 P_2 &= \left( 4k^4 + \frac{1}{4} (\Delta m)^4 k^2 \sigma_2 \sigma_3 t^2 \right) \left( \frac{-3}{4} (\Delta m)^4 k^2 t^2 \sigma_1 \sigma_4 - 2ik^3 (\Delta m)^2 (\sigma_1 + \sigma_4) t + 4k^4 \right) \left( \frac{\chi_1}{\chi_2} \right) \\ &= -\frac{1}{2} \frac{(2k^4 \sigma_1 - ik^3 (\Delta m)^2 \sigma_1 \sigma_4 t + 2k^4 \sigma_4) (\Delta m)^4 t^2}{k^2 \left( \frac{-3}{4} (\Delta m)^4 k^2 t^2 \sigma_1 \sigma_4 - 2ik^3 (\Delta m)^2 (\sigma_1 + \sigma_4) t + 4k^4 \right)}. \end{aligned}$$

lations and space-time foam it can only be a rudimentary approximation. The spinorial structure should be incorporated into the description. The usual discrete level descriptions of oscillation phenomena cannot suggest the natural way to incorporate the background and this leads to consideration of the Dirac equation in the presence of a stochastic gravitational background. For definiteness we will take neutrinos to be described by two flavors and by massive Dirac spinors  $\Psi$ ; also a term is introduced which incorporates in mean field the role of a medium that leads to the MSW effect. The neutrinos will interact via the weak interactions with electrons produced via evaporation of microscopic black holes. Any rigorous discussion of such a process would involve a full theory of QG which is not available currently. In the next section some semiclassical arguments from black-hole physics are summarized which

motivate this possibility. Of course for such a medium it is also necessary to incorporate fluctuations and this will be investigated at length in the next section through the introduction of a  $\Lambda_2$  with a specific double commutator structure.

As in the scalar case only weak fluctuations  $h^{\mu\nu}$  around the flat metric  $\eta^{\mu\nu}$  are considered and as for that case we will consider the form of  $g^{\mu\nu}$  in (2.4). The Lagrangian  $\mathcal{L}_f$  for a Dirac particle of mass  $m_f$  (in standard notation) is (see, for example, [35])

$$\mathcal{L}_f = \bar{\Psi} \left[ \left( 1 + \frac{1}{2}h \right) (i\gamma^\mu \partial_\mu - m_f) \right] \Psi - \frac{i}{2} \bar{\Psi} h^{\mu\nu} \gamma_\mu \partial_\nu \Psi - \frac{i}{4} \bar{\Psi} (\partial_\nu h^{\mu\nu}) \gamma_\mu \bar{\Psi} + \frac{i}{4} \bar{\Psi} (\partial_\mu h) \gamma^\mu \Psi, \quad (3.1)$$

where  $h = h^{\mu\nu} \eta_{\mu\nu} (= a_1^2 - a_2^2 - a_3^2 + a_4^2 + 2(a_1 + a_4))$ . The total Lagrangian will have contributions from electron and muon neutrino spinor fields  $\Psi_e$  and  $\Psi_\mu$  in the form of (3.1) together with a Dirac mass mixing term (proportional to  $m_{e\mu}$ ) and a MSW interaction. On writing

$$\Psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix}, \quad (3.2)$$

where  $\chi$  and  $\phi$  represent Weyl spinors, our total Lagrangian, including the mixing and MSW terms, becomes [36]

$$\begin{aligned} \mathcal{L} = & \left( 1 + \frac{1}{2}h \right) (\chi_e^\dagger i\partial_0 \chi_e + \chi_e^\dagger \sigma_1 i\partial_1 \chi_e + \phi_e^\dagger i\partial_0 \phi_e - \phi_e^\dagger \sigma_1 i\partial_1 \phi_e) - \frac{i}{2} (\chi_e^\dagger (b_1 \mathbf{1} - b_3 \sigma_1) \partial_0 \chi_e + \chi_e^\dagger (b_3 \mathbf{1} - b_2 \sigma_1) \partial_1 \chi_e) \\ & - \frac{i}{2} (\phi_e^\dagger (b_1 \mathbf{1} + b_3 \sigma_1) \partial_0 \phi_e + \phi_e^\dagger (b_3 \mathbf{1} + b_2 \sigma_1) \partial_1 \phi_e) + \{e \rightarrow \mu\} - \left( 1 + \frac{1}{2}h \right) (m_{e\mu} (\chi_e^\dagger \phi_\mu + \phi_\mu^\dagger \chi_e + \chi_\mu^\dagger \phi_e + \phi_e^\dagger \chi_\mu) \\ & + V \phi_e^\dagger \phi_e) - \left( 1 + \frac{1}{2}h \right) m_e (\chi_e^\dagger \phi_e + \phi_e^\dagger \chi_e) + \{e \rightarrow \mu\}. \end{aligned} \quad (3.3)$$

Here  $V$  is the coupling which represents an MSW effect and is proportional to the density of the microscopic black-hole density. Moreover, for convenience, we have made the definitions

$$\begin{aligned} b_1 & \equiv a_1^2 + 2a_1 - a_2^2, & b_2 & \equiv a_3^2 - a_4^2 - 2a_4, \\ b_3 & \equiv a_1 a_3 + a_3 - a_2 a_4 - a_2. \end{aligned} \quad (3.4)$$

We follow the basic procedure presented in [36] but now in the presence of a stochastic gravitational background. In the absence of  $V$  the mixing matrix  $U$  has the same form as in the last section with

$$\tan(2\theta) = \frac{2m_{e\mu}}{m_\mu - m_e} \quad (3.5)$$

and so

$$\begin{pmatrix} \phi_e \\ \phi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad (3.6)$$

and

$$\begin{pmatrix} \chi_e \\ \chi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \quad (3.7)$$

This results in

$$\begin{aligned} \mathcal{L} = & \left( 1 + \frac{1}{2}h \right) (\chi_1^\dagger (i\partial_0 + i\sigma_1 i\partial_1) \chi_1 + \chi_2^\dagger (i\partial_0 + \sigma_1 i\partial_1) \chi_2 + \phi_1^\dagger (i\partial_0 - \sigma_1 i\partial_1) \phi_1 + \phi_2^\dagger (i\partial_0 - \sigma_1 i\partial_1) \phi_2 \\ & - m_1 (\chi_1^\dagger \phi_1 + \phi_1^\dagger \chi_1) - m_2 (\chi_2^\dagger \phi_2 + \phi_2^\dagger \chi_2) - V (\cos\theta \phi_1^\dagger + \sin\theta \phi_2^\dagger) (\cos\theta \phi_1 + \sin\theta \phi_2) \\ & - \frac{i}{2} (\chi_1^\dagger (b_1 \mathbf{1} - b_3 \sigma_1) \partial_0 \chi_1 + \chi_2^\dagger (b_1 \mathbf{1} - b_3 \sigma_1) \partial_0 \chi_2 + \chi_1^\dagger (b_3 \mathbf{1} - b_2 \sigma_1) \partial_1 \chi_1 + \chi_2^\dagger (b_3 \mathbf{1} - b_2 \sigma_1) \partial_1 \chi_2) \\ & - \frac{i}{2} (\phi_1^\dagger (b_1 \mathbf{1} + b_3 \sigma_1) \partial_0 \phi_1 + \phi_2^\dagger (b_1 \mathbf{1} + b_3 \sigma_1) \partial_0 \phi_2 + \phi_1^\dagger (b_3 \mathbf{1} - b_2 \sigma_1) \partial_1 \phi_1 + \phi_2^\dagger (b_3 \mathbf{1} - b_2 \sigma_1) \partial_1 \phi_2). \end{aligned} \quad (3.8)$$

Owing to translation invariance for the MSW medium in mean field  $V$  is constant and we make an expansion of the fields in terms of helicity eigenstates

$$\begin{aligned}\phi_i &= \sum_k e^{ik \cdot x} \{ (P_\alpha^i(k, t) + N_\alpha^i(k, t)) \alpha(k) + (P_\beta^i(k, t) \\ &+ N_\beta^i(k, t)) \beta(k) \}, \\ \chi_i &= \sum_k e^{ik \cdot x} \{ (Q_\alpha^i(k, t) + M_\alpha^i(k, t)) \alpha(k) + (Q_\beta^i(k, t) \\ &+ M_\beta^i(k, t)) \beta(k) \},\end{aligned}\quad (3.9)$$

where the motion is in the  $x$ -direction,  $P_\mu^i, Q_\mu^i$  (with  $\mu = \alpha, \beta$ ) are positive frequency and  $N_\mu^i, M_\mu^i$  are negative

frequency field components. The properties of the helicity eigenstates can be summarized by the relations [36]

$$\begin{aligned}\sigma_1 k \beta(k) &= -k \beta(k) \Rightarrow \sigma_1 \beta(k) = -\beta(k), \\ \sigma_1 k \alpha(k) &= k \alpha(k) \Rightarrow \sigma_1 \alpha(k) = \alpha(k).\end{aligned}\quad (3.10)$$

