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# Production of *P*-wave charmed mesons in hadronic *B* decays

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Production of even-parity charmed mesons in hadronic $B$ decays is studied. Specifically, we focus on the Cabibbo-allowed decays $\bar{B} \to D^{**}\pi$ and $\bar{D}_s^{**}D^{(*)}$, where $D^{**}$ denotes generically a $P$-wave charmed meson. While the measured color-allowed decays $\bar{B}^0 \to D^{**+}\pi^-$ are consistent with the theoretical expectation, the experimental observation of $B^- \to D^{**0}\pi^-$ for the broad $D^{**}$ states is astonishing as it requires that the color-suppressed contribution dominates over the color-allowed one, even though the former is $1/m_b$ suppressed in the heavy quark limit. In order to accommodate the data of $\bar{B} \to D^{**}\pi^-$, it is found that the real part of $a_2/a_1$ has a sign opposite to that in $\bar{B} \to D\pi$ decays, where $a_1$ and $a_2$ are the effective parameters for color-allowed and color-suppressed decay amplitudes, respectively. The decay constants and form factors for $D^{**}$ and the Isgur-Wise functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are extracted from the data of $B \to D^{**}\pi$ decays. The Isgur-Wise functions calculated in the covariant light-front quark model are in good agreement with experiment. The neutral modes $\bar{B}^0 \to D^{**0}\pi^0$ for $D^{**} = D_0^*(2400)$, $D_1'(2430)$, and $\bar{B}^0 \to D_1'^0(2430)\omega$ are predicted to have branching ratios of order $10^{-4}$ which are also supported by the isospin argument. The decay constants of $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ are inferred from the measurements of $\bar{B} \to D_s^{**-}D$ to be 58–86 MeV and 130–200 MeV, respectively. Contrary to the decay constants $f_{D_0^*}$ and $f_{D_1'}$ which are similar in size, the large disparity between $f_{D_{s0}^*}$ and $f_{D_{s1}'}$ is surprising and unexpected.

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## I. INTRODUCTION

The spectroscopy for the $P$-wave charmed mesons has entered a new and exciting era since 2003. First of all, *BABAR* [1] has discovered a new narrow and light resonance $D_{s0}^*(2317)$. The existence of a second narrow resonance $D_{sJ}(2460)$ which can be identified with the $J^P = 1^+$ state was first hinted at by *BABAR* [1] and then observed and established by CLEO [2] and Belle [3]. Second, the broad $D_0^*$ and $D_1$ resonances, which are the counterpart of $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ in the nonstrange charm sector, were explored by Belle [4] in charged $B$ to $D^+\pi^-\pi^-$ and $D^{*+}\pi^-\pi^-$ decays and by FOCUS [5] in photoproduction experiment.

Just like the light scalar mesons $f_0(980)$, $a_0(980)$, $\sigma(600)$, the underlying structure of the above-mentioned $P$-wave charmed mesons is not well established theoretically. Recall that, before 2003, the $P$-wave states $D_{s0}^*$ and $D_{s1}'$ with $j_q = 1/2$ ($j_q$ being the angular momentum of the light degrees of freedom) were predicted to be broad and decay into $DK$ and $D^*K$, respectively, in the conventional quark model [6]. However, the observed $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ states are below the $DK$ and $DK^*$ thresholds, respectively, and hence are very narrow. This unexpected and surprising disparity between theory and experiment has sparked a flurry of theory studies. It has been advocated that this new state is a $DK$ molecular [7] or a $D_s\pi$ atom [8] or a four-quark bound state (first proposed in [9], followed by [10,11]).[1] On the contrary, it has been put forward that, based on heavy quark effective theory (HQET) and chiral perturbation theory, the newly observed $D_s(2317)$ is a $0^+$ $c\bar{s}$ state and that there is a $1^+$ chiral partner with the same mass splitting with respect to the $1^-$ state as that between the $0^+$ and $0^-$ states [12,13], namely, $m_{D_{s1}'} - m_{D_s^*} = m_{D_{s0}^*} - m_{D_s}$.

The spectra and strong, radiative decays of the $P$-wave charmed mesons have been studied extensively (for a review, see [14–16]). The measurements of two-body $B$ decays into $D^{**}$ and $D_s^{**}$, where $D^{**}$ denotes a generic even-parity charmed meson, provide very useful information on the decay constants and form factors of the excited charmed meson and the ratios of radiative to hadronic decay rates, for example, $\Gamma(D_{s0}^* \to D_s^*\gamma)/\Gamma(D_{s0}^* \to D_s\pi^0)$. Moreover, they provide an opportunity to test heavy quark effective theory. As we shall see below, the decay amplitudes of $B \to D^{**}\pi$ in the heavy quark limit are governed by the Isgur-Wise (IW) functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$.

In the present work, we will study even-parity charmed meson production in $B$ decays. Specifically, we focus on the Cabibbo-allowed decays $\bar{B} \to D^{**}\pi$ and $\bar{D}_s^{**}D^{(*)}$.[2] The decay $B^- \to D^{**0}\pi^-$ receives color-allowed and color-suppressed contributions, characterized by the effective

[1] An issue for the four-quark model is whether the conventional $c\bar{s}$ and $c\bar{q}$ states exist. A nonobservation of a heavier and broad $0^+$ $c\bar{s}$ state will not support the four-quark interpretation of $D_{s0}^*(2317)$. For nonstrange charmed mesons, it has been argued that $D_0^*(2308)^0$ observed by Belle [4] is a four-quark state, whereas $D_0^*(2405)^{0,+}$ measured by FOCUS [5] is a normal $c\bar{q}$ state [11]; that is, the $D_0^*$ state observed by Belle and FOCUS may not be the same.

[2] These decays have been studied previously in [17–27].

 

Wilson parameters $a_1$ and $a_2$, respectively. It is important to know whether the interference between the $a_1$ and $a_2$ terms is constructive or destructive. In the naive factorization approach, $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$. In the late 1980s and early 1990s, the large-$N_c$ approach was very popular and successful in explaining the hadronic weak decays of charmed mesons [28]. It was widely believed by most practitioners in the field that the $1/N_c$ expansion applies equally well to the weak decays of $B$ mesons. Since $a_2/a_1 \approx c_2/c_1 \sim -0.25$ at the renormalization scale $\mu = m_B$ in the leading $1/N_c$ expansion, $\bar{B}^0 \to D^+\pi^-$ is naively expected to have a larger rate than $B^- \to D^0\pi^-$ due to the destructive interference in the latter. However, the CLEO measurements of $B \to D\pi$ imply the opposite [29]. This is a very stunning result. In order to accommodate the $B \to D\pi$ data, the ratio $a_2/a_1$ is found to be of the order of 0.20–0.25. In the early 2000s, the color-suppressed mode $\bar{B}^0 \to D^0\pi^0$ is found to be significantly larger than the theoretical expectation based on naive factorization. For example, the measurement $\mathcal{B}(\bar{B}^0 \to D^0\pi^0) = (2.91 \pm 0.28) \times 10^{-4}$ [30] is larger than the theoretical prediction, $(0.58–1.13) \times 10^{-4}$ [31], by a factor of 2–4. Moreover, the three $B \to D\pi$ amplitudes form a nonflat triangle, indicating nontrivial relative strong phases between them. As a consequence, $a_2/a_1 \approx (0.45–0.65)e^{\pm i60^\circ}$ is inferred from the $B \to D\pi$ measurements including the neutral mode $\bar{B}^0 \to D^0\pi^0$ [31–34]. The question is then why the magnitude and phase of $a_2/a_1$ are so different from the model expectation. To resolve this difficulty, it has been shown in [35,36] that the enhancement of the effective $a_2$ and its strong phase can be ascribed to final-state interactions.

For $B \to D^{**}\pi$ decays, we will pay great attention to the relative sign of the decay constants and form factors of the $P$-wave mesons. It turns out that, in order to explain the larger rate of $B^- \to D^{**0}\pi^-$ than $\bar{B}^0 \to D^{**+}\pi^-$ for $D^{**} = D_0^*$ and $D_1'$, the real part of $a_2/a_1$ for $B \to D^{**}\pi$ has to be *negative* with a large magnitude. Moreover, the color-suppressed contribution has to dominate over the color-allowed one, in contrast to the naive expectation that the color-suppressed amplitude is $1/m_b$ suppressed in the heavy quark limit. This is a third surprise for the ratio $a_2/a_1$. The question to be addressed is why the sign of the real part of $a_2/a_1$ flips when $B \to D\pi$ is replaced by $B \to D^{**}\pi$.

This work is organized as follows. In Sec. II, the decay constants of $D^{**}$ and $B \to D^{**}$ form factors within the covariant light-front (CLF) quark model are summarized. The decays $\bar{B} \to D^{**}\pi$ and $\bar{B} \to \bar{D}_s^{**}D$ are studied in Secs. III and IV, respectively. Conclusions are presented in Sec. V.

## II. DECAY CONSTANTS AND FORM FACTORS

In the quark model, the even-parity mesons are conventionally classified according to the quantum numbers $J$, $L$, $S$: the scalar and tensor mesons correspond to ${}^{2S+1}L_J = {}^3P_0$ and ${}^3P_2$, respectively, and there exist two different axial-vector meson states, namely, ${}^1P_1$ and ${}^3P_1$, which can undergo mixing if the two constituent quarks do not have the same masses. For heavy mesons, the heavy quark spin $S_Q$ decouples from the other degrees of freedom in the heavy quark limit, so that $S_Q$ and the total angular momentum of the light quark $j$ are, separately, good quantum numbers. The total angular momentum $J$ of the meson is given by $\vec{J} = \vec{j} + \vec{S}_Q$ with $\vec{S} = \vec{s} + \vec{S}_Q$ being the total spin angular momentum. Consequently, it is more natural to use $L_J^j = P_2^{3/2}, P_1^{3/2}, P_1^{1/2}$, and $P_0^{1/2}$ to classify the first excited heavy meson states, where $L$ here is the orbital angular momentum of the light quark. It is obvious that the first and last of these states are ${}^3P_2$ and ${}^3P_0$, while [37]

$$|P_1^{3/2}\rangle = \sqrt{\frac{2}{3}}|{}^1P_1\rangle + \sqrt{\frac{1}{3}}|{}^3P_1\rangle,$$
$$|P_1^{1/2}\rangle = -\sqrt{\frac{1}{3}}|{}^1P_1\rangle + \sqrt{\frac{2}{3}}|{}^3P_1\rangle. \tag{2.1}$$

In the heavy quark limit, the physical eigenstates with $J^P = 1^+$ are $P_1^{3/2}$ and $P_1^{1/2}$ rather than ${}^3P_1$ and ${}^1P_1$.

The masses and decay widths of even-parity (or $P$-wave) charmed mesons $D_J^*$ and $D_{sJ}^*$ are summarized in Table I. It is known that, in the noncharm scalar meson sector, the quark model cannot explain why the scalar strange meson

TABLE I. The masses and decay widths of even-parity charmed mesons [30]. The four $P$-wave charmed meson states are denoted by $D_0^*$, $D_1'$, $D_1$, and $D_2^*$. In the heavy quark limit, $D_1'$ has $j = 1/2$ and $D_1$ has $j = 3/2$, with $j$ being the total angular momentum of the light degrees of freedom.

| State | Mass (MeV) | Width (MeV) |
|---|---|---|
| $D_0^*(2400)^0$ [a] | $2352 \pm 50$ | $261 \pm 50$ |
| $D_0^*(2400)^\pm$ [a] | $2403 \pm 14 \pm 35$ | $283 \pm 24 \pm 34$ |
| $D_1(2420)^0$ [b] | $2421.8 \pm 0.8$ | $20.3^{+1.9}_{-1.8}$ |
| $D_1(2420)^\pm$ | $2427 \pm 5$ | $26 \pm 8$ |
| $D_1'(2430)^\pm$ | $2427 \pm 26 \pm 25$ | $384^{+107}_{-75} \pm 74$ |
| $D_2^*(2460)^0$ [c] | $2462.7 \pm 0.8$ | $44 \pm 2$ |
| $D_2^*(2460)^\pm$ | $2464.9 \pm 3.0$ | $29 \pm 5$ |
| $D_{s0}^*(2317)$ | $2317.3 \pm 0.4 \pm 0.8$ | $<3.8$ [d] |
| $D_{s1}'(2460)$ | $2458.9 \pm 0.9$ | $<3.5$ [d] |
| $D_{s1}(2536)$ | $2535.35 \pm 0.31$ | $<2.3$ |
| $D_{s2}^*(2573)$ | $2573.5 \pm 1.7$ | $15^{+5}_{-4}$ |

[a]While the mass and the width of $D_0^*(2400)^0$ arise from the average of Belle [4] and FOCUS [5] measurements, the mass and the width of $D_0^*(2400)^\pm$ are solely due to FOCUS [5].
[b]Including the most recent CDF measurements $m(D_1^0) = 2421.7 \pm 0.7 \pm 0.6$ MeV and $\Gamma(D_1^0) = 20.0 \pm 1.7 \pm 1.3$ MeV [38] to the 2005 PDG average [30].
[c]Including the most recent CDF measurements $m(D_2^{*0}) = 2463.3 \pm 0.6 \pm 0.8$ MeV and $\Gamma(D_2^{*0}) = 49.2 \pm 2.3 \pm 1.3$ MeV [38] to the 2005 PDG average [30].
[d]The width limit from *BABAR* [39].

