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Two-photon width of η_c and η_c' from heavy-quark spin symmetry

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We evaluate the two-photon width of the pseudoscalar charmonia, $\Gamma_{\gamma\gamma}(\eta_c)$ and $\Gamma_{\gamma\gamma}(\eta_c')$, within a Heavy-Quark Spin-Symmetry setting and show that whereas the former width agrees with experiment, the latter is more than twice larger than the recent measurement by CLEO. When binding-energy effects are included in the η_c' case, the discrepancy is worse, pointing out at a possible anomaly in the η_c' decay.

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I. INTRODUCTION

Whereas heavy-quarkonium production is still a great source of debates (see [1,2] for recent reviews), the physics of quarkonium decay seems to be better understood within the conventional framework of QCD. However, a recent estimation of the ratio of the two-photon width of the η_c' to that of the η_c by the CLEO collaboration [3] seems to contradict most of the theoretical predictions [4–6]. Indeed, by assuming $\mathcal{B}(\eta_c \to KK\pi) = \mathcal{B}(\eta_c' \to KK\pi)$, they have obtained $\Gamma_{\gamma\gamma}(\eta_c') = 1.3 \pm 0.6$ keV, whereas the predictions of [4–6] range from 3.7 to 5.7 keV.

It is our purpose here to have another look at this problem using an effective Lagrangian procedure satisfying heavy-quark spin symmetry and including binding energy or equally mass effects, which would take into account features typical of radially-excited states.

Indeed, the η_c' is the first radially-excited pseudoscalar charmonium, labeled in the spectroscopic notation by 2^1S_0, with a mass $M_{\eta_c'} = 3638 \pm 5$ MeV [7], that is noticeably higher than for the ground states η_c and J/ψ. Its first observation was done by the Belle collaboration [8] in $B \to KK_SK^-\pi^+$ decay and was further confirmed by *BABAR* [9].

As a consequence of C-conservation, η_c', like η_c, can decay into two photons, which is from a theoretical point of view a rather clean channel to analyze. There have been several calculations of the $\eta_c' \to \gamma\gamma$ in the literature, some following Bethe-Salpeter equation [10], following Salpeter equation or relativistic quark models [4,5,11,12], and some based on the nonrelativistic results (see for instance [13]) but taking into account differences in the singlet and triplet wave function at the origin [6,14]. In particular, nonrelativistic calculations can only be done by considering the η_c' as a $2S$ state with the same mass, $2m_c$, as the $1S$ state with the result that the calculated decay rate differs from the η_c one only through the wave function at the origin, see Eq (3.17) of [13], or the long distance matrix element of NRQCD, see Eqs. (4.17) and (4.19) of [1].

Since the η_c' is more than 600 MeV above the η_c, the mass effects on the decay rate could be important. A better approach, which would allow the inclusion of such effects, would be to use relativistic kinematics in the calculation of the width. For this purpose, we need to construct an effective Lagrangian for the process $c\bar{c} \to \gamma\gamma$ by expanding the charm-quark propagator in powers of q^2/m_c^2, with $q = p_c - p_{\bar{c}}$, and neglecting terms of $\mathcal{O}(q^2/m_c^2)$ terms. The propagator will now depend only on the charm-quark mass and the binding energy of the charmonium state [15,16].

The effective Lagrangian derived in our approach will then allow a calculation of the decay amplitude in terms of the matrix element of a local operator. The latter is, for the two-photon decay width of η_c and η_c', the matrix element for the axial-vector current $\bar{c}\gamma_\mu\gamma_5 c$ between the vacuum and η_c or η_c'.

The nonperturbative parameters are here the decay constant $f_{n^1S_0}$ and $f_{n^3S_1}$ which can be given by the spatial wave function at the origin $\psi(0)$ [17]. It should be stressed here that our approach differs from the traditional approach in an important way. We express the decay amplitude in terms of the matrix element of a local operator which could be measured or extracted from measured physical quantities, like the leptonic-decay constant or could also be computed via sum rules [18] or lattice simulations [19].

We shall rely on the heavy-quark spin-symmetry (HQSS) relations [20–22] which state the equality between f_{η_c} and $f_{J/\psi}$ and between $f_{\eta_c'}$ and $f_{\psi'}$. The derivation of these relations is based on the fact that, in our approach, the flavor-conserving charm-quark currents $\bar{c}\gamma_\mu c$ and $\bar{c}\gamma_\mu\gamma_5 c$ take the form of an effective current in which c and $\bar{c}$ are replaced by static heavy-quark field operator and the $\mathcal{O}(1/(2m_c))$ terms are neglected.

