Strange quark mass from e^+e^- revisited and present status of light quark masses

Stephan Narison
Laboratoire de Physique Théorique et Astrophysiques, CNRS and Université de Montpellier II, Place Eugène Bataillon 34095—Montpellier Cedex 05, France
(Received 13 April 2006; published 17 August 2006)

We reconsider the determinations of the strange quark mass m_s from e^+e^- into hadrons data using a new combination of finite energy sum rules (FESR) and revisiting the existing τ-like sum rules by including nonresonant contributions to the spectral functions. To order α_s^3 and including the tachyonic gluon mass λ^2 contribution, which phenomenologically parametrizes the UV renormalon effect into the perturbative series, we obtain the invariant mass $\hat{m}_s = (119 \pm 17)$ MeV leading to $\bar{m}_s(2\ \text{GeV}) = (104 \pm 15)$ MeV. Combining this value with the recent and independent phenomenological determinations from some other channels, to order α_s^3 and including λ^2, we deduce the weighted average $\bar{m}_s(2\ \text{GeV}) = (96.1 \pm 4.8)$ MeV. The positivity of the spectral functions in the (pseudo)scalar (resp. vector) channels leads to the lower (resp. upper) bounds of $\bar{m}_s(2\ \text{GeV})$: $(71 \pm 4)\ \text{MeV} \leq \bar{m}_s(2\ \text{GeV}) \leq (151 \pm 14)$ MeV, to order α_s^3. Using the chiral perturbation theory (ChPT) mass ratio $r_3 \equiv 2m_s/(m_u + m_d) = 24.2 \pm 1.5$, and the average value of m_s, we deduce $(\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) = (7.9 \pm 0.6)$ MeV, consistent with the pion sum rule result, which, combined with the ChPT value for m_u/m_d, gives $\bar{m}_d(2\ \text{GeV}) = (5.1 \pm 0.4)$ MeV and $\bar{m}_u(2\ \text{GeV}) = (2.8 \pm 0.2)$ MeV. Finally, using $(\bar{m}_u + \bar{m}_d)$ from the pion sum rule and the average value of $\bar{m}_s$ (without the pion sum rule), the method gives $r_3 = 23.5 \pm 5.8$, in perfect agreement with the ChPT ratio, indicating the self-consistency of the sum rule results. Using the value $\bar{m}_b(\bar{m}_b) = (4.23 \pm 0.06)$ GeV, we also obtain the scale-independent mass ratio $m_b/m_s = 50 \pm 3$, which is useful for model-buildings. Absolute values of the light quark masses from QCD spectral sum rules reported in this paper are the most accurate determinations to date.

DOI: 10.1103/PhysRevD.74.034013 PACS numbers: 11.30.Rd, 12.38.Lg, 13.66.Bc, 14.65.Bt

I. INTRODUCTION

The determination of the strange quark mass is of prime importance for low-energy phenomenology, for *CP* violation, and for beyond standard model-buildings. Since the advent of QCD, where a precise meaning for the definition of the running quark masses within the $\overline{MS}$ scheme [1] has been provided, a large number of efforts have been devoted to the determinations of the strange quark mass[1] using QCD spectral sum rules (QSSR)[2] à la SVZ [6], in the pseudoscalar [4,5,7–10], the scalar [11,12], and the e^+e^- [4,5,13–17] channels, as well as tau-decay data [18–20] and lattice simulations [21–24], while some bounds have also been derived from the positivity of the spectral functions [4,5,15,25] and from the extraction of the quark condensate [4,5,26].

In the following, we reconsider the determinations of m_s from e^+e^- into hadrons data by using new combinations of FESR [20] and by revisiting the analysis done in [15]. In so doing, we take into account more carefully the small nonresonant contributions into the spectral functions, which, though negligible in the individual sum rules, become important in the combinations sensitive to leading order to m_s. We also present a new combination of sum rules used in [20] that we confront with previous sum rules presented in [15]. We conclude the paper with a comparison of recent different determinations of m_s from QCD spectral sum rules from which we extract the average. This average being confronted to lattice calculations.

[1]For reviews, see e.g. [2–5].
[2]For a review, see e.g. [5].

II. NORMALIZATIONS AND NOTATIONS

We shall be concerned with the transverse two-point correlator:

$$\Pi^{\mu\nu}_{ab}(q) \equiv i \int d^4x e^{iqx} \langle 0|\mathcal{T} J^\mu_a(x)(J^\nu_b(0))^\dagger|0\rangle = -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_{ab}(q^2), \quad (1)$$

built from the $SU(3)$ component of the local electromagnetic current,

$$J^\mu_{\text{EM}} = V^\mu_3(x) + \frac{1}{\sqrt{3}} V^\mu_8(x), \quad (2)$$

where

$$V^\mu_a(x) \equiv \sqrt{\frac{1}{2}} \bar{\psi}(x) \lambda_a \gamma^\mu \psi(x); \quad (3)$$

λ_a are the diagonal flavor $SU(3)$ matrices:

$$\lambda_3 = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \qquad \lambda_8 = \sqrt{\frac{1}{6}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}, \quad (4)$$

acting on the basis defined by the up, down, and strange quarks:

$$\psi(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}. \quad (5)$$

In terms of the diagonal quark correlator,

$$\Pi^{\mu\nu}_{jj}(q) \equiv i \int d^4x e^{iqx} \langle 0|\mathcal{T} J^\mu_j(x)(J^\nu_j(0))^\dagger|0\rangle$$
$$= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_{jj}(q^2) \qquad j = u, d, s, \quad (6)$$

where $J^\mu_j(x) \equiv \bar{\psi}_j \gamma^\mu \psi_j$, the previous $SU(3)$ flavor components of the electromagnetic correlator read[3]

$$\Pi_{33} = \frac{1}{2}\cdot\frac{1}{2}(\Pi_{uu} + \Pi_{dd}),$$
$$\Pi_{88} = \frac{1}{2}\cdot\frac{1}{6}(\Pi_{uu} + \Pi_{dd} + 4\Pi_{ss}), \quad (7)$$
$$\Pi_{38} = \frac{1}{4\sqrt{3}}(\Pi_{uu} - \Pi_{dd}).$$