$K_0^*(1430)$ with a mass $1412 \pm 6$ MeV [30] is lighter than the nonstrange one $a_0(1450)$ with a mass $1474 \pm 19$ MeV [30].[3] Likewise, it is clear from Table I that the relation $m(D_s^{**}) > m(D^{**})$ holds except for $D_{s0}^*$ and $D_0^*$ for which we have $m(D_{s0}^*) < m(D_0^*)$. This is the place where the conventional quark model seems not to work[4] and a four-quark structure for $D_0^*$ and $D_{s0}^*$ is preferable [11].

We shall use $1'^+$ and $1^+$ or $D_1'$ and $D_1$ to distinguish between two different physical axial-vector charmed meson states.[5] The physical $1'^+$ state is primarily $P_1^{1/2}$, while $1^+$ is predominately $P_1^{3/2}$. This is because, in the heavy quark limit, the physical mass eigenstates $D_1'$ and $D_1$ can be identified with $P_1^{1/2}$ and $P_1^{3/2}$, respectively. However, beyond the heavy quark limit, there is a mixing between $P_1^{1/2}$ and $P_1^{3/2}$, denoted by $D_1^{1/2}$ and $D_1^{3/2}$, respectively,

$$\begin{aligned} D_1'(2430) &= D_1^{1/2}\cos\theta + D_1^{3/2}\sin\theta, \\ D_1(2420) &= -D_1^{1/2}\sin\theta + D_1^{3/2}\cos\theta. \end{aligned} \tag{2.2}$$

Likewise, for strange axial-vector charmed mesons,

$$\begin{aligned} D_{s1}'(2460) &= D_{s1}^{1/2}\cos\theta_s + D_{s1}^{3/2}\sin\theta_s, \\ D_{s1}(2536) &= -D_{s1}^{1/2}\sin\theta_s + D_{s1}^{3/2}\cos\theta_s. \end{aligned} \tag{2.3}$$

Since $D_1^{1/2}$ is much broader than $D_1^{3/2}$, the decay width of $D_1(2420)$ is sensitive to the mixing angle $\theta$. The $D_1^{1/2} - D_1^{3/2}$ mixing angle was reported to be

$$\theta = 0.10 \pm 0.03 \pm 0.02 \pm 0.02 \text{ rad} = (5.7 \pm 2.4)^\circ \tag{2.4}$$

by Belle through a detailed analysis of $B \to D^*\pi\pi$ [4].

Since the decay $B^- \to D^{**0}\pi^-$ receives color-allowed and color-suppressed contributions, it is important to know whether the interference is constructive or destructive. In [43] we have computed the decay constants and form factors for the ground-state $S$-wave and low-lying $P$-wave mesons within the framework of a covariant light-front approach.[6] In our approach, we first fix the vertex functions (i.e. Feynman rules for the meson-quark-antiquark vertices) for both $S$-wave and $P$-wave mesons. Then we are able to compute their decay constants and form factors. Hence, the relative sign and the factors of $i$ between two-body and three-body matrix elements can be determined. We then adopt two different approaches to elaborate on the heavy quark limit behavior of physical quantities: one from top to bottom and the other from bottom to top. In the top-to-bottom approach, we derive the decay constants and form factors in the covariant light-front model within HQET and obtain model-independent heavy quark symmetry (HQS) relations. In the bottom-to-top approach, we study the heavy quark limit behavior of the decay constants and transition form factors of heavy mesons and show that they do match the covariant model results based on HQET [43].

### A. Decay constants

The decay constants of scalar and pseudoscalar mesons are defined by[7]

$$\langle 0|A_\mu|P(q)\rangle = if_P q_\mu, \qquad \langle 0|V_\mu|S(q)\rangle = f_S q_\mu. \tag{2.7}$$

It is known that the decay constants of noncharm light scalar mesons are smaller than that of pseudoscalar mesons, as they vanish in the SU(3) limit. The decay constants of the axial-vector charmed mesons are defined by

$$\begin{aligned} \langle 0|A_\mu|D_1^{1/2}(q,\varepsilon)\rangle &= f_{D_1^{1/2}} m_{D_1^{1/2}} \varepsilon_\mu, \\ \langle 0|A_\mu|D_1^{3/2}(q,\varepsilon)\rangle &= f_{D_1^{3/2}} m_{D_1^{3/2}} \varepsilon_\mu. \end{aligned} \tag{2.8}$$

It is known that, in the heavy quark limit [44],

$$f_{D_1^{1/2}} = f_{D_0^*}, \qquad f_{D_1^{3/2}} = 0. \tag{2.9}$$

Since the decay constant of $D_2^*$ vanishes irrespective of heavy quark symmetry (see below), the charmed mesons within the multiplet $(0^+, 1^+)$ or $(1'^+, 2^+)$ thus have the same decay constant.

The polarization tensor $\varepsilon_{\mu\nu}$ of a tensor meson satisfies the relations

$$\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}, \quad \varepsilon^\mu{}_\mu = 0, \quad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0. \tag{2.10}$$

Therefore,

$$\langle 0|(V-A)_\mu|D_2^*(\varepsilon,p)\rangle = a\varepsilon_{\mu\nu}p^\nu + b\varepsilon^\nu{}_\nu p_\mu = 0. \tag{2.11}$$

[3]Recently it has been advocated in [40] that the puzzle with the relative masses of $K_0^*(1430)$ and $a_0(1450)$ can be solved provided that the observed scalar nonet in the mass range 1–2 GeV is a tetraquark nonet plus a glueball. The yet-to-be observed $q\bar{q}$ scalar nonet lies around 1.1 GeV.

[4]There are some attempts to understand the mass relation $m(D_{s0}^*) < m(D_0^*)$ by modifying the conventional potential model. For example, one loop chiral corrections to the potential model is considered in [41], while the potential model in [42] takes into account negative energy states of a heavy quark in a bound state.

[5]The notation for $D_1'(2430)$ and $D_1(2420)$ is opposite to that in [31].

[6]There are many typos in the printed version of [43] but not in the archive version, hep-ph/0310359.

[7]Sometimes the decay constant of the pseudoscalar meson is defined as $\langle 0|A_\mu|P(q)\rangle = f_P q_\mu$ in the literature (see e.g. [26]). This corresponds to choosing a non-Hermitian vertex function $\Gamma = \gamma_5$ for the pseudoscalar meson or redefining the phase of the pseudoscalar field, namely $|P\rangle \to \exp(i\phi)|P\rangle$ with $\phi = \pi$. The two-body matrix element $\langle P|V_\mu|B\rangle$ is still given by Eq. (2.15). However, the usual soft-pion theorem

$$\lim_{q\to 0}\langle \pi(q)|V_\mu|B\rangle = -i\frac{\sqrt{2}}{f_\pi}\langle 0|A_\mu|B\rangle \tag{2.5}$$

has to be modified to

$$\lim_{q\to 0}\langle \pi(q)|V_\mu|B\rangle = \frac{\sqrt{2}}{f_\pi}\langle 0|A_\mu|B\rangle. \tag{2.6}$$

The above relation, in general, follows from Lorentz covariance and parity considerations. Hence the decay constant of the tensor meson vanishes; that is, the tensor meson $D_2^*$ cannot be produced from the $V - A$ current.

Using $f_D = 200$ MeV, $f_{D_s} = 230$ MeV, and $f_{D_s^*} = 230$ MeV as input, the decay constants (in units of MeV) of $P$-wave charmed mesons are found to be [43]

$$f_{D_0} = 86, \qquad f_{D_1^{1/2}} = 130, \qquad f_{D_1^{3/2}} = -36,$$
$$f_{D_{s0}} = 71, \qquad f_{D_{s1}^{1/2}} = 122, \qquad f_{D_{s1}^{3/2}} = -38 \tag{2.12}$$

in the covariant light-front model, where we have used the constituent quark masses

$$m_{u,d} = 0.26\ \text{GeV}, \qquad m_s = 0.37\ \text{GeV},$$
$$m_c = 1.40\ \text{GeV}, \qquad m_b = 4.64\ \text{GeV}. \tag{2.13}$$

Notice that, although $D_s$ has a decay constant larger than that of $D$, as expected, it is the other way around for the scalar mesons, namely, $f_{D_0} > f_{D_{s0}^*}$. This can be seen from the light-front quark model expression [43]

$$f_{D_s(D_{s0}^*)} \propto \int dx_2 \cdots [m_c x_2 \pm m_s(1 - x_2)]. \tag{2.14}$$

Since the momentum fraction $x_2$ of the strange quark in the $D_s$ ($D_{s0}^*$) meson is small, its effect of being constructive in the case of $D_s$ and destructive in $D_{s0}^*$ is sizable and explains why $f_{D_{s0}^*}/f_{D_s} \sim 0.3$ and $f_{D_{s0}^*} < f_{D_0}$.

In principle, the decay constants of the $P$-wave strange charmed meson $D_s^{**}$ can be extracted from the hadronic decays $B \to \bar{D} D_s^{**}$ since they proceed dominantly via external $W$ emission. In Sec. IV we shall extract the decay constants of $D_{s0}^*$ and $D_{s1}'$ from experiment.

There are other model calculations of the $P$-wave charmed meson decay constants. In general, these estimates are larger than ours, (2.12). For example, the QCD sum rule approach in [45] yields $f_{D_0^*} = 170 \pm 20$ MeV, while the quark model in [46] predicts $f_{D_0^*} = 139 \pm 30$ MeV and $f_{D_{s0}^*} = 110 \pm 18$ MeV. As we shall see below, some of the decay constants for $P$-wave charmed mesons can be phenomenologically extracted from $B \to D^{**}\pi$ and $B \to D_s^{**} D$ decays and compared with model predictions. It turns out that, while our prediction for $f_{D_0^*}$ is smaller than experiment, our result for $f_{D_{s0}^*}$ is in agreement with the data.

### B. Form factors

Form factors for $B \to M$ transitions with $M$ being a parity-odd meson are given by [47]

$$\langle P(p)|V_\mu|B(p_B)\rangle = \left((p_B + p)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right) F_1^{BP}(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0^{BP}(q^2),$$
$$\langle V(p, \varepsilon)|V_\mu|B(p_B)\rangle = \frac{2}{m_B + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_B^\alpha p^\beta V^{BV}(q^2), \tag{2.15}$$
$$\langle V(p, \varepsilon)|A_\mu|B(p_B)\rangle = i\left\{(m_B + m_V)\varepsilon_\mu^* A_1^{BV}(q^2) - \frac{\varepsilon^* \cdot p_B}{m_B + m_V}(p_B + p)_\mu A_2^{BV}(q^2) - 2m_V \frac{\varepsilon^* \cdot p_B}{q^2} q_\mu [A_3^{BV}(q^2) - A_0^{BV}(q^2)]\right\},$$

where $\epsilon_{0123} = 1$, $q = p_B - p$, $F_1^{BP}(0) = F_0^{BP}(0)$, $A_3^{BV}(0) = A_0^{BV}(0)$, and

$$A_3^{BV}(q^2) = \frac{m_B + m_V}{2m_V} A_1^{BV}(q^2) - \frac{m_B - m_V}{2m_V} A_2^{BV}(q^2). \tag{2.16}$$

For $B \to D^{**}$ transitions, we use

$$\langle D_0^*(p)|A_\mu|B(p_B)\rangle = -i\left[\left((p_B + p)_\mu - \frac{m_B^2 - m_{D_0}^2}{q^2} q_\mu\right) F_1^{BD_0}(q^2) + \frac{m_B^2 - m_{D_0}^2}{q^2} q_\mu F_0^{BD_0}(q^2)\right],$$
$$\langle D_1(p, \varepsilon)|V_\mu|B(p_B)\rangle = -i\left\{(m_B - m_{D_1})\varepsilon_\mu^* V_1^{BD_1}(q^2) - \frac{\varepsilon^* \cdot p_B}{m_B - m_{D_1}}(p_B + p)_\mu V_2^{BD_1}(q^2)\right.$$
$$\left. - 2m_{D_1} \frac{\varepsilon^* \cdot p_B}{q^2}(p_B - p)_\mu [V_3^{BD_1}(q^2) - V_0^{BD_1}(q^2)]\right\},$$
$$\langle D_1(p, \varepsilon)|A_\mu|B(p_B)\rangle = \frac{2}{m_B - m_{D_1}} \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p_B^\rho p^\sigma A^{BD_1}(q^2),$$
$$\langle D_2^*(p, \varepsilon)|V_\mu|B(p_B)\rangle = h(q^2)\epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu\alpha}(p_B)_\alpha (p_B + p)^\rho (p_B - p)^\sigma,$$
$$\langle D_2^*(p, \varepsilon)|A_\mu|B(p_B)\rangle = -i[k(q^2)\varepsilon_{\mu\nu}^* p_B^\nu + b_+(q^2)\varepsilon_{\alpha\beta}^* p_B^\alpha p_B^\beta (p_B + p)_\mu + b_-(q^2)\varepsilon_{\alpha\beta}^* p_B^\alpha p_B^\beta (p_B - p)_\mu], \tag{2.17}$$

with

$$V_3^{BD_1}(q^2) = \frac{m_B - m_{D_1}}{2m_{D_1}} V_1^{BD_1}(q^2) - \frac{m_B + m_{D_1}}{2m_{D_1}} V_2^{BD_1}(q^2) \tag{2.18}$$

and $V_3^{BD_1}(0) = V_0^{BD_1}(0)$.

