PHYSICAL REVIEW D **74,** 034501 (2006)

Exploring twisted mass lattice QCD with the Clover term

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(Received 19 May 2006; published 1 August 2006)

It has been shown that in the twisted mass formulation of Lattice QCD at maximal twist large cutoff effects are generated when the quark mass becomes of $\mathcal{O}(a\Lambda_{\rm QCD}^2)$. In general, these effects can be suppressed in two ways: either by choosing the critical quark mass in an "optimal way" or by adding the Clover term to the twisted action. We investigate the second option by performing a quenched lattice QCD simulation with twisted Clover fermions and pion masses as low as 280 MeV. We show that the Clover term is indeed efficient in reducing the large cutoff effects. In particular, the so-called bending phenomenon observed in the determination of the pion decay constant is cured in this way. In addition, by using maximally twisted Clover fermions, we provide a nonperturbative determination of the vector current renormalization constant Z_V as well as of the nonperturbatively renormalized light quark masses. Finally, we calculate the connected contribution to the charged-neutral pseudoscalar meson mass splitting, finding that the introduction of the Clover term in the twisted action is also beneficial, in the quenched approximation, in reducing cutoff effects related to the isospin symmetry breaking at finite lattice spacing.

DOI: 10.1103/PhysRevD.74.034501 PACS numbers: 12.38.Gc

I. INTRODUCTION

The twisted mass formulation of Lattice QCD (tmLQCD) [1,2] offers a number of interesting advantages. The absence of real unphysical zero modes, which affect the standard Wilson discretization of the fermionic action, prevents the appearance in this framework of the exceptional configurations in quenched and partially quenched numerical simulations. This allows the investigation of QCD dynamics with much lighter valence quarks. In addition, twisting the Wilson term in the action (in the physical basis) considerably simplifies the renormalization pattern of several local operators. Notable examples in this respect are the determination of the pion decay constant, which can be performed without introducing any renormalization factor [1], and of the kaon B_K parameter, which does not require subtraction of wrong chirality operators [1,3].[1] Furthermore, tmLQCD with a maximal twist angle ($\alpha = \pi/2$), denoted as maximally twisted mass LQCD (Mtm-LQCD), ensures automatic (or almost automatic) $\mathcal{O}(a)$ improvement for the physically interesting quantities computed on the lattice [6].

The drawback, however, is that with tmLQCD both parity and flavor symmetries are explicitly broken at finite lattice spacing. The breaking of parity generates a mixing with wrong parity states which may affect, in some relevant cases, the large time behavior of lattice correlation functions. The breaking of flavor symmetry is responsible for generating a mass splitting between otherwise degenerate isospin hadronic partners. Although the breaking effects are parametrically of $\mathcal{O}(a^2)$ at maximal twist, this splitting has been found to be numerically significant in the case of the neutral and charged pseudoscalar mesons [7].

Another important issue of practical interest for Mtm-LQCD is the presence of large cutoff effects which are generated when the quark mass m_q becomes of $\mathcal{O}(a\Lambda_{\rm QCD}^2)$ or smaller. This phenomenon has been the subject of recent numerical and theoretical investigations. From the point of view of the Symanzik expansion, these large cutoff effects have been studied in Ref. [8]. In correlation functions they are related to multiple insertions of the dimension-five operator $\mathcal{L}_5$ entering the Symanzik effective Lagrangian for Mtm-LQCD close to the continuum limit. Since, at maximal twist, $\mathcal{L}_5$ has the quantum numbers of the neutral pion, its insertions can create from the vacuum Goldstone bosons with zero momentum, thus generating discretization effects which are proportional, at the leading order, to $(a/M_\pi^2)^2 \sim (a/m_q)^2$. In this situation, the value of the quark mass is limited to the region $m_q \gtrsim a\Lambda_{\rm QCD}^2$. Being proportional to $(a/m_q)^2$, such lattice artifacts also have been called "infrared divergent" (IRD) discretization effects. Note, however, that they actually do not represent any infrared divergence, since the chiral limit can be taken only after the continuum extrapolation has been performed.

A further step forward in understanding the IRD cutoff effects has been made in Ref. [9], where it is shown that their contribution to the Symanzik expansion can be explicitly resummed using twisted mass chiral perturbation

[1]For an alternative approach which avoids this subtraction even when working with ordinary Wilson fermions, see Refs. [4,5].

theory (tmχPT) [10]. For a simple quantity like the pion decay constant f_π, the leading order result of tmχPT reads

$$f_\pi = f\frac{\mu}{\sqrt{\mu^2+\delta m^2}} \simeq f\left[1-\frac{1}{2}\left(\frac{\delta m}{\mu}\right)^2+\ldots\right], \quad (1)$$

where f is the pion decay constant in the chiral limit and μ and δm represent the (renormalized) twisted and nontwisted quark masses, respectively. Since at maximal twist δm is a quantity of $\mathcal{O}(a)$ introduced by a discretization error in the determination of the critical mass, Eq. (1) shows that the deviation of f_π from its value in the chiral limit has the form of an IRD cutoff effect. The leading correction is proportional to $(\delta m/\mu)^2 \sim (a/M_\pi^2)^2$, and its contribution becomes sizable when the twisted mass μ is small and comparable to δm, i.e., $\mu \sim \mathcal{O}(a\Lambda_{\rm QCD}^2)$.

