Casimir force in a Lorentz violating theory

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We study the effects of the minimal extension of the standard model including Lorentz violation on the Casimir force between two parallel conducting plates in the vacuum. We provide explicit solutions for the electromagnetic field using scalar field analogy, for both the cases in which the Lorentz violating terms come from the *CPT*-even or *CPT*-odd terms. We also calculate the effects of the Lorentz violating terms for a fermion field between two parallel conducting plates and analyze the modifications of the Casimir force due to the modifications of the Dirac equation. In all cases under consideration, the standard formulas for the Casimir force are modified by either multiplicative or additive correction factors, the latter case exhibiting different dependence on the distance between the plates.

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I. INTRODUCTION

The search for a theory beyond the minimal $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model (SM) is motivated by the fact that, although phenomenologically successful, the SM suffers from some theoretical inconsistencies, and from some long-standing unresolved problems. More general scenarios exist, in which the SM is viewed as a low-energy limit of a more fundamental theory, which should be able to provide a quantum description of gravitation.

An interesting alternative at the Planck scale is the possibility that the new physics scenario involves a violation of Lorentz symmetry. In particular, in the context of string theories, it has been shown that spontaneous breaking of Lorentz symmetry may occur with Lorentz-covariant dynamics [1]. In these theories, interactions are triggered by nonzero expectation values for Lorentz tensors, because spontaneous breaking of the higher-dimensional Lorentz invariance is expected in any realistic Lorentz-covariant fundamental theory involving more than four space-time dimensions. If the breaking extends into the four macroscopic space-time dimensions, the Lorentz symmetry violation could appear at the level of the SM. Because the breaking is spontaneous, Lorentz symmetry remains a property of the underlying fundamental theory. Another important property of the spontaneous breaking is that the theory remains invariant under *observer* Lorentz transformations, i.e., under rotations and boosts of an observer's inertial frame. In addition to that, conventional quantization, hermiticity, gauge invariance, power-counting renormalizability, and the expected microcausality and positivity of the energy are also maintained. Note also that there are other ways to break Lorentz symmetry. It can, for example, occur dynamically in quantum field theories [2], or via a *CPT* anomaly in compact spaces [3].

Such a framework for examining the effects of spontaneous Lorentz breaking in the context of a low-energy effective theory, known as the standard-model extension (SME), has been developed explicitly by Colladay and Kostelecký first without gravity [4] and later by Kostelecký with gravity [5]. Gravitationally coupled SME is described in non-Minkowski space-times and leads to space-time-dependent coefficients; and recently, the pure gravity sector of the model has been studied [5]. In this paper, we confine the framework into the minimal SME without gravity.[1] General reviews of the model and the status of the recent experimental investigations can be found in Refs. [6,7]. An extensive analysis of the implications of the model for electrostatics and magnetostatics in SME, as well as discussion of its QED sector can be found in [8–10].

The SME [4] has been the focus of various experimental studies, including ones with photons, from radiative corrections, photon splitting and vacuum Čerenkov and synchrotron radiation [9–11], electrons [12–14], protons and neutrons, including baryosynthesis and nucleosynthesis in [15–17], *CPT* phenomena in mesons [18], muons [19], neutrinos [20], and Higgs bosons [21]. Though no evidence for Lorentz violation has yet been found, only less than a half of the possible tests involving light and ordinary matter (electrons, protons, and neutrons) have been performed, while some of the other sectors remain virtually unexplored.

Our goal here is to provide additional theoretical predictions regarding the quantum vacuum in this extended theory, in particular, we concentrate on calculating the effects of the minimal SME on the Casimir force.

Zero-point fluctuations in quantum fields give rise to macroscopically observable forces between material bodies, the so-called Casimir forces [22]. In general, the

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[1]From now on, we drop the phrase ''minimal'' and use only ''SME'' to refer to the minimal version.

Casimir effect can be defined as the stress (force per unit area) on bounding surfaces when a quantum field is confined in a finite volume of space. The boundaries can be material media, interfaces between two phases of the vacuum, or topologies of space (such as in higher-dimensional theories). The simplest test is measuring the force between two parallel conducting plates. Simply imagined, the vacuum is a sea of waves of energy of all possible lengths, while between the plates only waves whose wavelengths exactly fit between the plates are present. One calculates all of the zero-point energy between the plates, and the result (which is divergent) is regulated by subtracting the value of the energy when no boundaries are present [23]. Experiments testing the Casimir force are very precise tests of the predictions of the field theory. Several experimental attempts have been pursued during various decades since its theoretical prediction, for unambiguously verifying Casimir's formula [24]. Early searches have been successful only in a particular geometry, namely, in a cavity constructed by a plane surface opposing a spherical one. Pioneering measurements by van Blokland and Overbeek [25] in such a configuration resulted in the observation of the associated Casimir force, and in its detailed comparison to the Lifshitz theory [26], by taking into account finite conductivity effects. More recently, these measurements have been performed using torsion balances [27], atomic force microscopes [28], and high precision capacitance bridges [29]. The latter two experiments have reached 1% precision, more precise determinations being limited by theoretical uncertainties. Sparnaay [30] has attempted the first experiments with parallel plates configuration, while Bressi *et al.* [31] have recently reported a measurement of the coefficient of the Casimir force between parallel conducting surfaces in the 0.5-3.0 μm range with 15% precision: $K_C = (1.22 \pm 0.18) \times 10^{-27}$ N m^2.