Note that, except for the form factors $h$, $b_+$, $b_-$, all the other form factors are dimensionless. In principle, it is better to parametrize the form factors in such a way that they are all positively defined. This is the case for $B$ to $S$-wave meson transitions, but not for all $B \to D^{**}$ transition form factors. At any rate, the signs of various form factors can be checked via heavy quark symmetry shown below in Eq. (2.21). For example, a factor of $-i$ is needed in the $B \to S$ transition in order for the $B \to D_0^*$ form factors $F_{1,0}^{BD_0}$ to be positive.

Given the Feynman rules for the meson-quark-antiquark vertices (see Table I of [43]) in the framework of the CLF quark model, we are able to compute the form factors in the spacelike momentum transfer $q^2 \leq 0$. Form factors at $q^2 > 0$ can be obtained by first recasting them as explicit functions of $q^2$ in the spacelike region and then analytically continuing them to the timelike region. We find that, except for the form factor $V_2^{BD_1^{3/2}}$, the momentum dependence of form factors in the spacelike region can be well parametrized and reproduced in the three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}, \tag{2.19}$$

for $B \to M$ transitions. The form factor $V_2^{BD_1^{3/2}}$ approaches zero at very large $-|q^2|$ where the three-parameter parametrization (2.19) becomes questionable. To overcome this difficulty, we will fit this form factor to the form

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_B^2)[1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2]} \tag{2.20}$$

and achieve a substantial improvement.

Form factors for $B \to \pi$ and $B \to D^{**}$ transitions calculated in the CLF model are listed in Table II. For comparison, form factors evaluated in the Isgur-Scora-Grinstein-Wise (ISGW) quark model [48] are exhibited in Table III. Before our work, the ISGW quark model was the only model that could provide a systematical estimate of the transition of a ground-state $S$-wave meson to a low-lying $P$-wave meson. This model is based on the nonrelativistic constituent quark picture. In general, the form factors evaluated in the original version of the ISGW model

TABLE II. Form factors for $B \to \pi, D^{**}$ transitions obtained in the covariant light-front model [43] and fitted to the three-parameter form Eq. (2.19) except for the form factor $V_2$ denoted by * for which the fit formula Eq. (2.20) is used. All the form factors are dimensionless except for $h$, $b_+$, $b_-$ with dimensions $\mathrm{GeV}^{-2}$.

| $F$ | $F(0)$ | $F(q^2_{\rm max})$ | $a$ | $b$ | $F$ | $F(0)$ | $F(q^2_{\rm max})$ | $a$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|
| $F_1^{B\pi}$ | 0.25 | 1.16 | 1.73 | 0.95 | $F_0^{B\pi}$ | 0.25 | 0.86 | 0.84 | 0.10 |
| $F_1^{BD_0^*}$ | 0.24 | 0.34 | 1.03 | 0.27 | $F_0^{BD_0^*}$ | 0.24 | 0.20 | −0.49 | 0.35 |
| $A^{BD_1^{1/2}}$ | −0.12 | −0.14 | 0.71 | 0.18 | $V_0^{BD_1^{1/2}}$ | 0.08 | 0.13 | 1.28 | −0.29 |
| $V_1^{BD_1^{1/2}}$ | −0.19 | −0.13 | −1.25 | 0.97 | $V_2^{BD_1^{1/2}}$ | −0.12 | −0.14 | 0.67 | 0.20 |
| $A^{BD_1^{3/2}}$ | 0.23 | 0.33 | 1.17 | 0.39 | $V_0^{BD_1^{3/2}}$ | 0.47 | 0.70 | 1.17 | 0.03 |
| $V_1^{BD_1^{3/2}}$ | 0.55 | 0.51 | −0.19 | 0.27 | $V_2^{BD_1^{3/2}}$ | −0.09* | −0.17* | 2.14* | 4.21* |
| $h$ | 0.015 | 0.024 | 1.67 | 1.20 | $k$ | 0.79 | 1.12 | 1.29 | 0.93 |
| $b_+$ | −0.013 | −0.021 | 1.68 | 0.98 | $b_-$ | 0.011 | 0.016 | 1.50 | 0.91 |

TABLE III. Form factors of $B \to D^{**}$ transitions calculated in the ISGW2 model [43].

| $F$ | $F(0)$ | $F(q^2_{\rm max})$ | $a$ | $b$ | $F$ | $F(0)$ | $F(q^2_{\rm max})$ | $a$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|
| $F_1^{BD_0^*}$ | 0.18 | 0.24 | 0.28 | 0.25 | $F_0^{BD_0^*}$ | 0.18 | −0.008 | ⋯ | ⋯ |
| $A^{BD_1^{1/2}}$ | −0.16 | −0.21 | 0.87 | 0.24 | $V_0^{BD_1^{1/2}}$ | 0.18 | 0.23 | 0.89 | 0.25 |
| $V_1^{BD_1^{1/2}}$ | −0.19 | 0.006 | ⋯ | ⋯ | $V_2^{BD_1^{1/2}}$ | −0.18 | −0.24 | 0.87 | 0.24 |
| $A^{BD_1^{3/2}}$ | 0.16 | 0.19 | 0.46 | 0.065 | $V_0^{BD_1^{3/2}}$ | 0.43 | 0.51 | 0.54 | 0.074 |
| $V_1^{BD_1^{3/2}}$ | 0.40 | 0.32 | −0.60 | 1.15 | $V_2^{BD_1^{3/2}}$ | −0.12 | −0.19 | 1.45 | 0.83 |
| $h$ | 0.011 | 0.014 | 0.86 | 0.23 | $k$ | 0.60 | 0.68 | 0.40 | 0.68 |
| $b_+$ | −0.010 | −0.013 | 0.86 | 0.23 | $b_-$ | 0.010 | 0.013 | 0.86 | 0.23 |

are reliable only at $q^2 = q_m^2$, the maximum momentum transfer, because the form-factor $q^2$ dependence is proportional to $\exp[-(q_m^2 - q^2)]$ and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW2 model [49] in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term.

In the infinite quark mass limit, all the heavy-to-heavy mesonic decay form factors are reduced to three universal IW functions—$\xi(\omega)$ for $S$-wave to $S$-wave, and $\tau_{1/2}(\omega)$ as well as $\tau_{3/2}(\omega)$ for $S$-wave to $P$-wave transitions, first introduced in [37]. Specifically, the $B \to D_0^*$ and $B \to D_1^{1/2}$ form factors are related to $\tau_{1/2}(\omega)$, while $B \to D_1^{3/2}$ and $B \to D_2^*$ transition form factors are related to $\tau_{3/2}(\omega)$ [43]:

$$\begin{gathered}
\langle D(v')|V_\mu|B(v)\rangle = \xi(\omega)(v+v')_\mu, \quad \langle D^*(v',\varepsilon)|V_\mu|B(v)\rangle = -\xi(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,\\
\langle D^*(v',\varepsilon)|A_\mu|B(v)\rangle = i\xi(\omega)[(1+\omega)\varepsilon^*_\mu - (\varepsilon^*\cdot v)v'_\mu], \quad \langle D_0^*(v')|A_\mu|B(v)\rangle = i2\tau_{1/2}(\omega)(v-v')_\mu,\\
\langle D_1^{1/2}(v',\varepsilon)|V_\mu|B(v)\rangle = -i2\tau_{1/2}(\omega)[(1-\omega)\varepsilon^*_\mu + (\varepsilon^*\cdot v)v'_\mu], \quad \langle D_1^{1/2}(v',\varepsilon)|A_\mu|B(v)\rangle = -2\tau_{1/2}(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,\\
\langle D_1^{3/2}(v',\varepsilon)|V_\mu|B(v)\rangle = i\frac{1}{\sqrt{2}}\tau_{3/2}(\omega)\{(1-\omega^2)\varepsilon^*_\mu - (\varepsilon^*\cdot v)[3v_\mu + (2-\omega)v'_\mu]\},\\
\langle D_1^{3/2}(v',\varepsilon)|A_\mu|B(v)\rangle = \frac{1}{\sqrt{2}}\tau_{3/2}(\omega)(1+\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta, \quad \langle D_2^*(v',\varepsilon)|V_\mu|B(v)\rangle = \sqrt{3}\tau_{3/2}(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu\gamma}v_\gamma v'^\alpha v^\beta,\\
\langle D_2^*(v',\varepsilon)|A_\mu|B(v)\rangle = -i\sqrt{3}\tau_{3/2}(\omega)\{(1+\omega)\varepsilon^*_{\mu\nu}v^\nu - \varepsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\mu\},
\end{gathered} \tag{2.21}$$

where $\omega \equiv v\cdot v'$. For completeness, we have also included $B \to D, D^*$ transitions in terms of the Isgur-Wise function $\xi(\omega)$. Using the vertex functions in the heavy quark limit given by Eqs. (4.23) and (4.24) of [43] in conjunction with HQET, we have derived the IW functions in the light-front model. Their numerical expressions are given by [43]

$$\begin{gathered}
\xi(\omega) = 1 - 1.22(\omega-1) + 0.85(\omega-1)^2, \qquad \tau_{1/2}(\omega) = 0.31(1 - 1.18(\omega-1) + 0.87(\omega-1)^2),\\
\tau_{3/2}(\omega) = 0.61(1 - 1.73(\omega-1) + 1.46(\omega-1)^2).
\end{gathered} \tag{2.22}$$

They are in good agreement with the lattice results $\tau_{1/2}(1) = 0.38 \pm 0.05$ and $\tau_{3/2}(1) = 0.53 \pm 0.08$ [50].[8]

It is easily seen from Eq. (2.21) that the $B \to D^{**}$ matrix elements of weak currents vanish at the zero recoil point $\omega = 1$ owing to the orthogonality of the wave functions of $B$ and $D^{**}$. From Eqs. (2.15) and (2.21) it is clear that the $B \to D_0^*$ and $B \to D_1^{1/2}$ form factors in the heavy quark limit are related to $\tau_{1/2}(\omega)$ by

$$\begin{aligned}
\tau_{1/2}(\omega) &= \frac{\sqrt{m_B m_{D_0^*}}}{m_B - m_{D_0^*}} F_1^{BD_0^*}(q^2) = \frac{\sqrt{m_B m_{D_0^*}}}{m_B - m_{D_0^*}} \frac{F_0^{BD_0^*}(q^2)}{[1 - \frac{q^2}{(m_B - m_{D_0^*})^2}]} = -\frac{\sqrt{m_B m_{D_1^{1/2}}}}{m_B - m_{D_1^{1/2}}} A^{BD_1^{1/2}}(q^2) = \frac{\sqrt{m_B m_{D_1^{1/2}}}}{m_B - m_{D_1^{1/2}}} V_0^{BD_1^{1/2}}(q^2)\\
&= -\frac{\sqrt{m_B m_{D_1^{1/2}}}}{m_B - m_{D_1^{1/2}}} V_2^{BD_1^{1/2}}(q^2) = -\frac{\sqrt{m_B m_{D_1^{1/2}}}}{m_B - m_{D_1^{1/2}}} \frac{V_1^{BD_1^{1/2}}(q^2)}{[1 - \frac{q^2}{(m_B - m_{D_1^{1/2}})^2}]}.
\end{aligned} \tag{2.23}$$

Hence, in the heavy quark limit, we have $V_0^{BD_1^{1/2}}(q^2) = -V_2^{BD_1^{1/2}}(q^2) = -A^{BD_1^{1/2}}(q^2)$. Likewise, the $B \to D_1^{3/2}$ and $B \to D_2^*$ form factors are related to $\tau_{3/2}(\omega)$ via [43]

$$\begin{aligned}
\tau_{3/2}(\omega) &= -\sqrt{\frac{2}{m_B m_{D_1^{3/2}}}} \frac{\ell_{3/2}(q^2)}{\omega^2 - 1} = -\frac{1}{3}\sqrt{\frac{2m_B^3}{m_{D_1^{3/2}}}}(c_+^{3/2}(q^2) + c_-^{3/2}(q^2)) = \sqrt{\frac{2m_B^3}{m_{D_1^{3/2}}}} \frac{c_+^{3/2}(q^2) - c_-^{3/2}(q^2)}{\omega - 2} = 2\sqrt{\frac{m_B^3 m_{D_2^*}}{3}} h(q^2)\\
&= \sqrt{\frac{m_B}{3m_{D_2^*}}} \frac{k(q^2)}{1+\omega} = -\frac{2\sqrt{2}}{1+\omega}\sqrt{m_B m_{D_1^{3/2}}} q_{3/2}(q^2) = -\sqrt{\frac{m_B^3 m_{D_2^*}}{3}}(b_+(q^2) - b_-(q^2)),
\end{aligned} \tag{2.24}$$

with $\omega = (m_B^2 + m_{D^{**}}^2 - m_\pi^2)/(2m_B m_{D^{**}})$, $b_+(q^2) + b_-(q^2) = 0$, and

[8]Comparison with other model calculations of $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ is summarized in Table XV of [43]. For example, based on QCD sum rules, $\tau_{3/2}(1) = 0.74 \pm 0.15$ is obtained [51].

$$\begin{aligned}\ell_{3/2}(q^2) &= -(m_B - m_{D_1^{3/2}})V_1^{BD_1^{3/2}}(q^2),\\ q_{3/2}(q^2) &= -\frac{A^{BD_1^{3/2}}(q^2)}{m_B - m_{D_1^{3/2}}},\\ c_+^{3/2}(q^2) &= \frac{V_2^{BD_1^{3/2}}(q^2)}{m_B - m_{D_1^{3/2}}},\\ c_-^{3/2}(q^2) &= -2m_{D_1^{3/2}}\frac{V_3^{BD_1^{3/2}}(q^2) - V_0^{BD_1^{3/2}}(q^2)}{q^2}.\end{aligned} \qquad (2.25)$$

One can check that the signs of various form factors in Table II are in agreement with the heavy quark limit behavior of $B \to D^{**}$ transitions, Eqs. (2.23) and (2.24).

