Can the four-zero-texture mass matrix model reproduce the observed quark and lepton mixing angles and *CP*-violating phases?

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We reconsider a universal mass matrix model which has a seesaw-invariant structure with four-zero texture common to all quarks and leptons. The Cabibbo-Kobayashi-Maskawa (CKM) quark and Maki-Nakagawa-Sakata (MNS) lepton mixing matrices of the model are analyzed analytically. We show that the model can be consistent with all the experimental data of neutrino oscillation and quark mixings by tuning free parameters of the model. It is also shown that the model predicts a relatively large value for the (1, 3) element of the MNS lepton mixing matrix, $|(U_{\rm MNS})_{13}|^2 \simeq (0.041\text{–}9.6) \times 10^{-2}$. Using the seesaw mechanism, we also discuss the conditions for the components of the Dirac and the right-handed Majorana neutrino mass matrices which lead to the neutrino mass matrix consistent with the experimental data.

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I. INTRODUCTION

The discovery of neutrino oscillation [1] indicates that neutrinos have finite masses and mix one another with near bimaximal lepton mixings in contrast to small quark mixings. In order to explain the large lepton mixings and small quark mixings, mass matrix models with various structures such as zero texture [2–12], flavor 2 ↔ 3 symmetry [13–31], etc. have been investigated in the literature. We think that quarks and leptons should be unified. Therefore, it is an interesting approach to investigate a possibility that all the mass matrices of the quarks and leptons have the same form which can lead to the large lepton mixings and the small quark mixings simultaneously. Since the mass matrix model is intended to be embedded into a grand unified theory (GUT), it is desirable for the model to have the following features: (i) The structure is common to all the mass matrices, M_u, M_d, M_e, and M_ν for up quarks (u, c, t), down quarks (d, s, b), charged leptons (e, μ, τ), and neutrinos (ν_e, ν_μ, ν_τ), respectively. (ii) Since we assume the seesaw mechanism [32] for neutrino masses, the structure should conserve its form through the relation $M_\nu \simeq -M_D M_R^{-1} M_D^T$. We shall call this structure a seesaw-invariant form. Here M_D and M_R are, respectively, the Dirac and the right-handed Majorana type neutrino mass matrices, which are also assumed to have the same structure.

In this paper, as typical mass matrices which have the features mentioned above, we reconsider Hermitian mass matrices M_f for $f = u$, d, e, and D and symmetric mass matrices M_f for $f = \nu$ and R with a universal form given by

$$M_f = P_f^\dagger \hat{M}_f P_f, \quad \text{for } f = u, d, e, \text{ and } D, \tag{1.1}$$

$$M_f = P_f^\dagger \hat{M}_f P_f^*, \quad \text{for } f = \nu \text{ and } R. \tag{1.2}$$

Here P_f is a diagonal phase matrix given by

$$P_f = \mathrm{diag}(e^{i\alpha_{f1}}, e^{i\alpha_{f2}}, e^{i\alpha_{f3}}), \tag{1.3}$$

and the matrix $\hat{M}_f$ is defined by

$$\hat{M}_f \equiv \begin{pmatrix} 0 & a_f & 0 \\ a_f & b_f & c_f \\ 0 & c_f & d_f \end{pmatrix}, \tag{1.4}$$

for $f = u, d, e, \nu, D$, and R. In this seesaw-invariant type of four-zero-texture model, we have four real component parameters a_f, b_f, c_f, and d_f in $\hat{M}_f$ and phase parameters α_{fi} ($i = 1, 2, 3$) in P_f. If we fix three eigenvalues m_{fi} ($i = 1, 2,$ and 3) of $\hat{M}_f$ by the observed fermion masses, one free parameter is left in $\hat{M}_f$. So we shall choose d_f as the free parameter in this paper. Then we shall present analytical expressions for the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [33] and the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix [34] of the model in terms of m_{f1}, m_{f2}, m_{f3}, d_f and α_{fi}.

By taking a special value for this free parameter as $d_f = m_{f3} + m_{f1}$, the model with the same structure has been discussed in Ref. [35]. However, in this special choice, the model predicts a rather smaller value for the (1, 3) element of the CKM quark mixing matrix than the corresponding observed experimental data. In order to overcome this defect in the quark sector, we treat d_f as a free parameter in the present paper and show that the observed small CKM quark mixings as well as large MNS lepton mixings can be well derived by fine tuning of the free parameters.