On substituting the expansions (3.9) into the equations of motion (B2) and taking the projection of the equations of motion onto positive frequency and negative helicity states we obtain

$$\begin{aligned}\left(1 + \frac{1}{2}h\right) &((i\partial_0 - k - V \cos^2 \theta) P_\beta^1(k, t) - m_1 Q_\beta^1(k, t) - V \cos \theta \sin \theta P_\beta^2(k, t) - \frac{i}{2}(b_1 - b_3) \dot{P}_\beta^1(k, t) + \frac{k}{2}(b_3 - b_2) P_\beta^1(k, t) = 0, \\ &\left(1 + \frac{1}{2}h\right) (i \dot{Q}_\beta^1(k, t) + k Q_\beta^1(k, t) - m_1 P_\beta^1(k, t) - \frac{i}{2}(b_1 + b_3) \dot{Q}_\beta^1(k, t) + \frac{k}{2}(b_3 + b_2) Q_\beta^1(k, t) = 0, \\ \left(1 + \frac{1}{2}h\right) &((i\partial_0 - k - V \sin^2 \theta) P_\beta^2(k, t) - m_2 Q_\beta^2(k, t) - V \cos \theta \sin \theta P_\beta^1(k, t) - \frac{i}{2}(b_1 - b_3) \dot{P}_\beta^2(k, t) + \frac{k}{2}(b_3 - b_2) P_\beta^2(k, t) = 0, \\ &\left(1 + \frac{1}{2}h\right) ((i\partial_0 + k) Q_\beta^2(k, t) - m_2 P_\beta^2(k, t) - \frac{i}{2}(b_1 + b_3) \dot{Q}_\beta^2(k, t) + \frac{k}{2}(b_3 + b_2) Q_\beta^2(k, t) = 0.\end{aligned}\quad (3.11)$$

We seek solutions with time dependence  $e^{-iEt}$ . This leads to an eigenvalue equation for  $E$  (cf. Appendix B for details). As with the scalar case, to find the flavor oscillation probability it is necessary to compute  $\langle e^{i(\omega_1 - \omega_2)t} \rangle$ . Gaussian integration gives

$$\langle e^{i(\omega_1 - \omega_2)t} \rangle = \int d^4 a e^{-\bar{a} \cdot \mathbf{B} \cdot \bar{a} + i \bar{u} \cdot \bar{a}} = \frac{\pi^2 e^{i \bar{u} \cdot \mathbf{B}^{-1} \cdot \bar{u}}}{\sqrt{\det \mathbf{B}}}, \quad (3.12)$$

where, in our case,

$$\begin{aligned}B_{11} &= \frac{1}{\sigma_1} - it \left( \frac{m_1^2 - m_2^2}{k} - 4Vk \cos 2\theta \right), & B_{12} &= B_{21} = it \left( \frac{m_1^2 - m_2^2}{8k} - \frac{V}{2} \cos 2\theta \right), \\ B_{13} &= B_{31} = it \left( \frac{5(m_1^2 - m_2^2)}{8k} + V \cos 2\theta \right), & B_{14} &= B_{41} = it \left( \frac{m_1^2 - m_2^2}{2k} + V \cos 2\theta \right), \\ B_{22} &= \frac{1}{\sigma_2} + \frac{it}{2} \left( \frac{m_1^2 - m_2^2}{k} + V \cos 2\theta \right), & B_{23} &= B_{32} = \frac{it}{2} \left( V \cos 2\theta - \frac{m_1^2 - m_2^2}{2k} \right), & B_{24} &= B_{42} = \frac{it(m_1^2 - m_2^2)}{8k}, \\ B_{33} &= \frac{1}{\sigma_3} - \frac{i}{2} t V \cos 2\theta, & B_{34} &= B_{43} = -\frac{it}{2} \left( \frac{m_1^2 - m_2^2}{4k} + V \cos 2\theta \right), & B_{44} &= \frac{1}{\sigma_4}.\end{aligned}\quad (3.15)$$

These expressions have been obtained in the physically relevant limit  $k^2 \gg m_1^2, m_2^2$ , and  $|Y| \ll 1$  where  $Y = \frac{Vk}{m_1^2 - m_2^2}$ . On using these relations and substituting into Eq. (3.12) we find

$$\begin{aligned}\langle e^{i(\omega_1 - \omega_2)t} \rangle &= e^{i(z_0^+ - z_0^-)t/k} e^{-(1/2)(-i\sigma_1 t(m_1^2 - m_2^2)/k + V \cos 2\theta) + i(\sigma_2 t/2)(m_1^2 - m_2^2)/k + V \cos 2\theta - (i\sigma_3 t/2)V \cos 2\theta} \\ &\times e^{-(m_1^2 - m_2^2)^2/2k^2(9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) + (2V \cos 2\theta)(m_1^2 - m_2^2)/k(12\sigma_1 + 2\sigma_2 - 2\sigma_3)t^2},\end{aligned}\quad (3.16)$$

where

$$\begin{aligned}z_0^+ &= \frac{1}{2}(m_1^2 + Y(1 + \cos 2\theta)(m_1^2 - m_2^2) + Y^2(m_1^2 - m_2^2)\sin^2 2\theta), \\ z_0^- &= \frac{1}{2}(m_2^2 + Y(1 - \cos 2\theta)(m_1^2 - m_2^2) - Y^2(m_1^2 - m_2^2)\sin^2 2\theta).\end{aligned}\quad (3.17)$$

$$\begin{aligned}\vec{u} &= \left( i \frac{3(m_1^2 - m_2^2)}{2k} t + i2Vt \cos 2\theta, i \frac{(m_1^2 - m_2^2)}{2k} t \right. \\ &\left. + iVt \cos 2\theta, \right.\end{aligned}\quad (3.13)$$

$$\left. - i \frac{(m_1^2 - m_2^2)}{2k} t - iVt \cos 2\theta, i \frac{(m_1^2 - m_2^2)}{2k} t \right), \quad (3.14)$$

and the components of the symmetric matrix  $B$  are

There is again a suppression of the oscillations which is Gaussian with time and also the oscillation period is modified in an interesting way which depends both on the square of the mass differences, the mean density of microscopic black holes and the effects of backreaction on the gravitational metric.

Although not done explicitly here, the analysis of the effect of stochastic quantum fluctuations of the background space-time for the case of Majorana fermions leads to qualitatively similar results.

#### IV. SPACE-TIME FOAM MODELLED AFTER THE MSW EFFECT

##### A. MSW-like effects of stochastic space-time foam medium

In [23] the suggestion that the observed mass differences between neutrinos are generated by a sort of stochastic space-time foam has been proposed. If microscopic charged virtual black/white hole pairs were created out of the vacuum then information loss would be induced and the subsequent Hawking radiation would produce a medium with stochastically fluctuating electric charges. This radiation would have a preponderance of electron/positron pairs ( $e \bar{e}$ ) (over other charged particles (muons, etc.) from kinematics) and the “evaporating” white hole could then absorb, say, the positrons. According to the standard model of particle physics, the resultant electric current fluctuations would interact more strongly with  $\nu_e$  rather than  $\nu_\mu$ , and lead to flavor oscillations, and hence, effective mass differences, for the neutrinos. This parallels the celebrated MSW effect [29,30] for neutrinos in ordinary media.

From semiclassical calculations there is a significant difference between neutral and charged black holes. As neutral black holes evaporate they become less massive and there is an increase in the rate of evaporation. Consequently they have a short lifetime. The force on a neutrino  $\nu$  due to the emitted electron-positron pair is [37]  $\sum_\sigma G_{\sigma\nu} n_\sigma$  where  $n_\sigma$  is the particle density of species  $\sigma$  in the medium and

$$G_{\sigma\nu} = \frac{G_F}{\sqrt{2}} [(\delta_{\sigma e} - \delta_{\sigma \bar{e}})(\delta_{\nu \nu_e} - \delta_{\nu \bar{\nu}_e})(1 + 4\sin^2\theta_W)] + O\left(\frac{G_F}{m_W^2}\right) \quad (4.1)$$

and  $m_W$  is the mass of the charged weak boson and  $\theta_W$  is the weak angle. If  $n_e = n_{\bar{e}}$  then the force on a  $\nu_e$  would vanish to  $O(\frac{G_F}{m_W^2})$ . Similar subdominant terms are produced for other flavors of neutrinos and so neutral black holes would have an *equivalent* interaction with all flavors of neutrinos. On the other hand charged (Reissner-Nordstrom) black holes of charge  $Q$  and mass  $\mathcal{M}$  emit electron-positron pairs for  $\mathcal{M} > Q$  but as  $\mathcal{M} \rightarrow Q$ , the extremal black-hole limit, the surface gravity  $\kappa \rightarrow 0$  and evaporation ceases (see e.g. [38] and references therein).