The requirement $\mu \gtrsim a\Lambda_{\rm QCD}^2$ is in practice rather restrictive. For a typical inverse lattice spacing $a^{-1} \simeq$ 2 GeV, it implies pion masses $M_\pi \gtrsim 500$ MeV. In order to allow for simulations with smaller values of the quark masses (down to $\mu \gtrsim a^2\Lambda_{\rm QCD}^3$), and keep discretization effects under control, two strategies have been proposed [8]. Either one uses an "optimal" determination of the critical mass $m_{\rm cr}$ or the (twisted) Clover term is introduced in the quark action. In the latter case, the coefficient c_{SW} has to be fixed to the same value that guarantees on-shell $\mathcal{O}(a)$ improvement with ordinary Wilson fermions.

The idea behind the first strategy, namely, the optimal determination of the critical mass is to tune the $\mathcal{O}(a)$ contribution to $m_{\rm cr}$ in such a way that the dimension-five operator in the Symanzik effective Lagrangian has vanishing matrix element between the vacuum and the single pion state, i.e. $\langle 0|\mathcal{L}_5|\pi^0\rangle = 0$ [8]. Since the IRD cutoff effects have been shown to be proportional at the leading order to the previous matrix element, this procedure guarantees their suppression and allows much smaller values of the quark masses to be safely reached in the simulation. In practice, this procedure corresponds to fix the value of the critical mass by requiring the vanishing of the PCAC quark mass in the twisted basis, as it was anticipated in Refs. [11,12]. In the following, we will denote as $m_{\rm cr}^{\rm opt}$ this optimal determination of the critical mass.

The numerical studies reported in Refs. [13,14] show that such a strategy is indeed efficient. In particular, in agreement with theoretical expectations, it helps in suppressing the pronounced bending behavior of the pion decay constant, i.e., the appearance of large lattice artifacts in this quantity when the quark mass becomes of $\mathcal{O}(a\Lambda_{\rm QCD}^2)$ or smaller. Note, however, that one has to make a preliminary numerical study, with twisted fermions, to determine the optimal value of the critical mass. Such a determination may be computationally expensive, particularly in the case of unquenched simulations.

In this paper we explore the second strategy of Ref. [8], namely, the simulation of Mtm-LQCD with the inclusion of the Clover term in the action. In this case, in the Symanzik expansion of the lattice effective Lagrangian at $\mathcal{O}(a)$ only a term proportional to the square of the twisted quark mass survives [15], that is $\mathcal{L}_5 = \mathcal{O}(a\mu^2) = \mathcal{O}(aM_\pi^4)$. Thus, the introduction of the Clover term is expected to be efficient in cancelling the leading order IRD cutoff effects as much as the tuning of the critical mass in the twisted Wilson case.

From the previous discussion it should be clear that the Mtm-Clover approach does not require any specific prescription for the determination of the critical mass. We show here that this is indeed the case. We have performed two numerical simulations with Mtm-Clover fermions and with a critical quark mass determined in the nontwisted theory either by requiring the vanishing of the pion mass in the chiral limit (denoted in the following as $m_{\rm cr}^{\rm pion}$) or by imposing the vanishing of the PCAC mass ($m_{\rm cr}^{\rm PCAC}$). In both cases, we do not observe any bending phenomenon in the pion decay constant, even at values of quark masses smaller than $\mathcal{O}(a\Lambda_{\rm QCD}^2)$. We also compare our results for the pion mass and decay constant with those obtained by the χLF Collaboration [13] using Mtm-Wilson fermions with the optimal choice of the critical mass, finding a very good agreement. The appealing feature of the Mtm-Clover approach is that the determination of the critical mass does not require any specific tuning and can be obtained from a single simulation performed with nontwisted Clover fermions.

Besides investigating the effectiveness of Mtm-Clover fermions in suppressing the IRD cutoff effects, we also present a nonperturbative determination of the local vector current renormalization constant Z_V which renormalizes, in Mtm-LQCD, the local axial-vector current (in the physical basis). This constant is expected to be identical, up to $\mathcal{O}(a^2)$ effects, to the value determined by using nontwisted Clover fermions. Indeed, we find a very good agreement between our result and the nontwisted determination of Z_V performed in Ref. [16] with the nonperturbative RI-MOM method.

In addition, we present a calculation of the strange and of the average up/down quark masses using the value of the quark mass renormalization constant determined nonperturbatively in Ref. [16]. With respect to previous quenched determinations of light quark masses, here we benefit from the use of Mtm-LQCD with the Clover term to probe ever lower values of the quark masses and thereby reduce the errors due to the chiral extrapolation when determining the light up/down quark masses on the lattice.

Finally, we investigate the isospin symmetry breaking effects, induced at finite lattice spacing by the tmLQCD action, by studying the contribution of connected diagrams to the charged-neutral pion mass splitting. This connected contribution can be interpreted as the charged-neutral mass splitting between pseudoscalar mesons composed by quarks and antiquarks belonging to different, though degenerate, isospin doublets. By comparing the results obtained with Mtm-Wilson and Mtm-Clover fermions, we find that the introduction of the Clover term is beneficial also in reducing these cutoff effects, at least for the pure connected contribution considered in this study and in the quenched approximation.

