

Are preons dyons? Naturalness of three generationsC. R. Das^{1,*} and Larisa Laperashvili^{1,2,†}¹*The Institute of Mathematical Sciences, Chennai, India*²*The Institute of Theoretical and Experimental Physics, Moscow, Russia*

(Received 14 May 2006; published 17 August 2006)

In the present paper a model of preons and their composites is constructed in the framework of the superstring-inspired flipped $E_6 \times \tilde{E}_6$ gauge symmetry group which reveals a generalized dual symmetry. Here E_6 and \tilde{E}_6 are nondual and dual sectors of the theory with hyperelectric g and hypermagnetic \tilde{g} charges, respectively. We follow the old idea by J. Pati presuming that preons are dyons, which in our model are confined by hyper-magnetic strings—composite $\mathbf{N} = 1$ supersymmetric non-Abelian flux tubes created by the condensation of spreons near the Planck scale. Considering preons belonging to the 27-plet of E_6 , we have investigated the breakdown of E_6 (and \tilde{E}_6) near the Planck scale into the $SU(6) \times U(1)$ (and $\widetilde{SU(6)} \times \widetilde{U(1)}$) gauge group and shown that the six types of strings having fluxes $\Phi_n = n\Phi_0$ ($n = \pm 1, \pm 2, \pm 3$) produce three (and only three) generations of composite quark-leptons and bosons. The model predicts the existence of the Family replicated gauge group $[E_6]^3$ near the Planck scale. The runnings of the corresponding fine structure constants are investigated, and the critical coupling constants at the two phase transition points near the Planck scale are calculated. The model explains the hierarchies of masses in the standard model naturally. The compactification in a space-time with five dimensions and its influence on form-factors of composite objects are also briefly discussed.

DOI: [10.1103/PhysRevD.74.035007](https://doi.org/10.1103/PhysRevD.74.035007)

PACS numbers: 12.60.Rc, 11.25.Wx, 12.10.-g, 14.80.Hv

I. INTRODUCTION

During the last 20 years grand unified theories (GUTs) have been inspired by the ultimate theory of superstrings [1–3], which gives the possibility of unifying all fundamental interactions including gravity. It was shown in Ref. [4] that strings are free of gravitational and Yang-Mills anomalies if the gauge group is $SO(32)$ or $E_8 \times E_8$. The “heterotic” $E_8 \times E_8'$ superstring theory was suggested in Ref. [2] as a more realistic candidate for unification. This ten-dimensional Yang-Mills theory can undergo a spontaneous compactification for which the E_8 group is broken to E_6 in four dimensions. The group E_8' remains unbroken and plays the role of a hidden sector, which provides the spontaneous breakdown of SUGRA and a soft SUSY breaking mechanism. As a result, superstring theory leads to the SUSY-GUT scenario in four dimensions.

In the present paper we develop a new preonic model of composite quark-leptons and bosons starting with the “flipped” $E_6 \times \tilde{E}_6$ supersymmetric group. We show that a dual sector of the theory described by the group \tilde{E}_6 is broken in our world near the Planck scale $M_{\text{Pl}} \approx 1.22 \cdot 10^{19}$ GeV. The breakdown of the dual sector gives in our model a very specific type of “horizontal symmetry” predicting only three generations of the standard model.

We consider that preons are dyons confined by hypermagnetic strings, which are created by the condensation of spreons near the Planck scale.

Pati first [5] suggested the use of the strong $U(1)$ magnetic force to bind preons-dyons and form composite objects. This idea was developed in Refs. [6–8] (see also the review [9]) and is extended in our model in the light of recent investigations of composite non-Abelian flux tubes in SQCD [10–15] (see also Refs. [16–18] and the review [19]). A very interesting solution for a QCD string with quantized flux was found in Ref. [20].

A supersymmetric preon model based on the (nonflipped) $E_6 \times E_6'$ gauge group was investigated in Ref. [21] with different properties.

A preonic model of composite quarks and leptons in higher space-time dimensions was constructed in Ref. [22]. Their results are utilized in our present model of preons. In general, composite models were reviewed in Refs. [23–31].

The paper is organized as follows. In Sec. II we have presented a feasible chain of flipped models, which may exist between $M_{\text{GUT}} \sim 10^{16}$ GeV and $M_{\text{Pl}} \sim 10^{19}$ GeV ended by the flipped E_6 unification, and have considered the content of a 27-plet of the flipped E_6 . Sec. III contains an $\mathbf{N} = 1$ supersymmetric description of E_6 -theory in five space-time dimensions. Section IV presents a new preonic model of quark-leptons and bosons, in which it is suggested that preons are dyons confined by hyper-magnetic strings. The breakdown of E_6 and \tilde{E}_6 symmetries and the condensation of spreons near the Planck scale are discussed in this section. It is shown that the composite non-Abelian flux tubes constructed from preonic strings are very thin ($\sim 10^{-18}$ GeV⁻¹) and have an enormously large tension ($\sim 10^{38}$ GeV²). The naturalness of three generations (families) of quark-leptons and bosons is also

*Email: crdas@imsc.res.in†Email: laper@itep.ru, laper@imsc.res.in

demonstrated in Sec. IV to be a consequence of the quantized flux of tubes considered in our theory. In Sec. V the existence of the Family replicated gauge group $[E_6]^3$ is discussed, and the breakdown of $[E_6]^3$ and $[\tilde{E}_6]^3$ near the Planck scale is analyzed. Sec. VI is devoted to the solution of the problem of mass hierarchies established in the standard model. Also form factors indicating the compositeness of quark-leptons are discussed in this section. The results of the present investigation are summarized in our conclusions (Sec. VII).

II. FLIPPED E_6 -UNIFICATION OF GAUGE INTERACTIONS

In Ref. [32], where we consider that only “flipped” $SU(5)$ unifies $SU(3)_C$ and $SU(2)_W$ of the standard model (SM) at the GUT scale $M_{GUT} \sim 10^{16}$ GeV, we gave an explanation of the discrepancy between the unification scale M_{GUT} and string scale $M_{str} \sim 10^{18}$ GeV. We have assumed that there exists a chain of extra intermediate symmetries between M_{GUT} and M_{Pl} :

$$\begin{aligned}
 SU(5) \times U(1)_X &\rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \\
 &\rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6. \quad (1)
 \end{aligned}$$

Amongst the many articles devoted to the E_6 unification (see, for example, [33–36] and references therein) we have chosen only one example of the flipped E_6 unification from Ref. [32]. We have considered such Higgs boson contents of the $SU(5)$ and $SO(10)$ gauge groups which give the final flipped E_6 unification at the scale $\sim 10^{18}$ GeV and a decreased running of the inverse gauge coupling constant α^{-1} near the Planck scale. Such an example, presented in Fig. 1, suits the purposes of our new model of preons. Here and below we consider flipped models in which $SU(5)$ contains Higgs bosons h, h^c and H, H^c belonging to $5_h + \bar{5}_h$ and $10_H + \bar{10}_H$ representations of $SU(5)$, respectively, also a 24-dimensional adjoint Higgs field A and Higgs bosons belonging to additional higher representations. Correspondingly, the flipped $SO(10)$ (coming at the superGUT scale M_{SG}) contains $10_h + \bar{10}_h$ and $45_H + \bar{45}_H$, a 45-dimensional adjoint A and higher representations of Higgs bosons. As was shown in [32], such Higgs boson contents lead to the flipped E_6 final unification at the supersuperGUT scale $M_{SSG} \sim 10^{18}$ GeV.