It has been claimed [36–38] that four-zero-texture models for quarks are ruled out at the three σ level from the experimental data for $\sin 2\beta$. However, we shall show from an analysis with the use of the free parameter d_f that the quark mixing angles and CP violating phase δ_q in our model are consistent with the data at one σ level, so that the $\sin 2\beta$ is also consistent at the same level.

This article is organized as follows. In Sec. II, we discuss the diagonalization of the mass matrix of our model. In Sec. III, approximations we use are presented. The analytical expressions of the quark mixing matrix of the model are given in Sec. IV. In Sec. V, the lepton mixing matrix of the model is given. Section VI is devoted to a summary.

II. DIAGONALIZATION OF THE MASS MATRIX

We now discuss a diagonalization of the mass matrix M_f. First we argue a diagonalization of $\hat{M}_f$ given by

$$\hat{M}_f = \begin{pmatrix} 0 & a_f & 0 \\ a_f & b_f & c_f \\ 0 & c_f & d_f \end{pmatrix}. \quad (2.1)$$

This is diagonalized by an orthogonal matrix O_f as

$$O_f^T \hat{M}_f O_f = \mathrm{diag}(m_{f1}, m_{f2}, m_{f3}), \quad (2.2)$$

where m_{f1}, m_{f2}, and m_{f3} are eigenvalues of M_f. Here we have four component parameters in $\hat{M}_f$, namely, a_f, b_f, c_f, and d_f. If we fix the m_{fi} by the observed quark and/or lepton mass, we have one free parameter left. Therefore we choose d_f as the free parameter. Then, we derive explicit expressions of the orthogonal matrix O_f in terms of m_{f1}, m_{f2}, m_{f3}, and d_f as

$$O_f = \begin{pmatrix} \sqrt{\frac{(d_f-m_{f1})m_{f2}m_{f3}}{R_{f1}d_f}} & \sqrt{\frac{(d_f-m_{f2})m_{f3}m_{f1}}{R_{f2}d_f}} & \sqrt{\frac{(d_f-m_{f3})m_{f1}m_{f2}}{R_{f3}d_f}} \\ -\sqrt{-\frac{(d_f-m_{f1})m_{f1}}{R_{f1}}} & \sqrt{-\frac{(d_f-m_{f2})m_{f2}}{R_{f2}}} & \sqrt{-\frac{(d_f-m_{f3})m_{f3}}{R_{f3}}} \\ \sqrt{\frac{m_{f1}(d_f-m_{f2})(d_f-m_{f3})}{R_{f1}d_f}} & -\sqrt{\frac{m_{f2}(d_f-m_{f3})(d_f-m_{f1})}{R_{f2}d_f}} & \sqrt{\frac{m_{f3}(d_f-m_{f1})(d_f-m_{f2})}{R_{f3}d_f}} \end{pmatrix}, \quad (2.3)$$

where R_{fi} ($i = 1$, 2, and 3) are defined by

$$R_{f1} = (m_{f1} - m_{f2})(m_{f1} - m_{f3}), \quad (2.4)$$

$$R_{f2} = (m_{f2} - m_{f3})(m_{f2} - m_{f1}), \quad (2.5)$$

$$R_{f3} = (m_{f3} - m_{f1})(m_{f3} - m_{f2}). \quad (2.6)$$

The expressions of the components a_f, b_f, and c_f in terms of m_{f1}, m_{f2}, m_{f3}, and d_f are presented as

$$a_f = \sqrt{-\frac{m_{f1}m_{f2}m_{f3}}{d_f}}, \quad (2.7)$$

$$b_f = m_{f1} + m_{f2} + m_{f3} - d_f, \quad (2.8)$$

$$c_f = \sqrt{-\frac{(d_f - m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}}. \quad (2.9)$$

From the condition that a_f, b_f, and c_f are real, we have the allowed region of d_f given by

$$|m_{f1}| < d_f < |m_{f3}|. \quad (2.10)$$

The cases in which $0 < d_f < |m_{f1}|$ or $|m_{f3}| < d_f$ are not allowed. We also have the following sign assignments for the eigenmass m_{fi}:

$$0 < m_{f1} < -m_{f2} < m_{f3} \quad \text{for} \quad |m_{f1}| < d_f < |m_{f2}|, \quad (2.11)$$

$$0 < -m_{f1} < m_{f2} < m_{f3} \quad \text{for} \quad |m_{f2}| < d_f < |m_{f3}|. \quad (2.12)$$

Namely m_{f2} should be taken negative while m_{f1} and m_{f3} are positive for the case in which $|m_{f1}| < d_f < |m_{f2}|$. On the other hand, m_{f1} should be taken negative while m_{f2} and m_{f3} are positive for $|m_{f2}| < d_f < |m_{f3}|$.

