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Gluonic excitation of nonexotic hybrid charmonium from lattice QCD

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The ground and first excited states of the hybrid charmonium $\bar{c}cg$, with nonexotic quantum numbers $J^{PC} = 0^{-+}$, 1^{--}, and 1^{++} are investigated using quenched lattice QCD. The excited states are completely ignored in the literature. However, we observe strong gluonic radial excitations in the first excited states; We find their masses are completely different from the first excited states of the corresponding conventional charmonium. Their relevance to the recent discovery of the $Y(4260)$ state and future experimental search for other states are also discussed.

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A hybrid meson $\bar{q}qg$ is a bound state of constituent quark q, antiquark $\bar{q}$ and excited gluon g. The existence of hybrids is one of the most important predictions of quantum chromodynamics (QCD). There has been a lot of experimental activity [1–4] in the search for hybrid mesons, for example: PEP-2 (*BABAR*), KEKB (Belle), 12 GeV Jefferson Lab upgraded, upgraded CLEO-c detector, and new BES3 detector.

For a conventional meson in the quark model, which is represented by the fermion bilinear $\bar{\psi}\Gamma\psi$, it can have the J^{PC} quantum numbers as $J = |L - S|, |L - S| + 1, \ldots, L + S$, $P = (-1)^{L+1}$, and $C = (-1)^{L+S}$, with L the relative angular momentum of the quark and antiquark, and S the intrinsic spin of the meson. For the gluon, the quantum numbers of the color electric field $\mathbf{E}$ and color magnetic field $\mathbf{B}$ are 1^{--} and 1^{+-} respectively. According to QCD, the operator of a hybrid meson is the gauge-invariant direct product of $\bar{\psi}\Gamma\psi$ and the color electric field $E_i^{c_1c_2} = F_{0i}^{c_1c_2}$ or color magnetic field $B_i^{c_1c_2} = \epsilon_{ijk}F_{jk}^{c_1c_2}$. Therefore, the quantum numbers of a hybrid meson could be either exotic, with $J^{PC} = 1^{-+}, 0^{+-}, 0^{--}, 2^{+-}\ldots$, inaccessible to conventional mesons, or nonexotic, with $J^{PC} = 0^{++}, 0^{-+}, 1^{--}, 1^{++}, 1^{+-}, 2^{++}, 2^{--}, 2^{-+}, \ldots,$ the same as conventional mesons.

Lattice gauge theory is the most reliable technique for computing hadron spectra. It involves discretization of the continuum theory on a space-time grid, and reduces to QCD when the lattice spacing a goes to zero. The implementation of the Symanzik program [5] with tadpole improvement [6] greatly reduces the discretization errors on very coarse and small lattices. Simulations on anisotropic lattices improve the signal in spectrum computations [7].

The 1^{-+}, 0^{+-}, and 2^{+-} exotic hybrid mesons have been extensively studied on the lattice. Reviews can be found in Refs. [8,9]. Recently, we computed the 0^{--} exotic hybrid charmonium mass [10]. However, the first excited states of the nonexotic hybrid mesons are completely ignored in the literature [11].

In this letter, we investigate the $J^{PC} = 0^{-+}$, 1^{--}, and 1^{++} nonexotic charmed hybrid mesons $\bar{c}cg$, employing quenched lattice QCD with tadpole improved gluon [12] and quark [13] actions on the anisotropic lattice. We observe, for the first time, very strong gluonic radial excitations in the first excited states.

It has been argued in Refs. [13,14] that such an Fermilab quark action [15] on the anisotropic works well in the charm quark regime and is valid even for heavier quarks $m_q a_s > 1$, with the lattice artifacts under well control. Our simulation parameters are listed in Table I. We also did simulations on $8^3 \times 48$ and $12^3 \times 48$ at $\beta = 2.6$, $12^3 \times 36$ at $\beta = 2.8$, and $16^3 \times 48$ at $\beta = 3.0$, but there and throughout the paper we just list the results from the largest volume, i.e., $16^3 \times 48$ at $\beta = 2.6$ and $\beta = 2.8$, and $20^3 \times 60$ at $\beta = 3.0$. At each β, three hundred independent configurations were generated with the improved gluonic action [12]. It is also important to check whether these lattice volumes are large enough. We found that when the spatial extent is greater than 2.2 fm, the finite volume effects become very small. At $\beta = 2.6$, e.g., the effect on the 1^{--} hybrid charmonium spectrum is less than 0.1% for the ground state, and 0.2% for the first excited state; For the 0^{-+} hybrid, the effect is less than 0.3%; For the heaviest one, i.e., the 1^{++} hybrid, the effect is about 0.9%, but still less than the errors.