Therefore, the $e^+e^- \to$ hadrons total cross-section reads

$$\sigma(e^+e^- \to \text{hadrons})_{u,d,s} = \frac{4\pi^2\alpha}{s} e^2 \frac{1}{\pi}\left\{\text{Im}\Pi_{33}(s) + \frac{1}{3}\text{Im}\Pi_{88}(s) + \frac{2}{\sqrt{3}}\text{Im}\Pi_{38}(s)\right\}. \quad (8)$$

In a narrow-width approximation (NWA), the resonance H contributions to the spectral functions can be introduced through

$$\langle 0|V^\mu_a|H\rangle = \epsilon^\mu \frac{M_H^2}{2\gamma_{Ha}}, \quad (9)$$

where the coupling γ_{Ha} is related to the meson leptonic width as

$$\Gamma_{H\to e^+e^-} = \frac{2}{3}\alpha^2\pi\frac{M_H}{2\gamma^2_{Ha}}, \quad (10)$$

which is itself related to the total cross section

$$\sigma(e^+e^- \to H) = 12\pi^2\frac{\Gamma_{H\to e^+e^-}}{M_H}\delta(s - M_H^2). \quad (11)$$

III. QCD CORRECTIONS AND RGI PARAMETERS

In order to account for the radiative corrections, one introduces the expressions of the running coupling and masses.

(i) To three-loop accuracy, the running coupling can be parametrized as [5,27]

$$a_s(\nu) = a_s^{(0)}\left\{1 - a_s^{(0)}\frac{\beta_2}{\beta_1}\log\log\frac{\nu^2}{\Lambda^2} + (a_s^{(0)})^2\left[\frac{\beta_2^2}{\beta_1^2}\log^2\log\frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2}\log\log\frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1}\right] + \mathcal{O}(a_s^3)\right\}, \quad (12)$$

with

$$a_s^{(0)} \equiv \frac{1}{-\beta_1\log(\nu/\Lambda)} \quad (13)$$

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β function in the $\overline{MS}$ scheme, which, for three flavors [5], read

$$\beta_1 = -9/2, \qquad \beta_2 = -8, \qquad \beta_3 = -20.1198. \quad (14)$$

(ii) The expression of the running quark mass in terms of the invariant mass $\hat{m}_i$ is [1,5]

$$\bar{m}_i(\nu) = \hat{m}_i(-\beta_1 a_s(\nu))^{-\gamma_1/\beta_1}\left\{1 + \frac{\beta_2}{\beta_1}\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2}\right)a_s(\nu) + \frac{1}{2}\left[\frac{\beta_2^2}{\beta_1^2}\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2}\right)^2 - \frac{\beta_2^2}{\beta_1^2}\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2}\right) + \frac{\beta_3}{\beta_1}\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_3}{\beta_3}\right)\right]a_s^2(\nu) + 1.951\,68 a_s^3(\nu)\right\}, \quad (15)$$

where γ_i are the $\mathcal{O}(a_s^i)$ coefficients of the quark-mass anomalous dimension, which, for three flavors [5], read

$$\gamma_1 = 2, \qquad \gamma_2 = 91/12, \qquad \gamma_3 = 24.8404. \quad (16)$$

(iii) The perturbative expression of the correlator, in terms of the running coupling evaluated at $Q^2 = \nu^2$ [5], reads

$$-Q^2\frac{d}{dQ^2}\Pi_{ss}(Q^2) = \frac{1}{4\pi^2}\left\{1 + \left(a_s \equiv \frac{\alpha_s(Q^2)}{\pi}\right) + 1.6398a_s^2 - \left[10.2839 - \left(\frac{\beta_1^2}{4}\right)\left(\frac{\pi^2}{3}\right)\right]a_s^3 + \cdots\right\}, \quad (17)$$

[3]We shall follow the normalization used in SN [15].

where the last extra term in the a_s^3 coefficient compared with the expression of the spectral function Im Π comes from the analytic continuation.

(iv) The $D = 2$ contribution reads to order α_s^3, in terms of the running mass and by including the tachyonic gluon mass λ^2 term [5,27–29],

$$Q^2\Pi_{ss}^{(D=2)}(Q^2) \simeq -\frac{1}{4\pi^2}\{1.05a_s\lambda^2 + 6\bar{m}_s^2(1 + 2.6667a_s + 24.1415a_s^2 + 250.4705a_s^3 + \mathcal{O}(a_s^4))\}. \quad (18)$$

The coefficient of the a_s^4 term like the ones of all unknown higher order terms will be mimicked by the λ^2 term [29–33] present in the $D = 2$ and $D = 4$ contributions. The presence of λ^2 in the operator product expansion helps in resolving the old puzzle of hierarchy scale [34] encountered in the sum rules analysis of the pion [4,5,7,8] and gluonia [35] channels. λ^2 also improves the determination of α_s and m_s from τ decay data [20,33]. In the present paper, the series converges slowly in the region where the analysis is performed. However, we expect that this slow convergence will not ruin the result, though it introduces a large error, as each correction is individually smaller than the lowest order term, while the size of the λ^2 contribution [see Eq. (19) below] introduced to mimic the resummation of the unknown higher order terms remains a correction of the lowest order one. Indeed, a naïve geometric estimate of the a_s^4 coefficient leads to a contribution of the order of $1000a_s^4$ [15], which is of the same order as the one of λ^2, and then justifies the arguments which motivate its introduction as a model for the unknown higher order terms.

(v) The $D = 4$ contributions read [5,6,27]

$$\begin{aligned} Q^4\Pi_{ss}^{(D=4)}(Q^2) \simeq \bigg\{&\frac{1}{12\pi}\left(1 - \frac{11}{18}a_s\right)\langle\alpha_s G^2\rangle + \left(1 - a_s - \frac{13}{3}a_s^2\right)\langle 2m_s\bar{s}s\rangle + \left(\frac{4}{3}a_s + \frac{59}{6}a_s^2\right)\langle 2m_s\bar{s}s\rangle \\ &+ \left[\frac{4}{27}a_s + \left(-\frac{257}{486} + \frac{4}{3}\zeta(3)\right)a_s^2\right]\sum_i\langle m_i\bar{\psi}_i\psi_i\rangle + \frac{1}{\pi^2}\left[-\frac{6}{7} + \frac{23}{28}a_s + \left(\frac{731}{56} - \frac{18}{7}\zeta(3)\right)a_s\right]\bar{m}_s^4 \\ &- \frac{8}{\pi^2}m_s^2 a_s\lambda^2\bigg\}, \end{aligned} \quad (19)$$

where the last term is due to the λ^2 term [29], and $\zeta(3) = 1.202\ldots$.