Figure 1 presents an example of the running of the inverse gauge coupling constants $\alpha_i^{-1}(\mu)$ (μ is the energy scale) for $i = 1, 2, 3, X, Z, X1, Z1, 5, 10$. It was shown that at the scale $\mu = M_{GUT}$ the flipped $SU(5)$ undergoes breakdown to the supersymmetric (MSSM) $SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X$ gauge group, which is the supersymmetric extension of the MSSM originating at the seesaw scale $M_{SS} \approx 10^{11}$ GeV, where heavy right-handed neutrinos appear. A singlet Higgs field S provides the following breakdown to the SM (see [37]):

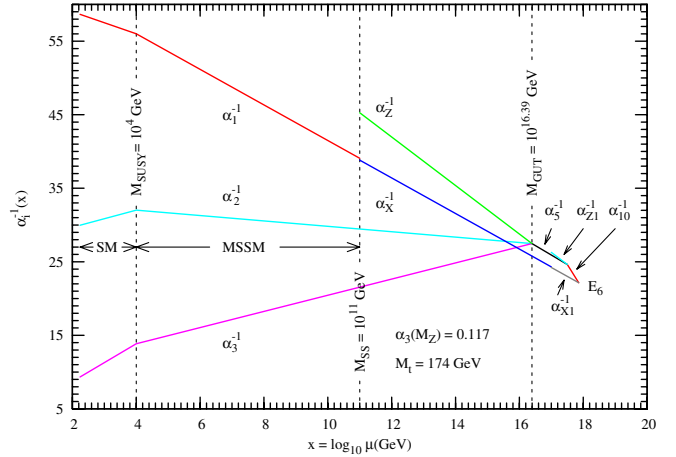


FIG. 1 (color online). This figure presents the running of the inverse gauge coupling constants $\alpha_i^{-1}(x)$ for $i = 1, 2, 3, X, Z, X1, Z1, 5, 10$ of the chain $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \rightarrow SU(5) \times U(1)_X \rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6$, corresponding to the breakdown of the flipped $SU(5)$ to the supersymmetric (MSSM) $SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X$ gauge symmetry group with Higgs bosons belonging to the $5_h + \bar{5}_h, 10_H + \bar{10}_H, 24$ -dimensional adjoint A and higher representations of the flipped $SU(5)$. The final unification group is flipped E_6 at the supersuperGUT scale $M_{SSG} \approx 10^{18}$ GeV with $\alpha^{-1}(M_{SSG}) \approx 22$ for $M_{SUSY} = 10$ TeV and seesaw scale $M_{SS} = 10^{11}$ GeV.

$$\begin{aligned}
 SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \\
 \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (2)
 \end{aligned}$$

Supersymmetry extends the conventional SM beyond the scale M_{SUSY} (in our example $M_{SUSY} = 10$ TeV).

The final E_6 unification assumes the existence of three 27-plets of E_6 containing three generations of quarks and leptons including the right-handed neutrinos N_i^c (here $i = 1, 2, 3$ is the generation index). Quarks and leptons of the fundamental 27 representation decompose under the $SU(5) \times U(1)_X$ subgroup as follows:

$$\begin{aligned}
 27 \rightarrow (10, 1) + (\bar{5}, -3) + (\bar{5}, 2) + (5, -2) + (1, 5) \\
 + (1, 0). \quad (3)
 \end{aligned}$$

The first and second quantities in the brackets of Eq. (3) correspond to the $SU(5)$ representation and $U(1)_X$ charge, respectively. We consider charges Q_X and Q_Z in units of $1/\sqrt{40}$ and $\sqrt{3/5}$, respectively, using the assignments: $Q_X = X$ and $Q_Z = Z$ (see [32] for details).

The conventional SM family, which contains doublets of left-handed quarks Q and leptons L , right-handed up and down quarks u^c, d^c , also e^c , is assigned to the $(10, 1) + (\bar{5}, -3) + (1, 5)$ representations of the flipped $SU(5) \times U(1)_X$, along with a right-handed neutrino N^c . These representations decompose under

$$SU(5) \times U(1)_X \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X, \quad (4)$$

and such a decomposition gives the following content:

$$(10, 1) \rightarrow Q = \begin{pmatrix} u \\ d \end{pmatrix} \sim \left(3, 2, \frac{1}{6}, 1\right),$$

$$d^c \sim \left(\bar{3}, 1, -\frac{2}{3}, 1\right), \quad (5)$$

$$N^c \sim (1, 1, 1, 1).$$

$$(\bar{5}, -3) \rightarrow u^c \sim \left(\bar{3}, 1, \frac{1}{3}, -3\right),$$

$$L = \begin{pmatrix} e \\ \nu \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}, -3\right), \quad (6)$$

$$(1, 5) \rightarrow e^c \sim (1, 1, 1, 5). \quad (7)$$

The remaining representations in Eq. (3) decompose as follows:

$$(5, -2) \rightarrow D \sim \left(3, 1, -\frac{1}{3}, -2\right),$$

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim \left(1, 2, \frac{1}{2}, -2\right). \quad (8)$$

$$(\bar{5}, 2) \rightarrow D^c \sim \left(\bar{3}, 1, \frac{1}{3}, 2\right),$$

$$h^c = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}, 2\right). \quad (9)$$

The light Higgs doublets are accompanied by coloured Higgs triplets D, D^c .

The singlet field S is represented by (1,0):

$$(1, 0) \rightarrow S \sim (1, 1, 2, 2). \quad (10)$$

It is necessary to notice that the flipping of our $SU(5)$:

$$d^c \leftrightarrow u^c, \quad N^c \leftrightarrow e^c, \quad (11)$$

distinguishes this group of symmetry from the standard Georgi-Glashow $SU(5)$ [38].

The multiplets (5) and (7) fit in the 16 spinorial representation of $SO(10)$:

$$F(16) = F(10, 1) + F(\bar{5}, -3) + F(1, 5). \quad (12)$$

Higgs chiral superfields belong to the 10 representation of $SO(10)$:

$$h(10) = h(5, -2) + h^c(\bar{5}, 2). \quad (13)$$

In Sec. V we predict that in the interval of energy $M_{SSG} \leq \mu \leq M_{\text{crit}}$ (see Fig. 2) the Family replicated gauge group of symmetry $[E_6]^3$ originates at the scale M_{FR} : $M_{SSG} < M_{\text{FR}} < M_{\text{crit}}$.

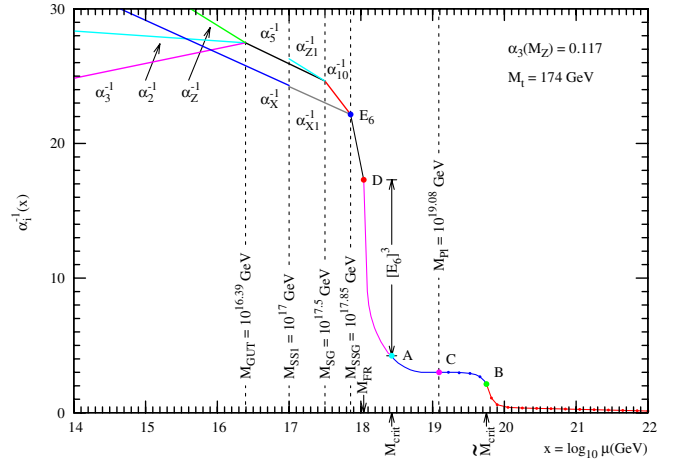


FIG. 2 (color online). The figure provides a qualitative description of the running of $\alpha^{-1}(x)$ near the Planck scale predicted by the present preonic model. The region AD corresponds to the Family replicated gauge group of symmetry $[E_6]^3$, which arises at the scale M_{FR} given by point D . The point A at the scale of energy $\mu = M_{\text{crit}}$ indicates that hyper-electric preonic strings exist for $\mu \geq M_{\text{crit}}$. The point B corresponds to the scale $\mu = \tilde{M}_{\text{crit}}$ and indicates that hyper-magnetic preonic strings exist for $\mu \leq \tilde{M}_{\text{crit}}$. The curve AB corresponds to the region of energies where spreons are condensed near the Planck scale, giving both hyper-electric and hyper-magnetic, preonic strings. For $\mu \geq \tilde{M}_{\text{crit}}$ we have the running of $\alpha^{-1}(x)$ for monopolic “quark-leptons.” The point C corresponds to the Planck scale and gives $\alpha^{-1}(M_{\text{Pl}}) = 3$.

III. SUPERSYMMETRIC E_6 CONTENT IN FIVE SPACE-TIME DIMENSIONS

Taking into account the role of compactification [39] we start, as in Ref. [22], with $N = 1$ supersymmetric gauge theory in 5D dimensions with a local symmetry gauge group G , which is equal to the flipped E_6 in our case. The matter fields transform according to one of the irreducible representations of G : for example, we have the 27-plet of E_6 given by Eqs. (5)–(10).

The supermultiplet $\mathcal{V} = (A^M, \lambda^\alpha, \Sigma, X^a)$ in 5D space (with $M = 0, 1, 2, 3, 4$ space-time indices) contains a vector field:

$$A^M = A^{MJ} T^J, \quad (14)$$

and a real scalar field:

$$\Sigma = \Sigma^J T^J, \quad (15)$$

where J runs over the E_6 group index values and T^J are generators of the E_6 algebra.

Two gauginos fields:

$$\lambda^\alpha = \lambda^{\alpha J} T^J, \quad (16)$$

form a decuplet under the R -symmetry group $SU(2)_R$ (with $\alpha = 1, 2$).