It is also worth noting that the same $\mathcal{O}(a^2)$ operator that provides the leading contribution to the pion mass splitting in Mtm-LQCD also affects the determination of both the charged and the neutral pion masses when working with ordinary Wilson/Clover fermions. In this respect, therefore, the use of Mtm-LQCD does not introduce any additional uncertainty.

The plan of the paper is as follows: In Sec. II we give the details of the lattice simulation and present our results for the pseudoscalar meson mass and decay constant; we show the effectiveness of the introduction of the Clover term in curing the bending phenomenon. We present the determination of the renormalization constant Z_V in Sec. III A and the calculation of the light quark masses in Sec. III B. In Sec. III C we evaluate the contribution of connected diagrams to the charged-neutral pion mass splitting and compare the results with those obtained by using Mtm-Wilson fermions with the optimal choice of the critical mass. Our conclusions are summarized in Sec. IV.

II. PSEUDOSCALAR MESON MASSES AND DECAY CONSTANTS WITH THE MTM-CLOVER ACTION

The numerical results presented in this study have been obtained from a set of 300 gauge configurations generated, in the quenched approximation, with the standard Wilson gauge action at $\beta = 6.0$, which corresponds to an inverse lattice spacing $a^{-1} \simeq 2.0$ GeV. The lattice volume is $16^3 \times 32$ so that the physical length of the lattice is about 1.6 fm in the spatial directions. Using this set of gauge configurations, quark propagators have been computed by implementing three different choices for the quark action:

(a) A simulation with Mtm-Clover fermions and the critical mass determined from the PCAC relation in the nontwisted theory, $m_{\rm cr} = m_{\rm cr}^{\rm PCAC}$. The value of the critical hopping parameter, $\kappa_{\rm cr}^{\rm PCAC} = 0.135217$, corresponding to $am_{\rm cr}^{\rm PCAC} \simeq -0.3022$, has been obtained by using the results of a previous simulation done by our collaboration with standard Clover fermions at the same value of the gauge coupling on the volume $24^3 \times 64$ [16].

(b) A simulation with Mtm-Clover fermions and the critical mass determined from the vanishing of the pion mass in the nontwisted theory, $m_{\rm cr} = m_{\rm cr}^{\rm pion}$. The value of the critical hopping parameter, $\kappa_{\rm cr}^{\rm pion} = 0.135293$, corresponds to $am_{\rm cr}^{\rm pion} \simeq -0.3043$ and has been determined from the same set of nontwisted Clover data used to determine the value of $m_{\rm cr}^{\rm PCAC}$ in simulation (a).

(c) A simulation with Mtm-Wilson fermions with the critical mass determined in the optimal way, that is $m_{\rm cr} = m_{\rm cr}^{\rm opt}$. We used the value $\kappa_{\rm cr}^{\rm opt} = 0.157409$ obtained by the χLF Collaboration from a simulation at $\beta = 6.0$ on the volume $16^3 \times 32$ [13].

In simulations (a) and (b) with Mtm-Clover fermions the coefficient of the Clover term has been fixed to the value determined nonperturbatively in Ref. [17], namely $c_{\rm SW} = 1.769$.

In all three sets of simulations we have computed quark propagators with the same nine values of the bare twisted quark mass $a\mu$ (cf. Table I), which are equal to those used in Ref. [13]. The range of chosen $a\mu$ values covers pion masses between approximately 280 MeV and 1.1 GeV. In order to invert the Dirac operator for such a broad set of quark masses we employed the multiple mass solver algorithm [18].

To examine the impact of cutoff effects when working with maximally twisted fermions at small values of the quark mass, we investigate whether or not the inclusion of the Clover term is efficient in curing the bending phenomenon observed in the determination of the pion decay constant with Mtm-Wilson fermions, when the critical mass is determined in a nonoptimal way.

Pseudoscalar meson masses and decay constants have been evaluated by studying the large time behavior of the

TABLE I. Values of the bare twisted quark mass $a\mu$, of the pseudoscalar meson mass aM_P, and of the pseudoscalar meson decay constant af_P from the three sets of simulations performed in this study. The quoted errors are purely statistical.

	(a) Mtm-Clover, $m_{\rm cr}^{\rm PCAC}$		(b) Mtm-Clover, $m_{\rm cr}^{\rm pion}$		(c) Mtm-Wilson, $m_{\rm cr}^{\rm opt}$	
$a\mu$	aM_P	af_P	aM_P	af_P	aM_P	af_P
0.0038	0.1359 (66)	0.0690 (30)	0.1308 (70)	0.0739 (34)	0.1233 (68)	0.0684 (29)
0.0057	0.1607 (52)	0.0710 (22)	0.1571 (51)	0.0741 (23)	0.1515 (52)	0.0696 (18)
0.0076	0.1811 (46)	0.0727 (18)	0.1787 (47)	0.0748 (19)	0.1751 (44)	0.0710 (14)
0.0113	0.2150 (39)	0.0754 (15)	0.2137 (39)	0.0767 (15)	0.2127 (36)	0.0739 (11)
0.0151	0.2449 (35)	0.0779 (13)	0.2444 (35)	0.0787 (13)	0.2445 (31)	0.0767 (10)
0.0302	0.3401 (26)	0.0865 (11)	0.3404 (26)	0.0868 (11)	0.3425 (22)	0.0861 (09)
0.0454	0.4182 (22)	0.0942 (11)	0.4187 (22)	0.0944 (11)	0.4223 (18)	0.0944 (08)
0.0605	0.4873 (20)	0.1012 (10)	0.4879 (20)	0.1014 (10)	0.4930 (16)	0.1019 (08)
0.0756	0.5512 (17)	0.1077 (10)	0.5519 (18)	0.1079 (10)	0.5585 (14)	0.1092 (08)