(vi) The $D = 6$ contributions read [6]

$$Q^6\Pi_{ss}^{(D=6)}(Q^2) = -\frac{1}{4\pi^2}\frac{896}{81}\rho\langle\bar{s}s\rangle^2, \quad (20)$$

where $\rho \simeq 2$–3 parametrizes the deviation from the vacuum saturation assumption of the four-quark condensate. We shall use as input $\Lambda_3 = (375 \pm 25)$ MeV for three flavors and [5,31,36,37]

$$\begin{aligned} (m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle &= -2m_\pi^2 f_\pi^2, \\ (m_s + m_u)\langle\bar{s}s + \bar{u}u\rangle &\simeq -2\times 0.7m_K^2 f_K^2, \\ \langle\bar{s}s\rangle/\langle\bar{u}u\rangle &\simeq 0.7 \pm 0.2, \\ a_s\lambda^2 &\simeq -(0.07 \pm 0.03)\ \text{GeV}^2, \\ \langle\alpha_s G^2\rangle &\simeq (0.07 \pm 0.01)\ \text{GeV}^4, \\ \rho\alpha_s\langle\bar{u}u\rangle^2 &\simeq (5.8 \pm 0.9)\times 10^{-4}\ \text{GeV}^6, \end{aligned} \quad (21)$$

where $f_\pi = 93.3$ MeV, $f_K = 1.2f_\pi$. We have taken into account a possible violation of kaon partial conservation of the axial current as suggested by the QSSR analysis [5,38].

IV. PARAMETRIZATION OF THE SPECTRAL FUNCTION

(i) For the resonances, we parametrize the spectral function within a NWA[4] by using the most recent data compiled in PDG [39] for the $\phi(1019.7)$ and $\phi'(1680)$ with

$$\begin{aligned} \Gamma_{\phi(1019.7)\to e^+e^-} &\simeq (1.27 \pm 0.02)\ \text{keV}, \\ \Gamma_{\phi(1680)\to e^+e^-} &\simeq (0.43 \pm 0.15)\ \text{keV}. \end{aligned} \quad (22)$$

(ii) For the nonresonant contributions in the region below $\sqrt{t} \leq 1.3$ GeV, we use the sum of the exclusive rates of the $I = 0$ channel compiled in [40] and a $SU(3)$ symmetry for keeping the ϕ component. An analogous parametrization has been used successfully for the accurate estimate of the hadronic contribution to the muon anomalous magnetic moment [41].

(iii) Above the $\phi(1680)$, we use a QCD parametrization of the spectral function as

[4] A parametrization using a Breit-Wigner form leads, within the errors, to the same result.

$$\frac{1}{\pi}\,\mathrm{Im}\Pi_{ss}(t \geq t_>) \simeq \theta(t - t_>)\frac{1}{4\pi^2} \times \{1 + a_s(t)(1 + 2m_s^2/t) + 1.6398a_s^2(t) - 10.284a_s^3(t)\}, \quad (23)$$

where $t_>$ is the QCD continuum threshold.

V. FESR

We shall use the combination of FESR introduced recently in [20] for extracting m_s from the $V + A$ component of the τ decay data[5]:

$$\mathcal{S}_{10} \equiv \mathcal{M}_0 - \frac{2}{t_c}\mathcal{M}_1 \equiv \int_0^{t_c} dt\left(1 - 2\frac{t}{t_c}\right)\frac{1}{\pi}\,\mathrm{Im}\Pi_{ss}(t), \quad (24)$$

which is sensitive, to leading order, to m_s^2 and λ^2. Unlike the individual sum rules, these combinations of sum rules are less sensitive to the high-energy tail of the spectral functions (effect of the t_c cut), as they are chosen such that at $t = t_c$ the integral vanishes.

A. Test of duality

In principle, the value of the t_c cut of the FESR integrals is a free parameter. We fix its optimal value by looking for the region where the phenomenological and QCD sides of the ratio of moments,

$$\mathcal{R}_{10} \equiv 2\frac{\mathcal{M}_1}{\mathcal{M}_0}, \quad (25)$$

are equal. We present this analysis in Fig. 1, by showing the value of t_c predicted by the sum rule versus t_c and by comparing the result with the exact solution $t_c = t_c$ expected to hold for all values of t_c because the continuum is parametrized by the QCD expression above t_c. From Fig. 1, one can deduce that QCD duality is best obtained at

$$t_c \simeq (6.0 \pm 0.5)\ \mathrm{GeV}^2, \quad (26)$$

where one expects to get the optimal value of m_s from FESR. In order to get this number, we have used the value of the invariant mass $\hat{m}_s = (56\text{–}145)$ MeV, which is, like λ^2, a tiny correction in this duality test analysis. Once we have fixed the value of t_c where the best duality from the two sides of FESR has been obtained, we can estimate m_s.

B. Estimate of $\hat{m}_s$ versus λ^2

In principle, $t_>$, the beginning of the QCD continuum parametrization, is a free parameter. We study the stability of the m_s output against the $t_>$ variation and find that, in the range $\sqrt{t_>} = (1.75 \pm 0.05)$ GeV, m_s is quite stable. We present the results of the invariant mass $\hat{m}_s$ for different values of λ^2 in Table I. Using the value of $a_s\lambda^2$ given in Eq. (21), we deduce the predictions

$$\hat{m}_s = (114 \pm 27 \pm 9 \pm 6 \pm 19)\ \mathrm{MeV} \longrightarrow \bar{m}_s(2\ \mathrm{GeV}) = (100 \pm 28_{\mathrm{exp}} \pm 20_{\mathrm{th}})\ \mathrm{MeV}, \quad (27)$$

where the last error in $\hat{m}_s$ is due to λ^2.[6] We have used the conversion scale

$$\bar{m}_s(2\ \mathrm{GeV}) \simeq 0.876\hat{m}_s. \quad (28)$$

One can notice that the result is perfectly consistent with the one from τ decay [20] (given in Table III using the same combination of FESR). Like in the τ-decay data, the theory with $\lambda^2 = 0$ tends to give too small a value of m_s, though the error is quite large.