Auxiliary fields:

$$X^a = X^{aJ}T^J \quad (17)$$

form a triplet of $SU(2)_R$ (with $a = 1, 2, 3$).

After compactification, these fields are combined into the $\mathbf{N} = 1$ 4D fields:

$$\text{vector supermultiplet } V = (A^\mu, \lambda^1, X^3)$$

(here $\mu = 0, 1, 2, 3$ are the space-time indices), and

$$\text{a chiral supermultiplet } \Phi = (\Sigma + iA^4, \lambda^2, X^1 + iX^2).$$

The matter fields are contained in the hypermultiplet:

$$\mathcal{H} = (h^\alpha, \Psi, F^\alpha),$$

where h^α is a doublet of $SU(2)_R$, the Dirac fermion field $\Psi = (\psi_1, \psi_2^+)^T$ is an $SU(2)_R$ singlet and the auxiliary fields F^α also form an $SU(2)_R$ doublet.

Then we have two $\mathbf{N} = 1$ 4D chiral multiplets:

$$H = (h^1, \psi_1, F^1) \quad \text{and} \quad H^c = (h^2, \psi_2, F^2)$$

transforming according to the representations R and anti- R of the gauge group $G = E_6$, respectively. Then the 5D E_6 -symmetric action (see [40]) is given as follows:

$$S = \int d^5x \int d^4\theta [H^c e^V H^{c+} + H^+ e^V H] \\ + \int d^5x \int d^2\theta \left[H^c \left(\partial_4 - \frac{1}{\sqrt{2}} \Phi \right) H + \text{H.c.} \right]. \quad (18)$$

This theory is anomaly-free [22,40]. The compactification results are given in Ref. [22], and we summarize them in the appendix.

IV. SUPERSYMMETRIC $E_6 \times \tilde{E}_6$ PREONIC MODEL OF COMPOSITE QUARK-LEPTONS AND BOSONS

Why do three generations exist in Nature? We suggest an explanation considering a new preonic model of composite SM particles. The model starts from the supersymmetric flipped $E_6 \times \tilde{E}_6$ gauge symmetry group for preons, where E_6 and \tilde{E}_6 correspond to nondual (with hyper-electric charge g) and dual (with hyper-magnetic charge \tilde{g}) sectors of the theory, respectively. We assume that preons are dyons confined by hyper-magnetic strings in the region of energy $\mu \leq \tilde{M}_{\text{crit}} \sim M_{\text{Pl}}$ (see Fig. 2).

A. Preons are dyons bound by hyper-magnetic strings

Considering the $\mathbf{N} = 1$ supersymmetric flipped $E_6 \times \tilde{E}_6$ gauge theory for preons in 4D space-time, we assume that preons P (antipreons P^c) are dyons with charges ng and $m\tilde{g}$ ($-ng$ and $-m\tilde{g}$), residing in the 4D hypermultiplets $\mathcal{P} = (P, P^c)$ and $\tilde{\mathcal{P}} = (\tilde{P}, \tilde{P}^c)$. Here “ \tilde{P} ” designates spreons, but not the belonging to \tilde{E}_6 .

The dual sector \tilde{E}_6 is broken in our world to some group \tilde{G} , where preons and spreons transform under the hyper-electric gauge group E_6 and hyper-magnetic gauge group \tilde{G} according to their fundamental representations:

$$P, \tilde{P} \sim (27, N), \quad P^c, \tilde{P}^c \sim (\overline{27}, \bar{N}), \quad (19)$$

where N is the N -plet of \tilde{G} group. We also consider scalar preons and spreons as singlets of E_6 :

$$P_s, \tilde{P}_s \sim (1, N), \quad P_s^c, \tilde{P}_s^c \sim (1, \bar{N}), \quad (20)$$

which are actually necessary for the entire set of composite quark-leptons and bosons [22].

The hyper-magnetic interaction is assumed to be responsible for the formation of E_6 fermions and bosons at the compositeness scale Λ_s . The main idea of the present investigation is the assumption that preons-dyons are confined by hyper-magnetic supersymmetric non-Abelian flux tubes, which are a generalization of the well-known Abelian ANO-strings [41,42] for the case of the supersymmetric non-Abelian theory developed in Refs. [10–19]. As a result, in the limit of infinitely narrow flux tubes (strings), we have the following bound states:

(i) quark-leptons (fermions belonging to the E_6 fundamental representation):

$$Q^a \sim P^{aA}(y) \left[\int_x^y \tilde{A}_\mu dx^\mu \right]_A^B (P_s^c)_B(x) \sim 27, \quad (21)$$

$$\bar{Q}_a \sim (P_s^c)^A(y) \left[\int_x^y \tilde{A}_\mu dx^\mu \right]_A^B P_{aB}(x) \sim \overline{27}, \quad (22)$$

where $a \in 27$ -plet of E_6 , $A, B \in N$ -plet of \tilde{G} , and $\tilde{A}_\mu(x)$ are dual hyper-gluons belonging to the adjoint representation of \tilde{G} ;

(ii) “mesons” (hyper-gluons and hyper-Higgses of E_6):

$$M_b^a \sim P^{aA}(y) \left[\int_x^y \tilde{A}_\mu dx^\mu \right]_A^B (P^c)_{bB}(x) \sim 1 + 78 \\ + 650 \quad \text{of } E_6, \quad (23)$$

$$S \sim (P_s)^A(y) \left[\int_x^y \tilde{A}_\mu dx^\mu \right]_A^B (P_s^c)_B(x) \sim 1, \quad (24)$$

$$\bar{S} \sim (P_s^c)^A(y) \left[\int_x^y \tilde{A}_\mu dx^\mu \right]_A^B (P_s)_B(x) \sim 1; \quad (25)$$

(iii) “baryons,” for \tilde{G} -triplet we have (see Sec. V):

$$D_1 \sim \epsilon_{ABC} P^{aA'}(z) P^{bB'}(y) P^{cC'}(x) \left[\int_X^z \tilde{A}_\mu dx^\mu \right]_{A'}^A \\ \times \left[\int_X^y \tilde{A}_\mu dx^\mu \right]_{B'}^B \left[\int_X^x \tilde{A}_\mu dx^\mu \right]_{C'}^C, \quad (26)$$

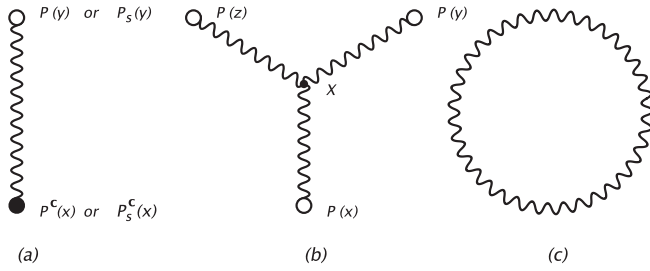


FIG. 3. Preons are bound by hyper-magnetic strings: (a), (b) correspond to the string configurations of composite particles belonging to the 27-plet of the flipped E_6 gauge symmetry group; (c) represents a closed string describing a graviton.

$$D_2 \sim \epsilon_{ABC} P^{aA'}(z) P^{bB'}(y) (P_s)^{C'}(x) \left[\int_X^z \tilde{A}_\mu dx^\mu \right]_{A'}^A \times \left[\int_X^y \tilde{A}_\mu dx^\mu \right]_{B'}^B \left[\int_X^x \tilde{A}_\mu dx^\mu \right]_{C'}^C, \quad (27)$$

and their conjugate particles.

The bound states (21)–(27) are shown in Fig. 3 as unclosed strings (a) and “baryonic” configurations (b). It is easy to generalize Eqs. (21)–(27) for the case of string constructions of superpartners—squark-sleptons, hyper-gluinos, and hyper-higgsinos. Closed strings—gravitons—are presented in Fig. 3(c). All these bound states belong to E_6 representations and they in fact form $\mathbf{N} = 1$ 4D superfields.

B. The breakdown of E_6 and \tilde{E}_6 groups at the planck scale

As was suggested in Ref. [43], E_6 can be broken by Higgses belonging to the 78-dimensional representation of E_6 :

$$E_6 \rightarrow SU(6) \times SU(2) \rightarrow SU(6) \times U(1), \quad (28)$$

where $SU(6) \times U(1)$ is the largest relevant invariance group of the 78.