The limiting behavior of near extremal charged-black holes can be made more precise from field theoretic studies of black holes [38], by actually bounding the number  $N_{\omega_0}$  of massless (scalar) particles (or pairs of particles/antiparticles) created in a state represented by a wave packet centered around an energy  $\omega_0$ :

$$N_{n\omega_0\ell m} \leq \frac{2c(\omega_0)|t(\omega_0)|^2}{(2n\pi)^{2k-1}}. \quad (4.2)$$

Here  $c(\omega_0)$  is a positive function,  $k > 0$  is an *arbitrary* but large power,  $\ell, m$  are orbital angular momentum quantum numbers (arising from spherical harmonics in the wavefunction of the packet), and  $2n\pi$ ,  $n$  being a positive integer, is a special representation of the retarded time in Kruskal coordinates [38]. The wave packet has a spread  $\epsilon$  in frequencies around  $\omega_0$ , and in fact it is the use of such wave packets that allows for a consistent calculation of the particle creation in the extremal black-hole case. From the expression (4.2), we observe that since  $2n\pi$  represents time, the rate of particle creation would drop to zero faster than any (positive) power of time at late times. The limit of extremality is obtained by means of certain analyticity properties of the particle creation number [38]. In the expression (4.2)  $t(\omega_0)$  denotes the transmission amplitude describing the fraction of the wave that enters the collapsing body, whose collapse produced the extreme black hole in [38].

In the case of space-time foam, we have currently no rigorous way of understanding the spontaneous formation of such black holes from the QG vacuum, and hence in our case, it is an assumption that the above results can be extrapolated to this case. Nevertheless, in the introduction, within a Liouville-string framework [8] in the context of brane worlds, we have mentioned some concrete attempts to discuss the formation of dynamical Planck (more precisely string-scale)-size horizons, with “real effects” on matter. We recall that in such models the rôle of black holes is played by pointlike defects in the bulk space-time, which cross the brane hypersurfaces, and interact stringy matter propagating on the brane worlds. Such situations can lead [8,10] to real (nonthermal) particle creation and decoherence, as a result of the quantum instability of the pertinent horizons, established through the study of the appropriate positive energy conditions for the associated space-times. On identifying the world-sheet Liouville zero mode with the target-space-time, it can be shown [9] that the decoherence effects of the recoiling  $D$ -particles, as a result of their scattering with matter (“ $D$ -particle foam”), can be quantified within a closed-time-path formalism on the world-sheet of the string, within a first quantization framework. It should be stressed that decoherent particle production is achieved only upon considering quantum fluctuations of the recoil velocity of the  $D$ -particle defect.

The above example should only be considered as a simplified model of space-time foam within string theory;



in general one may encounter more complicated situations, and so our understanding is very incomplete. For the phenomenological purposes of this work, therefore, we shall not specify the model of space-time foam that might characterize the QG vacuum but instead try to apply generic considerations.

In such a situation, then,  $t(\omega_0)$  in (4.2) would be a family of parameters describing the space-time foam medium. From the smooth connection of nonextremal black holes to the extremal ones, encountered in string theory [39], we can also conclude that near extremal black holes would be characterized by relatively small particle creation rate, as compared with their neutral counterparts. Hence black holes which are close to being extremal have long lifetimes. Furthermore when a charged black and white hole pair is produced, the absorption of the positron by the white hole leaves electrons to preferentially interact with the electron neutrinos. Hence the flavor-favoring medium is characterized by charged black/white hole configurations. This *flavor bias* of the foam medium, which could then be viewed as the “quantum-gravitational analogue” of the MSW effect in ordinary media. In this sense, the QG medium would be responsible for generating effective neutrino mass differences [24]. Since the charged-black holes lead to a stochastically fluctuating medium, we shall consider the formalism for the MSW effect in stochastically fluctuating media [31], where the density of electrons replaces the density of charged black hole/anti-black hole pairs. It should be stressed, however, that we have no way of rigorously checking the required extrapolation to microscopic black holes, with the present understanding of QG. However, we shall argue later in this paper, one can already place stringent bounds on the portion of the neutrino mass differences that may be due to QG foam, as a result of current neutrino data.

## B. Two generations of neutrinos

Following the MSW formalism, it was proposed in [24] that the stochastically fluctuating media caused by the space-time foam can give a mass square difference of the form:

$$\langle \Delta m_{\text{foam}}^2 \rangle \propto G_N \langle n_{\text{bh}}^c(r) \rangle k,$$

where  $k$  is the neutrino momentum scale and  $\langle n_{\text{bh}}^c(r) \rangle$  is the average number of virtual particles emitted from the foam. These flavor violating effects would contribute to the decoherence through quantum fluctuations of the foam-medium density by means of induced non-Hamiltonian terms in the density matrix time evolution. In this paper we model this foam/neutrino interaction by analogy to the MSW interaction Hamiltonian and follow corresponding procedures to calculate the relevant transition probabilities. Moreover, QG-induced Gaussian fluctuations of energy and oscillations lengths may be distinguished from the corresponding ones due to the conventional uncertainties

by their energy dependence: conventional effects decrease with increasing (neutrino) energy, while QG effects have exactly the opposite effects, increasing with energy.

In keeping with our analysis of the effects of  $\Lambda_1$ , and for simplicity, we restrict ourselves to the case of two generations of neutrinos which suffices for a demonstration of the generic properties of decoherence. We take the effective Hamiltonian to be of the form

$$H_{\text{eff}} = H + n_{\text{bh}}^c(r) H_I, \quad (4.3)$$

where  $H_I$  is a  $2 \times 2$  matrix whose entries depend on the interaction of the foam and neutrinos and  $H$  is the free Hamiltonian. For the purposes of this paper we take this matrix to be diagonal in flavor space. Although we leave the entries as general constants,  $a_{\nu_i}$ , we expect them to be of the form  $\propto G_N n_{\text{bh}}^c(r)$ ; so we write  $H_I$  as

$$H_I = \begin{pmatrix} a_{\nu_e} & 0 \\ 0 & a_{\nu_\mu} \end{pmatrix}. \quad (4.4)$$

where the foam medium is assumed to be described by Gaussian random variables [23]. We take the average number of foam particles,  $\langle n_{\text{bh}}^c(t) \rangle = n_0$  (a constant), and  $\langle n_{\text{bh}}^c(t) n_{\text{bh}}^c(t') \rangle \sim \Omega^2 n_0^2 \delta(t - t')$ . Following [31] we can deduce the modified time evolution of the density matrix as

$$\frac{\partial}{\partial t} \langle \rho \rangle = -i[H + n_0 H_I, \langle \rho \rangle] - \Omega^2 n_0^2 [H_I, [H_I, \langle \rho \rangle]], \quad (4.5)$$

where  $\langle \dots \rangle$  represents the average over the random variables of the foam. The double commutator is the *CPT* violating term since although it is *CP* symmetric it induces time-irreversibility. It is also important to note that  $\Lambda_2$  here is of the Markovian-Liouville-Lindblad form for a self-adjoint operator. This is as an appropriate form for decoherence for environments about which we have little *a priori* knowledge. In the *CPT* violating term we can require the density fluctuation parameter to be different for the antiparticle sector from that for the particle sector, i.e.  $\bar{\Omega} \neq \Omega$ , while keeping  $\langle n_{\text{bh}}^c(t) \rangle \equiv n_0$  the same in both sectors. Physically this means that neutrinos and antineutrinos with the same momenta, and hence interacting with the same amount of foam particles on average, will evolve differently; this is a result of *CPT* violation.

We expand the Hamiltonian and the density operator in terms of the Pauli spin matrices  $s_\mu$  (with  $\frac{s_0}{2} = \mathbf{1}_2$  the  $2 \times 2$  identity matrix) as follows

$$H_{\text{eff}} = \sum_{\mu=0}^3 (h_\mu + n_0 h'_\mu) \frac{s_\mu}{2}, \quad \rho = \sum_{\nu=0}^3 \rho_\nu \frac{s_\nu}{2} \quad (4.6)$$

(where  $H_{\text{eff}} = H + n_0 H_I$ ). We find that

$$h_\mu = \frac{m_1^2 + m_2^2}{4k} \delta_{\mu 0} + \frac{m_1^2 - m_2^2}{2k} \delta_{\mu 3} \quad (4.7)$$

and

$$n_0 h'_\mu = \frac{a_{\nu_e} + a_{\nu_\mu}}{2} \delta_{\mu 0} + (a_{\nu_e} - a_{\nu_\mu}) \sin 2\theta \delta_{\mu 1} + (a_{\nu_e} - a_{\nu_\mu}) \cos 2\theta \delta_{\mu 3}. \quad (4.8)$$

The master equation in (4.5) simplifies to

$$\dot{\rho}_l = \sum_{j=1}^3 \mathcal{L}_{lj} \rho_j. \quad (4.9)$$

for  $l = 1, \dots, 3$  (see Appendix C for further details). The pure state representing  $\nu_e$  is given by

$$\langle \rho \rangle^{(\nu_e)} = \frac{1}{2} \mathbf{1}_2 + \sin(2\theta) \frac{s_1}{2} + \cos(2\theta) \frac{s_3}{2} \quad (4.10)$$

and the corresponding state for  $\nu_\mu$  is

$$\langle \rho \rangle^{(\nu_\mu)} = \frac{1}{2} \mathbf{1}_2 - \sin(2\theta) \frac{s_1}{2} - \cos(2\theta) \frac{s_3}{2}. \quad (4.11)$$