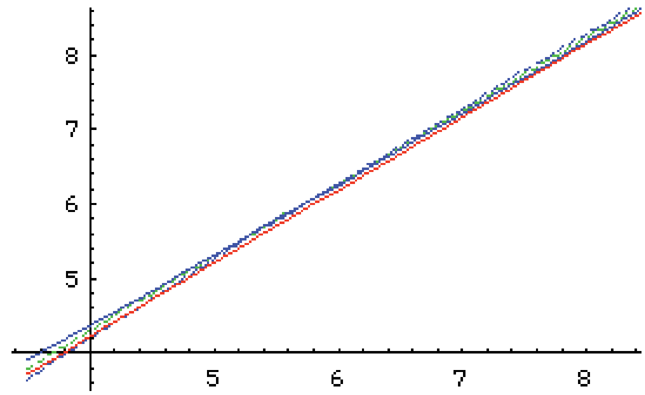

FIG. 1 (color online). FESR prediction of t_c versus t_c in GeV^2. The dashed curve corresponds to the central value of the data; this dashed curve is delimited by a continuous and a dashed curve corresponding to the larger and smaller values of the data; the straight line is the solution $t_c = t_c$, where the predictions from the left-hand side and right-hand side of the FESR should exactly coincide. The curves correspond to the value $t_> = 1.75$ GeV^2; $\hat{m}_s = (56\text{–}145)$ MeV and $a_s\lambda^2 = -(0.04\text{–}0.1)$ GeV^2.

TABLE I. $\hat{m}_s$ versus λ^2 for $t_c = 6$ GeV^2 and $t_> = 1.75$ GeV^2.

$-a_s\lambda^2$ in GeV^2	$\hat{m}_s$ in MeV
0.02	$77 \pm 55 \pm 15 \pm 8$
0.04	$94 \pm 36 \pm 10 \pm 7$
0.06	$108 \pm 30 \pm 9 \pm 6$
0.07	$114 \pm 27 \pm 9 \pm 6$
0.08	$120 \pm 25 \pm 6 \pm 5$
0.10	$131 \pm 22 \pm 5 \pm 4$
0.12	$142 \pm 21 \pm 5 \pm 4$

[5] More technical details of this sum rule can be found in [20].

[6] We have taken the central value of the asymmetrical errors due to λ^2.

VI. τ-LIKE SUM RULES REVISITED

τ-like sum rules have been proposed in SN [15] for extracting m_s from e^+e^- data, which have been exploited later on in [16,17] using the same or a slight variant of the SN sum rule. SN [15] results have been confirmed by [17] using the expected small effects of the $SU(2)$ breaking due to $\omega - \rho$ mixing. However, the central value of the results obtained in [15,17] are slightly higher than recent estimates, though consistent within the errors. In the following, we reconsider the original sum rules:

$$R_{\tau,\phi} \equiv \frac{3|V_{ud}|^2}{2\pi\alpha^2} S_{\rm EW} \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \times \frac{s}{M_\tau^2} \sigma_{e^+e^- \to \phi,\phi',\ldots}, \quad (29)$$

and the $SU(3)$-breaking combinations

$$\Delta_{1\phi} \equiv R_{\tau,1} - R_{\tau,\phi}. \quad (30)$$

Here, $S_{\rm EW} = 1.0194$ is the electroweak correction [42] and $|V_{ud}|^2 = 0.975$ is the Cabibbo-Kobayashi-Maskawa (CKM) mixing angle. The QCD expressions of these sum rules have been given in [15], except that we shall replace the contributions of the uncalculated a_s^4 and higher order terms of the PT series by the contribution of λ^2. We shall also use the computed coefficient of the $a_s^3 m_s^2$ term $k_3 = 250.4$ [28] instead of the estimated 218.55 used in [15]. Contrary to the previous sum rules, the τ-like sum rule is more precise near the real axis due to the presence of the threshold factor $(1 - s/M_\tau^2)^2$.

A. Upper bound on m_s from $R_{\tau,\phi}$

We shall use the positivity of $R_{\tau,\phi}$ and saturate the spectral function by the $\phi(1019.7)$ contribution. In this way, we derive a lower bound on $R_{\tau,\phi}$, which we show in Fig. 2 versus M_τ. From this figure, one can derive

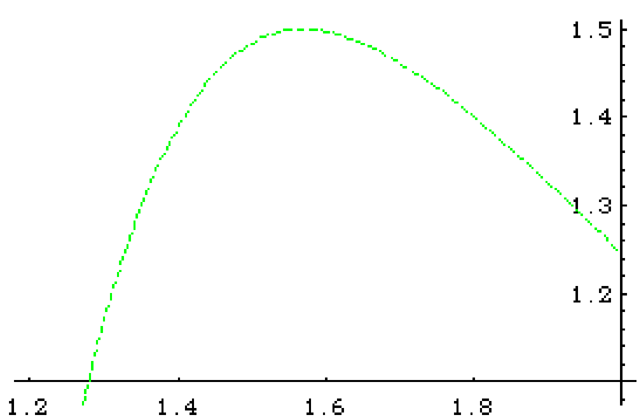

FIG. 2 (color online). Upper bound of $R_{\tau,\phi}$ versus M_τ in GeV for $a_s\lambda^2 = -0.07$ GeV^2, using the central value of the data.

TABLE II. Phenomenological estimates of $R_{\tau,I}$ and central values of $\hat{m}_s$ to order α_s^3.

M_τ	$\frac{9}{2}R_{\tau,\phi}$	$\hat{m}_s$
1.4	2.82 ± 0.47	...
1.6	1.73 ± 0.05	111
1.7	1.66 ± 0.05	128
1.8	1.65 ± 0.06	129
1.9	1.65 ± 0.06	127
2.0	1.80 ± 0.11	...

$$\hat{m}_s \leq (172 \pm 16)\ \text{MeV} \longrightarrow \bar{m}_s(2\ \text{GeV}) \leq (151 \pm 14)\ \text{MeV}. \quad (31)$$

This upper bound is comparable with the previous value $\bar{m}_s(2\ \text{GeV}) \leq (147 \pm 21)$ MeV obtained in [15] using the same method, where the improved accuracy comes mainly from a more precise value of the $\Gamma_{\phi\to e^+e^-}$ width. This bound is also comparable with the upper bound 148 MeV obtained from a direct estimate of the quark chiral condensate [26].