We input the bare quark mass m_{q0} and then computed quark propagators using the improved quark action [13], the conventional quarkonium correlation function using the operators $0^{-+} = \bar{\psi}^c\gamma_5\psi^c$, $1^{--} = \bar{\psi}^c\gamma_j\psi^c$, and $1^{++} = \bar{\psi}^c\gamma_5\gamma_j\psi^c$, and the hybrid meson correlation function us-

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TABLE I. Simulation parameters at largest volume. We employed the method in Ref. [13] to tune the parameters for the quark action. The last two columns are about the spatial lattice spacing and the lattice extent in physical units, determined from the charmonium mass splitting $m(\chi_{c1}(1^3P_1)) - m(1\bar{S})$, with the effective masses extracted by the method of Ref. [18].

$\beta = 6/g^2$	$\xi = a_s/a_t$	$L_s^3 \times L_t$	u_s	u_t	$a_t m_{q0}$		c_s	c_t	$a_s(1^3P_1 - 1S)$ [fm]	La_s[fm]
2.6	3	$16^3 \times 48$	0.81921	1	0.229	0.260	1.8189	2.4414	0.1856(84)	2.970
2.8	3	$16^3 \times 48$	0.83099	1	0.150	0.220	1.7427	2.4068	0.1537(101)	2.459
3.0	3	$20^3 \times 60$	0.84098	1	0.020	0.100	1.6813	2.3782	0.1128(110)	2.256

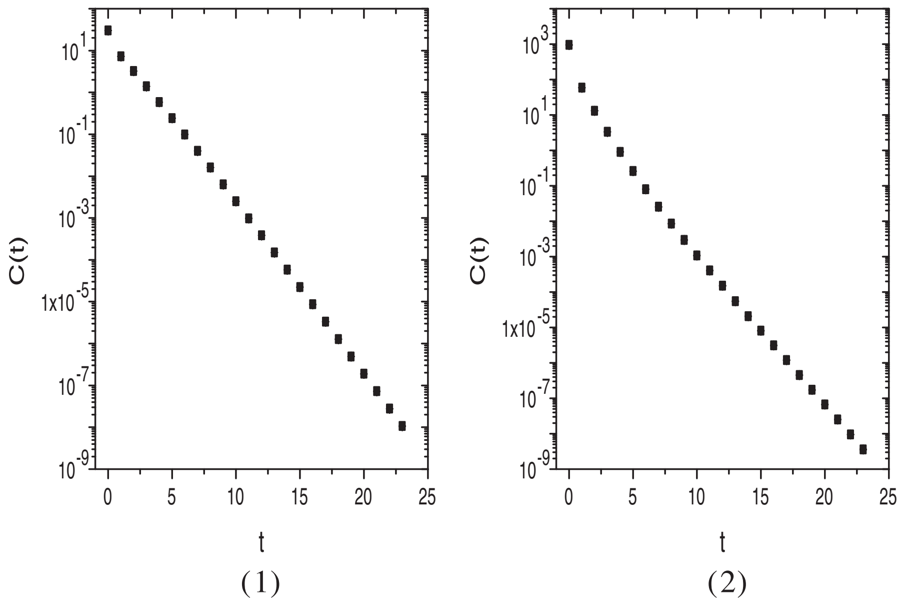

FIG. 1. (1) Correlation function for the conventional 1^{--} quarkonium at $\beta = 2.6$ and $a_t m_{q0} = 0.229$; (2) Same as (1), but for the 1^{--} hybrid meson.

ing the operators $0^{-+} = \epsilon_{ijk}\bar{\psi}^{c_1}\gamma_i\psi^{c_2}F_{jk}^{c_1c_2}$, $1^{--} = \epsilon_{ijk}\bar{\psi}^{c_1}\gamma_5\psi^{c_2}F_{jk}^{c_1c_2}$, and $1^{++} = \epsilon_{ijk}\bar{\psi}^{c_1}\gamma_j\psi^{c_2}F_{0k}^{c_1c_2}$ in Ref. [16]. Figure 1 shows the correlation function $C(t)$ of the conventional 1^{--} and hybrid mesons.