It turns out that, among the 14 $B \to D^{**}$ form factors, while the covariant light-front model predictions for $A^{BD_1^{1/2(3/2)}}$, $V_0^{BD_1^{1/2}}$, $V_2^{BD_1^{1/2}}$, $h$, $b_+$, $b_-$ are in good agreement with those in the heavy quark limit, the predictions for $F_{1,0}^{BD_0^*}$, $V_1^{BD_1^{1/2(3/2)}}$, and $k$ at zero recoil show a large deviation from the HQS expectation. Indeed, Eqs. (2.23) and (2.24) indicate that, except for $F_1^{BD_0^*}$, these form factors should approach to zero when $q^2$ reaches its maximum value, a feature not borne out in the covariant light-front calculations for finite quark masses. This may signal that $\Lambda_{\rm QCD}/m_Q$ corrections are particularly important in this case. Phenomenologically, it is thus dangerous to determine all the form factors directly from the IW functions and HQS relations since $1/m_Q$ corrections may play an essential role for some of them and the choice of the $\beta$ parameters[9] for $S$-wave and $P$-wave wave functions will affect the IW functions.

## III. $\bar{B} \to D^{**}\pi$ DECAYS

Given the decay constants and form factors discussed in Sec. II, we are ready to study the $B$ decays into $P$-wave charmed mesons. In this section we will focus on $\bar{B} \to D^{**}\pi$ decays. The experimental results for the product of the branching ratios $\mathcal{B}(B \to D^{**}\pi)$ and $\mathcal{B}(D^{**} \to$ two particles or three particles) are summarized in Table IV.

To determine the absolute branching ratios for $B \to D^{**}\pi$, we need some information on the branching fractions of $D^{**}$. The decay $D_0^*$ undergoes an $S$-wave hadronic decay to $D\pi$, while $D_1^{1/2}$ can decay into $D^*$ by $S$-wave and $D$-wave pion emissions, but only the former is allowed in the heavy quark limit $m_c \to \infty$. Hence, we shall assume that the $D_0^*$ and $D_1'$ widths are saturated by $D\pi$ and $D^*\pi$, respectively, so that

$$\mathcal{B}(D_0^{*0} \to D^+\pi^-) = \tfrac{2}{3}, \qquad \mathcal{B}(D_1'^0 \to D^{*+}\pi^-) = \tfrac{2}{3}. \qquad (3.1)$$

In the heavy quark limit where the total angular momentum $j$ of the light quark is conserved, $S$-wave $D_1^{3/2} \to D\pi$ is prohibited by heavy quark spin symmetry. Therefore, for $D_1(2420)$ we assume that the dominated strong decay modes are $(D_1 \to D^*\pi)_{d\text{-wave}}$, $(D\pi\pi)_{P\text{-wave}}$, $(D^*\pi\pi)_{P\text{-wave}}$. From Table IV it is clear that, among the possible strong decays of $D_1$, the three-body mode $D^*\pi\pi$ is suppressed relative to $D\pi\pi$. Moreover, the analysis of $D_1(2400) \to D\pi^+\pi^-$ by Belle [55] shows that the decay mode $D_1 \to D_0^*\pi$ gives the best description. Therefore,

$$\begin{aligned}&\mathcal{B}(D_1^0 \to D^0\pi^+\pi^-) \approx \tfrac{2}{3}\mathcal{B}(D_1^0 \to D_0^{*+}\pi^-),\\ &\mathcal{B}(D_1^0 \to D^{*+}\pi^-, D_0^{*+}\pi^-) = \tfrac{2}{3}.\end{aligned} \qquad (3.2)$$

The tensor meson $D_2^*$ decays into $D^*$ or $D$ via $D$-wave pion emission. Since the production of $D^*\pi\pi$ in $D_2^*$ decay is very suppressed, we take

$$\mathcal{B}(D_2^{*0} \to D^{*+}\pi^-, D^+\pi^-) = \tfrac{2}{3}. \qquad (3.3)$$

In heavy quark effective theory, it is expected that

$$\frac{\Gamma(D_2^{*0} \to D^+\pi^-)}{\Gamma(D_2^{*0} \to D^{*+}\pi^-)} = \frac{2}{3}\frac{m_D}{m_{D^*}}\left(\frac{p_c(D_2^* \to D\pi)}{p_c(D_2^* \to D^*\pi)}\right)^5 = 2.3, \qquad (3.4)$$

in excellent agreement with the direct measured value of $2.3 \pm 0.6$ [30]. Applying Eqs. (3.1), (3.2), and (3.3), the absolute branching ratios of $B \to D^{**}\pi$ are shown in Table V. Note that in the factorization approach it is expected that

$$\frac{\mathcal{B}(\bar{B}^0 \to D_2^{*+}K^-)}{\mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-)} = \sin\theta_C^2, \qquad (3.5)$$

with $\theta_C$ being the Cabibbo angle. From Table IV we see that this relation is well satisfied experimentally.

We shall study $B \to D^{**}\pi$ decays within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable contributions multiplied by the *universal* (i.e. process independent) effective parameters $a_i$ that are renormalization scale and scheme independent. Apart from a common factor of $G_F V_{cb} V_{ud}^*/\sqrt{2}$, the factorizable amplitudes for $B^- \to D^{**0}\pi^-$ read

[9] $\beta$ is a wave function parameter which governs the behavior of the phenomenological meson wave functions, $\phi \propto \exp(-|\vec{p}^2|/2\beta^2)$. It is expected to be of order $\Lambda_{\rm QCD}$.

TABLE IV. Experimental branching ratio products (in units of $10^{-4}$) of $B$ decays to $D^{**}\pi$, where $D_0^*$, $D_1'$, $D_1$, $D_2^*$ stand for the charmed mesons $D_0^*(2400)$, $D_1'(2430)$, $D_1(2420)$, and $D_2^*(2460)$, respectively. The Cabibbo-suppressed mode $\bar{B}^0 \to D_2^{*+}K^-$ is also included.

| Mode | *BABAR* [52,53] | Belle [4,54] | Average |
|---|---|---|---|
| $\mathcal{B}(B^- \to D_0^{*0}\pi^-)\mathcal{B}(D_0^{*0} \to D^+\pi^-)$ | | $6.1 \pm 0.6 \pm 0.9 \pm 1.6$ | $6.1 \pm 1.9$ |
| $\mathcal{B}(B^- \to D_1'^0\pi^-)\mathcal{B}(D_1'^0 \to D^{*+}\pi^-)$ | | $5.0 \pm 0.4 \pm 1.0 \pm 0.4$ | $5.0 \pm 1.1$ |
| $\mathcal{B}(B^- \to D_1^0\pi^-)\mathcal{B}(D_1^0 \to D^{*0}\pi^-\pi^+)$ | | $<0.06$ | $<0.06$ |
| $\mathcal{B}(B^- \to D_1^0\pi^-)\mathcal{B}(D_1^0 \to D^0\pi^-\pi^+)$ | | $1.85 \pm 0.29 \pm 0.35^{+0.00}_{-0.46}$ | $1.85^{+0.45}_{-0.65}$ |
| $\mathcal{B}(B^- \to D_1^0\pi^-)\mathcal{B}(D_1^0 \to D^{*+}\pi^-)$ | $5.9 \pm 0.3 \pm 1.1$ | $6.8 \pm 0.7 \pm 1.3 \pm 0.3$ | $6.2 \pm 0.9$ |
| $\mathcal{B}(B^- \to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0} \to D^{*0}\pi^-\pi^+)$ | | $<0.22$ | $<0.22$ |
| $\mathcal{B}(B^- \to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0} \to D^{*+}\pi^-)$ | $1.8 \pm 0.3 \pm 0.5$ | $1.8 \pm 0.3 \pm 0.3 \pm 0.2$ | $1.8 \pm 0.4$ |
| $\mathcal{B}(B^- \to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0} \to D^+\pi^-)$ | $2.9 \pm 0.2 \pm 0.5$ | $3.4 \pm 0.3 \pm 0.6 \pm 0.4$ | $3.1 \pm 0.4$ |
| $\mathcal{B}(\bar{B}^0 \to D_0^{*+}\pi^-)\mathcal{B}(D_0^{*+} \to D^0\pi^-)$ | | $0.60 \pm 0.13 \pm 0.15 \pm 0.22 < 1.20$ | $<1.20$ |
| $\mathcal{B}(\bar{B}^0 \to D_1'^+\pi^-)\mathcal{B}(D_1'^+ \to D^{*0}\pi^+)$ | | $0.14 \pm 0.13 \pm 0.12^{+0.00}_{-0.10} < 0.70$ | $<0.70$ |
| $\mathcal{B}(\bar{B}^0 \to D_1^+\pi^-)\mathcal{B}(D_1^+ \to D^{*+}\pi^-\pi^+)$ | | $<0.33$ | $<0.33$ |
| $\mathcal{B}(\bar{B}^0 \to D_1^+\pi^-)\mathcal{B}(D_1^+ \to D^+\pi^-\pi^+)$ | | $0.89 \pm 0.15 \pm 0.17^{+0.00}_{-0.26}$ | $0.89^{+0.23}_{-0.34}$ |
| $\mathcal{B}(\bar{B}^0 \to D_1^+\pi^-)\mathcal{B}(D_1^+ \to D^{*0}\pi^+)$ | | $3.68 \pm 0.60^{+0.71+0.65}_{-0.40-0.30}$ | $3.7^{+1.1}_{-0.8}$ |
| $\mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-)\mathcal{B}(D_2^{*+} \to D^{*+}\pi^-\pi^+)$ | | $<0.24$ | $<0.24$ |
| $\mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-)\mathcal{B}(D_2^{*+} \to D^{*0}\pi^+)$ | | $2.45 \pm 0.42^{+0.35+0.39}_{-0.45-0.17}$ | $2.4^{+0.7}_{-0.6}$ |
| $\mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-)\mathcal{B}(D_2^{*+} \to D^0\pi^+)$ | | $3.1 \pm 0.3 \pm 0.1^{+0.2}_{-0.0}$ | $3.1^{+0.4}_{-0.3}$ |
| $\mathcal{B}(\bar{B}^0 \to D_2^{*+}K^-)\mathcal{B}(D_2^{*+} \to D^0\pi^+)$ | $0.183 \pm 0.051$ | | $0.18 \pm 0.05$ |

$$\begin{aligned}
A(B^- \to D_0^{*0}\pi^-) &= -a_1 f_\pi (m_B^2 - m_{D_0}^2) F_0^{BD_0}(m_\pi^2) + a_2 f_{D_0}(m_B^2 - m_\pi^2) F_0^{B\pi}(m_{D_0}^2),\\
A(B^- \to D_1'^0\pi^-) &= 2(\varepsilon^* \cdot p_B)\{a_1 f_\pi V_0^{BD_1'}(m_\pi^2) m_{D_1'} - a_2 f_{D_1'} F_1^{B\pi}(m_{D_1'}^2) m_{D_1'}\},\\
A(B^- \to D_1^0\pi^-) &= 2(\varepsilon^* \cdot p_B)\{a_1 f_\pi V_0^{BD_1}(m_\pi^2) m_{D_1} - a_2 f_{D_1} F_1^{B\pi}(m_{D_1}^2) m_{D_1}\},\\
A(B^- \to D_2^{*0}\pi^-) &= -a_1 f_\pi \varepsilon^*_{\mu\nu} p_B^\mu p_B^\nu [k(m_\pi^2) + b_+(m_\pi^2)(m_B^2 - m_{D_2}^2) + b_-(m_\pi^2) m_\pi^2],
\end{aligned} \tag{3.6}$$

with

$$\begin{aligned}
V_0^{BD_1'} m_{D_1'} &= V_0^{BD_1^{1/2}} m_{D_1^{1/2}} \cos\theta + V_0^{BD_1^{3/2}} m_{D_1^{3/2}} \sin\theta, \qquad V_0^{BD_1} m_{D_1} = -V_0^{BD_1^{1/2}} m_{D_1^{1/2}} \sin\theta + V_0^{BD_1^{3/2}} m_{D_1^{3/2}} \cos\theta,\\
f_{D_1'} m_{D_1'} &= f_{D_1^{1/2}} m_{D_1^{1/2}} \cos\theta + f_{D_1^{3/2}} m_{D_1^{3/2}} \sin\theta, \qquad f_{D_1} m_{D_1} = -f_{D_1^{1/2}} m_{D_1^{1/2}} \sin\theta + f_{D_1^{3/2}} m_{D_1^{3/2}} \cos\theta.
\end{aligned} \tag{3.7}$$

The decay amplitudes for $\bar{B}^0 \to D^{**+}\pi^-$ can be obtained from $A(B^- \to D^{**0}\pi^-)$ by setting $a_2 = 0$.[10] Note that, except for $B^- \to D_2^{*0}\pi^-$, all other decay modes receive contributions from color-suppressed internal $W$ emission. In the heavy quark limit,

$$\begin{aligned}
A(B^- \to D_0^{*0}\pi^-) &= -2a_1 f_\pi \sqrt{m_B m_{D_0}}(m_B + m_{D_0})(\omega - 1)\tau_{1/2}(\omega) + 2a_2 f_{D_0} F_0^{B\pi}(m_{D_0}^2)(m_B^2 - m_\pi^2),\\
A(B^- \to D_1'^0\pi^-) &= 2(\varepsilon^* \cdot v)\{a_1 f_\pi \sqrt{m_B m_{D_1'}}(m_B - m_{D_1^{1/2}})\tau_{1/2}(\omega) - a_2 f_{D_1^{1/2}} F_1^{B\pi}(m_{D_1^{1/2}}^2) m_B m_{D_1^{1/2}}\},\\
A(B^- \to D_1^0\pi^-) &= \sqrt{2} a_1 (\varepsilon^* \cdot v) f_\pi \sqrt{m_B m_{D_1}}(m_B - m_{D_1^{3/2}})(\omega + 1)\tau_{3/2}(\omega),\\
A(B^- \to D_2^{*0}\pi^-) &= -\sqrt{3} a_1 f_\pi \varepsilon^*_{\mu\nu} v^\mu v^\nu \sqrt{m_B m_{D_2}}(m_B + m_{D_2})\tau_{3/2}(\omega),
\end{aligned} \tag{3.8}$$

where $\omega = (m_B^2 + m_{D^{**}}^2 - m_\pi^2)/(2m_B m_{D^{**}})$.