In this work, we investigate the Casimir effect in the vacuum of the SME. The Casimir force is a very sensitive measurement of the quantum fluctuations of the field at the macroscopic level. Since the stress between two parallel conducting plates separated by a distance of 1 μm is $K_C/(1\mu m)^4$, where K_C is of order of $\mathcal{O}(10^{-27}$ N m$^2)$, and with the hope that future experiments will improve the existing precision, this measurement should be tested against any new theory. We evaluate the stress between parallel plates, leaving evaluations of quantum fluctuations for dielectric media and different topologies for later discussions.

Our paper is organized as follows. In Sec. II we summarize the SME by giving only the relevant terms in the Lagrangian. In the subsequent sections, we calculate the Casimir force for scalar fields with two different frequencies of propagation, according to the SME (Sec. III) in the case of both *CPT*-even and *CPT*-odd Lorentz violation terms. We examine the correction for the fermion field (Sec. IV) and conclude and summarize our results in

Sec. V, where we also discuss the possibility of observing the deviations from the measured Casimir force.

II. THE LORENTZ VIOLATING MODEL

The minimal extension of the standard model (SME) as given in [4–7] contains all possible Lorentz violating terms that could arise from spontaneous symmetry breaking at a fundamental level, but that preserve $SU(3) \times SU(2) \times U(1)$ gauge invariance and power-counting renormalizability. All terms that are even or odd under *CPT* are explicitly given in Ref. [4] and will not be repeated here. We rather concentrate on the relevant part of the theory, namely, the extended quantum electrodynamics derived from SME.

The general form of a Lorentz violating term involves a part constructed from the basic fields in the standard model, whose strength is given by a coupling coefficient. This imposes various limitations on the possible structures of both the operators and the couplings. Taken together, these requirements place significant constraints on the form of allowed terms in the SME. Taken from the full SME that contains all known particles, the Lagrangian involving the Dirac field ψ of the electron and the electromagnetic field $F^{\mu\nu}$ can be written as

$$\mathcal{L}^{QED} = \frac{i}{2}\bar{\psi}\Gamma_\nu D^\nu \psi - \frac{1}{2}\bar{\psi}M\psi + \text{H.c.} - \frac{1}{4}F^{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu},$$

(2.1)

where H.c. is the Hermitian conjugate of the fermion part. The Lorentz violating parameters in the photon sector are included in the components of the tensors $(k_F)_{\kappa\lambda\mu\nu}$ (*CPT*-even) and $(k_{AF})^\kappa$ (*CPT*-odd). For the electron, the Lorentz violating parameters are encoded in Γ_ν and M as

$$\Gamma_\nu = \gamma_\nu + (c_{\mu\nu} + d_{\mu\nu}\gamma_5)\gamma^\mu + e_\nu + if_\nu\gamma_5 + \tfrac{1}{2}g_{\kappa\mu\nu}\sigma^{\kappa\mu},$$
$$M = m + (a_\mu + b_\mu\gamma_5)\gamma^\mu + \tfrac{1}{2}H_{\mu\nu}\sigma^{\mu\nu},$$ (2.2)

where the parameters in Γ_ν are dimensionless while the ones in M have dimension of mass. Similarly, in the photon sector, $(k_F)_{\kappa\lambda\mu\nu}$ is dimensionless and $(k_{AF})^\kappa$ has dimension of mass. The parameters e_ν, f_ν, and $g_{\kappa\mu\nu}$ which are not extractable directly from SME are taken to be zero or very suppressed due to the renormalizibilty and gauge invariance requirements. a_μ can also be set to zero since it is not physical (it can be absorbed by using gauge transformations). The parameters $c_{\mu\nu}$ and $d_{\mu\nu}$ are traceless[2] and $H_{\mu\nu}$ is antisymmetric. In the photon sector, $(k_F)_{\kappa\lambda\mu\nu}$ has the symmetries of a Riemann tensor plus a double traceless constraint, giving 19 independent components. Even

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[2]Any nonzero trace would not contribute to Lorentz violation and can be absorbed by a conventional field normalization of the usual kinetic operator for the matter fields.

though the components of $(k_{AF})^\kappa$ are strongly constrained by cosmological observations, we allow them to be non-zero in our theoretical investigation of Casimir force.