In this paper, we shall first derive the effective Lagrangian for the decay of singlet $1S$ state of charmonium into two photons. We then show that this effective Lagrangian, combined with HQSS, gives the same result as the traditional nonrelativistic approach and produces a decay rate for η_c in agreement with measurement. In the next section, we use this Lagrangian to determine the η_c' two-photon decay rate in terms of the ψ' leptonic width, as our main purpose here is to see whether our approach, which works for the η_c, could explain the observed decay rate of the η_c' into two photons.

II. EFFECTIVE LAGRANGIAN FOR $^1S_0$ DECAY INTO TWO PHOTONS

As announced, we now write down an effective Lagrangian for the coupling of the $c\bar{c}$ pair to two photons and to a dilepton pair $\ell\bar{\ell}$ (see Fig. 1):

$$\begin{aligned}\mathcal{L}^{\gamma\gamma}_{\rm eff} &= -ic_1(\bar{c}\gamma^\sigma\gamma^5 c)\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}A^\rho \\ \mathcal{L}^{\ell\bar{\ell}}_{\rm eff} &= -c_2(\bar{c}\gamma^\mu c)(\ell\gamma_\mu\bar{\ell})\end{aligned} \tag{1}$$

with $c_1 \simeq \frac{Q_c^2(4\pi\alpha_{em})}{M^2_{\eta_c}+b_{\eta_c}M_{\eta_c}}$ and $c_2 = \frac{Q_c(4\pi\alpha_{em})}{M^2_\psi}$.

The factor $1/(M^2_{\eta_c} + b_{\eta_c}M_{\eta_c})$ in c_1 contains the binding-energy effect [15,16] and is obtained from the denominator of the charm-quark propagator (k_1, k_2 being the outgoing-photon momenta):

$$\frac{1}{[(k_1 - k_2)^2/4 - m_c^2]} \tag{2}$$

by neglecting the term containing the relative momenta $q = p_c - p_{\bar{c}}$ of the quarks. For real photons, this factor can be written as

$$-\frac{1}{[(M^2 + bM)/2]} \tag{3}$$

with $b(= 2m_c - M)$, the bound-state binding energy and M the charmonium mass (in order to be consistent, we keep only term linear in b, since the $O(q^2/m_c^2)$ terms have been neglected in the propagator).

III. HEAVY-QUARK SPIN-SYMMETRY PREDICTION FOR $\Gamma_{\gamma\gamma}(\eta_c)$

First, we want to redo the calculation of $\Gamma_{\ell\bar{\ell}}(\psi)$ and $\Gamma_{\gamma\gamma}(\eta_c)$ through the simple application of heavy-quark spin symmetry (HQSS) and to show that the results are identical to those of nonrelativistic calculations.

Defining $\langle 0|\bar{c}\gamma^\mu c|\psi\rangle \equiv f_\psi M_\psi \varepsilon^\mu$, we have the following expression for the amplitude for $\psi \to \ell\bar{\ell}$:

$$\mathcal{M}_{\ell\bar{\ell}} = Q_c(4\pi\alpha_{em})\frac{f_\psi}{M_\psi}\varepsilon^\mu(\ell\gamma_\mu\bar{\ell}) \tag{4}$$

from which we obtain the width (neglecting the lepton masses):

$$\Gamma_{\ell\bar{\ell}}(\psi) = \frac{1}{64\pi^2 M_\psi}\int d\Omega|\mathcal{M}|^2 = \frac{4\pi Q_c^2\alpha_{em}^2 f_\psi^2}{3M_\psi}. \tag{5}$$

Using $M_\psi f_\psi^2 = 12|\psi(0)|^2$ [17], we recover the nonrelativistic result of Kwong *et al.* [13]. The experimental value for the leptonic width of the J/ψ ($\Gamma_{e^+e^-}(J/\psi) = 5.40 \pm 0.15 \pm 0.07$ keV [7]) and its mass (3.097 GeV) fixes—omitting NLO corrections for now—$f_{J/\psi}$ at 410 MeV. For the ψ', we correspondingly get $f_{\psi'}$ at 279 MeV for $\Gamma_{e^+e^-}(\psi') = 2.10 \pm 0.12$ keV [7] and $M_{\psi'} = 3.686$ GeV.

Similarly, with $\langle 0|\bar{c}\gamma^\mu\gamma^5 c|\eta_c\rangle \equiv if_{\eta_c}P^\mu$, the amplitude for $\eta_c \to \gamma\gamma$ is readily obtained:

$$\mathcal{M}_{\gamma\gamma} = -4iQ_c^2(4\pi\alpha_{em})\frac{f_{\eta_c}}{M^2_{\eta_c} + b_{\eta_c}M_{\eta_c}}\epsilon_{\mu\nu\rho\sigma}\varepsilon_1^\mu\varepsilon_2^\nu k_1^\rho k_2^\sigma \tag{6}$$

from which we obtain the $\eta_c(1S)$ width (with $b_{\eta_c} \simeq 0$):

$$\Gamma_{\gamma\gamma}(\eta_c) = \frac{1}{2}\frac{1}{64\pi^2 M_{\eta_c}}\int d\Omega|\mathcal{M}|^2 = \frac{4\pi Q_c^4\alpha_{em}^2 f_{\eta_c}^2}{M_{\eta_c}}, \tag{7}$$

the factor $\frac{1}{2}$ being the Bose-symmetry factor.