The effective masses of the ground and first excited states $a_t m_1$ and $a_t m_2$ are extracted by two different methods: (i) new correlation function method [17]; (ii) modified multiexponential fit [18]. The multiexponential fitting method has been widely used in the literature [11,13,14] for extracting the charmonium masses, and the results for the ground and first excited states are consistent with experiments; The MILC group [18] proposed an improved multiexponential fitting method, which chooses the best fit according to some criteria. The recently proposed method (i) has been successfully applied to the investigation of the Roper resonance of the nucleon [17], where $a_t m_1$ is obtained from $\ln(C(t)/C(t+1))$ in the large time interval $[t_i, t_f]$, and $a_t m_1 + a_t m_2$ from $\ln(C'(t)/C'(t+1))$ in the time interval $[t_i^*, t_f^*] < [t_i, t_f]$, with reasonable χ^2/d.o.f. and optimal confidence level. Here $C'(t) = C(t+1)C(t-1) - C(t)^2$. Two methods provide a cross-check of the results.

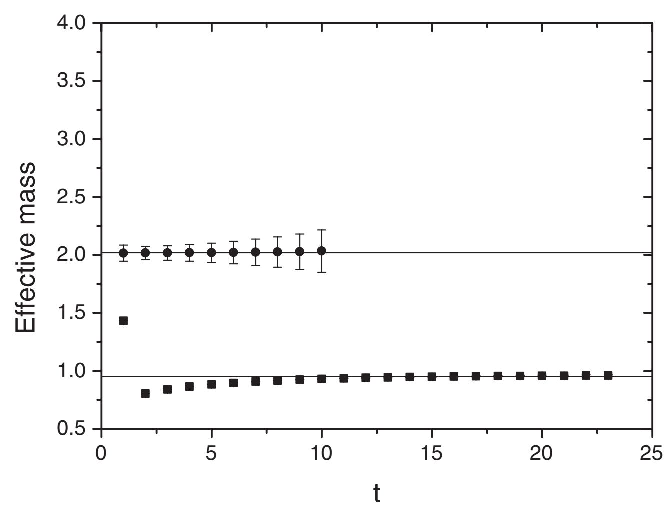

FIG. 2. Effective masses of the conventional 1^{--} quarkonium as a function of t for $\beta = 2.6$ and $a_t m_{q0} = 0.229$, using the new correlation function method [17]. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted, respectively, from the plateaux of the upper and lower curves, with $[t_i^*, t_f^*] = [1, 10]$ and $[t_i, t_f] = [11, 23]$.

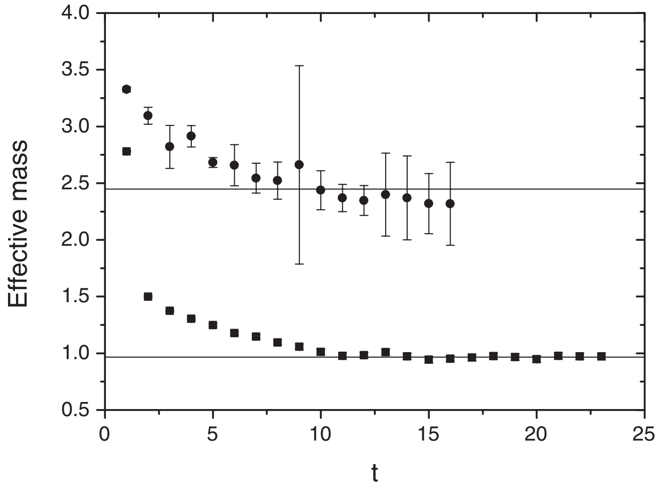

FIG. 3. Same as Fig. 2, but for the 1^{--} hybrid meson. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted, respectively, from the plateaus of the upper and lower curves, with $[t_i^*, t_f^*] = [6, 16]$ and $[t_i, t_f] = [17, 23]$.