After presenting the terms in the Lagrangian of the extended QED, we are concerned with the modified QED vacuum. So, one can reexamine the basic features of electromagnetic radiation in this framework. Working with the standard parametrization of the plane wave with wave 4-vector $p^\alpha = (p^0, \vec{p})$: $F_{\mu\nu}(x) = F_{\mu\nu}(p)e^{-ip_\alpha x^\alpha}$, the equation determining the dispersion relation and the electric field $\vec{E}$ in the SME is the modified Ampère Law

$$M^{jk}E^k \equiv [-\delta^{jk}p^2 - p^j p^k - 2(k_F)^{j\beta\gamma k}p_\beta p_\gamma$$
$$- 2i(k_{AF})_\beta \epsilon^{j\beta\gamma k}p_\gamma]E^k = 0. \qquad (2.3)$$

The dispersion relation is obtained by requiring the vanishing of the determinant of M^{jk}, and one obtains two modes for the radiation field [4]:

$$p^0_\pm = (1+\rho)|\vec{p}| \pm \sqrt{\sigma^2 \vec{p}^2 + \tau^2}, \qquad (2.4)$$

where

$$\rho = -\tfrac{1}{2}\tilde{k}_\alpha{}^\alpha, \qquad \sigma^2 = \tfrac{1}{2}(\tilde{k}_{\alpha\beta})^2 - \rho^2, \qquad \tau = (k_{AF})^\mu \hat{p}_\mu \qquad (2.5)$$

with

$$\tilde{k}^{\alpha\beta} = (k_F)^{\alpha\mu\beta\nu}\hat{p}_\mu \hat{p}_\nu, \qquad \hat{p}^\mu = p^\mu/|\vec{p}|. \qquad (2.6)$$

For vanishing k_{AF} coefficients, and to leading order in the coefficients k_F for Lorentz violation it becomes:

$$p^0_\pm = (1 + \rho \pm \sigma)|\vec{p}|. \qquad (2.7)$$

Here $\vec{p}^2\rho$ and $\vec{p}^2\sigma$ are observer Lorentz scalars, which implies ρ and σ are scalars under observer rotations. Note that in the leading order, $\hat{p}_\mu = (1, \vec{p}/|\vec{p}|)$ is assumed.

The dispersion relation (2.7) has two solutions, corresponding to electric field values $\vec{E}_\pm$. In conventional electrodynamics, the dispersion relation is $p^0 = |\vec{p}|$ and thus the propagation is independent of the polarization. However, in the present case the propagation is governed by two specific modes $\vec{E}_\pm$, with the general solution to (2.3) being any linear combination of the two, leading the phenomenon known as birefringence [4]. This represents a fundamentally different description of photon fields propagating through the vacuum from conventional QED. The vacuum Lorentz breaking extension of electrodynamics resembles more electrodynamics in moving media [4,8]. Thus we expect changes in the vacuum energy, which impact on the evaluation of the Casimir force.

III. THE SCALAR FIELD

Two main methods exist in the literature to calculate the vacuum energy of a field confined between parallel plates. In the simplest and most transparent method, one simply sums over all the modes of the fields in between the plates and then regularizes the result by subtracting the energy in the free space. A more rigorous technique uses consistent Green's functions and is presented in the following subsection. Summing the modes correctly produces the finite, observable force starting from a divergent formal expression, without any explicit subtractions, and is therefore of great utility in practice. In what follows, we assume that the photon field can be approximated by two scalar fields of frequency $\omega_\pm = (1 + \rho)|\vec{p}| \pm \sqrt{(\sigma^2 \vec{p}^2 + \tau^2)}$. Following normal procedure, we evaluate the correction to the usual Casimir energy by taking first $(k_{AF})^\kappa = 0$, $(k_F)_{\kappa\lambda\mu\nu} \neq 0$ (Case I), then $(k_{AF})^\kappa \neq 0$, $(k_F)_{\kappa\lambda\mu\nu} = 0$ (Case II).

A. Case I: $k_{AF} = 0$

1. Summing of the modes

For simplicity, the electromagnetic field between parallel, uncharged, perfectly conducting plates in the SME due to the *CPT*-even Lorentz violating parameter in the Lagrangian is seen as two massless scalar fields ϕ confined between two parallel plates separated by a distance D. In conventional QED, one accounts for the two polarization modes by simply multiplying the scalar result by 2. In SME, the two scalar fields will propagate with different frequencies, due to birefringence. The scalar field satisfies Dirichlet boundary conditions on the plates, that is:

$$\phi(0) = \phi(D) = 0. \qquad (3.1)$$

The Casimir force between the plates results from the zero-point energy per unit transverse area

$$u = \tfrac{1}{2}\sum \hbar\omega = \tfrac{1}{2}\sum_n \hbar(\omega_{n,1} + \omega_{n,2}), \qquad (3.2)$$

where $\omega_{n,1}$ and $\omega_{n,2}$ represent the two zero-point energies associated with the surface modes, labeled by the positive integer n [32] of the electromagnetic fields in the SME. The formalism bears some resemblance to the treatment of the force in a dispersive medium of effective "dielectric" constant ϵ where $\epsilon_\pm = (1 + \rho \pm \sigma)^2$.[3] The so-called surface modes are associated with the zero of the wave number:

$$\kappa^2_{1,2} = \vec{k}^2 - \omega^2_\pm = \vec{k}^2 - \epsilon_\pm \omega^2, \qquad (3.3)$$

where we set $\hbar = c = 1$, where $\vec{k}$ is the transverse momentum, for the simple case where $\omega_\pm \equiv p^0_\pm = (1 + \rho \pm \sigma)\omega$, ω being the frequency in the vacuum in the absence of dispersion. In this case (which corresponds to the conventional QED vacuum), the expression for the frequency