Figure 2 shows two points A and B near the Planck scale. The point A corresponds to the breakdown of E_6 according to the chain (28). The group E_6 is broken in the region of energies $\mu \geq M_{\text{crit}}$ producing hyper-electric strings between preons. The point B in Fig. 2 indicates the scale \tilde{M}_{crit} corresponding to the breakdown $\tilde{E}_6 \rightarrow \widetilde{SU}(6) \times U(1)$. At the point B hyper-magnetic strings are produced and exist in the region of energies $\mu \leq \tilde{M}_{\text{crit}}$ confining hyper-magnetic charges of preons. As a result, in the region $\mu \leq M_{\text{crit}}$ we see quark-leptons with charges ng ($n \in \mathbb{Z}$), but in the region $\mu \geq \tilde{M}_{\text{crit}}$ monopolic “quark-leptons”—particles with dual charges $m\tilde{g}$ ($m \in \mathbb{Z}$)—may exist. Since $\tilde{M}_{\text{crit}} > M_{\text{Pl}}$, monopoles are absent in our world.

In the region of energies $M_{\text{crit}} \leq \mu \leq \tilde{M}_{\text{crit}}$ around the Planck scale, both hyper-electric and hyper-magnetic strings come to play: preons, quarks, and monopolic “quarks” are totally confined giving heavy neutral parti-

cles with mass $M \sim M_{\text{Pl}}$, but closed strings—gravitons—survive there.

C. Condensation of spreons near the planck scale, and $\mathbf{N} = 1$ supersymmetric non-Abelian flux tubes confined preons

If the $SU(6) \times U(1)$ symmetry group works near the Planck scale, then we deal just with the theory of non-Abelian flux tubes in $\mathbf{N} = 1$ SQCD, which was developed recently in Refs. [10–19].

We assume the condensation of spreons at the Planck scale. One can combine the Z_6 center of $SU(6)$ with the elements $\exp(i\pi) \in U(1)$ to get topologically stable string solutions possessing both windings, in $SU(6)$ and $U(1)$. Henceforth we assume the existence of a dual sector of the theory described by $\widetilde{SU}(6) \times \widetilde{U}(1)$, which is responsible for hyper-magnetic fluxes. Then, according to the results obtained in Refs. [10–19], we have a nontrivial homotopy group:

$$\pi_1\left(\frac{SU(6) \times U(1)}{Z_6}\right) \neq 0, \quad (29)$$

and flux lines form topologically nontrivial Z_6 strings.

Besides $SU(6)$ and $U(1)$ gauge bosons, the model contains six scalar fields charged with respect to $U(1)$ which belong to the 6-plet of $SU(6)$. Considering scalar fields of spreons

$$\tilde{P} = \{\phi^{aA}\}, \quad (30)$$

which have indices a of $SU(6)$ and A of $\widetilde{SU}(6)$ fundamental multiplets, we construct a condensation of spreons in the vacuum:

$$\tilde{P}_{\text{vac}} = \langle \tilde{P}^{aA} \rangle = v \cdot \text{diag}(1, 1, \dots, 1), \quad a, A = 1, \dots, 6. \quad (31)$$

The vacuum expectation value (VEV) v is given in Refs. [10] as

$$v = \sqrt{\xi} \gg \Lambda_4, \quad (32)$$

where ξ is the Fayet-Iliopoulos D -term parameter in the $\mathbf{N} = 1$ supersymmetric theory and Λ_4 is its 4-dimensional scale. In our case:

$$v \sim M_{\text{Pl}} \sim 10^{19} \text{ GeV}, \quad (33)$$

where spreons are condensed.

Nontrivial topology (29) amounts to the winding of elements of the matrix (30), and we obtain string solutions:

$$\tilde{P}_{\text{string}} = v \cdot \text{diag}(e^{i\alpha(x)}, e^{i\alpha(x)}, \dots, 1, 1), \quad \text{where } x \rightarrow \infty. \quad (34)$$

Three types of string moduli space (see Ref. [11]):

$$\frac{SU(6)}{SU(5) \times U(1)},$$

$$\frac{SU(6)}{SU(4) \times SU(2) \times U(1)}, \quad \text{and} \quad \frac{SU(6)}{SU(3) \times SU(3) \times U(1)}$$

(35)

give us solutions for three types of Z_6 -flux tubes which form a non-Abelian analog of ANO-strings [41,42].

Assuming the existence of a preon P and antipreon P^c at the ends of strings with hyper-magnetic charges $n\tilde{g}$ and $-n\tilde{g}$, respectively, we obtain six types of string having their fluxes Φ_n quantized according to the Z_6 center group of $SU(6)$:

$$\Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3. \quad (36)$$

Indeed, Z_6 has six group elements:

$$Z_6 = \left\{ \exp\left(2\pi\frac{n}{6}i\right) \mid n \bmod 6 \right\}. \quad (37)$$

So far as n is given modulo 6, the fluxes of tubes corresponding to the solutions with $n = 4, 5$ are equal to the fluxes (36) with $n = -2, -1$, respectively, (see also Refs. [14]).

The string tensions of these non-Abelian flux tubes were also calculated in Refs. [10]. The minimal tension is

$$T_0 = 2\pi\xi, \quad (38)$$

which in our preonic model [see Eq. (32)] is equal to

$$T_0 = 2\pi v^2 \sim 10^{38} \text{ GeV}^2. \quad (39)$$

Such an enormously large tension means that preonic strings have almost infinitely small $\alpha' \rightarrow 0$, where $\alpha' = 1/(2\pi T_0)$ is the slope of trajectories in string theory [1]. Six types of preonic tubes give us three types of preonic k -strings having the following tensions:

$$T_k = kT_0, \quad \text{where } k = 1, 2, 3. \quad (40)$$

If the Fayet-Iliopoulos term ξ vanishes in the supersymmetric theory of preons, then the spreon condensate vanishes too, and the theory is in the Coulomb phase. But for a nonvanishing ξ the spreons develop their VEV, and the theory is in the Higgs phase. Then hyper-magnetic charges of preons and antipreons are confined by six strings which are oriented in opposite directions. For this reason, six strings have only three different tensions (40).

Also preonic strings are extremely thin. Indeed, the thickness of the flux tube depends on the mass of the dual gauge boson \hat{A}_μ acquired in the confinement phase:

$$m_V = gv. \quad (41)$$

As it is shown below, in the region of energies AB near the Planck scale where spreons are condensed (see Fig. 2) we have

$$\alpha = \frac{g^2}{4\pi} \approx 1, \quad g \approx 2\sqrt{\pi} \approx 3.5, \quad (42)$$

and the thickness of preonic strings given by the radius R_{str} of the flux tubes is very small:

$$R_{\text{str}} \sim \frac{1}{m_V} \sim \frac{1}{gv} \sim 10^{-18} \text{ GeV}^{-1}. \quad (43)$$

Such infinitely narrow non-Abelian supersymmetric flux tubes remind us of the superstrings in Superstring theory. Having in our preonic model supersymmetric strings with $\alpha' \rightarrow 0$ we obtain, according to the description [1], only massless ground states: spin 1/2 fermions (quarks and leptons), spin 1 hyper-gluons, and spin 2 massless graviton, as well as their superpartners. The excited states belonging to these strings are not realized in our world as they are very massive: they have mass $M > M_{\text{Pl}}$.

V. THREE GENERATIONS OF QUARK-LEPTONS AND BOSONS

The hyper-flavor ‘‘horizontal’’ symmetry was suggested first in Refs. [44,45] (see also interesting discussions of this problem in Refs. [22,46,47]). The previous Section gives a demonstration of a very specific type of ‘‘horizontal symmetry’’ (which resembles the idea of Refs. [48–50]): three, and only three, generations of fermions and bosons present in the superstring-inspired flipped E_6 theory, and also in each step given by Fig. 1 up to the standard model. This number ‘‘3’’ is explained by the existence of just three values of the hyper-magnetic flux (36) which bind the hyper-magnetic charges of preons-dyons. At the ends of these preonic strings there are placed hyper-magnetic charges $\pm\tilde{g}_0$, or $\pm 2\tilde{g}_0$, or $\pm 3\tilde{g}_0$, where \tilde{g}_0 is the minimal hyper-magnetic charge. Then all the bound states (5)–(10) form three generations—three 27-plets of E_6 corresponding to the three different tube fluxes. We also obtain three types of gauge boson A_μ^i ($i = 1, 2, 3$ is the generation index) belonging to the $27 \times \overline{27} = 1 + 78 + 650$ representations of E_6 . Figure 4 illustrates the formation of such hyper-gluons [Fig. 4(a)] and also hyper-Higgses [Fig. 4(b)].