Figure 2 shows effective masses for the conventional 1^{--} quarkonium, where $a_t m_1$ and $a_t m_1 + a_t m_2$ are extracted, respectively, from the plateaus of the lower and upper curves, using the new method [17]. Figure 3 shows those for the 1^{--} hybrid meson.

The data at two m_{q0} values were interpolated to the charm quark regime using $m(1\bar{S})_{\rm exp} = [m(\eta_c)_{\rm exp} + 3m(J/\psi)_{\rm exp}]/4 = 3067.6$ MeV. To extrapolate the quenched results to the continuum limit and determine the meson mass m in physical units, it is more convenient to consider the dimensionless ratio of effective masses $R = [a_t m]/[a_t m(1\bar{S})]$ or ratio of effective mass splittings $R' = [a_t m - a_t m(1\bar{S})]/[a_t m(\chi_{c1}(1^3P_1)) - a_t m(1\bar{S})]$. The ratio

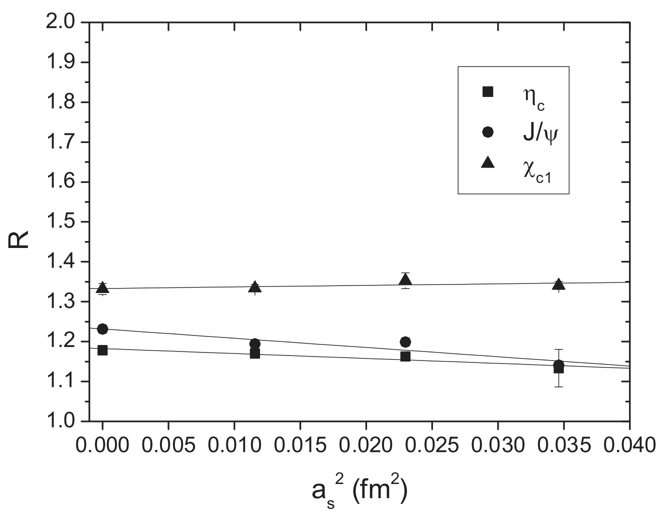

FIG. 4. Extrapolation of $R = [a_t m]/[a_t m(1\bar{S})]$ to the continuum limit. Here $[a_t m]$ is the effective mass of the first excited state of a conventional charmonium, extracted by the method of Ref. [17].

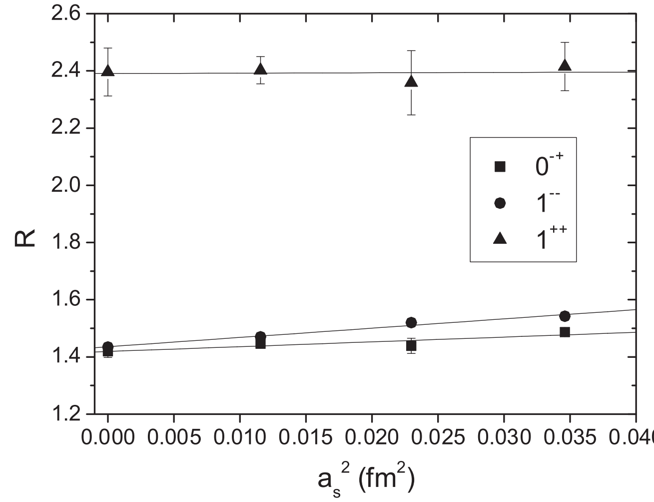

FIG. 5. The same as Fig. 4, but for the hybrid charmonium.

R for the first excited state of the conventional 0^{-+}, 1^{--}, and 1^{++} charmonium mesons as a function of a_s^2 is plotted in Fig. 4, and that for the hybrids is plotted in Fig. 5. They indicate the linear dependence of R on a_s^2. The continuum results are obtained by linearly extrapolating the data to $a_s^2 \to 0$.