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[3]This is a special case of the Lifschitz theory [26], where the Casimir force is evaluated in a dielectric medium between two arbitrary parallel dielectrics, in the limit where the plates are perfect conductors.

of the field would be $\omega = |\vec{k}|$ for both surface modes; while in SME the frequencies of the two surface modes are $\omega_{1,2} = \frac{\omega}{\sqrt{\epsilon_\pm}}$, and the energy of the zero modes becomes:

$$u = \frac{1}{2} \sum_{n=1}^{\infty} \int \frac{d^2k}{(2\pi)^2} \sqrt{k^2 + \frac{n^2\pi^2}{D^2}} \left[\frac{1}{(1+\rho+\sigma)} \right.$$
$$\left. + \frac{1}{(1+\rho-\sigma)} \right] \tag{3.4}$$
$$= \Lambda \sum_{n=1}^{\infty} \int \frac{d^2k}{(2\pi)^2} \sqrt{k^2 + \frac{n^2\pi^2}{D^2}},$$

where we denoted $\Lambda = \frac{1}{2}\left[\frac{1}{(1+\rho+\sigma)} + \frac{1}{(1+\rho-\sigma)}\right]$ and k_z is obtained from the boundary conditions.

To evaluate Eq. (3.4) we employ dimensional regularization. We let the number of the transverse dimensions to be d, which we will treat as a continuous, complex variable. We also employ the Schwinger proper-time representation for the square root, so that we have

$$u = \Lambda \sum_n \int \frac{d^dk}{(2\pi)^d} \int_0^\infty \frac{dt}{t} t^{-1/2} e^{-t(k^2+n^2\pi^2/D^2)} \frac{1}{\Gamma(-\frac{1}{2})}, \tag{3.5}$$

where we have used the Euler representation for the Gamma function. We follow the standard procedure and carry out the Gaussian integration over k, use the Euler representation, and use the definition of the Riemann zeta function ζ to obtain:

$$u = -\Lambda \frac{1}{2\sqrt{\pi}} \frac{1}{(4\pi)^{d/2}} \left(\frac{\pi}{D}\right)^{d+1} \Gamma\left(-\frac{d+1}{2}\right) \zeta(-d-1). \tag{3.6}$$

When d is an odd integer, this expression is indeterminate, but we use $\Gamma(\frac{z}{2})\zeta(z)\pi^{-z/2} = \Gamma(\frac{1-z}{2})\zeta(1-z)\pi^{(z-1)/2}$ to rewrite (3.6) as

$$u = -\Lambda \frac{1}{2^d \pi^{d/2+1}} \frac{1}{D^{d+1}} \Gamma\left(1+\frac{d}{2}\right) \zeta(2+d) \tag{3.7}$$

and obtain the final result for the energy per unit area in the transverse direction in SME

$$u = -\Lambda \frac{\pi^2}{720} \frac{1}{D^3}, \tag{3.8}$$

where we used $\zeta(4) = \pi^4/90$. The force per unit area between the plates is obtained by taking the negative derivative of u with respect to D:

$$f_s = -\frac{\partial}{\partial D} u = -\Lambda \frac{\pi^2}{240} \frac{1}{D^4}. \tag{3.9}$$

The above result represents the Casimir force due to two scalar fields of frequency ω_+ and ω_-. Comparing this to the classic result of Casimir [22]:

$$f_{\rm em} = -\frac{\pi^2}{240} \frac{1}{D^4}, \tag{3.10}$$

the deviation in the SME is given by the multiplicative correction factor Λ. For $\rho, \sigma \ll 1$, $\Lambda \cong 1 - \rho$, so the expression is simplified to

$$f_s^{CPT-\text{even}} \cong (1-\rho)f_{\rm em}. \tag{3.11}$$

Thus the lowest-order prediction of the SME is a reduction in the Casimir force by $\rho f_{\rm em}$.

2. Scalar Green's function

We derive the result of the previous subsection by the Green's function approach for a scalar field. A more general indexed Green's function for the photon in the SME framework, in the Coulomb gauge, has been given in [8]. We start from the equation of motion of a massless scalar field ϕ produced by a source K

$$-(\partial_t^2 \epsilon_\pm - \vec{\nabla}^2)\phi = K, \tag{3.12}$$

from which we deduce the equation satisfied by the corresponding Green's function

$$-(\partial_t^2 \epsilon_\pm - \vec{\nabla}^2)G(x, x') = \delta(x - x'). \tag{3.13}$$

We introduce a reduced Green's function $g(z, z')$ using the Fourier transformation

$$G(x, x') = \int \frac{d^dk}{(2\pi)^d} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} g(z, z'), \tag{3.14}$$

where we have suppressed the dependence of g on $\vec{k}$ and ω, and have allowed z on the right-hand side to represent the coordinate perpendicular to the plates. The reduced Green's function satisfies

$$\left(-\frac{\partial^2}{\partial z^2} - \lambda_\pm^2\right)g(z, z') = \delta(z - z'), \tag{3.15}$$

where $\lambda_\pm^2 = \omega_\pm^2 - k^2 \equiv \omega^2 \epsilon_\pm - k^2$ represent the two scalar modes. Equation (3.34) is to be solved subject to the boundary conditions (3.1), or

$$g(0, z') = g(D, z') = 0. \tag{3.16}$$

The form of the solution, obtained by the discontinuity method [23] for each of the two scalar modes, is

$$g(z, z') = \begin{cases} A \sin\lambda z, & 0 < z < z' < D, \\ B \sin\lambda(z - D), & D > z > z' > 0, \end{cases} \tag{3.17}$$

where g is continuous at $z = z'$, but its derivative has a discontinuity, which is:

$$A \sin\lambda_\pm z' - B \sin\lambda_\pm(z' - D) = 0, \tag{3.18}$$

$$A\lambda_\pm \cos\lambda_\pm z' - B\lambda \cos\lambda(z' - D) = 1. \tag{3.19}$$