If  $\langle \rho \rangle(0) = \langle \rho \rangle^{(\nu_e)}$  then the probability  $P_{\nu_e \rightarrow \nu_\mu}(t)$  of the transition  $\nu_e \rightarrow \nu_\mu$  is given by

$$P_{\nu_e \rightarrow \nu_\mu}(t) = \text{Tr}(\langle \rho \rangle(t) \langle \rho \rangle^{(\nu_\mu)}). \quad (4.12)$$

In order to study decoherence we will calculate the eigenvectors  $\vec{e}^{(i)}$  and corresponding eigenvalues  $\lambda_i$  of  $\mathcal{L}$  to leading order in  $\Omega^2$ . In terms of auxiliary variables  $\mathcal{U}$  and  $\mathcal{W}$  where

$$\mathcal{U} = (a_{\nu_e} - a_{\nu_\mu}) \cos(2\theta) + \frac{m_1^2 - m_2^2}{2k} \quad (4.13)$$

and

$$\mathcal{W} = (a_{\nu_e} - a_{\nu_\mu}) \sin(2\theta), \quad (4.14)$$

it is straightforward to show that

$$P_{\nu_e \rightarrow \nu_\mu}(t) = \frac{1}{2} \left[ 1 - \sin(2\theta) \{ \mathbf{b}_1 e_1^{(1)} e^{\lambda_1 t} + \mathbf{b}_2 (e_1^{(2)} e^{\lambda_2 t} + e_1^{(3)} e^{\lambda_3 t}) \} - \cos(2\theta) \{ \mathbf{b}_1 e_3^{(1)} e^{\lambda_1 t} + \mathbf{b}_2 (e_3^{(2)} e^{\lambda_2 t} + e_3^{(3)} e^{\lambda_3 t}) \} \right].$$

On writing  $\Delta = a_{\nu_e} - a_{\nu_\mu}$  and  $\delta_k = \frac{m_1^2 - m_2^2}{2k}$ ,  $P_{\nu_e \rightarrow \nu_\mu}(t)$  readily simplifies to give

$$P_{\nu_e \rightarrow \nu_\mu}(t) = \frac{\Gamma_1(t) + \Gamma_2(t)}{2(\Delta^2 + \delta_k^2 + 2\delta_k \Delta \cos(2\theta))}, \quad (4.21)$$

where

$$\Gamma_1(t) = (\Delta + \cos(2\theta)\delta_k)^2 (1 - e^{-\Omega^2 \sin^2(2\theta)\delta_k^2 t}) \quad (4.22)$$

and

$$\begin{aligned} \vec{e}^{(1)} &\simeq \left( \frac{\mathcal{W}}{\mathcal{U}}, 0, 1 \right), \\ \vec{e}^{(2)} &\simeq \left( -\frac{\mathcal{U}}{\mathcal{W}}, -i \frac{\sqrt{\mathcal{U}^2 + \mathcal{W}^2}}{\mathcal{W}}, 1 \right), \\ \vec{e}^{(3)} &\simeq \left( -\frac{\mathcal{U}}{\mathcal{W}}, i \frac{\sqrt{\mathcal{U}^2 + \mathcal{W}^2}}{\mathcal{W}}, 1 \right), \end{aligned} \quad (4.15)$$

and

$$\begin{aligned} \lambda_1 &\simeq -\Omega^2 (\mathcal{W} \cos(2\theta) - \mathcal{U} \sin(2\theta))^2, \\ \lambda_2 &\simeq -i \sqrt{\mathcal{U}^2 + \mathcal{W}^2} - \frac{\Omega^2}{2} (\mathcal{U}^2 + \mathcal{W}^2 + (\mathcal{U} \cos(2\theta) + \mathcal{W} \sin(2\theta))^2), \\ \lambda_3 &\simeq i \sqrt{\mathcal{U}^2 + \mathcal{W}^2} - \frac{\Omega^2}{2} (\mathcal{U}^2 + \mathcal{W}^2 + (\mathcal{U} \cos(2\theta) + \mathcal{W} \sin(2\theta))^2). \end{aligned} \quad (4.16)$$

In (4.9) the vector  $\vec{\rho}(0)$  can be decomposed as

$$\vec{\rho}(0) = \mathbf{b}_1 \vec{e}^{(1)} + \mathbf{b}_2 \vec{e}^{(2)} + \mathbf{b}_2 \vec{e}^{(3)} \quad (4.17)$$

with

$$\mathbf{b}_1 = \frac{\mathcal{U}^2 \cos(2\theta) + \mathcal{U} \mathcal{W} \sin(2\theta)}{\mathcal{U}^2 + \mathcal{W}^2} \quad (4.18)$$

and

$$\mathbf{b}_2 = \frac{\mathcal{W}^2 \cos(2\theta) - \mathcal{U} \mathcal{W} \sin(2\theta)}{2(\mathcal{U}^2 + \mathcal{W}^2)}. \quad (4.19)$$

Hence

$$\rho(t) = \frac{1}{2} (\mathbf{b}_1 e^{\lambda_1 t} \vec{e}^{(1)} + \mathbf{b}_2 \vec{e}^{(2)} + \mathbf{b}_2 \vec{e}^{(3)} + \mathbf{1}_2) \quad (4.20)$$

and so

$$\begin{aligned} \Gamma_2(t) &= \delta_k^2 \sin^2(2\theta) \\ &\times \left\{ \frac{1}{\exp \left[ -\frac{\Omega^2}{2} (2(\Delta + \delta_k \cos(2\theta))^2 + \delta_k^2 \sin^2(2\theta)) t \right]} - \cos(\sqrt{\Delta^2 + \delta_k^2 + 2\delta_k \Delta \cos(2\theta)} t) \right\}. \end{aligned} \quad (4.23)$$

Since we are concerned with relativistic neutrinos, we have  $t = x$  (in natural units) and we can use this to put our expression in terms of the oscillation length,  $L$ . The exponent in the damping factor in (4.21) has a generic form

$$\text{exponent} \propto \Omega^2 f(\theta) L$$

with  $f(\theta) = (\Delta + \delta_k \cos(2\theta))^2 + \frac{1}{2} \delta_k^2 \sin^2(2\theta)$  or  $\frac{\delta_k^2 \sin^2(2\theta)}{2}$ . Hence the damping is directly proportional to the stochas-

tic fluctuations in the medium. The limit  $\delta_k \rightarrow 0$  characterizes the situation where the dominant contribution to neutrino mass differences is due to space-time foam [23]. The damping exponent should then be independent of the mixing angle for consistency. Indeed we find the purely gravitational MSW to give exponent<sub>gravitational MSW</sub>  $\propto \Omega^2 \Delta^2 L$  which is independent of  $\theta$ . However this stochastic gravitational MSW effect, although capable of inducing neutrino mass differences, gives an oscillation probability which is suppressed by factors proportional to  $\delta_k^2$ . Hence the bulk of the oscillation is due to conventional flavor physics.

### C. Comparison with decoherence from conventional sources

In experiments with neutrino beams there is an uncertainty over the precise energy of the beam (and, in some cases, over the oscillation length), which can destroy coherence, as discussed in [32]. There are also small effects due to the wave packet nature of the incoming neutrino state. The coherence length associated with the latter is typically much larger than  $L$  and so a plane-wave approximation is sufficient. Below we first review the situation briefly, for the benefit of the inexpert.

In Refs. [32,33] the following expression for the neutrino transition probability has been considered:

$$\begin{aligned}
 P_{\alpha \rightarrow \beta} &\equiv P_{\alpha\beta}(L, E) \\
 &= \delta_{\alpha\beta} \\
 &\quad - 4 \sum_{a=1}^n \sum_{b=1a<b}^n \Re(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \sin^2\left(\frac{\Delta m_{ab}^2 L}{4E}\right) \\
 &\quad - 2 \sum_{a=1}^n \sum_{b=1a<b}^n \Im(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \sin^2\left(\frac{\Delta m_{ab}^2 L}{2E}\right), \\
 \alpha, \beta &= e, \mu, \tau, \dots,
 \end{aligned}$$

where  $L$  is the neutrino path length,  $E$  is the neutrino energy,  $n$  is the number of neutrino flavors, and  $\Delta m_{ab}^2$  ( $= m_a^2 - m_b^2$ ) and  $U_{\alpha a}$  as before is the mixing matrix. As there are uncertainties in the energy and oscillation length, in Refs. [32,33] a Gaussian average over the L/E dependence was taken. This average is defined by

$$\langle P \rangle = \int_{-\infty}^{\infty} dx P(x) \frac{1}{\sigma \sqrt{2\pi}} e^{-((x-l)^2/2\sigma^2)}.$$

where  $x = \frac{L}{4E}$ ,  $l = \langle x \rangle$ , and  $\sigma = \sqrt{\langle (x - \langle x \rangle)^2 \rangle}$ . Furthermore if  $L$  and  $E$  are independent then  $l = \langle L/E \rangle = \langle L \rangle / 4 \langle E \rangle$  (for highly peaked distributions) and one obtains for the averaged expression

$$\begin{aligned}
 P_{\alpha\beta}(L, E) &= \delta_{\alpha\beta} - 2 \sum_{a=1}^n \sum_{b=1a<b}^n \Re(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \\
 &\quad \times (1 - \cos(2l \Delta m_{ab}^2)) e^{-2\sigma^2 (\Delta m_{ab}^2)^2} \\
 &\quad - 2 \sum_{a=1}^n \sum_{b=1a<b}^n \Im(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \\
 &\quad \times \sin^2(2l \Delta m_{ab}^2) e^{-2\sigma^2 (\Delta m_{ab}^2)^2}, \\
 \alpha, \beta &= e, \mu, \tau, \dots
 \end{aligned} \tag{4.24}$$