following two-point correlation functions:

$$C_{PP}(t) \equiv \sum_{\vec{x}} \langle P_5(t,\vec{x}) P_5^\dagger(0)\rangle,$$
$$C_{AP}(t) \equiv \sum_{\vec{x}} \langle A_0(t,\vec{x}) P_5^\dagger(0)\rangle, \tag{2}$$

where $P_5 = \bar{u}\gamma_5 d$ and $A_\mu = \bar{u}\gamma_\mu\gamma_5 d$. Throughout this paper we choose to work in the physical basis and use mass-degenerate u and d quarks.

Using the completeness relation and considering large enough time separations, one gets

$$C_{PP}(t) \underset{t\gg 1}{\longrightarrow} \frac{Z_{PS}}{M_P} e^{-M_P T/2} \cosh(M_P(T/2 - t)),$$
$$C_{AP}(t) \underset{t\gg 1}{\longrightarrow} \frac{f_P M_P}{Z_V} \frac{\sqrt{Z_{PS}}}{M_P} e^{-M_P T/2} \sinh(M_P(T/2 - t)), \tag{3}$$

where M_P is the mass of the lightest pseudoscalar meson state, $\sqrt{Z_{PS}} = |\langle 0|P_5(0)|\pi\rangle|$ and $f_P M_P = Z_V \langle 0|A_0(0)|\pi\rangle$. Z_V is the renormalization constant of the local axial-vector current with Mtm-LQCD, which corresponds to the renormalization constant of the local vector current with the nontwisted quark action [6]. The pseudoscalar decay constant can therefore be extracted by studying the ratio $C_{AP}(t)/C_{PP}(t)$ at large time distances,

$$f_P = Z_V \frac{\sqrt{Z_{\rm PS}}}{M_P} \left[\frac{C_{AP}(t)}{C_{PP}(t)} \coth(M_P(T/2 - t))\right]_{t\gg 1}. \tag{4}$$

An alternative determination of the decay constant [1], which does not require the introduction of any renormalization constant, can be obtained by exploiting the consequences of the axial-vector Ward identity (AWI) for Mtm-LQCD. Up to $\mathcal{O}(a^2)$ correction, the relevant identity reads

$$Z_V \sum_{\vec{x}} \langle \partial_0 A_0(t,\vec{x}) P_5^\dagger(0)\rangle = 2\mu \sum_{\vec{x}} \langle P_5(t,\vec{x}) P_5^\dagger(0)\rangle, \tag{5}$$

where μ is the bare twisted mass. Combining the above relation with Eqs. (3) one gets

$$f_P = 2\mu \frac{\sqrt{Z_{\rm PS}}}{M_P^2}, \tag{6}$$

where both Z_{PS} and M_P can be determined from the correlation function $C_{PP}(t)$ of Eq. (3). We have evaluated the pseudoscalar meson decay constant f_P using both the "direct" method (4) and the "indirect" one (6), obtaining always a very good agreement within the statistical errors.[2] For the direct method, we used the nonperturbative RI-MOM determination $Z_V^{(\rm Clover)} = 0.772$ [16] and the one-loop boosted perturbative estimate $Z_V^{(\rm Wilson)} = 0.63$ [19,20] in the Mtm-Clover and Wilson case, respectively. In what follows, we only quote the results for f_P obtained from the indirect method (6).

Our result for the pseudoscalar meson masses and decay constants obtained from simulations (a), (b), and (c) are collected in Table I. These results are also shown in Figs. 1 and 2, as a function of the bare quark mass $a\mu$. The Mtm-Wilson results from our simulation (c) agree well with those presented in Ref. [13]. For comparison, we also show in these plots the results obtained in [13] by employing Mtm-Wilson fermions with the nonoptimal choice of the critical mass, as well as the results obtained with nontwisted Clover fermions (at $\beta = 6.0$) using data produced

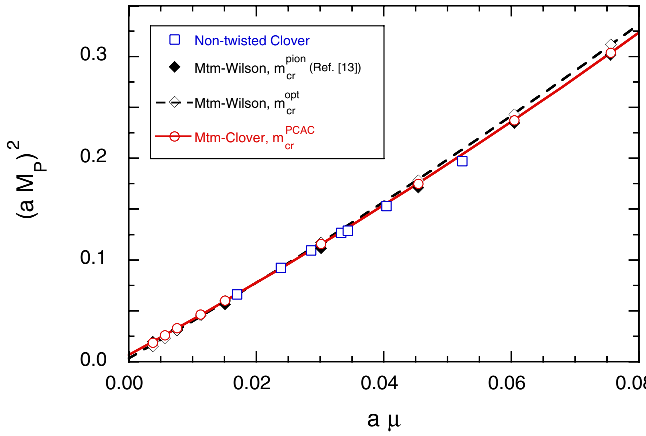

FIG. 1 (color online). Values of the pion mass squared obtained with different $\mathcal{O}(a)$-improved actions, as specified in the legend and in the text. The solid and dashed lines represent quadratic fits in $a\mu$.