The solution to this system of equations is

$$A = -\frac{1}{\lambda_\pm} \frac{\sin\lambda_\pm(z'-D)}{\sin\lambda_\pm D}, \tag{3.20}$$

$$B = -\frac{1}{\lambda_\pm} \frac{\sin\lambda_\pm z'}{\sin\lambda_\pm D}, \tag{3.21}$$

which gives for the reduced Green's function

$$g_\pm(z,z') = -\frac{1}{\lambda_\pm \sin\lambda_\pm D} \sin\lambda_\pm z_< \sin\lambda_\pm(z_> - D), \tag{3.22}$$

where $z_>$ ($z_<$) is the greater (lesser) of z and z'.

From the Green's function we can calculate the force on the bounding surfaces by evaluating the stress tensor. For a scalar field, the stress tensor is obtained from

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}\mathcal{L}, \tag{3.23}$$

where the Lagrange density is

$$\mathcal{L} = -\tfrac{1}{2}\partial_\lambda\phi\partial^\lambda\phi. \tag{3.24}$$

The normal-normal component of the Fourier transform of the stress tensor on the boundaries is

$$\langle T_{zz}\rangle = \frac{1}{2i}\partial_z\partial_{z'}g_\pm(z,z')\Big|_{z\to z'=0,D} = \frac{i}{2}\lambda_\pm \cot\lambda_\pm D. \tag{3.25}$$

However this expression will not give rise to a finite result [23], because we did not consider the discontinuity in T_{zz}. The corresponding normal-normal component of the stress tensor at $z = D$ is

$$\langle T_{zz}\rangle\Big|_{z=z'=D} = \frac{1}{2i}\partial_z\partial_{z'}g_\pm(z,z')\Big|_{z=z'=D} = \frac{\lambda_\pm}{2}. \tag{3.26}$$

Thus the correct expression for the force per unit area on the conducting surface is obtained integrating over all possible frequencies and momenta, and using complex frequencies $\omega \to i\xi$, $\lambda_\pm \to i\kappa_\pm$:

$$f_s = -\frac{1}{2}\int\frac{d^d k}{(2\pi)^d}\Bigg[\int\frac{d\xi}{2\pi}\kappa_+(\coth\kappa_+ D - 1) + \int\frac{d\xi}{2\pi}\kappa_-(\coth\kappa_- D - 1)\Bigg]. \tag{3.27}$$

We evaluate this integral using polar coordinates:

$$f_s = -\frac{1}{2}\left(\frac{1}{1+\rho+\sigma} + \frac{1}{1+\rho-\sigma}\right)\frac{A_{d+1}}{(2\pi)^{d+1}}$$

$$\times \int_0^\infty \kappa^d d\kappa \frac{\kappa}{e^{2\kappa D}-1}. \tag{3.28}$$

Here A_{d+1} is the surface area of a unit sphere in $d+1$ dimensions. The force per unit transverse area in the SME is

$$f_s = \Lambda(d+1)2^{-d-2}\pi^{-d/2-1}\frac{\Gamma(1+\frac{d}{2})\zeta(d+2)}{D^{d+2}} \tag{3.29}$$

which, for $d = 2$, using $\Gamma(2) = 1$ and $\zeta(4) = \frac{\pi^4}{90}$ gives

$$f_s^{CPT\text{-even}} = -\Lambda\frac{\pi^2}{240}\frac{1}{D^4}, \tag{3.30}$$

which confirms the formula obtained before, (3.9).

B. Case II: $k_{AF} \neq 0$, scalar Green's function

If one starts from the photon part of the Lagrangian (2.1) by setting $k_F = 0$, the equation of motion is modified to:

$$-\partial_\alpha F_\mu{}^\alpha + (k_{AF})^\alpha \epsilon_{\mu\alpha\beta\gamma}F^{\beta\gamma} = 0. \tag{3.31}$$