It should be noted that  $l$  has to do with the sensitivity of the experiment and  $\sigma$  the damping factor of neutrino oscillation probabilities. A pessimistic (less stringent) and an optimistic (more stringent) upper bound for  $\sigma$  (obtained from a first order Taylor expansion of  $x$  around  $\langle E \rangle$  and  $\langle L \rangle$ ) can be given [32]

- (i) pessimistic:  $\sigma \simeq \Delta x = \Delta \frac{L}{4E} \leq \Delta L \left| \frac{\partial x}{\partial L} \right|_{L=\langle L \rangle, E=\langle E \rangle} + \Delta E \left| \frac{\partial x}{\partial E} \right|_{L=\langle L \rangle, E=\langle E \rangle} = \frac{\langle L \rangle}{4 \langle E \rangle} \left( \frac{\Delta L}{\langle L \rangle} + \frac{\Delta E}{\langle E \rangle} \right)$
- (ii) optimistic:  $\sigma \leq \frac{\langle L \rangle}{4 \langle E \rangle} \sqrt{\left( \frac{\Delta L}{\langle L \rangle} \right)^2 + \left( \frac{\Delta E}{\langle E \rangle} \right)^2}$

For the case of two generations, using this procedure, the transition probability between flavor eigenstates is [32]

$$\langle P_{\nu_e \rightarrow \nu_\mu} \rangle = \frac{1}{2} \sin^2 2\theta \left( 1 - e^{-2\sigma^2 (\Delta m_{12}^2)^2} \cos\left(\frac{\Delta m_{12}^2 \langle L \rangle}{2 \langle E \rangle}\right) \right). \tag{4.25}$$

Owing to the averaging over Gaussian fluctuations, (4.25) shares one characteristic with the backreaction effects of  $\Lambda_1$  (discussed earlier) viz. the  $L^2$  dependence of the decohering decay and is dissimilar to the  $L$  dependence of the space-time foam (as modeled by the gravitational MSW effect). This clearly, in principle, is a way of distinguishing the MSW type effect. Although typically experimental data make allowances for systematics, it is interesting to consider whether for a given  $L$  the magnitude of the decoherence effect may be assigned to conventional sources. When one compares the damping factors of the conventional averaging and our MSW effect we get

$$2\sigma^2 (\Delta m_{12}^2)^2 = [\Omega^2 (\Delta + \delta_k \cos(2\theta))^2 + \frac{1}{2} \delta_k^2 \sin^2(2\theta)] L, \tag{4.26}$$

which we can express as

$$\Omega^2 (\Delta + \delta_k \cos(2\theta))^2 + \frac{1}{2} \delta_k^2 \sin^2(2\theta) = \frac{(\Delta m_{12}^2)^2}{8E^2} L r^2, \tag{4.27}$$

where  $r = \frac{\Delta L}{L} + \frac{\Delta E}{E}$  for the pessimistic case or  $r = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta E}{E}\right)^2}$  for the optimistic case. For decoherence due to standard matter effects with  $L \sim 12000$  Km,  $r \sim \mathcal{O}(1)$ ,  $E \sim \mathcal{O}(1)$  GeV,  $\Delta m_{12}^2 \sim \mathcal{O}(10^{-5})$  eV<sup>2</sup> and  $\sigma_{\text{atm}} \sim 1.5 \times 10^{22}$  GeV<sup>-1</sup> one obtains  $\gamma_{\text{atm, fake}} (= \frac{(\Delta m_{12}^2)^2}{8E^2} L r^2) < 10^{-24}$  GeV.

It is worth pointing out here that such a small order of magnitude is of a similar order to that found in quantum-gravity decoherence suppressed by a single power of Planck mass [9,19,40]. In [40] the cases for the decoherence damping factor being of the form  $\gamma = \gamma_0 (\frac{E}{\text{GeV}})^n$ , with  $\gamma_0$  as a constant, has been analyzed for the  $n = 0, -1, 2$  cases (a more pessimistic view is presented in [41] with  $\gamma = \frac{(\Delta m^2)^2}{E^2 M_{\text{QG}}}$ , for which there is no experimental sensitivity at least in the foreseeable future). An effect of a similarly miniscule order appears to characterize also cosmological decoherence, i.e. the decoherence due to the (future) horizon in de Sitter space, in the case of a Universe with a cosmological constant [7,42]).

In order to investigate experimental signals of quantum-gravitational decoherence it will be necessary to distinguish genuine quantum-gravity effects from the above “fake” ordinary-matter effects through the dependence of the respective transition probabilities on the energy and oscillation length. Indeed, it is expected, at least intuitively, that the fuzzyness of space-time caused by quantum-gravity-induced stochastic fluctuations of the metric tensor, would lead to effects that are enhanced by the energy of the probe, i.e. the higher the energy the greater the backreaction on the surrounding space-time fluid. Such an expectation is confirmed in detailed microscopic models of the so-called  $D$ -particle foam [8]. Then, in such cases we may write in a generic way

$$\frac{\Delta L}{L}, \quad \frac{\Delta E}{E} \sim \beta \left( \frac{E}{M_{\text{QG}}} \right)^\alpha \quad (4.28)$$

for some positive integer  $\alpha \geq 1$ , and some coefficient,  $\beta$ . For this case we would have  $r \sim \beta (\frac{E}{M_{\text{QG}}})^\alpha$  then from the Gaussian average we would have

$$\begin{aligned} & \Omega^2 (\Delta + \delta_k \cos(2\theta))^2 + \frac{1}{2} \delta_k^2 \sin^2(2\theta) \\ & \sim \frac{(\Delta m^2_{12})^2}{8E^2} \beta^2 \left( \frac{E}{M_{\text{QG}}} \right)^{2\alpha} L. \end{aligned} \quad (4.29)$$

For the specific model of  $D$ -particle foam of Ref. [8]  $\alpha = +1$ , and  $M_{\text{QG}} \sim M_s/g_s$  with  $M_s$  the string scale and  $g_s < 1$  the (weak) string coupling.

Since for the oscillation length  $L$ ,  $L^{-1} \sim \frac{\Delta m^2_{12}}{E}$ , from (4.29) and the above analysis, it becomes clear that genuine quantum-gravity effects in some models are characterized by damping factors which are proportional to  $E^{2\alpha}$ ,  $\alpha \geq 1$ , and thus are enhanced by the energy of the probe, leading to significantly more damped oscillations for high-energy probes as compared to the low-energy ones. This is to be contrasted with the conventional effects, due to the passage of neutrinos through matter, which are diminished with the energy [33].

Although in the presence of  $\Lambda_2$ , as shown in [15], the  $CPT$  operator cannot be defined, the  $CPT$  violating differ-

ence between neutrino and antineutrino sectors [33],  $\left( \frac{\Delta P_{\alpha\beta}^{CPT}}{P_{\beta\bar{\alpha}}^{CPT}} \right)^{(\text{decoh})} \equiv \frac{P_{\alpha\beta}^{(\text{decoh})}}{P_{\beta\bar{\alpha}}^{(\text{decoh})}} - 1$  vanishes unless the decoherence coefficients between particles and antiparticles are distinct, a case considered in [24]. Here the superscript *decoh* denotes the decohering piece of the relevant probability. In the case of different decoherence coefficients between particle and antiparticle sectors, the QG-induced difference  $\Delta P_{\alpha\beta}^{CPT}$  would either increase or decrease with energy, at least as fast as a Gaussian, depending on the relative magnitudes of the decoherence parameters in the neutrino and antineutrino sectors. In contrast the conventional matter induced  $CPT$  difference saturates with increasing  $E$ . In this way, at least in principle, the two effects can be disentangled. It must be noted, though that, as seen from (4.29) the proportionality coefficient  $\beta^2 (\Delta m^2_{12})^2$  accompanying  $(E/M_{\text{QG}})^{2\alpha} (L/E)^2$  in the decoherence exponents is very small (for natural values of  $\beta$ , although in principle this is another phenomenological parameter to be constrained by data). Hence, for this particular model of QG decoherence, appreciable effects might only be expected in situations involving very high-energy cosmological neutrinos. In view of this, the analysis of high-energy neutrinos performed in [28], which was based only on conventional Lindblad decoherence, needs to be repeated in order to incorporate the above effects.