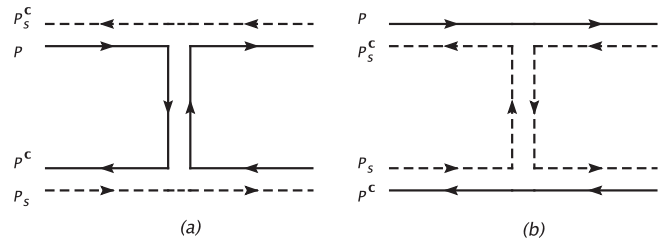


FIG. 4. Vector gauge bosons belonging to the 78 representation of the flipped E_6 and Higgs scalars—singlets of E_6 —are composite objects created (a) by fermionic preons P, P^c and (b) by scalar preons P_s, v . Both of them are confined by hyper-magnetic strings.

Such a description predicts the Family replicated gauge group of symmetry $[E_6]^3$ for quark-leptons and bosons, which works near the Planck scale. Here the number of families is equal to the number of generations $N_g = 3$. We assume that the $[E_6]^3$ symmetry holds in the region of energies $M_{\text{FR}} \leq \mu \leq M_{\text{crit}}$, where the scales M_{FR} and M_{crit} are indicated in Fig. 2 by the points D and A , respectively.

The symmetry breakdown $[E_6]^3 \rightarrow E_6$ at the scale M_{FR} is provided by several Higgses. An analogous mechanism is described in Refs. [51–53] (see also reviews [54,55]). Indeed, in the Family replicated gauge group we have three types of gauge bosons A_μ^i , which produce linear combinations:

$$A_{\mu,\text{diag}}^{(i)} = C_1^{(i)} A_\mu^{(1\text{st fam.})} + C_2^{(i)} A_\mu^{(2\text{nd fam.})} + C_3^{(i)} A_\mu^{(3\text{rd fam.})}, \quad (44)$$

$$i = 1, 2, 3.$$

The combination:

$$A_{\mu,\text{diag}} = \frac{1}{\sqrt{3}} (A_\mu^{(1\text{st fam.})} + A_\mu^{(2\text{nd fam.})} + A_\mu^{(3\text{rd fam.})}) \quad (45)$$

is the massless one, which corresponds to the 78 adjoint representation of hyper-gluons of E_6 . Two other combinations are massive and exist only in the region of energies $\mu \geq M_{\text{FR}}$.

The assumption that only $[E_6]^3$ exists in Nature may be not valid: we are not sure that the Family replicated gauge groups $[SO(10)]^3$ [56], $[SU(5)]^3$ [57–60], $[\text{SMG} \times U(1)_{(B-L)}]^3$ [51–54], or $[\text{SMG}]^3$ [61–64] do not survive at lower energies $\mu < M_{\text{FR}}$. Here we have used the following notation: SMG is the Standard Model group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

The values of $\alpha^{-1}(M_{\text{crit}})$ and $\alpha^{-1}(\tilde{M}_{\text{crit}})$ can be calculated by the method developed in Refs. [65,66]. If the $SU(N)$ group is broken by Abelian scalar particles belonging to its Cartan subalgebra $U(1)^{N-1}$, then the $SU(N)$ critical coupling constant α_N^{crit} is given by the following expression in the one-loop approximation (see [65]):

$$\alpha_N^{\text{crit}} \approx \frac{N}{2} \sqrt{\frac{N+1}{N-1}} \alpha_{U(1)}^{\text{crit}}, \quad (46)$$

where $\alpha_{U(1)}^{\text{crit}}$ is the critical coupling constant for the Abelian $U(1)$ theory.

Here it is necessary to distinguish the E_6 gauge symmetry group for preons from the E_6 for quark-leptons. The points A and B of Fig. 2 respectively correspond to the breakdowns of $[E_6]^3$ and $[\tilde{E}_6]^3$ in the region AB of spreon condensation. In that region we have the breakdown [43] of the preon (one family) E_6 (or \tilde{E}_6):

$$E_6 \rightarrow SU(6) \times U(1) \quad (\text{or } \tilde{E}_6 \rightarrow \widetilde{SU}(6) \times \widetilde{U}(1)),$$

and the scale M_{crit} (or \tilde{M}_{crit}) is the scale of breaking.

In our preonic model the group $SU(6)$ is broken by condensed Abelian scalar spreons belonging to the Cartan subalgebra $U(1)^5$ (compare with the results of Ref. [66]). Then for the one family of preons we have

$$\alpha_6^{\text{crit}} \approx 3.55 \alpha_{U(1)}^{\text{crit}}, \quad (47)$$

according to Eq. (46) for $N = 6$.

The behavior of the effective fine structure constants in the vicinity of the phase transition point ‘‘Coulomb-confinement’’ was investigated in the compact lattice $U(1)$ theory by Monte Carlo methods [67,68]. The following result was obtained:

$$\alpha_{\text{lat.}U(1)}^{\text{crit}} \approx 0.20 \pm 0.015, \quad \tilde{\alpha}_{\text{lat.}U(1)}^{\text{crit}} \approx 1.25 \pm 0.010. \quad (48)$$

The calculation of the critical coupling constants in the Higgs scalar monopole (dyon) model of dual $U(1)$ theory [65,69,70] gave the following results:

$$\alpha_{U(1)}^{\text{crit}} \approx 0.21, \quad (49)$$

$$\tilde{\alpha}_{U(1)}^{\text{crit}} \approx 1.20 \text{—in the Higgs monopole model}$$

and

$$\alpha_{U(1)}^{\text{crit}} \approx 0.19, \quad (50)$$

$$\tilde{\alpha}_{U(1)}^{\text{crit}} \approx 1.29 \text{—in the Higgs dyon model.}$$

According to Eqs. (48)–(50), the condensation of spreons leads to the following critical constants:

$$\alpha_6^{\text{crit}} \approx 3.55 \cdot 0.2 \approx 0.71, \quad \tilde{\alpha}_6^{\text{crit}} = (\alpha_6^{\text{crit}})^{-1} \approx 1.41. \quad (51)$$

For the point C shown in Fig. 2 we have

$$\alpha_6^{(\text{one fam.})}(M_{\text{Pl}}) = 1. \quad (52)$$

This result for one family of preons confirms the estimate (42). Here we have used the Dirac relation for non-Abelian theories (see the explanation in Ref. [66]):

$$g\tilde{g} = 4\pi n, \quad n \in \mathbb{Z}, \quad \alpha\tilde{\alpha} = 1. \quad (53)$$

From the phase transition result (51) for one family of preons, we obtain the following inverse critical coupling constant for the quark-lepton $[E_6]^3$ at the phase transition point A (see reviews [54,55]):

$$\alpha_A^{-1}(M_{\text{crit}}) = 3\tilde{\alpha}_6^{\text{crit}}(M_{\text{crit}}) \approx 3 \cdot 1.41 \approx 4.23. \quad (54)$$

At the phase transition point B , where we have the breakdown of $[\tilde{E}_6]^3$ into the confinement phase, the one-family value $\tilde{\alpha}_6^{\text{crit}}(\tilde{M}_{\text{crit}}) \approx 0.71$ gives the following result:

$$\alpha_B^{-1}(\tilde{M}_{\text{crit}}) \approx 3 \cdot 0.71 \approx 2.13. \quad (55)$$

The values (54) and (55) were used for the construction of the curve AB presented in Fig. 2. The point C corresponds

to the Planck scale and $\alpha^{-1}(M_{\text{Pl}}) = 3$, according to Eq. (52).

The condensation of spreons at the Planck scale predicts the existence of a second minimum of the effective potential $V_{\text{eff}}(\mu)$ at the scale $\mu = M_{\text{Pl}}$. The behavior of this potential and its relation with the Multiple Point Principle (see Refs. [63] and review [55]) will be considered in our future investigations.

The dotted curve in Fig. 2 describes the running of $\alpha^{-1}(\mu)$ for monopolic “quark-leptons” created by preons which are bound by supersymmetric hyper-electric non-Abelian flux tubes. We assume that such monopoles do not exist in our world. However, they can play an essential role in vacuum polarization [71] and in the solution of the cosmological constant problem [55,72].

VI. THE CONSEQUENCES OF THE PREONIC MODEL

In this Section, using our preonic model, we give a simple explanation of why quarks and leptons of three SM generations have such different masses. We show that the hierarchy of SM masses is connected with the string construction of preon bound states.