TABLE V. Experimental branching ratios for $B \to D^{**}\pi$ decays (in units of $10^{-4}$), where $D^{**}$ denotes a generic $P$-wave charmed meson.

| Mode | Expt. | Mode | Expt. |
|---|---|---|---|
| $B^- \to D_0^{*0}\pi^-$ | $9.2 \pm 2.9$ | $\bar{B}^0 \to D_0^{*+}\pi^-$ | $0.90 \pm 0.45 < 1.8$ |
| $B^- \to D_1'^0\pi^-$ | $7.5 \pm 1.7$ | $\bar{B}^0 \to D_1'^+\pi^-$ | $0.21^{+0.27}_{-0.30} < 1.1$ |
| $B^- \to D_1^0\pi^-$ | $13.5^{+1.7}_{-2.0}$ | $\bar{B}^0 \to D_1^+\pi^-$ | $7.6^{+1.7}_{-1.4}$ |
| $B^- \to D_2^{*0}\pi^-$ | $7.4 \pm 0.8$ | $\bar{B}^0 \to D_2^{*+}\pi^-$ | $8.3^{+1.2}_{-1.0}$ |

[10] It is customary to neglect the $W$-exchange contributions to $\bar{B}^0 \to D^{**+}\pi^-$ and $\bar{B}^0 \to D^{**0}\pi^0$.

The decay rates are given by[11]

$$\Gamma(B \to D_0^* \pi) = \frac{p_c}{8\pi m_B^2} |A(B \to D_0^* \pi)|^2,$$
$$\Gamma(B \to D_1^{(\prime)} \pi) = \frac{p_c^3}{8\pi m_{D_1}^2} |A(B \to D_1^{(\prime)} \pi)/(\varepsilon^* \cdot p_B)|^2,$$
$$\Gamma(B \to D_2^* \pi) = \frac{p_c^5}{12\pi m_{D_2}^2} \left(\frac{m_B}{m_{D_2}}\right)^2 |M(B \to D_2^* \pi)|^2, \tag{3.9}$$

where $A(B \to D_2^* \pi) = \varepsilon^*_{\mu\nu} p_B^\mu p_B^\nu M(B \to D_2^* \pi)$ and $p_c$ is the c.m. momentum of the pion. From Eqs. (3.8) and (3.9), we obtain

$$\Gamma(\bar{B}^0 \to D_0^{*+} \pi^-) = \Gamma(\bar{B}^0 \to D_1^{\prime +} \pi^-)$$
$$= \frac{G_F}{16\pi} |V_{cb} V_{ud}^*|^2 a_1^2 f_\pi^2 m_B^3 \frac{(1-r)^5(1+r)^3}{2r} |\tau_{1/2}(\omega)|^2,$$
$$\Gamma(\bar{B}^0 \to D_1^+ \pi^-) = \Gamma(\bar{B}^0 \to D_2^{*+} \pi^-)$$
$$= \frac{G_F}{16\pi} |V_{cb} V_{ud}^*|^2 a_1^2 f_\pi^2 m_B^3 \frac{(1-r)^5(1+r)^7}{16r^3} |\tau_{3/2}(\omega)|^2,$$
$$\Gamma(B^- \to D_1^0 \pi^-) = \Gamma(\bar{B}^0 \to D_1^+ \pi^-),$$
$$\Gamma(B^- \to D_2^{*0} \pi^-) = \Gamma(\bar{B}^0 \to D_2^{*+} \pi^-), \tag{3.10}$$

in the heavy quark limit, where $r = m_{D^{**}}/m_B$. It is evident from Table IV that the HQS relations $\Gamma(\bar{B}^0 \to D_2^{*+} \pi^-) = \Gamma(B^- \to D_2^{*0} \pi^-)$ and $\Gamma(\bar{B}^0 \to D_1^+ \pi^-) = \Gamma(\bar{B}^0 \to D_2^{*+} \pi^-)$ are respected by experiment, while the relation $\Gamma(\bar{B}^0 \to D_1^+ \pi^-) = \Gamma(B^- \to D_1^0 \pi^-)$ is not satisfied, implying the importance of color-suppressed contributions which vanish in the heavy quark limit.

From Eq. (3.6) we see that, apart from the coefficients $a_1$ and $a_2$, the color-allowed and color-suppressed amplitudes for $B^- \to \{D_0^{*0}, D_1^{\prime 0}, D_1^0\}\pi^-$ have opposite signs,[12] in contrast to the case of $B \to D\pi$ decays.

### A. Color-allowed $\bar{B} \to D^{**}\pi$ decays

We first discuss the color-allowed $\bar{B}^0 \to D^{**+}\pi^-$ decays governed by the parameter $a_1$. This is the place where the calculations are considered to be more robust. To proceed, we first fix $a_1$ to be 0.88. In Table VI we show the predictions of $\mathcal{B}(\bar{B} \to D^{**+}\pi^-)$ in the covariant light-front model for $B \to D^{**}$ form factors and its extension to the heavy quark limit with the IW functions given by Eq. (2.22).[13] It is evident that the color-allowed modes are "normal." To illustrate this point, we consider the decay amplitudes in the heavy quark limit given by Eq. (3.8). We see that the color-allowed $a_1$ amplitudes for $D_0^*$ and $D_1'$ production are suppressed relative to that for $D_1$ and $D_2^*$ production because of the smallness of $\tau_{1/2}(\omega)/\tau_{3/2}(\omega)$, $(\omega-1)/(\omega+1)$, and $(m_B - m_{D_1})/(m_B + m_{D_1})$. Note that the first three modes in Table VI prefer an $a_1$ smaller than unity, whereas the $D_2^{*+}\pi^-$ channel favors an $a_1$ close to unity.

TABLE VI. The predicted branching ratios for $\bar{B}^0 \to D^{**+}\pi^-$ decays (in units of $10^{-4}$) calculated in the covariant light-front model and its extension to the heavy quark limit (denoted by HQS). The parameter $a_1$ is taken to be $a_1 = 0.88$. Experimental results are taken from Table V.

| Mode | Theory | HQS | Expt. |
|---|---|---|---|
| $\bar{B}^0 \to D_0^{*+}\pi^-$ | 3.1 | 1.7 | $<1.8$ |
| $\bar{B}^0 \to D_1^{\prime +}\pi^-$ | 0.8 | 1.5 | $<1.1$ |
| $\bar{B}^0 \to D_1^{+}\pi^-$ | 10.4 | 11.1 | $7.6^{+1.7}_{-1.4}$ |
| $\bar{B}^0 \to D_2^{*+}\pi^-$ | 6.9 | 10.8 | $8.3^{+1.2}_{-1.0}$ |

If we treat the $B \to D_0^*, D_1', D_1$ transition form factors as unknown parameters, we can determine them from the data. In order to satisfy the constraint $\mathcal{B}(\bar{B}^0 \to D_0^{*+}\pi^-) < 1.8 \times 10^{-4}$, it follows that the $B \to D_0^*$ form factor is constrained to be

$$F_0^{BD_0^*}(0) \lesssim 0.18. \tag{3.11}$$

This is smaller than the CLF prediction, $F^{BD_0^*}(0) = 0.24$ (cf. Table II). From the measurements of $\bar{B}^0 \to D_1^{\prime +}\pi^-, D_1^+\pi^-$, we obtain

$$V_0^{BD_1'}(0) \lesssim 0.15, \qquad V_0^{BD_1}(0) = 0.39^{+0.05}_{-0.03}. \tag{3.12}$$

Taking $V_0^{BD_1'}(0) = 0.15$ and using the experimental central value for the $D_1' - D_1$ mixing angle $\theta = 5.7°$ [Eq. (2.4)], we find

$$V_0^{BD_1^{1/2}}(0) = 0.11^{+0.00}_{-0.04}, \qquad V_0^{BD_1^{3/2}}(0) = 0.41 \pm 0.04, \tag{3.13}$$

to be compared with the CLF model predictions (see Table II): $V_0^{BD_1^{1/2}}(0) = 0.08$ and $V_0^{BD_1^{3/2}}(0) = 0.47$.

The IW functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ can be extracted from the data of $\bar{B}^0 \to D^{**+}\pi^-$:

[11]Because the scalar resonances $D_0^*$ and $D_1'$ have widths of order 300 MeV, we have checked the finite width effects on their production in $B$ decays and found that the conventional narrow width approximation is accurate enough to describe the production of broad resonances owing to the large energy released in hadronic two-body decays of $B$ mesons [24].

[12]We disagree with [26] on the signs. If we follow [26] to define $\langle P(q)|A_\mu|0\rangle = f_P q_\mu$ and $\langle D_1^{1/2}(p,\varepsilon)|A_\mu|0\rangle = -f_{D_{1/2}} m_{D_{1/2}} \varepsilon_\mu$ for decay constants, we find that the matrix elements $\langle D_0^*|A_\mu|B\rangle$ and $\langle D_1^{1/2}|V_\mu|B\rangle$ should have signs opposite to that given in Eq. (28) of [26].

[13]The predicted rates for $D_0^{*+}\pi^-$ and $D_2^{*+}\pi^-$ are somewhat different in the covariant model and its heavy quark limit extension. This is because the form factors $F^{BD_0^*}(q^2)$ and $k(q^2)$ do not respect the HQS relations (2.23) and (2.24) satisfactorily.

$$\begin{aligned} \bar{B}^0 \to D_0^{*+}\pi^- &\Rightarrow \quad |\tau_{1/2}(1.36)| < 0.22, \\ \bar{B}^0 \to D_1'^{+}\pi^- &\Rightarrow \quad |\tau_{1/2}(1.32)| < 0.19, \\ \bar{B}^0 \to D_1^{+}\pi^- &\Rightarrow \quad |\tau_{3/2}(1.32)| = 0.30 \pm 0.03, \\ \bar{B}^0 \to D_2^{*+}\pi^- &\Rightarrow \quad |\tau_{3/2}(1.31)| = 0.33^{+0.02}_{-0.03}. \end{aligned} \tag{3.14}$$

As stressed in passing, the HQS relation $\Gamma(\bar{B}^0 \to D_1^+\pi^-) = \Gamma(B^- \to D_1^0\pi^-)$ is badly broken and hence $\tau_{3/2}(\omega)$ cannot be reliably extracted from $D_1^+\pi^-$ production. Our predictions [43]

$$\begin{aligned} \tau_{1/2}(1) = 0.31, \qquad & \tau_{1/2}(1.32) = 0.22, \\ \tau_{1/2}(1.36) = 0.21, \qquad & \tau_{3/2}(1) = 0.61, \\ \tau_{3/2}(1.31) = 0.37 & \end{aligned} \tag{3.15}$$

are in good agreement with the phenomenological determination (3.14) and the lattice calculations [50]. For comparison, the phenomenological determination of IW functions in [26] is given by

$$\begin{aligned} \tau_{1/2}(1) < 0.26, \qquad & \tau_{1/2}(1.32) < 0.20, \\ \tau_{3/2}(1) = 0.46 \pm 0.18, \qquad & \tau_{3/2}(1.31) = 0.31 \pm 0.12. \end{aligned} \tag{3.16}$$

In short, Eqs. (3.11), (3.12), (3.13), and (3.14) are the main results in this subsection.

### B. Class-III $B^- \to D^{**0}\pi^-$ decays

We next turn to the so-called class-III decays $B^- \to D^{**0}\pi^-$ that receive both color-allowed and color-suppressed contributions. The experimental observation that the production of broad $D^{**}$ states in charged $B$ decays is more than a factor of 5 larger than that produced in neutral $B$ decays (Table V) is astonishing as it is naively expected to be a factor of 2 difference at most. For example, $D_0^{*0}\pi^-$ and $D_0^{*+}\pi^-$ rates are predicted to be similar in [17,22], while $D_1'^0\pi^-$ is predicted to be even smaller than $D_1'^+\pi^-$ in [24]. The constructive interference in $B^- \to \{D_0^{*0}, D_1'^0\}\pi^-$ decays requests that the relative sign between the real parts of $a_1$ and $a_2$ be *negative*, as noticed in passing. Moreover, if we take the Belle measurements of $\bar{B} \to \{D_0^*, D_1'\}\pi^-$ seriously, they will imply a color-suppressed contribution larger than the color-allowed one, even though the former is $1/m_B$ suppressed in the heavy quark limit. Since the color-allowed modes have been shown to be normal before, anything unusual must arise from the color-suppressed contributions.