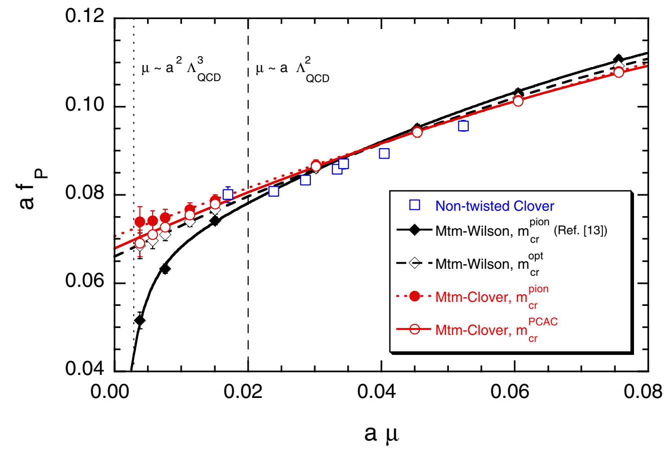

FIG. 2 (color online). Same as in Fig. 1, but for the pseudoscalar decay constant. The lines represent quadratic fit in $a\mu$ except for the results obtained with Mtm-Wilson fermions and the $m_{\rm cr}^{\rm pion}$ determination of the critical mass. In this case, the fit also takes into account the $tm\chi$PT prediction expressed by Eq. (1).

[2]This agreement is further improved if the factor M_P^2 in the denominator of Eq. (6) is replaced with $M_P \sinh(aM_P)/a$, where the sinh is originated from the discretized symmetric lattice version of the derivative in Eq. (5). Our results from Eq. (6) have been obtained with this replacement always implemented.

by our collaboration for the studies described in Refs. [16,21]. In the nontwisted case, the quark mass $a\mu$ in Figs. 1 and 2 represents the bare AWI quark mass $am^{\rm AWI}$ [21] multiplied by the axial-vector renormalization constant Z_A. Indeed, this is the quantity that renormalizes with the same renormalization constant $Z_\mu = 1/Z_P$ of the bare twisted mass $a\mu$ in the twisted formulation of LQCD.[3]

From the plots in Figs. 1 and 2 we observe that:

(i) There is a very good agreement among the results for the pseudoscalar meson masses calculated using various quark actions, all of which are $\mathcal{O}(a)$-improved. This may signal the smallness of discretization effects on this quantity at the value of the lattice spacing considered in the present study. Further investigations at different lattice spacings could better clarify this point.

(ii) IRD cutoff effects are clearly visible in the pseudoscalar meson decay constant at low quark masses ($a\mu \lesssim 0.02 \sim a^2\Lambda^2_{\rm QCD}$) when computed by using Mtm-Wilson fermions with the $m_{\rm cr}^{\rm pion}$ determination of the critical mass. This is the bending phenomenon observed in Ref. [13]. As shown in Fig. 2, these effects are strongly reduced either by choosing the optimal value of $m_{\rm cr}$ or by introducing the Clover term in the quark action. The reduction of IRD effects is almost the same within the two approaches. This is the main result of the present study.

(iii) As shown in Fig. 1, IRD cutoff effects are negligible in the case of the pseudoscalar meson mass. This is consistent with the observation that IRD cutoff effects in the determination of this mass are softened by an additional factor of M_P^2 [8].

(iv) The results for the pseudoscalar meson masses obtained with the Mtm-Clover action and the $m_{\rm cr}^{\rm pion}$ determination of the critical mass are not shown in Fig. 1, because they are practically indistinguishable from those obtained with $m_{\rm cr}^{\rm PCAC}$ (cf. Table I). In the case of the decay constant, instead, the two Mtm-Clover determinations with different choices of $m_{\rm cr}$ show a small spread at very low quark masses. This may signal the contribution of higher-order IRD cutoff effects that we cannot cure.

For illustrative purposes we quote the results for the pion decay constants obtained by quadratically extrapolating the lattice data to the physical pion mass. We obtain $f_\pi = \{138(8), 144(9), 138(8)\}$ MeV in simulations (a), (b), and (c), respectively.

[3] It also should be mentioned that the bare twisted quark mass $a\mu$ with Mtm-Clover fermions renormalizes differently from its counterpart with Mtm-Wilson fermions. However, the ratio of the corresponding renormalization constants, $Z_P^{(\rm Clover)}/Z_P^{(\rm Wilson)}$, is very close to unity (it is 0.97 when using the one-loop boosted perturbative expressions).