We also present a description of form factors related with the compositeness of quark-leptons and bosons, briefly discussing a problem of compactification in the framework of higher dimensional theories (see Ref. [22]).

A. Yukawa couplings and quark-lepton masses of three generations

New preon-antipreon pairs can be generated in the flux tubes between preons. Our assumption is that the mechanism of preon-antipreon pair production in these tubes is similar to that of e^+e^- pair production in a uniform

constant electric field considered in one space dimension [73].

Almost 30 years ago J. Schwinger [73] obtained the following expression for the rate per unit volume that a e^+e^- pair will be created in a constant electric field of strength \mathcal{E} :

$$W = \frac{(e\mathcal{E})^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{\pi m_e^2 n}{e\mathcal{E}}\right), \quad (56)$$

where e is the electron charge and m_e is its mass.

A number of authors have applied this result to hadronic production, considering the creation of quark-antiquark pairs in QCD tubes of color flux (see, for example, Refs. [74–76]).

Considering that the probability W_k of one preon-antipreon pair production in a unit space-time volume of the k -string tube with tension (40) is given by Schwinger’s expression:

$$W_k = \frac{T_k^2}{4\pi^3} \exp\left(-\frac{\pi M_k^2}{T_k}\right), \quad \text{where } T_k = kT_0, \quad (57)$$

$$k = 1, 2, 3,$$

we assume that the Yukawa couplings Y_k for the three generations of quarks (t , c and u) are proportional to the square root of the probabilities W_k . Then we obtain the following ratios:

$$Y_t:Y_c:Y_u::W_1^{1/2}:W_2^{1/2}:W_3^{1/2}. \quad (58)$$

In Eq. (57) the values M_k are the constituent masses of preons produced in the k -strings. The Yukawa couplings Y_k are proportional to the masses of the t , c , u quarks, and so we have

$$m_t:m_c:m_u::\frac{T_0}{2\pi^{3/2}} \exp\left(-\frac{\pi M_1^2}{2T_0}\right): \frac{2T_0}{2\pi^{3/2}} \exp\left(-\frac{\pi M_2^2}{4T_0}\right): \frac{3T_0}{2\pi^{3/2}} \exp\left(-\frac{\pi M_3^2}{6T_0}\right). \quad (59)$$

Assuming that M_k is proportional to k :

$$M_k = kM_0, \quad (60)$$

we get the following result:

$$m_t:m_c:m_u = m_0(w:2w^2:3w^3), \quad (61)$$

where

$$w = \exp\left(-\frac{\pi M_0^2}{2T_0}\right), \quad (62)$$

and m_0 is a mass parameter.

For the parameter value

$$w = w_1 \approx 2.9 \cdot 10^{-3} \quad (63)$$

we obtain the following values for the masses of the t , c ,

u -quarks:

$$m_t \approx 173 \text{ GeV}, \quad (64)$$

$$m_c \approx 1 \text{ GeV}, \quad \text{and} \quad m_u \approx 4 \text{ MeV}.$$

For b , s , d -quarks we have different parameters M_0 , m_0 , and $w = w_2$. The value:

$$w_2 \approx 1.7 \cdot 10^{-2} \quad (65)$$

gives the following masses for the b , s , d -quarks:

$$m_b \approx 4 \text{ GeV}, \quad (66)$$

$$m_s \approx 140 \text{ MeV}, \quad \text{and} \quad m_d \approx 4 \text{ MeV}.$$

The results (64) and (66) are in agreement with experimentally established quark masses published in Ref. [77]:

$$\begin{aligned}
 m_t &\approx 174 \pm 5.1 \text{ GeV}, \\
 m_c &\approx 1.15 \text{ to } 1.35 \text{ GeV}, \quad \text{and} \quad m_u \approx 1.5 \text{ to } 4 \text{ MeV}.
 \end{aligned}
 \tag{67}$$

and

$$\begin{aligned}
 m_b &\approx 4.1 \text{ to } 4.9 \text{ GeV}, \\
 m_s &\approx 80 \text{ to } 130 \text{ MeV}, \quad \text{and} \quad m_d \approx 4 \text{ to } 8 \text{ MeV}.
 \end{aligned}
 \tag{68}$$

The value of the w -parameter:

$$w_3 \approx 2.5 \cdot 10^{-2} \tag{69}$$

leads to the following values for the masses of the τ , μ -leptons, and electron:

$$\begin{aligned}
 m_\tau &\approx 2 \text{ GeV}, \\
 m_\mu &\approx 100 \text{ MeV}, \quad \text{and} \quad m_e \approx 3.5 \text{ MeV},
 \end{aligned}
 \tag{70}$$

which are comparable with the results of experiment [77]:

$$\begin{aligned}
 m_\tau &\approx 1.777 \text{ GeV}, \\
 m_\mu &\approx 105.66 \text{ MeV}, \quad \text{and} \quad m_e \approx 0.51 \text{ MeV}.
 \end{aligned}
 \tag{71}$$

We see that our preonic model explains the hierarchy of masses existing in the SM.

B. Form factors as an indication of the compositeness of quark-leptons

Non-point-like behavior of the fundamental quarks and leptons is related with the appearance of form factors describing the dependence of cross sections on 4-momentum squared q^2 for different elementary particle processes; for example, in the reactions:

$$e^+ e^- \rightarrow e^+ e^-, \quad \mu^+ \mu^-, \quad \tau^+ \tau^-, \quad \gamma\gamma,$$

or

$$e^+ e^- \rightarrow \text{hadrons},$$

or in deep inelastic lepton scattering experiments, etc.

Considering the form factor $F(q^2)$ as a power series in q^2 , we find:

$$F(q^2) = 1 + \frac{q^2}{\Lambda^2} + O\left(\frac{1}{\Lambda^4}\right), \quad \text{where} \quad q^2 \ll \Lambda^2. \tag{72}$$

If Λ ($\Lambda = \Lambda_4$, see Sec. IV) is the energy scale associated with the quark-lepton compositeness scale Λ_s of the preon interaction, then our model predicts:

$$\Lambda \sim \Lambda_s \sim 10^{18} \text{ GeV}. \tag{73}$$

Of course, such a scale is not experimentally available even for future high energy colliders.

From a phenomenological point of view it is quite important to understand whether the compactification scale lowers down Λ in Eq. (72) to energies accessible for future accelerators.

Proposing the presence of extra space-time dimensions at small distances [22] we have a hope that the radius R_C of compactification is larger than the radius R_s of compositeness:

$$R_C \gg R_s, \tag{74}$$

and

$$\Lambda \sim \Lambda_C \ll 10^{18} \text{ GeV}. \tag{75}$$

However, this problem leads to more detailed investigations in higher dimensional theories.

VII. CONCLUSIONS

- (i) In the present paper, starting with the idea that the most realistic model based on superstring theory, which leads to the unification of all fundamental interactions including gravity, is the heterotic string-derived flipped model, we have assumed that at high energies $\mu > 10^{16}$ GeV there exists the following chain of flipped models:

$$\begin{aligned}
 &SU(3)_C \times SU(2)_L \times U(1)_Y \\
 &\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \\
 &\rightarrow SU(5) \times U(1)_X \\
 &\rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \\
 &\rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6,
 \end{aligned}$$

ending in the flipped E_6 gauge symmetry group (see Fig. 1). We have chosen such Higgs boson contents of the $SU(5)$ and $SO(10)$ gauge groups, which give the final flipped E_6 unification at the scale $\sim 10^{18}$ GeV and the decreased running of $\alpha^{-1}(\mu)$ near the Planck scale given by Fig. 2.

- (ii) The final E_6 unification assumes the existence of three 27-plets of the flipped E_6 gauge symmetry group and we have considered the content of the flipped 27-plet.
- (iii) The supersymmetric E_6 content in five space-time dimensions was presented in Sec. III. The problem of compactification was briefly discussed.
- (iv) Suggesting an $\mathbf{N} = 1$ supersymmetric $E_6 \times \tilde{E}_6$ preonic model of composite quark-leptons and bosons, we have assumed that preons are dyons confined by hyper-magnetic strings in the region of energies $\mu \lesssim M_{\text{Pl}}$. This approach is an extension of the old idea by J. Pati [5] to use the strong magnetic forces which may bind preons-dyons in composite particles—quark-leptons and bosons. Our model is based on the recent theory of com-

posite non-Abelian flux tubes in SQCD, which was developed in Refs. [10–19].