Before proceeding, we need to specify the $a_2$ parameter. Recall that $|a_1| = 0.88 \pm 0.06$, $|a_2| = 0.47 \pm 0.06$, and $a_2/a_1 = (0.53 \pm 0.06)\exp(i59°)$ are obtained in [31] by a fit to the data of $B \to D\pi$. Owing to the missing $\bar{B}^0 \to D^{**0}\pi^0$ decays, one can only determine $|1 - xa_2/a_1|$ from the measurements of $B^- \to D^{**0}\pi^-$ and $\bar{B}^0 \to D^{**+}\pi^-$, where $x(D_0^*\pi) = f_{D_0}(m_B^2 - m_\pi^2)F_0^{B\pi}(m_{D_0}^2)/[f_\pi(m_B^2 - m_{D_0}^2)F_0^{BD_0}(m_\pi^2)]$, for example. That is, a determination of the relative strong phase between $a_1$ and $a_2$ has to await the measurement of the neutral mode $\bar{B}^0 \to D^{**+}\pi^-$. For the present purpose, we shall choose $a_1 = 0.88$ and $a_2 = -0.47$ without considering their relative strong phase. Using this set of effective Wilson coefficients, we obtain

$$\begin{aligned} \mathcal{B}(B^- \to D_2^{*0}\pi^-) &= 7.6 \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-) &= 6.9 \times 10^{-4}, \end{aligned} \tag{3.17}$$

for $\bar{B} \to D_2^*\pi^-$ decays, which are in agreement with experiment.

In order to accommodate the experimental observation $\mathcal{B}(B^- \to D_0^{*0}\pi^-) \gtrsim 5\mathcal{B}(\bar{B}^0 \to D_0^{*+}\pi^-)$, the decay constant of $D_0^*$ cannot be too small. A fit to the $B^- \to D_0^{*0}\pi^-$ rate yields

$$f_{D_0^*} = 148^{+40}_{-46}\ \text{MeV}, \tag{3.18}$$

where $F_0^{BD_0}(0)$ has been set to 0.18 [see Eq. (3.18)]. This value of the $D_0^*$ decay constant is larger than our CFL prediction $f_{D_0^*} = 86$ MeV [cf. Eq. (2.12)]. Putting the form factors (3.12) back into Eq. (3.6) [with $V_0^{BD_1'}(0) = 0.15$] and fitting to the measured rates of $B^- \to D_1'^0\pi^-, D_1^0\pi^-$ give rise to

$$f_{D_1'} = 151^{+27}_{-30}\ \text{MeV}, \qquad f_{D_1} = 73^{+21}_{-22}\ \text{MeV}. \tag{3.19}$$

Applying the experimental central value for the $D_1' - D_1$ mixing angle $\theta = 5.7°$, we find

$$f_{D_1^{1/2}} = 143^{+29}_{-32}\ \text{MeV}, \qquad f_{D_1^{3/2}} = 88^{+24}_{-25}\ \text{MeV}. \tag{3.20}$$

Equations (3.18), (3.19), and (3.20) are the main results in this subsection.

It should be stressed that, if $|a_2|$ is chosen to be smaller, say $a_2 = -0.30$, then the decay constants $f_{D_0}$, $f_{D_1'}$, and $f_{D_1}$ all have to be scaled up by a factor of 0.47/0.30. This will lead to a decay constant of $D_0^*$ larger than that of the $D$ meson. This is in contradiction to the observation that $f_{D_0^*}$ should be smaller than $f_D$ [see Eq. (2.14)]. Hence, $|a_2/a_1|$ is preferred to be larger than the naive expectation.

Now we can get into more detail about the relative strength of color-allowed and color-suppressed amplitudes in $B^- \to \{D_0^{*0}, D_1^0\}\pi^-$ decays. Since $f_{D_0}, f_{D_1'} \gtrsim f_\pi$, $F_0^{B\pi}(m_{D_0}^2) = 0.30 \gg F_0^{BD_0}(m_\pi^2) \lesssim 0.18$, and $F_1^{BD_1'}(m_{D_1^{\prime 2}}) = 0.37 \gg V_0^{BD_1'}(m_\pi^2) \lesssim 0.15$ it is evident that the color-suppressed amplitude is slightly larger than the color-allowed one for $|a_2/a_1| = 0.53$. In contrast, internal $W$ emission is suppressed for $D_1$ and $D_2^*$ productions. Under the factorization approximation, the color-suppression amplitude is prohibited in $B \to D_2^*\pi$. It also vanishes in $B \to D_1\pi$ in the heavy quark limit as $\theta \to 0$ and $f_{D_1^{3/2}} \to 0$. Therefore, it is expected that $\Gamma(\bar{B}^0 \to D_2^{*+}\pi^-) = \Gamma(B^- \to D_2^{*0}\pi^-)$ in general and $\Gamma(\bar{B}^0 \to D_1^+\pi^-) = \Gamma(B^- \to D_1^0\pi^-)$ in the heavy quark limit.

Comparing Eq. (3.20) with Table II and Eq. (2.12), it is clear that form factors and decay constants extracted from the data are consistent with the CLF model calculations except for the decay constant $f_{D_1^{3/2}}$ which needs to be positive. This can be seen from Eq. (3.6) that, in order to account for $\mathcal{B}(B^- \to D_1^0\pi^-) > \mathcal{B}(\bar{B}^0 \to D_1^+\pi^-)$, it is necessary to have a constructive interference between color-allowed and color-suppressed $W$-emission amplitudes. This, in turn, implies a positive decay constant for $D_1^{3/2}$, i.e. $f_{D_1^{3/2}} > 0$. (This is most obvious in the heavy quark limit where $\theta \to 0$.) It is not clear to us why our light-front model prediction with a negative $f_{D_1^{3/2}}$ does not work. Of course, it is possible that $a_2$ is process dependent and for some reason $a_2/a_1$ becomes positive for $B^- \to D_1^0\pi^-$. Then $f_{D_1^{3/2}}$ will be negative.

It is worth mentioning that the ratio

$$R = \frac{\mathcal{B}(B^- \to D_2^*(2460)^0\pi^-)}{\mathcal{B}(B^- \to D_1(2420)^0\pi^-)} \tag{3.21}$$

is measured to be $0.54 \pm 0.18$ by Belle [55].[14] In soft-collinear effective theory (SCET), the equality of branching ratios and strong phases

$$\frac{\mathcal{B}(\bar{B} \to D_2^* M)}{\mathcal{B}(\bar{B} \to D_1 M)} = 1, \qquad \phi^{D_2^* M} = \phi^{D_1 M} \tag{3.22}$$

holds in the heavy quark limit for both color-allowed and color-suppressed modes, where $M$ is a light meson [57]. The early prediction by Neubert [19] yields a value of 0.35.

### C. Color-suppressed $\bar{B}^0 \to D^{**0}\pi^0$ and $\bar{B}^0 \to D_1'^0\omega$ decays

The factorizable $\bar{B}^0 \to D^{**0}\pi^0$ amplitudes are given by

$$\begin{aligned} A(\bar{B}^0 \to D_0^{*0}\pi^0) &= -\frac{a_2}{\sqrt{2}} f_{D_0}(m_B^2 - m_\pi^2)F_0^{B\pi}(m_{D_0}^2), \\ A(\bar{B}^0 \to D_1'^0\pi^0) &= \sqrt{2}a_2 f_{D_1'} F_1^{B\pi}(m_{D_1'}^2) m_{D_1'}(\varepsilon^* \cdot p_B), \\ A(\bar{B}^0 \to D_1^0\pi^0) &= \sqrt{2}a_2 f_{D_1} F_1^{B\pi}(m_{D_1}^2) m_{D_1}(\varepsilon^* \cdot p_B). \end{aligned} \tag{3.23}$$

They satisfy the isospin triangle relation

$$\begin{aligned} A(\bar{B}^0 \to D^{**+}\pi^-) &= \sqrt{2}A(\bar{B}^0 \to D^{**0}\pi^0) \\ &\quad + A(B^- \to D^{**0}\pi^-). \end{aligned} \tag{3.24}$$

Assuming no relative strong phases between $D^{**+}\pi^-$, $D^{**0}\pi^-$, and $D^{**0}\pi^0$ amplitudes (i.e. these three amplitudes are parallel or antiparallel to each other) and using the experimental data from Table IV, then the above isospin relation leads to

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to D_0^{*0}\pi^0) &= (1.9 \pm 1.0) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_1'^0\pi^0) &= (2.4 \pm 0.9) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_1^0\pi^0) &= (3.0 \pm 2.9) \times 10^{-5}. \end{aligned} \tag{3.25}$$

Since in reality there should be some relative strong phases between the aforementioned three amplitudes, the above predictions for color-suppressed modes can be considered as the lower bounds and should be robust. At the 90% confidence level, we have

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to D_0^{*0}\pi^0) &> 0.6 \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_1'^0\pi^0) &> 1.1 \times 10^{-4}. \end{aligned} \tag{3.26}$$

Using $a_2 = -0.47$ and the decay constants given in (3.18) and (3.19), a direct calculation of the branching ratios of the neutral modes yields

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to D_0^{*0}\pi^0) &= (1.4^{+0.9}_{-0.7}) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_1'^0\pi^0) &= (1.4 \pm 0.5) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \to D_1^0\pi^0) &= (3.2^{+2.1}_{-1.6}) \times 10^{-5}, \end{aligned} \tag{3.27}$$

which are similar to the model-independent results (3.25) derived from the isospin argument. It is important to measure these modes to see if the production of the broad $D^{**0}$ states in neutral $B$ decays is not color and $1/m_b$ suppressed. Also the isospin relation Eq. (3.24) will enable us to determine the relative strong phases in $\bar{B} \to D^{**}\pi$ decays.

Two remarks are in order: (i) Contrary to $\bar{B} \to D\pi$ decays where $D^0\pi^0$ rates are suppressed by 1 order of magnitude compared to $D^+\pi^-$, the $D^{**0}\pi^0$ rates here are comparable to $D^{**+}\pi^-$ for broad $D^{**}$ states, as the color-suppressed amplitude is larger than the color-allowed one. (ii) Although the $\bar{B}^0$ decay into $D_2^{*0}\pi^0$ is prohibited under the factorization hypothesis, it can nevertheless be induced via final-state interactions (FSIs) and/or nonfactorizable contributions. In soft-collinear effective theory, this decay receives a factorizable contribution at the leading nonvanishing order in $\Lambda/m_B$ [57].

Although the class-III decays $\bar{B}^0 \to D^{**0}\pi^0$ have not been observed so far, there exists one neutral mode $\bar{B}^0 \to D_1'^0\omega$ that can be inferred from a recent measurement of $\bar{B}^0 \to D^{*+}\omega\pi^-$ decays by *BABAR* [58]. There is an enhancement for $D^*\pi$ masses broadly distributed around 2.5 GeV. Assuming that the enhancement is actually due to $\bar{B}^0 \to D_1'\omega$ followed by $D_1' \to D^{*+}\pi^-$, *BABAR* obtained [58]

$$\begin{aligned} &\mathcal{B}(\bar{B}^0 \to D_1'^0\omega)\mathcal{B}(D_1' \to D^{*+}\pi^-) \\ &\quad = (4.1 \pm 1.2 \pm 0.4 \pm 1.0) \times 10^{-4}. \end{aligned} \tag{3.28}$$

It follows from Eq. (3.1) that

$$\mathcal{B}(\bar{B}^0 \to D_1'^0\omega) = (6.2 \pm 2.4) \times 10^{-4}. \tag{3.29}$$

[14] The early Belle [4], *BABAR* [52], and CLEO [56] results, with $R = 0.77 \pm 0.15$, $0.80 \pm 0.07 \pm 0.16$, and $1.8 \pm 0.8$, respectively, did not take into account the contribution from $D_1(2400) \to D\pi^+\pi^-$.

Theoretically, the decay amplitude of $\bar{B}^0 \to D_1'^0 \omega$ is given by

$$A[\bar{B}^0 \to D_1'^0(\varepsilon_{D_1'}, p_{D_1'})\omega(\varepsilon_\omega, p_\omega)] = \frac{1}{\sqrt{2}}\varepsilon^{*\mu}_{D_1'}\varepsilon^{*\nu}_\omega[S_1 g_{\mu\nu} + S_2(p_B)_\mu(p_B)_\nu + iS_3\epsilon_{\mu\nu\alpha\beta}p^\alpha_{D_1'}p^\beta_\omega],$$

with (apart from a common factor of $G_F V_{cb}V^*_{ud}/\sqrt{2}$)

$$S_1 = a_2(m_B + m_\omega)m_{D_1'}f_{D_1'}A_1^{B\omega}(m^2_{D_1'}),$$
$$S_2 = -2\frac{a_2}{m_B + m_\omega}m_{D_1'}f_{D_1'}A_2^{B\omega}(m^2_{D_1'}), \quad (3.30)$$
$$S_3 = -2\frac{a_2}{m_B + m_\omega}m_{D_1'}f_{D_1'}V^{B\omega}(m^2_{D_1'}).$$

Then the helicity amplitudes $H_0$, $H_+$, and $H_-$ can be constructed as

$$H_0 = \frac{1}{2m_{D_1'}m_\omega}[(m_B^2 - m^2_{D_1'} - m^2_\omega)S_1 + 2m_B^2 p_c^2 S_2],$$
$$H_\pm = S_1 \pm m_B p_c S_3. \quad (3.31)$$

The decay rate reads

$$\Gamma(B \to D_1'\omega) = \frac{p_c}{8\pi m_B^2}(|H_0|^2 + |H_+|^2 + |H_-|^2). \quad (3.32)$$

It is found that

$$\mathcal{B}(\bar{B}^0 \to D_1'^0\omega) = 2.6 \times 10^{-4}, \quad (3.33)$$

where we have assumed that $B \to \omega$ form factors are the same as that for $B \to \rho$ transitions which we took from [43]. The above branching ratio prediction is slightly smaller than the *BABAR* measurement.