## V. CONCLUSIONS AND OUTLOOK: PRELIMINARY DATA COMPARISON

It is hoped that decoherence due to quantum gravity can be confirmed or ruled out by physical observation. We will make a few remarks concerning possible conclusions from data from reactors and the atmosphere. Different approaches have been used in examining transitions of atmospheric neutrinos. As mentioned above, more pessimistic expressions for damping factors such as  $\gamma = \frac{(\Delta m^2)^2}{E^2 M_p}$  have been presented [41]. However, more optimistic values can be obtained. In [40] a phenomenological analysis is done for the case of atmospheric neutrino transitions ( $\nu_\mu \leftrightarrow \nu_\tau$ ). They obtain upper bounds to the decoherence parameters and find that the Super-Kamiokande data can be a good probe into quantum-gravity-induced decoherence. They discuss three possible energy dependencies of the decoherence parameter, in particular  $\gamma = \gamma_0 (E/\text{GeV})^n$  with  $n = -1, 0, 2$ , with  $\gamma_0$  a constant, and the subsequent constraints. The controversial data obtained by LSND [43], if confirmed by future experiment (for instance MiniBOONE), could provide important data which may lead to evidence of space-time foam interacting with antineutrinos.

We would now like to mention briefly some preliminary attempts to constrain the models presented here by means of currently available neutrino data. In a recent work [44] we have presented a fit of a three-generation (completely

positive) Lindblad [17] decoherence model for neutrinos with mixing to all the available data, including the LSND result in the antineutrino sector. In contrast to the manifestly *CPT*-violating fit of [24], which attempted to explain the LSND result from the point of view of *CPT*-violating decoherence, in [44] it was assumed that the decoherence coefficients were the same in the particle and antiparticle sectors. The best fit that was obtained showed that only some of the oscillation terms in the three-generation probability formula had nontrivial damping factors; moreover over an oscillation length the exponent of such nontrivial damping,  $\mathcal{D} \cdot L$ , satisfied [44]:

$$\mathcal{D} = -\frac{1.3 \cdot 10^{-2}}{L}, \quad (5.1)$$

in units of 1/km with  $L = t$  the oscillation length.

In the light of (5.1) it is possible to analyze [44] the two types of theoretical models of space-time foam discussed in sections III and IV of the present paper. The conclusion is that models incorporating stochastically fluctuating MSW-like QG media as in (4.21) cannot provide the full explanation for the fit. Indeed if the decoherent result of the fit (5.1) was exclusively due to such a model, then the pertinent decoherent coefficient  $\mathcal{D}$  in the damping exponent, for, say, the KamLand experiment with an  $L \sim 180$  Km, would be  $|\mathcal{D}| = \Omega^2 \Delta^2 \sim 2.84 \cdot 10^{-21}$  GeV (note that the mixing angle part does not affect the order of the exponent). Smaller values are found for longer  $L$ , appropriate to atmospheric neutrino experiments. In this context the  $L$  independence of  $\mathcal{D} \cdot L$ , as required by (5.1), may be interpreted as follows: (4.21) suggests that we write  $\Delta = \xi \frac{\Delta m^2}{E}$ , where  $\xi \ll 1$  parametrizes the contributions of the foam to the induced neutrino mass differences. Hence, the damping exponent becomes in this case  $\xi^2 \Omega^2 (\Delta m^2)^2 \cdot L/E^2$ . Thus, for oscillation lengths  $L$  (since  $L^{-1} \sim \Delta m^2/E$ ) one is left with the following estimate for the dimensionless quantity  $\xi^2 \Delta m^2 \Omega^2/E \sim 1.3 \cdot 10^{-2}$ . This implies that the quantity  $\Omega^2$  is proportional to the probe energy  $E$ . Since backreaction effects, which affect the stochastic fluctuations  $\Omega^2$ , are expected to increase with probe energy  $E$ , this is not an unreasonable result in principle. However, due to the smallness of the quantity  $\Delta m^2/E$ , for energies of the order of a GeV,  $\Delta m^2 \sim 10^{-3}$  eV<sup>2</sup> and  $\xi \ll 1$ , we can conclude that  $\Omega^2$ , in this case, would be unrealistically large for a quantum-gravity effect in the model. We remark at this point that, in such a model, we can in principle bound independently the  $\Omega$  and  $\Delta$  parameters by also examining the period of oscillation. However in this example,  $\Delta a_{e\mu} \ll \Delta_{12}$  and so the modification in the period is too small to be detected.

The second model (3.16) of stochastic space-time can also be confronted with the data. In this case (5.1) would imply for the pertinent damping exponent

$$\begin{aligned} & \left( \frac{(m_1^2 - m_2^2)^2}{2k^2} (9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \right. \\ & \quad \left. + \frac{2V \cos 2\theta (m_1^2 - m_2^2)}{k} (12\sigma_1 + 2\sigma_2 - 2\sigma_3) \right) t^2 \\ & \sim 1.3 \cdot 10^{-2}. \end{aligned} \quad (5.2)$$

Ignoring, for simplicity, subleading MSW effects from  $V$ , and considering oscillation lengths  $t = L \sim \frac{2k}{(m_1^2 - m_2^2)}$ , we observe that the experimental fit (5.1), may be interpreted, in this case, as bounding the stochastic fluctuations of the metric (2.4) to  $9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \sim 1.3 \cdot 10^{-2}$ . Again, this is too large to be a quantum-gravity effect, which means that in this model the  $L^2$  contributions to the damping, (3.16), due to stochastic fluctuations of the space-time metric cannot be the sole explanation of the fit of [44].

The analysis of [44] also demonstrated that, at least as far as the *order of magnitude* of the effect in (5.1) is concerned, a reasonable explanation is provided by Gaussian-type energy fluctuations, due to standard physics effects, leading to decoherencelike damping of oscillation probabilities of the form (4.25). The order of magnitude of these fluctuations, consistent with the independence of the damping exponent on the oscillation length  $L$  (irrespective of the power of  $L$ ), is

$$\frac{\Delta E}{E} \sim 1.6 \cdot 10^{-1} \quad (5.3)$$

if one assumes that this is the principal reason for the result of the fit.

However, not even this can be the end of the story, given that the result (5.1) applies only to *some* but not all of the oscillation terms; this would not be the case expected for standard physics uncertainties (4.25). The fact that the best fit model includes terms which are not suppressed at all calls for a more radical explanation, and so the issue is still wide open. It is interesting, however, that the current neutrino data can already impose stringent constraints on quantum-gravity models, and exclude some of them from being the exclusive source of decoherence, as we have discussed above.

We reiterate that, within the classes of stochastic models discussed, one can safely conclude space-time foam can be at most responsible only for a small part of the observed neutrino mass difference, and certainly the foam-induced decoherence cannot be the primary reason for the result of the best fit (5.1), obtained from a global analysis of the currently available neutrino data. Of course, it is not possible to exclude other classes of theoretical models of quantum gravity, which could escape these constraints. At present, however, we are not aware of any such theory.

In the near future we plan to make a more complete and systematic comparison of our new formulae, especially those derived in Secs. II and III, with all experimental data available and perhaps arrive at new constraints.

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## APPENDIX A: SCALAR PARTICLE AVERAGES

For integration over metric fluctuations we shall use the formula

$$\int d^4 a e^{-\tilde{a} \cdot \mathbf{B} \cdot \tilde{a} + \tilde{u} \cdot \tilde{a}} = \frac{\pi^2 e^{\tilde{u} \cdot \mathbf{B}^{-1} \cdot \tilde{u}}}{\sqrt{\det \mathbf{B}}}.$$

(Here the  $a$ 's are assumed to be in the range  $(-\infty, \infty)$  and the form of  $\mathbf{B}$  must be such that "convergence" of the integral is assured.)

$$\mathbf{B} = \Xi - it(\mathbf{f}(m_1) - \mathbf{f}(m_2)).$$

For simplicity we define

$$\mathcal{F} = \mathbf{f}(m_1) - \mathbf{f}(m_2)$$

and

$$\begin{aligned} \tilde{d} &= \sqrt{k^2 + m_1^2} \sqrt{k^2 + m_2^2}, \\ \tilde{b} &= \sqrt{k^2 + m_1^2} - \sqrt{k^2 + m_2^2}, \\ \tilde{c} &= m_1^2(k^2 + m_1^2)^{-3/2} - m_2^2(k^2 + m_2^2)^{-3/2}. \end{aligned}$$

So we can write

$$\begin{aligned} \mathcal{F}_{11} &= \tilde{b}, & \mathcal{F}_{14} &= \frac{k^2 \tilde{b}}{2 \tilde{d}}, \\ \mathcal{F}_{22} &= \frac{m_1^2 + 2k^2}{2\sqrt{k^2 + m_1^2}} - \frac{m_2^2 + 2k^2}{2\sqrt{k^2 + m_2^2}} \\ &= \frac{1}{2} \left( \tilde{b} - k^2 \frac{\tilde{b}}{\tilde{d}} \right) = \frac{\tilde{b}}{2\tilde{d}} (\tilde{d} - k^2), \\ \mathcal{F}_{23} &= \frac{k^2 \tilde{b}}{2 \tilde{d}}, & \mathcal{F}_{44} &= \frac{1}{2} k^2 \tilde{c}, \end{aligned}$$

and the remaining  $\mathcal{F}_{ij} = 0$ .