- (v) Considering the breakdown of E_6 (or \tilde{E}_6) at the Planck scale into the $SU(6) \times U(1)$ (or $SU(6) \times U(1)$) gauge group, we have shown that six types of k -strings—composite $N = 1$ supersymmetric non-Abelian flux tubes—are created by the condensation of spreons near the Planck scale.
- (vi) It was shown that the six types of strings-tubes, having six fluxes quantized according to the Z_6 center group of $SU(6)$:

$$\Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3,$$

produce three (and only three) generations of composite quark-leptons and bosons. We have obtained a specific type of “horizontal symmetry” explaining flavor.

- (vii) It was shown that in the present model preonic strings are very thin, with radius

$$R_{\text{str}} \sim 10^{-18} \text{ GeV}^{-1},$$

and their tension is enormously large:

$$T \sim 10^{38} \text{ GeV}^2.$$

These strings are similar to the superstrings of superstring theory.

- (viii) The model predicts the existence of three families of 27-plets and also gauge bosons A_μ^i (with $i = 1, 2, 3$) belonging to the 78-plets of E_6 . Then near the Planck scale we have the Family replicated gauge group of symmetry $[E_6]^3$ for quark-leptons. In the present paper we have assumed that the breakdown $[E_6]^3 \rightarrow E_6$ occurs near the Planck scale leading to the E_6 unification at the scale $\sim 10^{18} \text{ GeV}$.
- (ix) We have considered the condensation of spreons near the Planck scale, which gives phase transitions at the scales M_{crit} and \tilde{M}_{crit} shown in Fig. 2. The points A (or B) of Fig. 2 correspond to the breakdown of E_6 (or \tilde{E}_6) for preons:

$$E_6 \rightarrow SU(6) \times U(1) \quad (\text{or} \quad \tilde{E}_6 \rightarrow \widetilde{SU(6)} \times \widetilde{U(1)}).$$

We have investigated the idea that hyper-magnetic strings are produced and exist at $\mu \leq \tilde{M}_{\text{crit}}$, and hyper-electric strings are created and exist at $\mu \geq M_{\text{crit}}$ (the positions of M_{crit} and \tilde{M}_{crit} are shown in Fig. 2 by the points A and B , respectively). As a result, in our world we have quark-leptons and gauge bosons A_μ (in the region of energies $\mu \leq M_{\text{pl}}$), while monopolic “quark-leptons” and dual gauge fields \tilde{A}_μ exist in the region $\mu \geq M_{\text{pl}}$. We have calculated the critical values of the gauge

coupling constants at the points A, B :

$$\alpha^{-1}(M_{\text{crit}}) \approx 4.23, \quad \text{and} \quad \alpha^{-1}(\tilde{M}_{\text{crit}}) \approx 2.13.$$

- (x) Assuming the existence of the three types of hyper-magnetic fluxes and using Schwinger’s formula, we have given an explanation of the hierarchies of masses established in the SM. The following values of the masses were obtained in our preonic model:

$$m_t \approx 173 \text{ GeV}, \quad m_c \approx 1 \text{ GeV}, \quad \text{and} \\ m_u \approx 4 \text{ MeV},$$

$$m_b \approx 4 \text{ GeV}, \quad m_s \approx 140 \text{ MeV}, \quad \text{and} \\ m_d \approx 4 \text{ MeV},$$

$$m_\tau \approx 2 \text{ GeV}, \quad m_\mu \approx 100 \text{ MeV}, \quad \text{and} \\ m_e \approx 3.5 \text{ MeV}.$$

They are comparable with the experimentally known results published in Ref. [77].

- (xi) We have considered form factors describing the compositeness of quark-leptons. In our preonic model the scale Λ_s of the quark-lepton compositeness is very large:

$$\Lambda_s \sim 10^{18} \text{ GeV},$$

which is not accessible even for future high energy colliders. We have concluded that only the compactification procedure of extra space-time dimensions can help to lower the scale Λ_s . Then quark-lepton form factors might be experimentally observed.

ACKNOWLEDGMENTS

C. R. D. thanks Professor J. Pati a lot for interesting discussions during the Conference WHEPP-9 (Bhubaneswar, India, January, 2006). L. L. sincerely thanks the Institute of Mathematical Sciences (Chennai, India) and personally the Director of IMSc Professor R. Balasubramanian and Professor N. D. Hari Dass for the wonderful hospitality and financial support. The authors also deeply thank Professor J. L. Chkareuli, Professor F. R. Klinkhamer and Professor G. Rajasekaran for fruitful discussions and advice. We are thankful of Professor C. D. Froggatt for his great help.

APPENDIX: COMPACTIFICATION OF EXTRA DIMENSIONS IN PREONIC MODEL

Compactifying the extra fifth dimension x^4 in Eq. (18) on a circle of radius R_C , the authors of Ref. [22] impose the Scherk-Schwarz [39] boundary conditions to the preonic

superfields:

$$\begin{aligned}
 P(x^m, x^4 + 2\pi R_C, \theta) &= e^{i2\pi q_P} P(x^m, x^4, e^{i\pi(q_P + q_{\bar{P}})} \theta), \\
 P^c(x^m, x^4 + 2\pi R_C, \theta) &= e^{i2\pi q_{\bar{P}}} P^c(x^m, x^4, e^{i\pi(q_P + q_{\bar{P}})} \theta), \\
 P_s(x^m, x^4 + 2\pi R_C, \theta) &= e^{i2\pi q_s} P_s(x^m, x^4, e^{i\pi(q_s + q_{\bar{s}})} \theta), \\
 P_s^c(x^m, x^4 + 2\pi R_C, \theta) &= e^{i2\pi q_{\bar{s}}} P_s^c(x^m, x^4, e^{i\pi(q_s + q_{\bar{s}})} \theta),
 \end{aligned}
 \tag{A1}$$

where $q_P, q_{\bar{P}}$ and $q_s, q_{\bar{s}}$ are the R charges of preons P, P^c and P_s, P_s^c , respectively. The following condition was

obtained in Ref. [22]:

$$q_P + q_{\bar{P}} = q_s + q_{\bar{s}}. \tag{A2}$$

Then the R charges for the composite states (21)–(24) are:

$$\begin{aligned}
 Q &\sim q_P + q_{\bar{s}}, & \bar{Q} &\sim q_{\bar{P}} + q_s, \\
 M &\sim q_P + q_{\bar{P}}, & S &\sim q_s + q_{\bar{s}}.
 \end{aligned}
 \tag{A3}$$

As a result of compactification, all fermionic preons are massive in 4D space-time. Supersymmetry is broken by the boundary conditions (A1).