## IV. $\bar{B} \to \bar{D}_s^{**} D$ DECAYS

Under the factorization hypothesis, the decays $\bar{B} \to \bar{D}_s^{**}D$ receive contributions only from the external $W$-emission diagram, as the penguin contributions to them are negligible. More precisely, their factorizable amplitudes are given by

$$A(\bar{B} \to \bar{D}_s^{**}D) = \frac{G_F}{\sqrt{2}}\{(V_{cb}V^*_{cs}a_1 - V_{tb}V^*_{ts}(a_4 + a_{10})) \times \langle \bar{D}_s^{**}|(\bar{s}c)|0\rangle\langle D|(\bar{c}b)|\bar{B}\rangle + 2V_{tb}V^*_{ts}(a_6 + a_8)\langle \bar{D}_s^{**}|\bar{s}(1+\gamma_5)c|0\rangle \times \langle D|\bar{c}(1-\gamma_5)b|\bar{B}\rangle\}, \quad (4.1)$$

where $(\bar{q}_1 q_2) \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. Since the tensor meson cannot be produced from the $V - A$ current, the $B$ decay into $\bar{D}^*_{s2}D^{(*)}$ is prohibited under the factorization hypothesis. Except for $D^*_{s2}$, the measurement of $\bar{B} \to \bar{D}_s^{**}D$ can be used to determine the decay constant of $D_s^{**}$ to the approximation that penguin contributions are negligible.

The current measurements of $\bar{B} \to \bar{D}_s^{**}D$ are summarized in Table VII. They are consistent with the relation $\Gamma(\bar{B}^0 \to D_s^{**-}D^+) = \Gamma(B^- \to D_s^{**0}D^+)$, but it is obvious

TABLE VII. Experimental branching ratio products (in units of $10^{-4}$) of $B$ decays to $D_s^{**}\pi$, where $D^*_{s0}$, $D'_{s1}$ stand for the strange charmed mesons $D^*_{s0}(2317)$ and $D'_{s1}(2460)$, respectively.

| Mode | *BABAR* [59] | Belle [60] | Average |
|---|---|---|---|
| $\mathcal{B}(B^- \to D^{*-}_{s0}D^0)\mathcal{B}(D^{*-}_{s0} \to D^{*-}_s\gamma)$ | | <6.6 | <6.6 |
| $\mathcal{B}(B^- \to D^{*-}_{s0}D^0)\mathcal{B}(D^{*-}_{s0} \to D^-_s\pi^0)$ | $10.0 \pm 3.0 \pm 1.0^{+4.0}_{-2.0}$ | $9.8^{+2.1}_{-1.9} \pm 2.9$ | $9.9^{+2.9}_{-2.6}$ |
| $\mathcal{B}(B^- \to D^{*-}_{s0}D^{*0})\mathcal{B}(D^{*-}_{s0} \to D^-_s\pi^0)$ | $9.0 \pm 6.0 \pm 2.0^{+3.0}_{-2.0}$ | <9.3 | $9.0^{+7.0}_{-6.6}$ |
| $\mathcal{B}(B^- \to D'^-_{s1}D^0)\mathcal{B}(D'^-_{s1} \to D^-_s\pi^+\pi^-)$ | | <2.7 | <2.7 |
| $\mathcal{B}(B^- \to D'^-_{s1}D^0)\mathcal{B}(D'^-_{s1} \to D^-_s\pi^0)$ | | <2.6 | <2.6 |
| $\mathcal{B}(B^- \to D'^-_{s1}D^0)\mathcal{B}(D'^-_{s1} \to D^{*-}_s\pi^0)$ | $27 \pm 7 \pm 5^{+9}_{-6}$ | $11.6^{+3.9}_{-3.4} \pm 3.5$ | $14.1^{+4.7}_{-4.6}$ |
| $\mathcal{B}(B^- \to D'^-_{s1}D^0)\mathcal{B}(D'^-_{s1} \to D^{*-}_s\gamma)$ | | <5.8 | <5.8 |
| $\mathcal{B}(B^- \to D'^-_{s1}D^0)\mathcal{B}(D'^-_{s1} \to D^-_s\gamma)$ | $6.0 \pm 2.0 \pm 1.0^{+2.0}_{-1.0}$ | $5.9^{+1.1}_{-1.0} \pm 1.8$ | $5.9^{+1.7}_{-1.6}$ |
| $\mathcal{B}(B^- \to D'^-_{s1}D^{*0})\mathcal{B}(D'^-_{s1} \to D^{*-}_s\pi^0)$ | $76 \pm 17 \pm 18^{+26}_{-16}$ | $22.3^{+9.8}_{-8.1} \pm 6.7$ | $28.0^{+10.7}_{-10.5}$ |
| $\mathcal{B}(B^- \to D'^-_{s1}D^{*0})\mathcal{B}(D'^-_{s1} \to D^-_s\gamma)$ | $14.0 \pm 4.0 \pm 3.0^{+5.0}_{-3.0}$ | $9.8^{+3.4}_{-2.9} \pm 2.9$ | $11.1^{+3.7}_{-3.5}$ |
| $\mathcal{B}(\bar{B}^0 \to D^{*-}_{s0}D^+)\mathcal{B}(D^{*-}_{s0} \to D^{*-}_s\gamma)$ | | <10.3 | <10.3 |
| $\mathcal{B}(\bar{B}^0 \to D^{*-}_{s0}D^+)\mathcal{B}(D^{*-}_{s0} \to D^-_s\pi^0)$ | $18.0 \pm 4.0 \pm 3.0^{+6.0}_{-4.0}$ | $10.3^{+2.3}_{-2.0} \pm 3.1$ | $12.0^{+3.4}_{-3.3}$ |
| $\mathcal{B}(\bar{B}^0 \to D^{*-}_{s0}D^{*+})\mathcal{B}(D^{*-}_{s0} \to D^-_s\pi^0)$ | $15.0 \pm 4.0 \pm 2.0^{+5.0}_{-3.0}$ | $5.6^{+2.9}_{-2.3} < 13.5$ | $7.1^{+2.7}_{-2.1}$ |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^+)\mathcal{B}(D'^-_{s1} \to D^-_s\pi^+\pi^-)$ | | <2.7 | <2.7 |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^+)\mathcal{B}(D'^-_{s1} \to D^-_s\pi^0)$ | | <3.3 | <3.3 |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^+)\mathcal{B}(D'^-_{s1} \to D^{*-}_s\pi^0)$ | $28 \pm 8 \pm 5^{+10}_{-6}$ | $16.9^{+4.3}_{-3.6} \pm 5.1$ | $19.3^{+6.0}_{-5.5}$ |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^+)\mathcal{B}(D'^-_{s1} \to D^{*-}_s\gamma)$ | | <6.3 | <6.3 |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^+)\mathcal{B}(D'^-_{s1} \to D^-_s\gamma)$ | $8.0 \pm 2.0 \pm 1.0^{+3.0}_{-2.0}$ | $7.1^{+1.4}_{-1.2} \pm 2.1$ | $7.4^{+2.1}_{-1.9}$ |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^{*+})\mathcal{B}(D'^-_{s1} \to D^{*-}_s\pi^0)$ | $55 \pm 12 \pm 10^{+19}_{-12}$ | $28.8^{+10.1}_{-8.6} \pm 8.6$ | $35.0^{+11.7}_{-10.4}$ |
| $\mathcal{B}(\bar{B}^0 \to D'^-_{s1}D^{*+})\mathcal{B}(D'^-_{s1} \to D^-_s\gamma)$ | $23 \pm 3 \pm 3^{+8}_{-5}$ | $14.3^{+3.0}_{-2.7} \pm 4.3$ | $17.0^{+4.5}_{-4.0}$ |

that one needs more improved data to test the above relation.

Since $D_{s0}^*(2317)$ is below the $DK$ threshold, the only allowed hadronic decay is the isospin-violating one, namely, $D_s^+\pi^0$. Thus far, no other modes have been observed (see e.g. [39]). The allowed radiative mode is $D_{s0}^* \to D_s^*\gamma$, which is constrained to be $\Gamma(D_{s0}^* \to D_s^*\gamma)/\Gamma(D_{s0}^* \to D_s\pi^0) < 0.059$ by CLEO [2]. Therefore, $0.94 \lesssim \mathcal{B}[D_{s0}^*(2317) \to D_s\pi^0] \lesssim 1.0$. It follows from Table VII and (4.1) that

$$a_1 f_{D_{s0}^*} = \begin{cases} 58\text{–}83 \text{ MeV} & \text{from } B^- \text{ decays,} \\ 63\text{–}86 \text{ MeV} & \text{from } \bar{B}^0 \text{ decays.} \end{cases} \tag{4.2}$$

Hence, our prediction $f_{D_{s0}^*} = 71$ MeV is in good agreement with experiment.

Various model predictions on the branching ratios of $D_{s1}'(2460)$ are shown in Table VIII. In general, the models [12,61] differ in the predictions of the branching fractions for the hadronic and radiative decays of $D_{s1}'$ which, in turn, will lead to different absolute branching ratios for $\bar{B} \to D_{s1}'D$. Very recently *BABAR* has been able to measure $\mathcal{B}(\bar{B} \to D_{s1}'D)$ without any assumption on the decays of $D_{s1}'$ [62]. What *BABAR* has measured are the decays $\bar{B} \to D_{\rm meas}D_X$, with $D_{\rm meas}$ being a fully reconstructed charmed meson. The mass and momentum of the $D_X$ are then inferred from the kinematics of the two-body $B$ decay. By selecting final states with $D_X = D_{s1}(2460)^-$, *BABAR* measurements of $\mathcal{B}(\bar{B} \to D_{s1}'D^{(*)})$ are summarized in Table IX. Combining with the *BABAR* results for branching ratio products of $B$ decays to $D_{s1}'\pi$ (Table VII), the absolute branching ratios of $D_{s1}'(2460)$ are determined to be [62]

$$\mathcal{B}(D_{s1}' \to D_s^*\pi^0) = 0.56 \pm 0.13 \pm 0.09, \quad \mathcal{B}(D_{s1}' \to D_s\gamma) = 0.16 \pm 0.04 \pm 0.03. \tag{4.3}$$

It is clear from Table VIII that the model in [12] is more preferred. It is interesting to note that the sum $\mathcal{B}(D_{s1}' \to D_s^*\pi^0 + D_s\gamma) \approx 0.70$ turns out to be the same in both models and is consistent with the measurement $0.72 \pm 0.17$ derived from Eq. (4.3).

TABLE IX. Branching ratios (in units of $10^{-4}$) of $B$ decays to $D_{s1}'D^{(*)}$ measured by *BABAR* [62].

| Mode | Br | Mode | Br |
|---|---|---|---|
| $B^- \to D_{s1}'^- D^0$ | $43 \pm 16 \pm 13$ | $B^- \to D_{s1}'^- D^{*0}$ | $112 \pm 26 \pm 20$ |
| $\bar{B}^0 \to D_{s1}'^- D^+$ | $26 \pm 15 \pm 7$ | $\bar{B}^0 \to D_{s1}'^- D^{*+}$ | $88 \pm 20 \pm 14$ |

The decay constant of $D_{s1}'$ is then found to be

$$a_1 f_{D_{s1}'} = \begin{cases} 188^{+40}_{-54} \text{ MeV} & \text{from } B^- \text{ decays,} \\ 152^{+43}_{-62} \text{ MeV} & \text{from } \bar{B}^0 \text{ decays.} \end{cases} \tag{4.4}$$

For comparison, the decay constants

$$a_1 f_{D_{s0}^*} > 74 \pm 11 \text{ MeV}, \qquad a_1 f_{D_{s1}'} > 166 \pm 20 \text{ MeV} \tag{4.5}$$

are obtained in [63]. Comparing (4.4) with (4.2) we see that $f_{D_{s1}'}$ is about 2 times $f_{D_{s0}^*}$, whereas $f_{D_0^*}$ and $f_{D_1'}$ are very similar in size [cf. Eqs. (3.19) and (4.4)]. It is not clear to us why the HQS relation for the decay constants is satisfied for the sector $\{D_0^*, D_1'\}$ but badly broken for $\{D_{s0}^*, D_{s1}'\}$.

The decays $\bar{B} \to D_s^{**-}D$ have also been studied in [27]. However, the branching ratios $\mathcal{B}(\bar{B} \to D_{s0}^{*-}D) = (3.0\text{–}3.8) \times 10^{-3}$, $\mathcal{B}(\bar{B} \to D_{s1}'^- D) = (7.2\text{–}8.9) \times 10^{-3}$, and $\mathcal{B}(\bar{B} \to D_{s1}'^- D^*) = (2.5\text{–}2.9) \times 10^{-2}$ predicted in [27] are too large compared to experiment (see Tables VII and IX).