This expression is similar to the one obtained in the analysis of the Casimir effect in two dimensions (the so-called Maxwell-Chern-Simons Casimir effect [23]). The gauge field is effectively massive, and it satisfies the equation:

$$[-\partial^2 + (k_{AF})^2]\epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta = 0 \tag{3.32}$$

with the factor k_{AF} playing the role of the mass. The reduced Green's function

$$G(x,x') = \int\frac{d^d k}{(2\pi)^d}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}g(z,z'), \tag{3.33}$$

satisfies the equation:

$$\left(-\frac{\partial^2}{\partial z^2} - \lambda_{I,II}^2 + (k_{AF})2^2\right)g_{I,II}(z,z') = \delta(z-z'), \tag{3.34}$$

where $\lambda_{I,II}^2 = \omega_{I,II}^2 - k^2$, $\omega_{I,II} = (1 \pm \tau)p^0$. This gives, for the reduced Green's function:

$$g_{I,II}(z,z') = -\frac{1}{\kappa_{1,2}\sin\kappa_{I,II}D}\sin\kappa_{1,2}z_<\sin\kappa_{I,II}(z_> - D), \tag{3.35}$$

where $z_>$ ($z_<$) is the greater (lesser) of z and z' and $\kappa_{I,II} = \lambda_{I,II}^2 - (k_{AF})^2$. The Casimir force becomes (with the usual Schwinger change $\omega \to i\xi$, $\kappa \to i\rho$):

$$f_s = -\frac{1}{2}\int\frac{d^d k}{(2\pi)^d}\Bigg[\int\frac{d\xi}{2\pi}\rho_I(\coth\rho_I D - 1)\Bigg]$$

$$-\frac{1}{2}\int\frac{d^d k}{(2\pi)^d}\Bigg[\int\frac{d\xi}{2\pi}\rho_{II}(\coth\rho_{II}D - 1)\Bigg]. \tag{3.36}$$

Neglecting terms of higher order in σ, the equation can be transformed, working in polar coordinates, into:

$$f_s = -\frac{4\pi^{(d+1)/2}}{\Gamma(\frac{(d+1)}{2})(2\pi)^{(d+1)}}\int\rho^d\frac{d^d\rho\sqrt{\rho^2+(k_{AF})^2}}{e^{2a\sqrt{\rho^2+(k_{AF})^2}}-1}. \tag{3.37}$$

Setting $x = 2D\sqrt{\rho^2 + (k_{AF})^2}$, $d = 2$, and approximating,

for small values of $(k_{AF}D)^2 \ll 1$, $(x^2 - 4(k_{AF})^2D^2)^{1/2} \cong x - \frac{2(k_{AF})^2D^2}{x}$:

$$f_s \cong -\frac{1}{16\pi^2 D^4}\left[\int_0^\infty \frac{x^3 dx}{e^x - 1} - 2(k_{AF})^2 D^2 \int_0^\infty \frac{x dx}{e^x - 1}\right],$$
(3.38)

which gives, expressed in terms of the usual Riemann zeta functions:

$$f_s \cong -\frac{1}{16\pi^2 D^4}[\Gamma(4)\zeta(4) - 2(k_{AF})^2 D^2 \Gamma(2)\zeta(2)]$$

$$= -\frac{\pi^2}{240D^4} + \frac{(k_{AF})^2}{48D^2}$$
(3.39)

or

$$f_s^{CPT\text{-odd}} = -\frac{\pi^2}{240D^4} + \frac{(k_{AF})^2}{48D^2}.$$
(3.40)

One recognizes in the first term the usual scalar Casimir force, while the second represents the correction from the *CPT*-odd terms in the photon Lagrangian, which has a different dependence on the separation between the plates $(1/D^2)$. However, it is very unlikely to have any observable effect since k_{AF} is constrained to have very tiny values and appears squared in f_s; also the weaker separation dependence makes the sensitivity to k_{AF} even weaker.

IV. THE FERMION FIELD

By taking the fermion part of the Lagrangian (2.1) and setting e_ν, f_ν, and $g_{\kappa\mu\nu}$ to zero, the modified Dirac equation can be obtained as:

$$\{i[\gamma^\mu + \gamma_\nu(c^{\mu\nu} + d^{\mu\nu}\gamma_5)]D_\mu - [m_e + V(r) + a_\mu\gamma^\mu$$
$$+ b_\mu\gamma_5\gamma^\mu + H_{\mu\nu}\sigma^{\mu\nu}]\}\psi = 0, \quad (4.1)$$

with $D_\mu = i\partial_\mu + ieA_\mu$, and we assume a conventional photon sector.

To reproduce the boundary set for the Casimir effect between two plates for massless fermions, take $V = V_0\theta(x)$ and $A_\mu = 0$, and set $\psi(r, t) = \psi(r)e^{i\omega t}$,

$$\{-in_i[\gamma^i + (c^{\nu i} + d^{\nu i}\gamma_5)\gamma_\nu] - 1\}V_0\psi(r) = 0, \quad (4.2)$$

where n_μ represents an outward normal at a boundary surface. Note that neither a_μ nor b_μ appears in the boundary condition. Taking $V_0 \to \infty$, we obtain the boundary condition corresponding to the Dirac equation for the SME:

$$\{1 + in_i[\gamma^i + (c^{\nu i} + d^{\nu i}\gamma_5)\gamma_\nu]\}\psi(r)|_{z=0,D} = 0. \quad (4.3)$$