As suggested by HQSS, let us now suppose the equality between $f_{J/\psi}$ and f_{η_c}, enabling us the following evaluation, $\Gamma_{\gamma\gamma}(\eta_c) = 7.46$ keV.

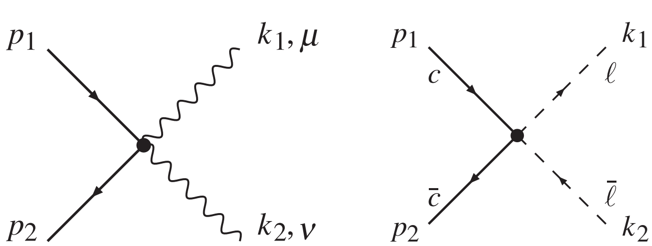
When NLO corrections are taken into account [13],

$$\begin{aligned}\Gamma^{NLO}(^3S_1) &= \Gamma^{LO}\left(1 - \frac{\alpha_s}{\pi}\frac{16}{3}\right) \\ \Gamma^{NLO}(^1S_0) &= \Gamma^{LO}\left(1 - \frac{\alpha_s}{\pi}\frac{(20 - \pi^2)}{3}\right),\end{aligned} \tag{8}$$

with $\alpha_s = 0.26$, $\Gamma_{\gamma\gamma}(\eta_c)$ is shifted to 9.66 keV. The latter agrees with the world-average value $7.4 \pm 0.9 \pm 2.1$ keV [7] in view of the large statistic and systematic uncertainties in the measured value. This indicates that our effective Lagrangian approach can also successfully predict the η_c two-photon width. The agreement with experiment also suggests that there is no large spin-symmetry breaking term in the charm vector and axial-vector current matrix elements. We now use the same effective Lagrangian and HQSS to compute the η_c' two-photon width.

IV. HQSS PREDICTIONS FOR $\Gamma_{\gamma\gamma}(\eta_c')$

We now turn to the excited states. Extrapolating HQSS to $2S$ states, i.e. $f_{\psi'} = f_{\eta_c'}$, and neglecting binding-energy effects, we obtain $\Gamma_{\gamma\gamma}(\eta_c') = \Gamma_{\gamma\gamma}(\eta_c)\frac{f^2_{\psi'}}{f^2_{J/\psi}} = 3.45$ keV, which is more than twice larger than the evaluation by CLEO (1.3 ± 0.6 keV) although nearly in agreement with Ackleh *et al.* [4] (3.7 keV), Kim *et al.* [5] (4.44 ± 0.48 keV), Ahmady *et al.* [6] ($5.7 \pm 0.5 \pm 0.6$ keV).



(a) $cc \to \gamma\gamma$ (b) $cc \to \ell\bar{\ell}$

FIG. 1. Effective coupling between a $c\bar{c}$ pair and two photons (a) and a dilepton pair (b).

TABLE I. Summary of experimental measurements and theoretical predictions for $\Gamma_{\gamma\gamma}(\eta_c)$ and $\Gamma_{\gamma\gamma}(\eta'_c)$. (All values are in units of keV).

$\Gamma_{\gamma\gamma}$	Experiments	This paper	Ackleh [4]	Kim [5]	Ahmady [6]	Münz [11]	Chao [10]	Ebert [12]
η_c	$7.4 \pm 0.9 \pm 2.1$ (PDG [7])	7.5–10	4.8	7.14 ± 0.95	$11.8 \pm 0.8 \pm 0.6$	3.5 ± 0.4	5.5	5.5
η'_c	1.3 ± 0.6 (CLEO [3])	3.5–4.5	3.7	4.44 ± 0.48	$5.7 \pm 0.5 \pm 0.6$	1.38 ± 0.3	2.1	1.8