B. Extraction of m_s from $R_{\tau,\phi}$

Parametrizing the spectral function by the $\phi(1019.7)$, $\phi(1680)$, and the nonresonant contributions below 1.39 GeV, we deduce, for the QCD parameters in Eq. (21), the results in Table II. Comparing these results with the previous ones in [15], we notice that the inclusion of the nonresonant states' contributions has slightly increased the value of $R_{\tau,\phi}$. However, this small change affects the value of m_s, which is also a correction in the QCD expression of $R_{\tau,\phi}$. Taking as an optimal estimate of m_s the value which is stable in the change of $M_\tau \simeq (1.8 \pm 0.1)$ GeV,[7] we obtain

$$\hat{m}_s = (129 \pm 15_{\rm exp} \pm 25_{\rm th})\ \text{MeV} \longrightarrow \bar{m}_s(2\ \text{GeV}) = (113 \pm 13_{\rm exp} \pm 22_{\rm th})\ \text{MeV}. \quad (32)$$

This value is consistent with the result in Eq. (27). Like the previous FESR, this prediction is also affected to leading order by λ^2, where the value of m_s increases with $-a_s\lambda^2$. For $\lambda^2 = 0$, the corresponding value of $\hat{m}_s$ is 104 MeV, which is still consistent with the other determinations, though on the lower side. Therefore, one can notice that, contrary to the τ-sum rule, this sum rule cannot differentiate between the two cases $\lambda^2 = 0$ and $\lambda^2 \neq 0$.

C. Extraction of m_s from $\Delta_{1\phi}$

Here, we analyze the $\Delta_{1\phi}$ sum rule. Unlike $R_{\tau,\phi}$, $\Delta_{1,\phi}$ is not sensitive to leading order to λ^2. However, like the

[7]Notice that the inclusion of higher states has slightly shifted the position of the optimum from 1.6 GeV (Fig. 2) to 1.8 GeV (Table II).

$SU(3)$-breaking sum rule, $\Delta_{1,\phi}$ has the disadvantage of involving the difference of two large independent channels (isovector-isoscalar here). Using the optimal value of $R_{\tau,\phi}$ at $M_\tau = 1.8$ GeV from Table II, and using the value of $R_{\tau,1} = 1.78 \pm 0.025$ at this scale from τ-decay data [43], one can deduce

$$\Delta_{1\phi}(1.8\ \text{GeV}) = 0.13 \pm 0.06, \quad (33)$$

leading to

$$\hat{m}_s = (113 \pm 26_{\text{exp}} \pm 4_{\text{th}})\ \text{MeV} \longrightarrow \bar{m}_s(2\ \text{GeV}) = (99 \pm 23_{\text{exp}} \pm 4_{\text{th}})\ \text{MeV}, \quad (34)$$

which is in good agreement with the former determinations.

D. Final value of m_s from e^+e^-

One can see in Table III that there is good agreement between results from e^+e^- data from alternative forms of the sum rules, indicating the reliability of the results. Taking the average of the results in Eqs. (27), (32), and (34), we deduce the final value from e^+e^- data:

TABLE III. Recent phenomenological determinations of $\bar{m}_s$ (2 GeV) to order α_s^3, including the tachyonic gluon mass λ^2 which parametrizes the UV renormalon contributions into the PT series.

Channels	Refs.	$\bar{m}_s$ (2 GeV) in MeV
e^+e^- data		
FESR	This work	$100 \pm 28_{\text{exp}} \pm 20_{\text{th}}$
$R_{\tau,\phi}$	This work	$113 \pm 13_{\text{exp}} \pm 22_{\text{th}}$
$\Delta_{1\phi}$	This work	$99 \pm 23_{\text{exp}} \pm 4_{\text{th}}$
Average	This work	104.3 ± 15.4
τ-decay data		
FESR	[20]	93 ± 30
$SU(3)$ breaking SR	[18,19]	81 ± 22
Pseudoscalar		
Pion SR + ChPT	[4,5,29]	105 ± 26
Kaon FESR	[9]	100 ± 12
Kaon exponential SR	[10][a]	103 ± 9
Scalar		
K_0^* SR	[12][b]	88 ± 8
Chiral condensate		
N, $B^* - B$, $D \to Kl\nu$	[4,5,26][c]	131 ± 18
Final average[d]	Weighted	96.10 ± 4.80
	Arithmetic	96.30 ± 17.5

[a]Result at order α_s^3 quoted in [10]. α_s^4 corrections increase the value by 2 MeV.
[b]Result to order α_s^4 quoted in [12]. α_s^4 corrections are expected, like in the pseudoscalar channel, to be small.
[c]Not included in the average as known to order α_s.
[d]We have assumed that the different determinations are independent from each other.

$$\hat{m}_s = (119 \pm 17)\ \text{MeV} \longrightarrow \bar{m}_s(2\ \text{GeV}) = (104 \pm 15)\ \text{MeV}. \quad (35)$$

This value is consistent within the errors (though in its higher side) with the old value: $\bar{m}_s(2\ \text{GeV}) = (125 \pm 14_{\text{exp}} \pm 20_{\text{th}})$ MeV in [15] and (139 ± 31) MeV in [17].

VII. PRESENT STATUS OF LIGHT QUARK MASSES

In this part, we present the status of the recent determinations of the light quark masses from different sum rule channels, to the same order α_s^3 and including the λ^2 term, which we also compare with the lattice results including dynamical fermions.

A. The (pseudo)scalar channels

These channels are, in principle, the best place for extracting the value of the light quark masses because these masses enter as the leading overall coefficients in the corresponding QCD correlators, precisely known up to order α_s^4 or alternatively to order α_s^3 plus the λ^2 term, which mimics the unknown higher order terms.