Because the parameter a_μ is not physical and can be eliminated (which is equivalent to shifting the zero-point energy) and that Dirac equation is still complicated and difficult to disentangle when all parameters are switched on at the same time, we consider the simplest nontrivial case

by analyzing the effect of nonzero b_μ. Under these simplifying conditions, the Dirac equation becomes:

$$i\gamma^\mu(\partial_\mu + b_\mu\gamma_5)\psi(r) = 0 \quad (4.4)$$

and the boundary condition on the Dirac field ψ

$$(1 + in \cdot \gamma)\psi|_{z=0,D} = 0. \quad (4.5)$$

For the situation of parallel plates at $z = 0$ and $z = D$, this boundary condition becomes

$$(1 \mp i\gamma^3)\psi|_{z=0,D} = 0. \quad (4.6)$$

A. Summing the modes

The easiest, but not the most rigorous method, is to sum the modes. We introduce a Fourier transform in time and the transverse spatial directions,

$$\psi(x) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \int \frac{(d\vec{k})}{(2\pi)^2} e^{i\vec{k}\cdot\vec{x}}\psi(z; \vec{k}, \omega). \quad (4.7)$$

In what follows we choose a representation of the Dirac matrices in which $i\gamma_5$ is diagonal. In 2×2 block form, the representation for the γ_5, γ_0 is:

$$i\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad (4.8)$$

and the explicit form of all the other Dirac matrices follows from $\vec{\gamma} = i\gamma^0\gamma_5\vec{\sigma}$. We obtain two equations (taking for simplicity)

$$\left[\omega + ib_z \pm \left(ib_0 + i\frac{\partial}{\partial z}\right)\right]u_\pm + (ib_+ \mp k_+)v_\pm = 0, \quad (4.9)$$

$$(ib_- \mp k_-)u_\pm + \left[\omega - ib_z \pm \left(-i\frac{\partial}{\partial z} + ib_0\right)\right]v_\pm = 0, \quad (4.10)$$

where the subscripts indicate the eigenvalues of $i\gamma_5$ and u and v are eigenvectors of σ^3 with eigenvalue $+1$ or -1, respectively. In the above, we used the notation $k_\pm = k_x \pm ik_y$, $b_\pm = b_x \pm ib_y$. The equations simplify considerably if we take $\vec{k}$ along the y axis and $b = (0, b_x, 0, 0)$, (or similarly, $\vec{k}$ along the x axis, and $\vec{b}$ along the y axis) and we obtain:

$$\left(\lambda^2 + \frac{\partial^2}{\partial z^2}\right)\psi = 0, \quad (4.11)$$

where $\lambda^2 = \omega^2 - k^2 + b_x^2$. From the boundary conditions

$$\lambda D = (n + 1/2)\pi, \qquad \omega = \sqrt{k^2 - b_x^2 + (n + 1/2)^2 \frac{\pi^2}{D^2}}. \quad (4.12)$$

When we compute the zero-point energy, we must sum over odd integers, remembering that there are two modes, and taking into account the minus sign (for fermion energy):

$$u = -2\frac{1}{2}\sum_{n=0}^{\infty}\int\frac{(d\vec{k})}{(2\pi)^2}\sqrt{k^2 - b_x^2 + (n+1/2)^2\frac{\pi^2}{D^2}}$$

$$= \frac{1}{2\sqrt{\pi}}\frac{1}{4\pi}\sum_{n=0}^{\infty}\int_0^{\infty}\frac{dt}{t}t^{-3/2}e^{-t[-b_x^2+(n+1/2)^2\pi^2/D^2]}$$

$$= \frac{1}{8\pi^{3/2}}\Gamma\left(-\frac{3}{2}\right)\sum_{n=0}^{\infty}\left(\left(n+\frac{1}{2}\right)^2\frac{\pi^2}{D^2} - b_x^2\right)^{3/2}. \quad (4.13)$$

Using $\frac{1}{8}\sum_{n=0}^{\infty}(2n+1)^3 = -\frac{7}{8}\zeta(-3)$ and $\zeta(-3) = -B_4/4 = 1/120$, we obtain the Casimir energy for a fermion in the SME:

$$u = \frac{7}{8}\frac{\pi^2}{720D^3} + \frac{1}{48}\frac{b_x^2}{D} \quad (4.14)$$

and correspondingly, the Casimir force

$$f_f = -\frac{\partial}{\partial D}u = -\frac{7}{8}\frac{\pi^2}{240D^4} - \frac{1}{48}\frac{b_x^2}{D^2}. \quad (4.15)$$

The first term in this expression represents the conventional fermion Casimir force. The second term represents the shift due to the Lorentz violating terms in the SME.

B. Green's function method

A more controlled calculation starts from the equation satisfied by the Dirac Green's function,

$$\gamma_\mu(\partial^\mu + b^\mu\gamma_5)G(x, x') = i\delta(x - x'), \quad (4.16)$$

subject to the boundary condition

$$(1 + i\vec{n}\cdot\vec{\gamma})G|_{z=0,D} = 0. \quad (4.17)$$

We introduce a reduced, Fourier-transformed, Green's function,

$$G(x, x') = \int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\int\frac{(d\vec{k})}{(2\pi)^2}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}g(z, z'; \vec{k}, \omega), \quad (4.18)$$

which satisfies

$$\left[-\gamma^0\omega + \vec{\gamma}\cdot(\vec{k} + \vec{b}\gamma_5) - i\gamma^3\frac{\partial}{\partial z}\right]g(z, z') = \delta(z - z'). \quad (4.19)$$