Binding-energy effects are easily taken into account by introducing a correcting factor such that $\Gamma_{\gamma\gamma}(\eta'_c)$ can be written as as a function of $\Gamma_{\gamma\gamma}(\eta_c)$, $\Gamma_{e^+e^-}(J/\psi)$ and $\Gamma_{e^+e^-}(\psi')$ as follows

$$\Gamma_{\gamma\gamma}(\eta'_c) = \Gamma_{\gamma\gamma}(\eta_c)\left(\left(\frac{M^2_{\eta_c} + b_{\eta_c}M_{\eta_c}}{M^2_{\eta'_c} + b_{\eta'_c}M_{\eta'_c}}\right)^2 \frac{M^3_{\eta'_c}}{M^3_{\eta_c}}\right) \times \left(\frac{\Gamma_{e^+e^-}(\psi')}{\Gamma_{e^+e^-}(J/\psi)}\frac{M_{\psi'}}{M_{J/\psi}}\right). \quad (9)$$

This gives

$$\Gamma_{\gamma\gamma}(\eta'_c) = 4.1 \text{ keV}, \quad (10)$$

therefore the introduction of differences in the mass of η_c and η'_c increases the discrepancy with the experimental result obtained by CLEO. Note that, up to corrections due to differences in the scale of α_s, the radiative corrections are canceled in Eq. (9) as well as in the formula giving the first quoted value, 3.45 keV. If one wanted to introduce relativistic corrections in the spirit of NRQCD, one would expect them to cancel also, following Eqs (4.3c), (4.3d), (A31), (A32c), (A34) and (A35a) of Bodwin *et al.* [23].

It has to be noted however that the experimental values of $\Gamma_{\gamma\gamma}(\eta_c)$ are affected by a large systematic uncertainty related to the branching $\mathcal{B}(\eta_c \to KK\pi)$ and the evaluation of $\Gamma_{\gamma\gamma}(\eta'_c)$ done by CLEO was realised by assuming $\mathcal{B}(\eta_c \to KK\pi) = \mathcal{B}(\eta'_c \to KK\pi)$ which is only to hold approximately. This assumption also allows an extraction of $\mathcal{B}(B \to K\eta'_c)$ from the Belle measurement of the ratio $(\mathcal{B}(B \to K\eta'_c) \times \mathcal{B}(\eta'_c \to KK\pi))/(\mathcal{B}(B \to K\eta_c) \times \mathcal{B}(\eta_c \to KK\pi)$ [8]. The value of the ratio $\mathcal{B}(B \to K\eta'_c)/\mathcal{B}(B \to K\eta_c)$ thus obtained seems to agree with a theoretical prediction using QCD factorisation model for color-suppressed B decays with a charmonium in the final state [24]. Thus the assumption of the approximate equality between the $\eta'_c \to KK\pi$ and $\eta_c \to KK\pi$ branching ratio seems to be justified to some extent. In this case, the CLEO low value for the ratio $(\mathcal{B}(\eta'_c \to \gamma\gamma)\mathcal{B}(\eta'_c \to KK\pi))/(\mathcal{B}(\eta_c \to \gamma\gamma)\mathcal{B}(\eta_c \to KK\pi))$ would imply the small $\eta'_c \to \gamma\gamma$ decay rate quoted above.

There exist however models that are able to reproduce correctly $\Gamma_{\gamma\gamma}(\eta'_c)$ but, in general, they tend to underestimate $\Gamma_{\gamma\gamma}(\eta_c)$. Indeed, Münz [11] predicts 3.5 ± 0.4 keV for η_c and 1.38 ± 0.3 keV for η'_c, Chao *et al.* [10] 5.5 keV for η_c and 2.1 keV for η'_c, and Ebert *et al.* [12] 5.5 keV for η_c and 1.8 keV for η'_c (see also the results of [25]). This clearly points at a specificity not yet understood of the η'_c decay. All the theoretical predictions and the experimental measurements can be found in Table I.

V. CONCLUSION

Whereas heavy quarkonia are supposed to be reasonably described by nonrelativistic approximations, some works (e.g. [26–29]) have pointed out that nonstatic effects within quarkonium (especially in radially-excited states) should not be neglected without further considerations.

In the case of η_c, we have seen that the simple application of HQSS gives a reasonable estimate of the width compared to the world-average experimental measurements. For η'_c, we have obtained the same discrepancy as other models. On the other hand, we have shown here that the introduction of binding energy in this calculation introduces a correction of about 20% but worsens the comparison with the CLEO measurement.

When one considers the ratio of the two decay widths, radiative corrections cancel out, up to effects due to changes in the renormalization scale. This might slightly affect the results, but not sufficiently to recover agreement with data. Of course, heavy-quark spin symmetry (or, equivalently, the equality between the decay constant for the $^3S_1$ and the $^1S_0$) could be broken for excited states, but it is quite unlikely that it could be so badly broken to explain such a discrepancy.

Since many other works have shown difficulties to reproduce both η_c and η'_c two-photon widths, we are looking forward for a confirmation of the CLEO measurements, especially through a better understanding of their branching ratio in $KK\pi$.

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