(i) Estimates of the sum of the light quark masses $(m_u + m_d)$ [7,8] have been updated in [4,5,29] by including the λ^2 and a_s^3 corrections, and by using the parametrization of the 3π spectral function [8] which satisfies the ChPT constraints. The result is

$$(\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) \simeq (8.6 \pm 2.1)\ \text{MeV}, \quad (36)$$

where λ^2 decreases the value of the sum by 5%. Combined with the ChPT ratio [2],

$$r_3 \equiv \frac{2m_s}{(m_u + m_d)} = 24.4 \pm 1.5, \quad (37)$$

one can deduce the value of m_s given in Table III.[8]

(ii) Direct extractions of m_s from kaon sum rules also exist in the literature [9,10]. Here, the analysis suffers from the unmeasured value of the kaon radial excitations $K(1460)$ and $K(1830)$ decay constants which play an important role at the scale where the sum rules are optimized. We expect that the errors induced by this model dependence have not yet been properly included in the quoted small errors of the estimated decay constants.

(iii) The scalar channel has been revisited in [12] using $K\pi$ phase shift data,[9] with the resulting value of the quark mass given in Table III. Here, the phenomenological side of the scalar sum rule is better known

[8]The kaon sum rule also gives an analogous value [4,5,7] but the corresponding spectral function is less controlled by ChPT than the one of the pion.

[9]For some recent discussions on the scalar channels from the sum rules, see e.g. [44].

due to the availability of the $K\pi$-phase shift data combined with the constraints from ChPT.

(iv) Lower bounds on the light quark masses have also been derived from the (pseudo)scalar channels using the positivity of the spectral function [25]. These bounds have been updated in [4,5] by including the effect of λ^2 and the order α_s^3 PT contributions. The best updated bounds from pseudoscalar kaon and pion sum rules given in [4,5] including the α_s^3 and $1/q^2$ terms are

$$\bar{m}_s(2\ \text{GeV}) \geq (71 \pm 4)\ \text{MeV}, \quad (\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) \geq (5.9 \pm 0.3)\ \text{MeV}. \tag{38}$$

The inclusion of the α_s^4 term obtained in [45] decreases this value by about 2 MeV. However, slightly different central values without error bars have been given in [45] which agree with ours within our quoted errors.

B. τ-decay data

τ-decay data have been employed using different methods for extracting m_s:

(i) In [18,19], $SU(3)$-breaking moment sum rules involving the difference of the nonstrange and strange $V + A$ components of τ-decay data have been used. These sum rules have the advantage of being unaffected by λ^2 to leading order, but have the disadvantage of involving a strong cancellation of the two independent channels $\bar{u}d$ and $\bar{u}s$. The value of m_s is given in Table III.

(ii) Alternatively, a combination of FESR involving only the $\Delta S = -1$, but sensitive to λ^2 to leading order, has been proposed in [20]. This sum rule has been used for studying the effect of λ^2 on the value of m_s. Using the value of λ^2 in Eq. (21), one obtains the value in Table III.

C. Direct extraction of the chiral condensate $\langle\bar{\psi}\psi\rangle$

Extraction of the chiral condensate $\langle\bar{\psi}\psi\rangle$ has been used in [26] for estimating and bounding m_s. The nucleon and $B^* - B$ sum rules give [26]

$$\langle\bar{\psi}\psi\rangle(M_N) \simeq [-(225 \pm 9)\ \text{MeV}]^3, \tag{39}$$

which, combined with the Gell-Mann-Oakes-Renner relation and the ChPT mass ratio in Eq. (37), leads to

$$(\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) \simeq (10.8 \pm 1.3)\ \text{MeV} \quad \text{and} \quad \bar{m}_s(2\ \text{GeV}) \simeq (131 \pm 18)\ \text{MeV}. \tag{40}$$

Bounds have also been derived from the $D \to K^* l\nu$ decays [26]:

$$0.6 \leq \langle\bar{\psi}\psi\rangle(1\ \text{GeV})/[-229\ \text{MeV}]^3 \leq 1.5, \tag{41}$$

giving

$$6.8\ \text{MeV} \leq (\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) \leq 11.4\ \text{MeV}. \tag{42}$$

Combined with the ChPT mass ratio r_3 in Eq. (37), it gives

$$82\ \text{MeV} \leq \bar{m}_s(2\ \text{GeV}) \leq 138\ \text{MeV}, \tag{43}$$

which is comparable with the lower bound from the pseudoscalar sum rule given in Eq. (38) and the upper bound from e^+e^- data given in Eq. (31). However, as the result is obtained to order α_s, we will not consider these bounds in our final estimate. Instead, we use the allowed region in order to deduce the inaccurate estimate given in Table III.

D. Final value of the strange quark mass and m_b/m_s from QSSR

(i) As a final result, we consider the weighted average given in Table III which emphasizes the contributions of the most accurate results from (pseudo)scalar channels, which give more weight in the averaging procedure:

$$\bar{m}_s(2\ \text{GeV}) = (96.10 \pm 4.80)\ \text{MeV}. \tag{44}$$

(ii) As discussed in the previous subsection, the precisions from these two channels can be qualitatively understood because the square of the strange quark mass enters as the leading overall coefficient in the analysis of the correlator associated with the divergence of the axial-vector (resp. of the vector) currents, while in the vector- and tau-decay channels m_s enters as m_s^2/q^2 corrections in the corresponding two-point correlator. The accuracy of the phenomenological side of the pseudoscalar sum rule is more questionable due to the lack of data and to the accuracy of the radial excitation decay constants which play a crucial role in the analysis. The phenomenological side of the scalar sum rule is in better shape due to the availability of the $K\pi$-phase shift data and to the constraints from ChPT. A confirmation of the accuracy obtained from the (pseudo)scalar channels requires an independent analysis of these channels.

(iii) However, it is difficult to quantify with good precision the systematic errors of the different sum rule approaches; though, in each analysis, the different authors have used their own estimate of such errors by studying the effects of external parameters (sum rule scale, continuum threshold,...) based on optimization and/or stability procedures or duality tests. Because of the remarkable good agree-

ment of the different results given in Table III within about 1σ, we might expect that the quoted error in Eq. (44) is quite realistic though relatively small. This value of m_s, is consistent with the older sum rule (arithmetic) average (117.4 ± 23.4) MeV quoted in [5,15], though on the lower side. Such an agreement indicates the stability of the sum rule results with time and then their reliability.
The more conservative value from the arithmetic average obtained in Table III,

$$\bar{m}_s(2\ \text{GeV}) = (96.3 \pm 17.5)\ \text{MeV}, \tag{45}$$

can be translated into the range of m_s values allowed by the sum rules analysis:

$$79\ \text{MeV} \leq \bar{m}_s(2\ \text{GeV}) \leq 114\ \text{MeV}, \tag{46}$$

which can be compared with the rigorous lower (resp. upper) bound coming from the positivity of the spectral functions in the pseudoscalar (resp. ϕ) sum rules updated to order α_s^3[10]:

$$(71 \pm 4)\ \text{MeV} \leq \bar{m}_s(2\ \text{GeV}) \leq (151 \pm 14)\ \text{MeV}. \tag{47}$$