Putting this information together we find

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sigma_1} - i\tilde{b}t & 0 & 0 & -\frac{i\tilde{b}}{2\tilde{d}} k^2 t \\ 0 & \frac{1}{\sigma_2} - \frac{i\tilde{b}}{2\tilde{d}} (\tilde{d} - k^2) & \frac{-ik^2 \tilde{b}t}{2\tilde{d}} & 0 \\ 0 & \frac{-ik^2 \tilde{b}t}{2\tilde{d}} & \frac{1}{\sigma_3} & 0 \\ \frac{-i\tilde{b}}{2\tilde{d}} k^2 t & 0 & 0 & \frac{1}{\sigma_4} - \frac{1}{2} ik^2 \tilde{c}t \end{pmatrix},$$

$$u_1 = -it\tilde{b}, \quad u_4 = -it\frac{\tilde{b}}{\tilde{d}}k^2,$$

$$\text{i.e. } \tilde{u} = it\tilde{b} \left( -1, 0, 0, -\frac{k^2}{\tilde{d}} \right),$$

$$\det \mathbf{B} = \frac{1}{16\sigma_1\sigma_2\sigma_3\sigma_4\tilde{d}^4} P_1 P_2,$$

where

$$\begin{aligned} P_1 &= 4\tilde{d}^2 + 2i\tilde{d}\tilde{b}\sigma_2 k^2 t - 2i\tilde{b}\sigma_2 \tilde{d}^2 t + \tilde{b}^2 k^4 t^2 \sigma_2 \sigma_3 \\ &= 4\tilde{d}^2 + 2i\tilde{d}\tilde{b}\sigma_2 (k^2 - \tilde{d})t + \tilde{b}^2 k^4 \sigma_2 \sigma_3 t^2, \end{aligned}$$

$$\begin{aligned} P_2 &= 4\tilde{d}^2 - 2i\tilde{d}^2 (k^2 \tilde{c} \sigma_4 + 2\tilde{b} \sigma_1) t \\ &\quad + \tilde{b} k^2 \sigma_1 \sigma_4 (\tilde{b} k^2 - 2\tilde{d}^2 \tilde{c}) t^2, \end{aligned}$$

$$\det \Xi = \frac{1}{\sigma_1 \sigma_2 \sigma_3 \sigma_4}.$$

So we obtain

$$\left( \frac{\det \Xi}{\det \mathbf{B}} \right)^{1/2} = \left( \frac{16\tilde{d}^4}{P_1 P_2} \right)^{1/2} = \frac{4\tilde{d}^2}{(P_1 P_2)^{1/2}},$$

$$\mathbf{B}^{-1} \tilde{u} = (v_1, v_2, v_3, v_4).$$

Now

$$\begin{aligned} v_1 &= \frac{2\sigma_1 \tilde{b} t (k^2 \sigma_4 (\tilde{b} k^2 - \tilde{d}^2 \tilde{c}) t - 2i\tilde{d}^2)}{4\tilde{d}^2 + \tilde{b} \sigma_1 k^2 \sigma_4 (\tilde{b} k^2 - 2i\tilde{d}^2) t^2 - 2i(k^2 \tilde{c} \sigma_4 + 2\tilde{b} \sigma_1) \tilde{d}^2}, \\ v_2 &= 0, \quad v_3 = 0, \\ v_4 &= \frac{-2(\tilde{b} \sigma_1 t + 2i)\tilde{b} \tilde{d} \sigma_4 k^2 t}{4\tilde{d}^2 - 2i\tilde{d}^2 (k^2 \tilde{c} \sigma_4 + 2\tilde{b} \sigma_1) t + \tilde{b} k^2 \sigma_1 \sigma_4 t^2 (\tilde{b} k^2 - 2\tilde{c} \tilde{d}^2)}, \end{aligned}$$

$$\exp(\tilde{u} \cdot \tilde{v}) = \exp \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix},$$

where

$$\begin{aligned} \chi_1 &= -2(2\tilde{d}^2 \sigma_1 - i\tilde{d}^2 k^2 \tilde{c} \sigma_1 \sigma_4 t + 2\sigma_4 k^4) \tilde{b}^2 t^2, \\ \chi_2 &= 4\tilde{d}^2 - (2i\tilde{d}^2 k^2 \tilde{c} \sigma_4 + 4i\tilde{d}^2 \tilde{b} \sigma_1) t \\ &\quad + \tilde{b} k^2 \sigma_1 \sigma_4 (\tilde{b} k^2 - 2\tilde{d}^2 \tilde{c}). \end{aligned}$$

## APPENDIX B: DIRAC PARTICLE AVERAGES

The equations of motion which follow from (3.9) are

$$\begin{aligned}
\left(1 + \frac{1}{2}h\right)(i\partial_0\phi_1 - (i\sigma_1\partial_1\phi_1 + m_1\chi_1)) - V\cos\theta(\cos\theta\phi_1 + \sin\theta\phi_2) - \frac{i}{2}((b_1\mathbf{1} + b_3\sigma_1)\partial_0\phi_1 + (b_3\mathbf{1} + b_2\sigma_1)\partial_1\phi_1) &= 0, \\
\left(1 + \frac{1}{2}h\right)(i\partial_0\chi_1 + i\sigma_1\partial_1\chi_1 - m_1\phi_1) - \frac{i}{2}((b_1\mathbf{1} - b_3\sigma_1)\partial_0\chi_1 + (b_3\mathbf{1} - b_2\sigma_1)\partial_1\chi_1) &= 0, \\
\left(1 + \frac{1}{2}h\right)(i\partial_0\phi_2 - i\sigma_1\partial_1\phi_2 - m_2\chi_2 - V\sin\theta(\cos\theta\phi_1 + \sin\theta\phi_2)) - \frac{i}{2}((b_2\mathbf{1} + b_3\sigma_1)\partial_0\phi_2 + (b_3\mathbf{1} + b_2\sigma_1)\partial_1\phi_2) &= 0, \\
\left(1 + \frac{1}{2}h\right)(i\partial_0\chi_2 + i\sigma_1\partial_1\chi_2 - m_2\phi_2) - \frac{i}{2}((b_2\mathbf{1} - b_3\sigma_1)\partial_0\chi_2 + (b_3\mathbf{1} - b_2\sigma_1)\partial_1\chi_2) &= 0.
\end{aligned} \tag{B1}$$

On using (3.9) in (B2) we have a

$$\mathbf{M} \begin{pmatrix} \tilde{P}_\beta^1(k, E) \\ \tilde{Q}_\beta^1(k, E) \\ \tilde{P}_\beta^2(k, E) \\ \tilde{Q}_\beta^2(k, E) \end{pmatrix} = 0, \tag{B2}$$

where  $\mathbf{M}$  is a  $4 \times 4$  matrix with components

$$\begin{aligned}
M_{11} &= E\left(1 + \frac{1}{2}h - \frac{1}{2}(b_1 - b_3)\right) - \left(1 + \frac{1}{2}h\right)k - \left(1 + \frac{1}{2}h\right)V\cos^2(\theta), & M_{12} &= -\left(1 + \frac{1}{2}h\right)m_1, \\
M_{13} &= -V\left(1 + \frac{1}{2}h\right)\sin(\theta)\cos(\theta), & M_{14} &= 0, & M_{21} &= -\left(1 + \frac{1}{2}h\right)m_1, \\
M_{22} &= E\left(1 + \frac{1}{2}h - \frac{1}{2}(b_1 + b_3)\right) + k\left(1 + \frac{1}{2}h + \frac{1}{2}(b_2 + b_3)\right), & M_{23} &= M_{24} = 0, \\
M_{31} &= -\left(1 + \frac{1}{2}h\right)V\cos(\theta)\sin(\theta), & M_{32} &= 0, \\
M_{33} &= E\left(1 + \frac{1}{2}h - \frac{1}{2}(b_1 - b_3)\right) - k\left(1 + \frac{1}{2}h + \frac{1}{2}(b_2 - b_3)\right) - \left(1 + \frac{1}{2}h\right)V\sin^2(\theta), & M_{34} &= -\left(1 + \frac{1}{2}h\right)m_2, \\
M_{41} = M_{42} &= 0, & M_{43} &= -m_2\left(1 + \frac{1}{2}h\right), & M_{44} &= E\left(1 + \frac{1}{2}h - \frac{1}{2}(b_1 + b_3)\right) + k\left(1 + \frac{1}{2}h + \frac{1}{2}(b_2 + b_3)\right).
\end{aligned}$$