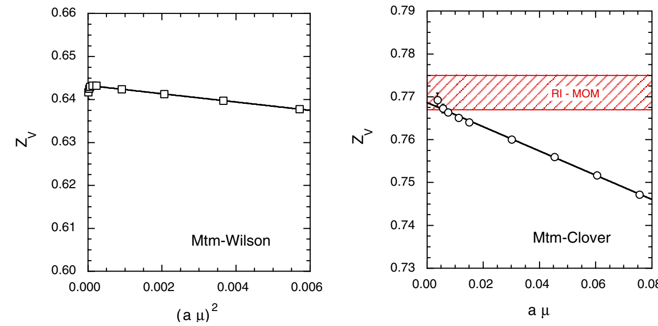

FIG. 3 (color online). Values of the renormalization constant Z_V as a function of the bare twisted quark mass as obtained by using the Mtm-Wilson action (left) and the Mtm-Clover action (right). Note that the square mass $(a\mu)^2$ and the mass $(a\mu)$ are plotted, respectively, on the x axis in the two cases. Solid lines are fits to the lattice data. The hatched area in the right plot shows the nonperturbative RI-MOM estimate of Z_V obtained from nontwisted Clover fermions in Ref. [16].

III. MORE NUMERICAL RESULTS

A. The renormalization constant Z_V

A nonperturbative determination of the renormalization constant Z_V[4] can be obtained using the two-point AWI of Eq. (5) as

$$Z_V^{-1} = \frac{(C_{AP}(t+1) - C_{AP}(t-1))/2}{2\mu C_{PP}(t)}, \qquad (7)$$

where in the numerator we used the symmetric ($\mathcal{O}(a)$-improved) version of the lattice time derivative.

We have computed Z_V from Eq. (7) using both the Wilson and the Clover action at maximal twist. In the Clover case, the results obtained from simulations (a) and (b), which only differ by the choice of the critical mass, are in good agreement within the statistical errors. For this reason, only data from simulation (a) and (c) will be discussed below. Our results for Z_V are shown in Fig. 3 as a function of the bare twisted quark mass.

The dependence of Z_V on the quark mass is induced by discretization effects, which are predicted to be of $\mathcal{O}(a^2)$ at maximal twist. Thus one can expect both a quadratic dependence on the quark mass, due to contributions of $\mathcal{O}(a^2\mu^2)$ and a linear dependence induced by terms of $\mathcal{O}(a^2\Lambda_{\rm QCD}\mu)$.[5] From the results shown in Fig. 3, it can be seen clearly that terms of $\mathcal{O}(a^2\Lambda_{\rm QCD}\mu)$ are dominant in the Mtm-Clover case, while the quadratic contributions of

[4] In this paper we follow the standard practice to denote with Z_V the renormalization constant of the axial-vector current in Mtm-LQCD. The reason is that the axial-vector current corresponds to the vector current in the twisted basis, which also implies that Z_V coincides with the vector current renormalization constant of the standard nontwisted theory.

[5] We thank R. Frezzotti and G. Rossi for having drawn our attention to this point.

$\mathcal{O}(a^2\mu^2)$ are larger in the Wilson case, except that at very small values of the quark mass.

In both the Wilson and Clover cases, we can correct for mass-dependent discretization effects by extrapolating the results for Z_V to the chiral limit. In this way, we obtained the estimates

$$Z_V^{(\mathrm{Wilson})} = 0.643(1), \qquad Z_V^{(\mathrm{Clover})} = 0.768(1). \tag{8}$$

Our result for $Z_V^{(\mathrm{Wilson})}$ agrees with the one reported in Ref. [22], namely, $Z_V^{(\mathrm{Wilson})} = 0.6424(4)$.

The determinations in Eq. (8) can be compared also to the estimates of Z_V obtained in the corresponding non-twisted theories (for which $\mu = 0$), since renormalization constants are independent of the quark mass. For the standard Clover action, nonperturbative determinations of Z_V have been obtained using several approaches. In Ref. [16], by using the RI-MOM method the estimate $Z_V^{(\mathrm{Clover})} = 0.772(2)(2)$ has been obtained, in good agreement with our result in Eq. (8) (this comparison is also illustrated in Fig. 3). Similar estimates have been obtained with the other approaches [23–25]. The determinations of Z_V with standard Wilson fermions are affected, instead, by much larger uncertainties, since the action is not improved at $\mathcal{O}(a)$. Typically, the nonperturbative estimates lie in the range $Z_V^{(\mathrm{Wilson})} \simeq 0.6 \div 0.8$ [25–27]. At variance with these estimates, however, we remind the reader that our determination in Eq. (8) is improved at $\mathcal{O}(a)$. Another interesting comparison is provided by the estimates $Z_V^{(\mathrm{Wilson})} \simeq 0.63$ and $Z_V^{(\mathrm{Clover})} \simeq 0.81$ obtained using one-loop tadpole-improved boosted perturbation theory [19,20].[6]

B. Light quark masses

In the quenched approximation, one of the main advantages of tmLQCD is the possibility to perform simulations at small values of the quark masses, without encountering the problem of exceptional configurations. This can be very beneficiary for the extrapolations towards the chiral limit. A notable example, in this respect, is the calculation of the light quark masses and, in particular, of the average up/down quark mass.