(iv) The final average value of m_s obtained from phenomenological methods given in Eq. (44) is inside the range of values quoted by PDG [39]:

$$\bar{m}_s(2\ \text{GeV}) = (80\text{–}130)\ \text{MeV}. \tag{48}$$

(v) One can also compare the value in Eq. (44) with the different lattice results [21–23]. The recent lattice results including dynamical fermions are

$$\begin{aligned} \bar{m}_s(2\ \text{GeV})|_{n_f=2} &= (100\text{–}130)\ \text{MeV}, \\ \bar{m}_s(2\ \text{GeV})|_{n_f=2+1} &= (70\text{–}90)\ \text{MeV}, \end{aligned} \tag{49}$$

which appear to depend on the number of flavors. The difference of the results for $n_f = 2$ [22] and $n_f = 2 + 1$ [23] (see, however, [24]) and the slightly higher prediction of the ChPT mass ratio $r_3 = 27.4 \pm 4.2$ defined in Eq. (37) may indicate that it is premature, at present, to extract a precise value of m_s from the lattice calculations, without a reliable control of the systematic errors, higher order terms, and some other effects.

(vi) Running $\bar{m}_s$ until $\bar{m}_b(\bar{m}_b) = (4.23 \pm 0.06)$ GeV [4,5,39,46], by taking care of the threshold effects, one can deduce the useful scale-independent quantity for model-buildings:

$$r_5 \equiv \frac{m_b}{m_s} = 50 \pm 3. \tag{50}$$

E. Implied values of the up and down quark masses from QSSR + ChPT

(i) Using the previous value of $\bar{m}_s(2\ \text{GeV})$ in Eq. (44) together with the ChPT mass ratio in Eq. (37), we can deduce

$$(\bar{m}_u + \bar{m}_d)(2\ \text{GeV}) = (7.9 \pm 0.6)\ \text{MeV}, \tag{51}$$

in nice agreement with the result from the pion sum rule in Eq. (36). Using again the ChPT mass ratio [2][11]

$$\frac{m_u}{m_d} = 0.553 \pm 0.043, \tag{52}$$

we obtain

$$\begin{aligned} \bar{m}_d(2\ \text{GeV}) &= (5.1 \pm 0.4)\ \text{MeV}, \\ \bar{m}_u(2\ \text{GeV}) &= (2.8 \pm 0.2)\ \text{MeV}. \end{aligned} \tag{53}$$

(ii) Taking the average value of $\bar{m}_s(2\ \text{GeV}) = (95.8 \pm 4.9)$ MeV, by excluding the pion sum rule result in Table III, and using the prediction of $(m_u + m_d)$ from the pion sum rule in Eq. (36), one can deduce the ratio

$$r_3 \equiv \frac{2m_s}{(m_u + m_d)} = 23.5 \pm 5.8, \tag{54}$$

in perfect agreement with the ChPT mass ratio in Eq. (37).

VIII. CONCLUSIONS

We have revisited the estimate of the strange quark mass from e^+e^- data. Including the α_s^3 plus a phenomenological

[10]Stronger lower and upper bounds from a direct extraction of the chiral condensate have been obtained in Eq. (43), but they are only known to order α_s.

[11]The different values of $(m_u + m_d)$ and $(m_d - m_u)$ from, respectively, the pseudoscalar and scalar sum rules [4,5,7,11] exclude the possibility to have $m_u = 0$.

estimate of the UV renormalon contributions parametrized by the tachyonic gluon mass λ^2, we deduce the final value from e^+e^- data in Eq. (35) and report it in Table III. We compare this value with recent determinations from different channels in Table III, known to the same level of approximations. Our final result, coming from a weighted average of different determinations from Table III, is given in Eq. (44). The updated lower and upper bounds for the strange quark mass to order α_s^3 are summarized in Eq. (47). The value and range of m_s given in Eq. (44)–(47) are inside the PDG values quoted in Eq. (48) and agree within the errors with the recent lattice calculations in Eq. (49), including dynamical fermions. Combining the final average result of m_s in Eq. (44) with the ChPT mass ratios, we deduce the value of the running u and d quark masses in Eqs. (51) and (53), while we also predict, in Eqs. (50) and (54), the useful scale-independent mass ratios $m_s/(m_u + m_d)$ and m_b/m_s. Absolute values of the light quark masses from QSSR reported in this paper are the most accurate determinations to date.