Introducing the components of g corresponding to the $+1$ or -1 eigenvalues of $i\gamma_5$,

$$g = \begin{pmatrix} g_{++} & g_{+-} \\ g_{-+} & g_{--} \end{pmatrix}, \quad (4.20)$$

we insert these Green's functions into the vacuum expectation value of the energy-momentum tensor. The latter is

$$T^{\mu\nu} = \frac{1}{4i}\psi\gamma^0(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi + g^{\mu\nu}\mathcal{L}, \quad (4.21)$$

where $\mathcal{L}$ is the fermionic part of $\mathcal{L}^{\text{QED}}$ in (2.1) with only b_μ nonzero. We take the vacuum expectation value by the usual replacement $\psi\psi\gamma^0 \to -iG$, where G is the fermionic Green's function, and obtain

$$\langle T_{zz}\rangle = i\lambda\tan\lambda D, \quad (4.22)$$

where $\lambda^2 = \omega^2 - k^2 + b_x^2 \equiv -\xi^2 - k^2 + b_x^2$. The force per unit area is

$$f_f = \int\frac{d^2k}{(2\pi)^2}\int\frac{d\omega}{2\pi}i\lambda\tan\lambda D. \quad (4.23)$$

Introducing polar coordinates $k = \kappa\sin\theta$ and $(\xi^2 - b_x^2)^{1/2} = \kappa\cos\theta$,

$$dkd\xi = \frac{\kappa^2\cos\theta}{\xi}d\kappa d\theta = \frac{\kappa d\kappa d\theta}{(1 + \frac{b_x^2}{\kappa^2\cos^2\theta})^{1/2}}$$

$$\cong \kappa d\kappa d\theta\left(1 - \frac{1}{2}\frac{b_x^2}{\kappa^2\cos^2\theta}\right), \quad (4.24)$$

we obtain:

$$f_f = \int\frac{d^2k}{(2\pi)^2}\int\frac{d\xi}{2\pi}\kappa\tanh\kappa D$$

$$= \frac{1}{2\pi^2}\int_0^{\infty}d\theta d\kappa\,\kappa^3\left[1 - \frac{2}{e^{2\kappa D}+1}\right]\left(1 - \frac{1}{2}\frac{b_x^2}{\kappa^2\cos^2\theta}\right). \quad (4.25)$$

As in t, we omit the first term in the last square bracket, as the same term is present in the vacuum energy outside the plates. Using

$$\int_0^{\infty}\frac{x^{s-1}dx}{e^x + 1} = (1 - 2^{1-s})\zeta(s)\Gamma(s), \quad (4.26)$$

we find

$$f_f = -\frac{7}{8}\frac{2A_3}{(2\pi)^3}\frac{1}{(2D)^3}\zeta(4)\Gamma(4) - \frac{b_x^2(4\pi)}{(2\pi)^3}\frac{1}{(2D)^2}\zeta(2)\Gamma(2) \quad (4.27)$$

with A_3 the area of the unit sphere in 3 dimensions; which is, indeed,

$$f_f = -\frac{\partial}{\partial D}u = -\frac{7}{8}\frac{\pi^2}{240D^4} - \frac{1}{48}\frac{b_x^2}{D^2}, \quad (4.28)$$

the same as (4.15).

V. SUMMARY AND CONCLUSION

We have presented an analysis of the Casimir force in the extended QED derived from the minimal version of the standard model (SME) which includes Lorentz violating terms, for both the scalar and fermion fields. Thinking of the two polarizations of the photon as two scalar fields,

propagating with different frequencies, we calculated the leading order of the deviation from the classical expression for the Casimir force for two parallel conducting plates. In the case where the Lorentz violation is induced by *CPT*-even effects only, the correction term on the Casimir force is multiplicative and of the form $(1 - \rho)f_{\text{em}}$, with f_{em} the conventional Casimir force and ρ a small parameter characterizing the Lorentz violation. Since the Casimir force is measured to only 15% precision, no useful bounds on the ρ parameter can be obtained from such an expression. In the case in which Lorentz violation is induced by the *CPT*-odd terms in the Lagrangian, the effective Lagrangian gives rise to an additive term as correction; additionally this term has a different dependence on the distance D between the plates from the usual Casimir force: $1/D^2$ versus $1/D^4$. Unfortunately in this case the bounds obtained on the k_{AF} parameter are too weak to have an experimental significance at this time. For the case of the fermion field, the correction term in the SME is also additive, and also of the form b^2/D^2, where b

is the measure of Lorentz violation. In all cases, the leading order contribution is quadratic in Lorentz violating parameters, which makes the experimental sensitivity much weaker.

In conclusion, we calculated the deviation of the Casimir force in the Lorentz violating extension of the standard model from its standard quantum electrodynamics value. We have shown that, while small, it does not contradict any experimental measurements, unlike in some theories with extra dimensions [33]. The deviation predicted is of theoretical interest, and would only be useful in setting any significant constraints on the parameters of the model only if the precision of the experimental measurements will increase significantly.

ACKNOWLEDGMENTS

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