Moreover, the determination of quark masses with Mtm-LQCD is simpler than with the standard Wilson and/or Clover actions. To emphasize this point we remind the reader that the renormalized quark mass in Mtm-LQCD is obtained from the twisted mass parameter μ as $\hat{m}_q = Z_\mu \mu$, with $Z_\mu = 1/Z_P$. Thus μ plays a role similar to the bare m^{AWI} in the standard, nontwisted, case. But contrary to m^{AWI}, which is obtained from the ratio of the correlation functions and thus suffers from both statistical and systematic errors, the bare twisted mass μ is known without uncertainties. In addition to the automatic $\mathcal{O}(a)$ improvement of the results obtained by using either the Mtm-Wilson or the Mtm-Clover action, this makes the calculation of the light quark mass particularly attractive.

To exemplify the benefits of using the Mtm-LQCD we now apply the same strategy explained in detail in Ref. [21] and determine the values of the strange and the average up/down quark masses. Concerning the renormalization constant, in the Mtm-Clover case we use the nonperturbative estimate $Z_P^{\mathrm{RI/MOM}}(2\ \mathrm{GeV}) = 0.525$ [16], which corresponds to $Z_P^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 0.624$. In the Mtm-Wilson case, instead, we will use the one-loop boosted perturbative result, namely, $Z_P^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 0.596$.

The physical values of strange and of the average up/down quark masses have been extracted by fitting the results for the pseudoscalar meson masses collected in Table I as a function of the twisted quark mass. We find that these results are consistent with a quadratic dependence of the form

$$(aM_P)^2 = P_1 + P_2(a\mu) + P_3(a\mu)^2. \tag{10}$$

The coefficient P_1 parameterizes small, though nonvanishing, $\mathcal{O}(a^2)$-discretization effects: we obtain $P_1 = \{6(2), 5(2), 3(2)\} \cdot 10^{-3}$ in simulations (a), (b), and (c), respectively. In the range of our simulated quark masses we do not observe any deviation from the simple quadratic dependence of M_P^2 on the quark masses which could signal

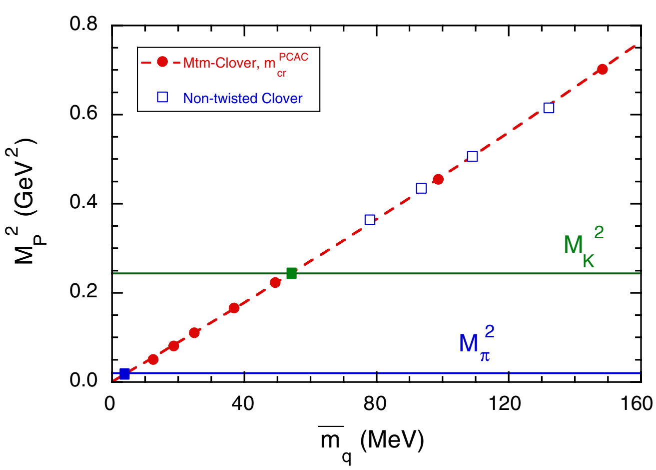

FIG. 4 (color online). Pseudoscalar meson masses squared, in physical units, as a function of the quark mass $\bar{m}_q$ renormalized in the $\overline{\mathrm{MS}}$ scheme at the scale $\mu = 2$ GeV. The dashed line represents the result of the fit to Eq. (10). In the M_P^2 values shown in the plot the pure lattice artifact contribution represented by the coefficient P_1 in Eq. (10) has been subtracted. Solid lines indicate the physical values of the pion and kaon masses. The results obtained with the nontwisted Clover action in Ref. [21] are also shown for comparison.

[6]We choose the boosted coupling obtained by inverting the perturbative series of the logarithm of the plaquette,

$$\ln\langle P\rangle = -\frac{1}{3}\tilde{g}^2(3.40/a)\left[1 - 1.1905\frac{\tilde{g}^2(3.40/a)}{4\pi}\right]. \tag{9}$$

TABLE II. Values of the average up/down (m_ℓ) and of the strange (m_s) quark masses renormalized in the $\overline{\text{MS}}$ scheme at the scale of 2 GeV as obtained using the various Mtm-LQCD actions at $\beta = 6.0$.

Mtm-LQCD	m_ℓ (MeV)	m_s (MeV)
Mtm-Clover, $m_{\text{cr}}^{\text{PCAC}}$	4.3 (2)	105 (5)
Mtm-Clover, $m_{\text{cr}}^{\text{pion}}$	4.2 (2)	103 (5)
Mtm-Wilson, $m_{\text{cr}}^{\text{opt}}$	4.2 (2)	103 (6)

the contribution of nonanalytical chiral logs. In Fig. 4 we show that the Mtm-LQCD indeed allows a much better control over the extrapolation to the physical pion mass than it was the case with the standard nontwisted Wilson/Clover action.

Our final results for the average up/down and strange quark masses are collected in Table II. The determinations obtained with the various Mtm-LQCD actions are consistent with each other and in agreement with most of the existing quenched estimates available in literature (for a recent review see e.g. Ref. [28]).