-
- [1] J. Schwarz, Phys. Rep. **89**, 223 (1982); M. B. Green, Surv. High Energy Phys. **3**, 127 (1984).
- [2] D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 253 (1985); **B267**, 75 (1986).
- [3] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B 258**, 46 (1985).
- [4] M. B. Green and J. H. Schwarz, Phys. Lett. **B 149**, 117 (1984); **151**, 21 (1985).
- [5] J. C. Pati, Phys. Lett. **B 98**, 40 (1981).
- [6] J. C. Pati, A. Salam, and J. Strathdee, Nucl. Phys. **B185**, 416 (1981).
- [7] P. Senjanovic, Fizika (Zegreb) **15**, 165 (1983); M. Blagojevic, S. Meljanac, and P. Senjanovic, Phys. Lett. **B 131**, 111 (1983).
- [8] Dan-di Wu, Phys. Rev. D **34**, 280 (1986).
- [9] M. Blagojevic and P. Senjanovic, Phys. Rep. **157**, 233 (1988).
- [10] M. Shifman and A. Yung, Phys. Rev. D **72**, 085017 (2005); **70**, 045004 (2004); **70**, 025013 (2004).
- [11] A. Gorsky, M. Shifman, and A. Yung, Phys. Rev. D **71**, 045010 (2005).
- [12] R. Auzzi, M. Shifman, and A. Yung, Phys. Rev. D **73**, 105012 (2006).
- [13] V. Markov, A. Marshakov, and A. Yung, Nucl. Phys. **B709**, 267 (2005).
- [14] M. A. C. Kneipp, Phys. Rev. D **69**, 045007 (2004); M. A. C. Kneipp and P. Brockill, Phys. Rev. D **64**, 125012 (2001).
- [15] D. Tong, Phys. Rev. D **69**, 065003 (2004); A. Hanany and D. Tong, J. High Energy Phys. **04 (2004)** 066; **07 (2003)** 037.
- [16] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. Lett. **96**, 161601 (2006).
- [17] Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. D **71**, 065018 (2005); M. Eto, T. Fujimori, Y. Isozumi, M. Nitta, K. Ohashi, K. Ohta, and N. Sakai, Phys. Rev. D **73**, 085008 (2006); M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. D **73**, 125008 (2006).
- [18] M. Eto, M. Nitta, and N. Sakai, Nucl. Phys. **B701**, 247 (2004).
- [19] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, J. Phys. A **39**, R315 (2006).
- [20] G. S. Iroshnikov, hep-th/0508129.
- [21] S. Sen, Phys. Rev. D **37**, 1296 (1988).
- [22] M. Chaichian, J. L. Chkareuli, and A. Kobakhidze, Phys. Rev. D **66**, 095013 (2002).
- [23] H. Harari, Phys. Rep. **104**, 159 (1984).
- [24] R. E. Marshak, Phys. Rep. **137**, 73 (1986).
- [25] R. R. Volkas and R. C. Joshi, Phys. Rep. **159**, 303 (1988).
- [26] L. B. Okun, *Leptons and Quarks* (North-Holland Physics Publishing, Amsterdam, 1982).
- [27] L. Lyons, Prog. Part. Nucl. Phys. **10**, 227 (1983); **7**, 157 (1981).
- [28] J. L. Chkareuli, in Proceedings of Quarks-82, Sukhumi, USSR, 1982 (Institute of Nuclear Research, USSR Academy of Sciences, Moscow, 1983).
- [29] H. Stremnitzer, in *Proceedings to the Workshop at Bled, Slovenia, 1998* (DMFA, Ljubljana, Slovenia, 1999), p. 45.
- [30] J. J. Dugne, S. Fredriksson, J. Hansson, and E. Predazzi, in *Beyond the Desert 1999*, edited by H. V. Klapdor-Kleingrothaus and I. Krivosheina (Institute of Physics Publishing, Bristol and Philadelphia, 2000), p. 285.
- [31] S. Fredriksson, in *Beyond the Desert 2003*, edited by H. V. Klapdor-Kleingrothaus (Springer, Berlin, 2004), p. 211.
- [32] C. R. Das and L. V. Laperashvili, hep-ph/0604052.
- [33] V. Barger, N. G. Deshpande, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **33**, 1912 (1986).
- [34] J. L. Hewett and T. G. Rizzo, Phys. Rep. **183**, 193 (1989).
- [35] B. Stech and Z. Tavartkiladze, Phys. Rev. D **70**, 035002 (2004).
- [36] S. F. King, S. Moretti, and R. Nevzorov, Phys. Lett. **B 634**, 278 (2006); hep-ph/0601269.
- [37] C. R. Das, C. D. Froggatt, L. V. Laperashvili, and H. B. Nielsen, Mod. Phys. Lett. **A 21**, 1151 (2006).
- [38] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
- [39] J. Scherk and J. H. Schwarz, Phys. Lett. **B 82**, 60 (1979).
- [40] N. Arkani-Hamed, T. Gregoire, and J. Wacker, J. High Energy Phys. **03 (2002)** 055; D. Marti and A. Pomarol, Phys. Rev. D **64**, 105025 (2001).
- [41] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).
- [42] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
- [43] S. Cecotti, J. P. Derendinger, S. Ferrara, and L. Girardello, Phys. Lett. **B 156**, 318 (1985).

- [44] J. L. Chkareuli, JETP Lett. **32**, 671 (1980); **34**, 453 (1981); **36**, 493 (1983).
- [45] Z. G. Berezhiani, J. L. Chkareuli, Sov. J. Nucl. Phys. **37**, 618 (1983).
- [46] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Nucl. Phys. **B626**, 307 (2002).
- [47] T. Appelquist, Y. Bai, and M. Piai, Phys. Rev. D **72**, 036005 (2005).
- [48] Chan Hong-Mo and Tsou Sheung Tsun, Phys. Rev. D **56**, 3646 (1997); **57**, 2507 (1998).
- [49] Chan Hong-Mo, J. Faridani, and Tsou Sheung Tsun, Phys. Rev. D **51**, 7040 (1995); **52**, 6134 (1995); **53**, 7293 (1996).
- [50] Sheung Tsun Tsou, Int. J. Mod. Phys. A **18S2**, 1 (2003).
- [51] H. B. Nielsen and Y. Takanishi, Nucl. Phys. **B604**, 405 (2001); Phys. Lett. B **507**, 241 (2001); **543**, 249 (2002).
- [52] C. D. Froggatt, H. B. Nielsen, and Y. Takanishi, Nucl. Phys. **B631**, 285 (2002).
- [53] C. D. Froggatt and H. B. Nielsen, *Proceedings of Winter School of Physics at the Moscow, Russia, 2003*, Session XXXI ITEP, edited by H. B. Nielsen; Surv. High Energy Phys. **18**, No. 55 (2003).
- [54] C. D. Froggatt, L. V. Laperashvili, H. B. Nielsen, and Y. Takanishi, in *Proceedings of the Fifth International Conference on Symmetry in Nonlinear Mathematical Physics, Kiev, Ukraine, 2003*, edited by A. G. Nikitin, V. M. Boyko, R. O. Popovich, and I. A. Yehorchenko (Institute of Mathematics of NAS of Ukraine, Kiev, 2004), Vol. 50, Chap. Part 2, p. 737.
- [55] C. R. Das and L. V. Laperashvili, Int. J. Mod. Phys. A **20**, 5911 (2005).
- [56] F. S. Ling and P. Ramond, Phys. Rev. D **67**, 115010 (2003).
- [57] S. Rajpoot, Nucl. Phys. Proc. Suppl. A **51**, 50 (1996).
- [58] S. Dimopoulos, G. Dvali, R. Rattazzi, and G. F. Giudice, Nucl. Phys. **B510**, 12 (1998).
- [59] L. V. Laperashvili, H. B. Nielsen, and D. A. Ryzhikh, Int. J. Mod. Phys. A **18**, 4403 (2003); Phys. At. Nucl. **65**, 353 (2002); **66**, 2070 (2003).
- [60] L. V. Laperashvili and D. A. Ryzhikh, in *Proceedings Bled 2000/2001, What Comes Beyond the Standard Model*, edited by M. Breskvar *et al.* (DMFA, Zaloznistvo, Ljubljana, 2002), Vol. 1, pp. 83–100; hep-th/0112142.
- [61] D. L. Bennett, H. B. Nielsen, and I. Picek, Phys. Lett. B **208**, 275 (1988).
- [62] C. D. Froggatt and H. B. Nielsen, *Origin of Symmetries* (World Sci., Singapore, 1991).
- [63] D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A **9**, 5155 (1994); D. L. Bennett, C. D. Froggatt, and H. B. Nielsen, in *Proceedings of the 27th International Conference on High Energy Physics, Glasgow, Scotland, 1994*, edited by P. Bussey and I. Knowles (IOP Publishing Ltd., Bristol 1995), p. 557; in *Perspectives in Particle Physics '94*, edited by D. Klabučar, I. Picek, and D. Tadić (World Scientific, Singapore, 1995), p. 255.
- [64] L. V. Laperashvili, Phys. At. Nucl. **57**, 471 (1994); **59**, 162 (1996).
- [65] L. V. Laperashvili, H. B. Nielsen, and D. A. Ryzhikh, Int. J. Mod. Phys. A **16**, 3989 (2001).
- [66] C. R. Das, L. V. Laperashvili, and H. B. Nielsen, hep-ph/0511267 [Int. J. Mod. Phys. A (to be published)].
- [67] J. Jersak, T. Neuhaus, and P. M. Zerwas, Phys. Lett. B **133**, 103 (1983); Nucl. Phys. **B251**, 299 (1985).
- [68] J. Jersak, T. Neuhaus, and H. Pfeiffer, Phys. Rev. D **60**, 054502 (1999).
- [69] L. V. Laperashvili and H. B. Nielsen, Int. J. Mod. Phys. A **16**, 2365 (2001).
- [70] C. R. Das and L. V. Laperashvili, hep-ph/0511067 [Mod. Phys. Lett. A (to be published)].
- [71] F. R. Klinkhamer and G. E. Volovik, JETP Lett. **81**, 551 (2005).
- [72] C. Froggatt, L. Laperashvili, R. Nevzorov, and H. B. Nielsen, Phys. At. Nucl. **67**, 582 (2004).
- [73] J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, MA, 1973).
- [74] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979).
- [75] E. G. Gurvich, Phys. Lett. B **87**, 386 (1979).
- [76] N. K. Glendenning and T. Matsui, Phys. Rev. D **28**, 2890 (1983).
- [77] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).