Using these equations one can eliminate  $\tilde{Q}_\beta^{1,2}$  by substitution to obtain

$$\mathcal{N} \begin{pmatrix} \tilde{P}_\beta^1 \\ \tilde{P}_\beta^2 \end{pmatrix} = 0, \tag{B3}$$

where

$$\begin{aligned}
\mathcal{N}_{11} &= M_{11} + \frac{M_{12}}{M_{22}}m_1\left(1 + \frac{1}{2}h\right), \\
\mathcal{N}_{12} &= -V\sin\theta\cos\theta\left(1 + \frac{1}{2}h\right), \\
\mathcal{N}_{21} &= M_{31}, & \mathcal{N}_{22} &= M_{33} - \frac{m_2^2(1 + \frac{1}{2}h)^2}{M_{44}}.
\end{aligned} \tag{B4}$$

We take the momentum  $k$  to be very large, and so we write  $E \simeq k + \frac{m^2}{2k}$ . We make the substitution

$$m^2 = z_0 + \sum_i z_i a_i + \sum_{ij} z_{ij} a_i a_j \tag{B5}$$

and expand the components of  $\mathbf{N}$  in terms of the stochastic parameters  $a_i$ . This allows us to use the condition  $\det N = 0$  to find the  $z_i$  terms. There are two solutions of  $m^2$  labeled by  $z_0^\pm$  and  $z_i^\pm$ .

We use (A) to evaluate

$$\langle e^{i(\omega_1 - \omega_2)t} \rangle \equiv \int d^4 a \exp(-\vec{a} \cdot \vec{\Xi} \cdot \vec{a}) e^{i(\omega_1 - \omega_2)t} \frac{\det \vec{\Xi}}{\pi^2} \tag{B6}$$

with

$$\vec{u} = -\frac{it}{2k}(z_1^+ - z_1^-, z_2^+ - z_2^-, z_3^+ - z_3^-, z_4^+ - z_4^-)$$

and

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sigma_1} - i(z_{11}^+ - z_{11}^-) \frac{1}{k} & -\frac{i}{2k}(z_{12}^+ - z_{12}^-) & -\frac{i}{2k}(z_{13}^+ - z_{13}^-) & -\frac{i}{2k}(z_{14}^+ - z_{14}^-) \\ -\frac{i}{2k}(z_{12}^+ - z_{12}^-) & \frac{1}{\sigma_2} - i(z_{22}^+ - z_{22}^-) \frac{1}{k} & -\frac{i}{2k}(z_{23}^+ - z_{23}^-) & -\frac{i}{2k}(z_{24}^+ - z_{24}^-) \\ -\frac{i}{2k}(z_{13}^+ - z_{13}^-) & -\frac{i}{2k}(z_{23}^+ - z_{23}^-) & \frac{1}{\sigma_3} - i(z_{33}^+ - z_{33}^-) \frac{1}{k} & -\frac{i}{2k}(z_{34}^+ - z_{34}^-) \\ -\frac{i}{2k}(z_{14}^+ - z_{14}^-) & -\frac{i}{2k}(z_{24}^+ - z_{24}^-) & -\frac{i}{2k}(z_{34}^+ - z_{34}^-) & \frac{1}{\sigma_4} - i(z_{44}^+ - z_{44}^-) \frac{1}{k} \end{pmatrix}. \quad (\text{B7})$$

On substituting the detailed expressions for  $z_0^+$  and  $z_i^+$  it is straightforward to obtain the forms in (3.14) and (3.16).

### APPENDIX C: LINDBLAD DECOHERENCE

A useful generic form of the Lindblad master equation for a  $N \times N$  density matrix  $\rho$  is

$$\frac{d}{dt}\rho = \mathcal{L}\rho, \quad (\text{C1})$$

where [17]

$$\mathcal{L}\rho = -i[H, \rho] + \frac{1}{2} \sum_{k,l=1}^{N^2-1} c_{kl} ([F_k \rho, F_l] + [F_k, \rho F_l]). \quad (\text{C2})$$

The complex  $N \times N$  matrices  $F_l (= F_l^\dagger)$ ,  $l = 1, \dots, N^2 - 1$ , together with the identity matrix  $1_N (= F_0)$  form a basis for a space of complex  $N \times N$  matrices and so any operator  $\mathcal{D}$  can be written as  $\mathcal{D} = \sum_{\mu=0}^{N^2-1} \mathcal{D}_\mu F_\mu$ . If  $\{c_{kl}\}$  is a non-negative matrix,  $\text{Tr}(F_l) = 0$ , and  $\text{Tr}(F_i F_j) = \frac{1}{2} \delta_{ij}$ , then the density matrix  $\rho$  evolves in the space of physical density matrices [18] and so probabilities are non-negative. On writing  $H = \sum_{\mu=0}^8 h_\mu F_\mu$  we have

$$\mathcal{L}\rho = -i \sum_{j,k=1}^{N^2-1} h_j [F_j, \rho_k F_k] + \frac{1}{2} \sum_{k,l=1}^{N^2-1} c_{kl} n_{kl}, \quad (\text{C3})$$

where

$$n_{kl} = \frac{1}{2} \left( [F_k, [\rho, F_l]] + \{F_k, [\rho, F_l]\} + [[F_k, \rho], F_l] + \{[F_k, \rho], F_l\} + 2\{\rho, [F_k, F_l]\} \right). \quad (\text{C4})$$

For  $N = 2$ ,  $F_j = \frac{s_j}{2}$  (where  $s_j$  are the Pauli matrices)  $\mathcal{D}_0 = \frac{1}{2} \text{Tr}(\mathcal{D})$  and  $\mathcal{D}_j = \text{Tr}(\mathcal{D} s_j)$ . The master equation of [(4.5)] becomes

$$\frac{\partial}{\partial t} \langle \rho \rangle = -i[H + n_0 H_I, \langle \rho \rangle] + \Omega^2 n_0^2 ([H_I \langle \rho \rangle, H_I] + [H_I, \langle \rho \rangle H_I]) \quad (\text{C5})$$

on noting that

$$[H_I, [H_I, \langle \rho \rangle]] = -([H_I \langle \rho \rangle, H_I] + [H_I, \langle \rho \rangle H_I]). \quad (\text{C6})$$

The nonzero elements of the associated  $c$  matrix for (C5) are

$$\begin{aligned} c_{11} &= 2\Omega^2 (a_{\nu_e} - a_{\nu_\mu})^2 \sin^2 2\theta, \\ c_{13} &= c_{31} = 2\Omega^2 (a_{\nu_e} - a_{\nu_\mu})^2 \sin 2\theta \cos 2\theta, \\ c_{33} &= 2\Omega^2 (a_{\nu_e} - a_{\nu_\mu})^2 \cos^2 2\theta. \end{aligned} \quad (\text{C7})$$

On using (4.6)

$$\begin{aligned} [H_0 + n_0 H_I, \langle \rho \rangle] &= i \sum_{j,l=1}^3 (\varepsilon_{1jl} n_0 h'_1 + i \varepsilon_{3jl} (n_0 h'_3 + h_3)) \\ &\quad \times \rho_j \frac{s_l}{2}. \end{aligned} \quad (\text{C8})$$

Also

$$c_{pl} n_{pl} = -\frac{1}{2} c_{pl} \sum_{j,r=1}^3 (2\delta_{jr} \delta_{pl} - \delta_{jp} \delta_{rl} - \delta_{jl} \delta_{pr}) \rho_j \frac{s_r}{2}. \quad (\text{C9})$$

$\rho_0$  is independent of time from the structure of (C5) whereas  $\rho_q$  ( $q = 1, 2, 3$ ) satisfies

$$\begin{aligned} \frac{d}{dt} \rho_q &= \sum_{j=1}^3 (n_0 h'_1 \varepsilon_{1jq} + [n_0 h'_3 + h_3] \varepsilon_{3jq}) \rho_j - \frac{\Omega^2}{2} \\ &\quad \times \sum_{p,l,j=1}^3 c_{pl} (2\delta_{jq} \delta_{pl} - \delta_{jp} \delta_{ql} - \delta_{jl} \delta_{pq}) \rho_j. \end{aligned} \quad (\text{C10})$$

Using this it is straightforward to show that the  $\mathcal{L}$  corresponding to (4.9) is

$$\begin{pmatrix} -\Omega^2 \Delta^2 \cos^2(2\theta) & -\mathcal{U} & \Omega^2 \Delta^2 \sin(2\theta) \cos(2\theta) \\ \mathcal{U} & -\Omega^2 \Delta^2 & -\mathcal{W} \\ \Omega^2 \Delta^2 \sin(2\theta) \cos(2\theta) & \mathcal{W} & -\Omega^2 \Delta^2 \sin^2(2\theta) \end{pmatrix},$$

where  $\mathcal{U}$  and  $\mathcal{W}$  are defined in (4.13) and (4.14).



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