After extrapolation, we determine m by inputting the experimental data $m(1\bar{S})_{\rm exp}$ in R, or $m(\chi_{c1}(1^3P_1))_{\rm exp} - m(1\bar{S})_{\rm exp}$ and $m(1\bar{S})_{\rm exp}$ in R'. Tables II and III list, respectively, the results for the ground and first excited states, obtained by R or R', with effective masses extracted by the method of Ref. [18] or Ref. [17].

As shown in the last line of Table II, in the continuum limit, the masses of the 0^{-+}, 1^{--}, and 1^{++} charmonium ground states are consistent with their experimental values 2.9804, 3.0969, and 3.5106 for $\eta_c(1S)$, J/ψ, and $\chi_{c1}(1^3P_1)$. The results in Table II also show that the ground state for the nonexotic hybrid charmonium is degenerate with the conventional charmonium with the same quantum numbers. This might mislead people into giving up further study of the nonexotic hybrids.

The last line of Table III shows in the continuum limit the first excited state masses of the conventional charmonium and nonexotic hybrid charmonium. The results for the conventional 0^{-+} and 1^{--} charmonium are in good agreement with the experimental data 3.638 and 3.686 for $\eta_c(2S)$ and $\psi(2S)$, which supports the reliability of the methods. Although there has not been experimental input for $\chi_{c1}(2^3P_1)$, our result is consistent with earlier lattice calculations [13,14]. The minor differences between the data and experiments might be due to the quenched approximation used in the paper.

What new is that the first excited states of nonexotic charmonium hybrids are completely different from the conventional ones. The results in last line of Table III show the masses of the 0^{-+} and 1^{--} hybrids to be about 0.7 GeV heavier, and the 1^{++} about 3.2 GeV heavier. These

TABLE II. Conventional and hybrid charmonium meson spectrum (GeV) for the ground state, in the continuum limit. They were obtained, respectively, from: (1) R and (2) R' computed by the method of Ref. [18]; (3) the same as (1), and (4) the same as (2) but using the method of Ref. [17]. The last line (5) is the average of the results from these four methods.

η_c	J/ψ	χ_{c1}	0^{-+}	1^{--}	1^{++}	
3.026(7)	3.074(7)	3.461(10)	3.035(9)	3.154(7)	3.541(24)	(1)
3.036(11)	3.082(9)	3.502(40)	3.034(11)	3.151(12)	3.515(73)	(2)
3.039(7)	3.091(8)	3.516(9)	3.068(7)	3.132(8)	3.457(145)	(3)
3.040(8)	3.092(8)	3.533(22)	3.065(8)	3.133(8)	3.471(150)	(4)
3.035(8)	3.085(8)	3.503(41)	3.051(9)	3.143(9)	3.496(98)	(5)

TABLE III. The same as Table II, but for the first excited state.

η_c	J/ψ	χ_{c1}	0^{-+}	1^{--}	1^{++}	
3.543(8)	3.619(10)	4.187(246)	4.335(95)	4.337(141)	7.229(109)	(1)
3.505(44)	3.608(53)	4.149(100)	4.335(96)	4.353(185)	7.253(172)	(2)
3.615(37)	3.778(20)	4.086(43)	4.357(68)	4.400(137)	7.351(256)	(3)
3.658(23)	3.752(25)	4.101(48)	4.390(108)	4.426(71)	7.436(198)	(4)
3.580(28)	3.689(27)	4.131(109)	4.354(92)	4.379(134)	7.317(184)	(5)

are very strong indications of gluonic excitations. This implies that radial excitations of the charmonium hybrids are completely different from the conventional nonhybrid ones, although their ground states overlap. This is clearly demonstrated in Figs. 1–5.

This also teaches a very important lesson. One should carefully study not only the ground state, but also the excited states. Sometimes, the excited states show more fundamental properties of a hadron.

To check whether the conventional operator has overlap with the hybrid one, a simple method is to compute a nondiagonal correlation function $C_{12}(t) = \langle O_1^\dagger(t) O_2(0)\rangle$ between the conventional and hybrid meson operators. We found that although the mass of the ground state is consistent with the conventional one, the mass of the first excited state is completely different from the hybrid state. For 0^{-+}, 1^{--}, and 1^{++}, the mass of the first excited state extracted from the method of Ref. [18] is 323(85) MeV, 611(182) MeV, and 2008(264) MeV less than the corresponding hybrid state; Averaging the results over four methods as in Tables II and III, the value of the mass is 4.114(64) GeV, 3.747(132) GeV, and 5.369(131) GeV, respectively.