C. Isospin breaking effects

The twisted mass formulation of LQCD breaks both parity and flavor symmetries at finite lattice spacing. At maximal twist, these breakings are expected to be of $\mathcal{O}(a^2)$.

A clear manifestation of isospin breaking effects is a difference between the neutral and the charged pion mass. In the Mtm-Wilson case, this splitting has been investigated in Ref. [7]. The main observations are (i) isospin breaking effects are numerically large, even for reasonably fine lattice spacings; quantitatively, it is found $r_0^2(M_{\pi^0}^2 - M_{\pi^+}^2) = c(a/r_0)^2$ with $c \approx 10$; (ii) the contribution of disconnected diagrams helps in significantly reducing this splitting; without adding the disconnected piece, the value of c is about twice bigger; (iii) the optimal choice of the critical mass, which minimizes parity breaking effects, does not help in reducing discretization effects related to isospin breaking.

We cannot make the study along the lines presented in Ref. [7], since we explore the benefits of using the Mtm-Clover action at single lattice spacing. Furthermore, we neglect the contribution of disconnected diagrams, the calculation of which is computationally expensive and statistically noisy. One observes that the pure connected contribution provides by itself the neutral-charged mass splitting between pseudoscalar mesons composed by different, though mass-degenerate, isospin doublets, namely $P^i = \bar{q}\frac{\tau^i}{2}\gamma_5 q'$ with $q \neq q'$. In the absence of isospin breaking effects the neutral and charged mesons would be degenerate, and thus their splitting still represents a measure of isospin breaking discretization effect of $\mathcal{O}(a^2)$.

In Fig. 5 we compare the results for the neutral-charged mass splitting obtained by using the Mtm-Wilson and the Mtm-Clover actions, with the optimal and the PCAC determination of the critical mass, respectively. In the Mtm-Clover case, the results obtained with the two different choices for the critical mass ($m_{\text{cr}}^{\text{PCAC}}$ and $m_{\text{cr}}^{\text{pion}}$) do not show significant differences. From Fig. 5, it appears that in the Mtm-Clover case the isospin breaking effects are significantly reduced with respect to the situation in which the Mtm-Wilson action is used, at least for the pure connected contribution considered in this study and in the quenched approximation.

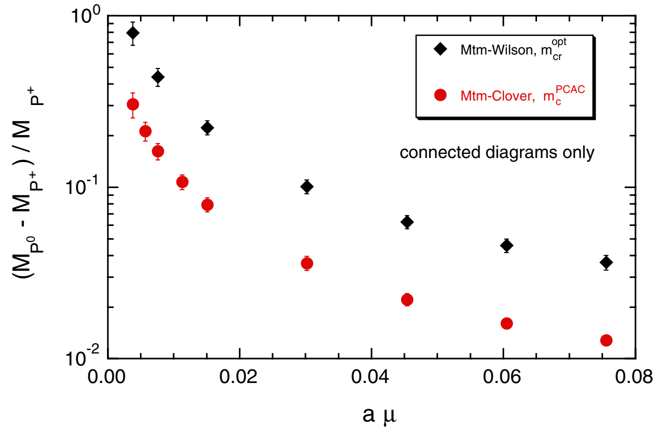

FIG. 5 (color online). Relative mass splitting between charged and neutral pseudoscalar mesons as obtained by using the Mtm-Wilson and the Mtm-Clover actions. The results are presented as a function of the bare quark mass. For both actions, the calculation of the neutral meson mass only includes the contribution of connected diagrams (see text for explanations).

IV. CONCLUSIONS

In this paper we have presented an exploratory numerical study of Mtm-LQCD with the Clover term in the action. We have performed quenched simulations with both Wilson and Clover twisted fermions at $\beta = 6.0$, with pseudoscalar meson masses ranging from about 1.1 GeV down to approximately 280 MeV. We have investigated, in particular, the role of the Clover term in reducing the large (IRD) cutoff effects which affect, in general, simulations with Mtm-LQCD when the quark mass becomes of $\mathcal{O}(a\Lambda_{\text{QCD}}^2)$. As anticipated in Ref. [8], we find that adding the Clover term to the twisted mass quark action is very effective in reducing these effects, as much as using Mtm-Wilson fermions with the optimal choice of the critical mass.

By using Mtm-Clover fermions, we have also performed other interesting numerical studies. We have computed the renormalization constant Z_V, finding good agreement with the estimates obtained with the standard Clover action. We have presented a calculation of the strange and the average up/down quark masses, which uses nonperturbative renormalization and much lighter valence quark masses that could not have been simulated with standard Wilson/Clover fermions, due to the problem of exceptional configurations. Finally, we have calculated the contribution of connected diagrams to the charged-neutral pion mass splitting, finding that the use of the Mtm-Clover action is also beneficial in reducing cutoff effects related to isospin breaking, at least for this pure connected contribution in the quenched approximation. We believe that all these results encourage the pursuit of this project in exploring the unquenched QCD dynamics at small quark masses.

ACKNOWLEDGMENTS

We warmly thank R. Frezzotti, M. Papinutto, C. Pena, and G. Rossi for useful discussions on the subject of this paper. We also thank the χLF Collaboration for having provided us with their numerical results obtained in the simulation with twisted Wilson fermions at maximal twist.

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