[1] E. G. Floratos, S. Narison, and E. de Rafael, Nucl. Phys. **B155**, 115 (1979).
[2] For reviews, see e.g J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982); H. Leutwyler, Nucl. Phys. B, Proc. Suppl. **94**, 108 (2001).
[3] E. de Rafael, Dubrovnik's School, Report No. QCD161:T64:1983, 1983 (unpublished).
[4] For recent reviews, see e.g. S. Narison, Nucl. Phys. B, Proc. Suppl. **86**, 242 (2000) and [5].
[5] For reviews, see e.g. S. Narison, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **17**, 1 (2002); *QCD Spectral Sum Rules*, World Sci. Lect. Notes Phys. **26**, 1 (1989); Acta Phys. Pol. **B26**, 687 (1995); Riv. Nuovo Cimento **10 N2**, 1 (1987); Phys. Rep. **84**, 263 (1982) and references therein.
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).
[7] C. Becchi, S. Narison, E. de Rafael, and F. J. Ynduràìn, Z. Phys. C **8**, 335 (1981); S. Narison and E. de Rafael, Phys. Lett. **103B**, 57 (1981); C. A. Dominguez and M. Loewe, Phys. Rev. D **31**, 2930 (1985); C. Ayala, E. Bagan, and A. Bramon, Phys. Lett. B **189**, 347 (1987); C. A. Dominguez and E. de Rafael, Ann. Phys. (N.Y.) **174**, 372 (1987); S. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. **135B**, 457 (1984); S. Narison, Phys. Lett. B **216**, 191 (1989); Riv. Nuovo Cimento **10 N2**, 1 (1987); T. Bhattacharya, R. Gupta, and K. Maltman, Phys. Rev. D **57**, 5455 (1998).
[8] J. Bijnens, J. Prades, and E. de Rafael, Phys. Lett. B **348**, 226 (1995).
[9] J. Kambor and K. Maltman, Phys. Rev. D **62**, 093023 (2000).
[10] K. Chetrykin and A. Khodjamirian, hep-ph/0512295.
[11] S. Narison, N. Paver, E. de Rafael, and D. Treleani, Nucl. Phys. **B212**, 365 (1983); C. A. Dominguez, C. van Gend, and N. Paver, Phys. Lett. B **253**, 241 (1991); P. Colangelo, F. De Fazio, G. Nardulli, and N. Paver, Phys. Lett. B **408**, 340 (1997); K. G. Chetyrkin, C. A. Dominguez, D. Pirjol, and K. Schilcher, Phys. Rev. D **51**, 5090 (1995); M. Jamin and M. Münz, Z. Phys. C **66**, 633 (1995).
[12] M. Jamin, J. A. Oller, and A. Pich, Eur. Phys. J. C **24**, 237 (2002).
[13] S. Narison and E. de Rafael, Nucl. Phys. **B169**, 253 (1980); S. Narison, Z. Phys. C **22**, 161 (1984).
[14] L. J. Reinders and H. Rubinstein, Phys. Lett. **145B**, 108 (1984).
[15] S. Narison, Phys. Lett. B **358**, 113 (1995); **466**, 345 (1999).
[16] K. Maltman, Phys. Lett. B **428**, 179 (1998).
[17] M. Eidemuller, M. Jamin, and F. Schwab, hep-ph/0310185.
[18] E. Gámiz *et al.*, Phys. Rev. Lett. **94**, 011803 (2005); Nucl. Phys. B, Proc. Suppl. **144**, 59 (2005) and references therein.
[19] P. A. Baikov, K. G. Chetyrkin, and J. H. Kühn, Phys. Rev. Lett. **95**, 012003 (2005).
[20] S. Narison, Phys. Lett. B **626**, 101 (2005).
[21] For a recent review on lattice calculations, see e.g. S. Hashimoto, Int. J. Mod. Phys. A **20**, 5133 (2005); see also S. Gupta, hep-lat/0502005.
[22] M. Della Morte *et al.*, Nucl. Phys. **B729**, 117 (2005); M. Gockeler *et al.*, Proc. Sci., LAT2005 (2005) 078 [hep-lat/0509159]; Phys. Rev. D **73**, 054508 (2006); D. Becirevic *et al.*, Nucl. Phys. **B734**, 138 (2006).
[23] C. Aubin *et al.*, Phys. Rev. D **70**, 031504 (2004); Q. Mason *et al.*, Phys. Rev. D **73**, 114501 (2006).
[24] T. Ishikawa *et al.*, Proc. Sci., LAT2005 (2005) 057.
[25] L. Lellouch, E. de Rafael, and J. Taron, Phys. Lett. B **414**, 195 (1997); E. de Rafael, Nucl. Phys. B, Proc. Suppl. **64**, 258 (1998); F. J. Yndurain, Nucl. Phys. **B517**, 324 (1998).
[26] H. G. Dosch and S. Narison, Phys. Lett. B **417**, 173 (1998).
[27] E. Braaten, S. Narison, and A. Pich, Nucl. Phys. **B373**, 581 (1992).
[28] P. A. Baikov, K. G. Chetyrkin, and J. H. Kühn, Nucl. Phys. B, Proc. Suppl. **135**, 243 (2004); K. Chetyrkin (private communication).
[29] K. Chetyrkin, S. Narison, and V. I. Zakharov, Nucl. Phys. **B550**, 353 (1999).
[30] For reviews, see e.g. V. A. Zakharov, hep-ph/0509114; hep-ph/0309301; hep-ph/0309178; Nucl. Phys. B, Proc. Suppl. **74**, 392 (1999); F. V. Gubarev, M. I. Polikarpov, and V. I. Zakharov, hep-th/9812030.
[31] S. Narison, Phys. Lett. B **300**, 293 (1993); **361**, 121 (1995); S. Narison and V. I. Zakharov, Phys. Lett. B **522**, 266 (2001).
[32] K. Chetyrkin and S. Narison, Phys. Lett. B **485**, 145 (2000).
[33] For a review, see e.g. S. Narison, hep-ph/0508259.

[34] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B191**, 301 (1981).
[35] S. Narison, Nucl. Phys. **B509**, 312 (1998); Nucl. Phys. B, Proc. Suppl. **64**, 210 (1998); Z. Phys. C **26**, 209 (1984).
[36] S. Narison, Phys. Lett. B **361**, 121 (1995); **387**, 162 (1996).
[37] G. Launer, S. Narison, and R. Tarrach, Z. Phys. C **26**, 433 (1984).
[38] S. Narison, Phys. Lett. **104B**, 485 (1981); Riv. Nuovo Cimento **10 N2**, 1 (1987); Phys. Lett. B **216**, 191 (1989); **358**, 113 (1995).
[39] C. Caso *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[40] S. Dolinsky *et al.*, Phys. Rep. **202**, 99 (1991).
[41] S. Narison, Phys. Lett. B **513**, 53 (2001); **526**, 414(E) (2002); hep-ph/0108065; hep-ph/0203053.
[42] W. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).
[43] R. Barate *et al.* (ALEPH Collaboration), Eur. Phys. J. C **4**, 409 (1998); R. Barate *et al.*, Eur. Phys. J. C **11**, 599 (1999); T. Coan *et al.* (CLEO Collaboration), Phys. Lett. B **356**, 580 (1995); R. A. Briere *et al.*, Phys. Rev. Lett. **90**, 181802 (2003); K. Ackerstaff *et al.* (OPAL Collaboration), Eur. Phys. J. C **7**, 571 (1999).
[44] S. Narison, Phys. Rev. D **73**, 114024 (2006).
[45] P. A. Baikov, K. G. Chetyrkin, and J. H. Kühn, Phys. Rev. Lett. **96**, 012003 (2006).
[46] S. Narison, Phys. Lett. B **341**, 73 (1994); **216**, 191 (1989).