There is the issue as to whether the excited hybrid states extracted correspond to actual resonances or multiparticle scattering states. One important step is to show the volume dependence of each energy level. We have previously mentioned that when $L_s a$ is greater than 2.2 fm, the finite volume effects on the charmonium hybrids become very small.[1] This is the same for the masses extracted from $C_{12}(t)$. At $\beta = 2.6$, e.g., the effect on the first excited state is about 0.3% for 1^{--}, 0.05% for 0^{-+}, and 0.5% for 1^{++}. However, the spectral weights of the scattering states are very sensitive to the spatial volume. If they were scattering states, the spectral weights[2] would be proportional to $1/L_s^3$. Let $L_s^{\rm small}$ and $L_s^{\rm large}$ denote smaller and larger spatial lattice extent, respectively. The averaged spectral weight ratio $W(L_s^{\rm small})/W(L_s^{\rm large})$ is, respectively, 1.05(40), 1.22(45), and 1.14(33) for the excited state of 0^{-+}, 1^{--}, and 1^{++} hybrid, and 1.02(5), 0.97(12), and 0.91(29) for those extracted from $C_{12}(t)$. Examples for $W(L_s^{\rm small})/W(L_s^{\rm large})$ are shown in Figs. 6–9. These confirm their nature of the resonance (bound) states.

Therefore, we have identified the ground state and three excited states in each of the three J^{PC} channels considered. One or some of the excited states might be candidates for nonexotic hybrids.

Finally, we discuss the new state $Y(4260)$, recently observed by the *BABAR* experiment [21] in the $J/\psi\pi^+\pi^-$ channel. It has the quantum numbers $J^{PC} = 1^{--}$. The discovery has attracted broad interest.

There have been several phenomenological descriptions [22–27] of this state: as tetra-quarks, a molecule of two mesons, $\psi(4S)$, or as a hybrid meson; However, most these assumptions were not based on QCD spectrum computations.

[1] The energy of a multiparticle state needs only vary with L_s if the constituent particles are moving relative to the center of mass of the multiparticle state. In quark model language, the energy of an S-wave scattering state need not have any more volume dependence than a resonance.

[2] For detailed discussions about the volume dependence of the scattering states, please refer to Refs. [19,20].

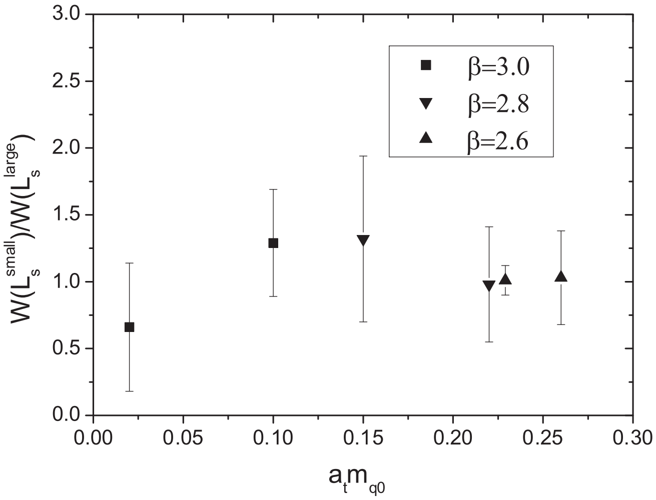

FIG. 6. Ratio of spectral weights for the first excited 0^{-+} hybrid state at different $a_t m_{q0}$ and β.