For decays $B^- \to D_s^{**-}D^{*0}$, their factorizable amplitudes are given by

$$A(B^- \to D_{s0}^{*-}D^{*0}) = 2(\varepsilon^* \cdot p_B)a_1 f_{D_s^*} m_{D^*} A_0^{BD^*}(m^2_{D_{s0}^*}),$$
$$A[B^- \to D_{s1}'^-(\varepsilon_{D_{s1}}, p_{D_{s1}})D^{*0}(\varepsilon_{D^*}, p_{D^*})] = \varepsilon_{D_1}^{*\mu}\varepsilon_{D^*}^{*\nu}[S_1 g_{\mu\nu} + S_2(p_B)_\mu(p_B)_\nu + iS_3\epsilon_{\mu\nu\alpha\beta}p^\alpha_{D_{s1}}p^\beta_{D^*}], \tag{4.6}$$

with

TABLE VIII. The predicted branching ratios for $D_{s1}'^+(2460)$. Branching ratios in parentheses are with respect to the decay mode $D_s^{*+}\pi^0$, and so are the experimental results.

| Mode | [12] | [61] | *BABAR* [39,59] [a] | Belle [3,60] | CLEO [2] |
|---|---|---|---|---|---|
| $D_s^{*+}\pi^0$ | 56% | 43% | | | |
| $D_s^+\gamma$ | 13%(0.24) | 27%(0.63) | $0.337 \pm 0.036 \pm 0.038$ [b] | $0.63 \pm 0.15 \pm 0.15$ | <0.49 |
| | | | $0.275 \pm 0.045 \pm 0.020$ [c] | $0.43 \pm 0.08 \pm 0.04$ | |
| $D_s^{*+}\gamma$ | 12%(0.22) | 24%(0.56) | <0.24 | <0.31 | <0.16 |
| $D_s^+\pi^+\pi^-$ | 11%(0.20) | 7%(0.16) | $0.077 \pm 0.013 \pm 0.008$ | <0.13 | <0.08 |
| $D_{s0}^{*+}\gamma$ | 7%(0.13) | $0.05\%(1.2 \times 10^{-3})$ | <0.25 | | <0.58 |
| $D_{s0}^{*+}\pi$ | 0.002% | | | | |

[a]The *BABAR* results are for the branching ratios with respect to $D_{s1}' \to D_s\pi^0\gamma$ arising from $D_s^*\pi^0$ and $D_{s0}^*\gamma$. The *BABAR* data are consistent with the decay $D_{s1}' \to D_s\pi^0\gamma$ proceeding entirely through $D_s^*\pi^0$.
[b]From continuum $e^+e^- \to c\bar{c}$.
[c]From $B$ decay.

TABLE X. The predicted ratios $\Gamma(\bar{B} \to D_s^{**} D^*)/\Gamma(\bar{B} \to D_s^{**} D)$ for $D_s^{**} = D_{s0}^*$ and $D'_{s1}$. Experimental results are taken from Tables VII and IX.

| Mode | Theory | Expt. |
|---|---|---|
| $D_{s0}^{*-} D^{*0}/D_{s0}^{*-} D^0$ | 0.49 | $0.91 \pm 0.73$ |
| $D_{s0}^{*-} D^{*+}/D_{s0}^{*-} D^+$ | 0.49 | $0.59 \pm 0.26$ |
| $D_{s1}'^{-} D^{*0}/D_{s1}'^{-} D^0$ | 3.6 | $3.4 \pm 2.4$ |
| $D_{s1}'^{-} D^{*+}/D_{s1}'^{-} D^+$ | 3.6 | $2.6 \pm 1.5$ |

$$\begin{aligned} S_1 &= a_1(m_B + m_{D^*}) m_{D'_{s1}} f_{D'_{s1}} A_1^{BD^*}(m^2_{D'_{s1}}), \\ S_2 &= -2\frac{a_1}{m_B + m_{D^*}} m_{D'_{s1}} f_{D'_{s1}} A_2^{BD^*}(m^2_{D'_{s1}}), \\ S_3 &= -2\frac{a_1}{m_B + m_{D^*}} m_{D'_{s1}} f_{D'_{s1}} V^{BD^*}(m^2_{D'_{s1}}). \end{aligned} \tag{4.7}$$

We consider the ratios $\Gamma(\bar{B} \to D_s^{**-} D^*)/\Gamma(\bar{B} \to D_s^{**-} D)$ for $D_s^{**} = D_{s0}^*(2317)$ and $D'_{s1}(2460)$ which have the advantage of being $a_1 f_{D_s^{**}}$ independent. The results are exhibited in Table X where uses of the $B \to D^*$ form factors from [43] have been made. It is evident that the predictions are consistent with experiment.

## V. $\bar{B} \to D_s^{**} K$ DECAYS

The experimental branching ratio products of $\bar{B}^0$ decays to $D_s^{**+} K^-$ and $D_s^{**-}\pi^+$ are summarized in Table XI. The decay $\bar{B}^0 \to D_s^{**+} K^-$ can only proceed through the $W$-exchange diagram, whereas $\bar{B}^0 \to D_s^{**-}\pi^+$ does receive an external $W$-emission contribution which is, however, Cabibbo-Kobayashi-Maskawa suppressed. From Table XI it is obvious that $\Gamma(\bar{B}^0 \to D_{s0}^{*+} K^-) \gtrsim \Gamma(\bar{B}^0 \to D_s^+ K^-)$. These two decays can only proceed via a short-distance $W$-exchange process or through the long-distance final-state rescattering processes $\bar{B}^0 \to D^+\pi^- \to D_s^+ K^-$ and $\bar{B}^0 \to D_0^{*+}\pi^- \to D_{s0}^{*+} K^-$. (In fact, the rescattering process has the same topology as $W$ exchange.) Since $\mathcal{B}(\bar{B}^0 \to D^+\pi^-) \approx 2.8 \times 10^{-3} \gg \mathcal{B}(\bar{B}^0 \to D_s^+ K^-)$, it is thus expected that the decay $\bar{B}^0 \to D_s^+ K^-$ is dominated by the long-distance rescattering process. As $\mathcal{B}(\bar{B}^0 \to D_0^{*+}\pi^-) < 1.8 \times 10^{-4}$ [30], we will naively conclude that $\Gamma(\bar{B}^0 \to D_{s0}^{*+} K^-)/\Gamma(\bar{B}^0 \to D_s^+ K^-) \approx \Gamma(\bar{B}^0 \to D_0^{*+}\pi^-)/\Gamma(\bar{B}^0 \to D^+\pi^-) < 0.06$, where we have assumed that the rescattering effects of $D^+\pi^- \to D_s^+ K^-$ and $D_0^{*+}\pi^- \to D_{s0}^{*+} K^-$ are similar. This is obviously in contradiction to experiment. Nevertheless, if $D_{s0}^*(2317)^+$ is a bound state of $c\bar{s}d\bar{d}$ [9], then a tree diagram will contribute and this may allow us to understand why $\Gamma(\bar{B}^0 \to D_{s0}^{*+} K^-) \gtrsim \Gamma(\bar{B}^0 \to D_s^+ K^-)$.

For $\bar{B}^0 \to D_s^{**-}\pi^+$ decays, we obtain

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to D_{s0}^{*-}\pi^+) &= (2.6^{+0.9}_{-0.7}) \times 10^{-6}, \\ \mathcal{B}(\bar{B}^0 \to D_{s1}'^{-}\pi^+) &= (1.5 \pm 0.4) \times 10^{-5}. \end{aligned} \tag{5.1}$$

They are consistent with the experimental limits: $\mathcal{B}(\bar{B}^0 \to D_{s0}^{*-}\pi^+) < 2.5 \times 10^{-5}$ and $\mathcal{B}(\bar{B}^0 \to D_{s1}'^{-}\pi^+) < 2.7 \times 10^{-5}$, where use has been made of Eq. (4.3).

## VI. CONCLUSIONS

We have studied the production of even-parity charmed mesons in hadronic $B$ decays, namely, the Cabibbo-allowed decays $\bar{B} \to D^{**}\pi$ and $\bar{D}_s^{**} D^{(*)}$. The main conclusions are the following:

(i) We have shown that the measured color-allowed decays $\bar{B}^0 \to D^{**+}\pi^-$ are consistent with the theoretical expectation. However, the experimental observation of $B^- \to D^{**0}\pi^-$ for the broad $D^{**}$ states is rather astonishing as it requires that a color-suppressed decay amplitude be larger than the color-allowed one, although the former is $1/m_b$ suppressed in the heavy quark limit.

(ii) It is found that, in order to accommodate the data of $\bar{B} \to D^{**}\pi$, the real part of $a_2/a_1$ has a sign opposite to that in $\bar{B} \to D\pi$ decays, where $a_1$ and $a_2$ are the effective parameters for color-allowed and color-suppressed decay amplitudes, respectively. This indicates that $a_2$ is process dependent and the nonfactorizable contributions to color-suppressed amplitudes are not universal; that is, they are process dependent.

(iii) The decay constants and form factors for $D^{**}$ and the Isgur-Wise functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are extracted from the $B \to D^{**}\pi$ data. The Isgur-Wise functions calculated in the covariant light-front quark model are in good agreement with experiment.

TABLE XI. Experimental branching ratio products (in units of $10^{-4}$) of $B$ decays to $D_s^{**+} K^-$ and $D_s^{**-}\pi^+$ where $D_{s0}^*$, $D'_{s1}$ stand for the strange charmed mesons $D_{s0}^*(2317)$ and $D'_{s1}(2460)$, respectively.

| Mode | *BABAR* [64] | Belle [65] | Average |
|---|---|---|---|
| $\mathcal{B}(\bar{B}^0 \to D_s^+ K^-)$ | $0.25 \pm 0.04 \pm 0.04$ | $0.24^{+0.10}_{-0.08} \pm 0.7$ | $0.25 \pm 0.06$ |
| $\mathcal{B}(\bar{B}^0 \to D_s^- \pi^+)$ | $0.13 \pm 0.03 \pm 0.02$ | $0.46^{+0.12}_{-0.11} \pm 0.13$ | $0.14 \pm 0.04$ |
| $\mathcal{B}(\bar{B}^0 \to D_{s0}^{*+} K^-)\mathcal{B}(D_{s0}^{*+} \to D_s^+\pi^0)$ | | $0.44 \pm 0.08 \pm 0.06 \pm 0.11$ | $0.44 \pm 0.15$ |
| $\mathcal{B}(\bar{B}^0 \to D_{s1}'^{+} K^-)\mathcal{B}(D_{s1}'^{+} \to D_s^+\gamma)$ | | $<0.086$ | $<0.086$ |
| $\mathcal{B}(\bar{B}^0 \to D_{s0}^{*-}\pi^+)\mathcal{B}(D_{s0}^{*-} \to D_s^-\pi^0)$ | | $<0.25$ | $<0.25$ |
| $\mathcal{B}(\bar{B}^0 \to D_{s1}'^{-}\pi^+)\mathcal{B}(D_{s1}'^{-} \to D_s^-\gamma)$ | | $<0.04$ | $<0.04$ |

(iv) The color-suppressed modes $\bar{B}^0 \to D^{**0}\pi^0$ for broad $D^{**}$ states and $\bar{B}^0 \to D_1'^0(2430)\omega$ are predicted to have branching ratios of order $10^{-4}$. Robust lower bounds on $\bar{B}^0 \to \{D_0^*, D_1'^0\}\pi^0$ can be derived from the isospin argument.

(v) The decay constants of $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ are inferred from the measurements of $\bar{B} \to D_s^{**-}D$ to be 58–86 MeV and 130–200 MeV, respectively. The large disparity between $f_{D_{s0}}$ and $f_{D_{s1}'}$ is surprising and unexpected. The ratios $\Gamma(\bar{B} \to D_{s0}^{*-}D^*)/\Gamma(\bar{B} \to D_{s0}^{*-}D)$ smaller than unity and $\Gamma(\bar{B} \to D_{s1}^-D^*)/\Gamma(\bar{B} \to D_{s1}^-D)$ larger than unity are confirmed by experiment.

(vi) The observation that the production of $D_{s0}^{*+}K^-$ is larger than $D_s^+K^-$ may imply a four-quark structure for the scalar charmed meson $D_{s0}^*$.

There are two main puzzles concerning $\bar{B} \to D^{**}\pi$ decays, namely, why the color-suppressed amplitude dominates in the broad $D^{**}$ production in charged $B$ decays, and why the sign of $a_2/a_1$ is different from that in $\bar{B} \to D\pi$ decays. Thus far, we have analyzed the $\bar{B} \to D^{**}\pi$ and $D_s^{**-}D$ decays within the phenomenological framework of generalized factorization. In the QCD factorization approach, the parameter $a_2$ is not calculable, owing to the presence of infrared divergence caused by the gluon exchange between the emitted $D^{**}$ meson and the $(\bar{B}\pi)$ system. That is, the nonfactorizable contribution to $a_2$ is dominated by nonperturbative effects.

In soft-collinear effective theory, the color-suppressed $\bar{B}^0 \to D^{**0}\pi^0$ decay is proved to be factorizable. More precisely, its amplitude factors into a pion light-cone wave function and a $B \to D^{**}$ soft distribution function rather than being like the naive $a_2$ factorization as shown in Eq. (3.23) [57,66]. However, it is difficult to make a quantitative prediction in SCET at this stage and it is not clear to us if SCET can provide an explanation of why the sign of $a_2/a_1$ flips from $\bar{B} \to D\pi$ to $\bar{B} \to D^{**}\pi$.

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