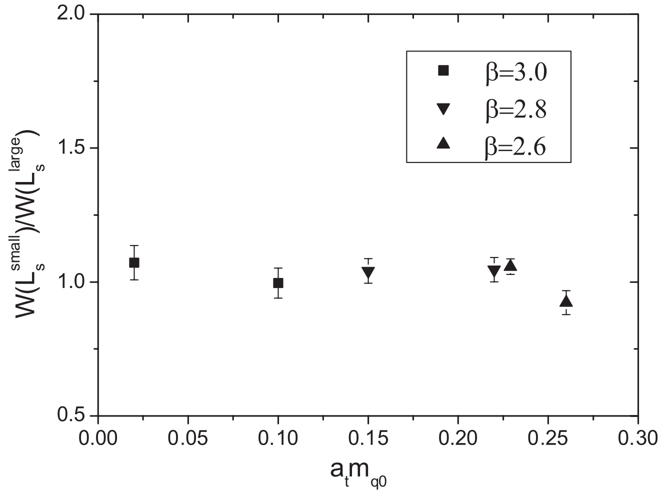

FIG. 8. The same as Fig. 6, but for the first excited 0^{-+} state, extracted from nondiagonal correlation function $C_{12}(t)$ between the conventional and hybrid meson operators.

If $Y(4260)$ is a hybrid meson, from the last line of Table II, it could certainly not be identified as the ground state of the 1^{--} hybrid meson. However, from our lattice QCD spectrum calculations (the last line of Table III), it is most probably the first excited state of the 1^{--} hybrid charmonium. Further experimental study of the decay modes will clarify this issue.

From the last line of Table III, one sees that the first excited state mass of the 0^{-+} hybrid charmonium is about the same as that of the 1^{--} hybrid charmonium, but much lighter than the first excited state of the 1^{++} hybrid charmonium. It should not be very difficult to find it in future experiment.

After submission of the manuscript, we noticed that the CLEO Collaboration announced their new experimental measurements [28] of $Y(4260)$, which strongly support the interpretation of a 1^{--} hybrid state.

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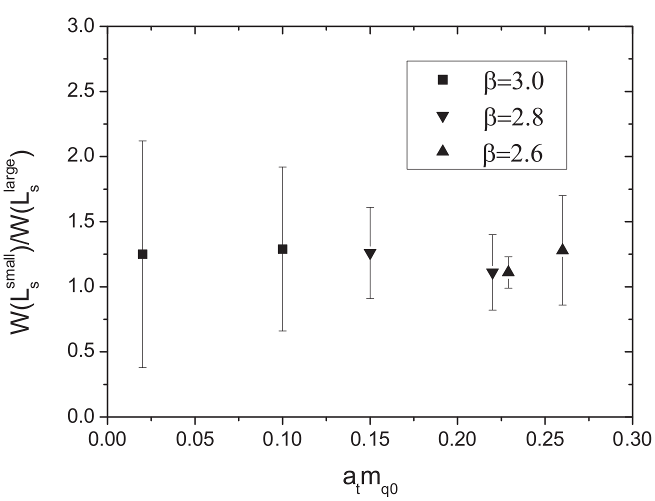

FIG. 7. The same as Fig. 6, but for the 1^{--} hybrid.

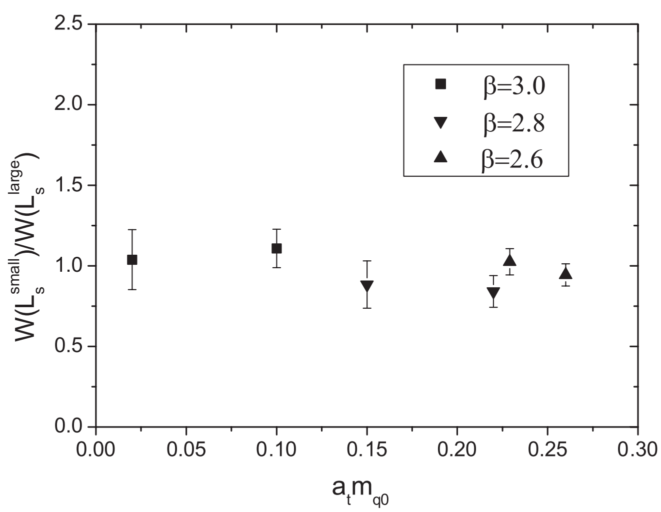

FIG. 9. The same as Fig. 8, but for the 1^{--} state.

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