III. APPROXIMATIONS

We present approximated expressions of the orthogonal matrix O_f for the normal hierarchy, inverse hierarchy, and quasidegenerate cases for the masses m_{fi}. Here we introduce a x_f parameter, instead of using d_f, defined by

$$x_f = \frac{d_f}{m_{f3}}. \quad (3.1)$$

The approximated expressions are obtained as follows:

Case (a): For $|m_{f1}| \ll m_{f2} \ll d_f < m_{f3}$ (normal hierarchy 1), we have

$$O_f \simeq \begin{pmatrix} 1 & \sqrt{\frac{|m_{f1}|}{m_{f2}}} & \sqrt{\frac{|m_{f1}|m_{f2}}{m_{f3}^2}\frac{1-x_f}{x_f}} \\ -\sqrt{\frac{|m_{f1}|}{m_{f2}}x_f} & \sqrt{x_f} & \sqrt{1-x_f} \\ \sqrt{\frac{|m_{f1}|}{m_{f2}}(1-x_f)} & -\sqrt{1-x_f} & \sqrt{x_f} \end{pmatrix}. \quad (3.2)$$

Case (b): For $|m_{f1}| < m_{f2} \ll d_f < m_{f3}$ (normal hierarchy 2), we have

$$O_f \simeq \begin{pmatrix} \sqrt{\frac{|m_{f2}|}{|m_{f1}|+m_{f2}}} & \sqrt{\frac{|m_{f1}|}{|m_{f1}|+m_{f2}}} & \sqrt{\frac{|m_{f1}|m_{f2}}{m_{f3}^2}\frac{1-x_f}{x_f}} \\ -\sqrt{\frac{|m_{f1}|}{|m_{f1}|+m_{f2}}x_f} & \sqrt{\frac{m_{f2}}{|m_{f1}|+m_{f2}}x_f} & \sqrt{1-x_f} \\ \sqrt{\frac{|m_{f1}|}{|m_{f1}|+m_{f2}}(1-x_f)} & -\sqrt{\frac{m_{f2}}{|m_{f1}|+m_{f2}}(1-x_f)} & \sqrt{x_f} \end{pmatrix}. \tag{3.3}$$

Case (c): For $m_{f1} \ll d_f < |m_{f2}| \simeq m_{f3}$ (inverse hierarchy), we have

$$O_f \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_{f3}m_{f1}(d_f+|m_{f2}|)}{|m_2|(|m_{f2}|+m_{f3})d_f}} & \sqrt{\frac{m_{f1}|m_{f2}|(d_f+|m_{f2}|)}{|m_3|(|m_{f2}|+m_{f3})d_f}} \\ -\sqrt{\frac{m_{f1}d_f}{|m_{f2}|m_{f3}}} & \sqrt{\frac{d_f+|m_{f2}|}{|m_{f2}|+m_{f3}}} & \sqrt{\frac{m_{f3}-d_f}{|m_{f2}|+m_{f3}}} \\ \sqrt{\frac{m_{f1}(d_f+|m_{f2}|)(m_{f3}-d_f)}{|m_2|m_{f3}d_f}} & -\sqrt{\frac{m_{f3}-d_f}{|m_{f2}|+m_{f3}}} & \sqrt{\frac{d_f+|m_{f2}|}{|m_{f2}|+m_{f3}}} \end{pmatrix}. \tag{3.4}$$

Case (d): For $m_f = |m_{f1}| < m_{f2} < d_f < m_{f3}$ (quasidegenerate 1), we have

$$O_f \simeq \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}\frac{d_f-m_{f2}}{m_{f3}-m_{f2}}} & \sqrt{\frac{1}{2}\frac{m_{f3}-d_f}{m_{f3}-m_{f2}}} \\ -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}\frac{d_f-m_{f2}}{m_{f3}-m_{f2}}} & \sqrt{\frac{1}{2}\frac{m_{f3}-d_f}{m_{f3}-m_{f2}}} \\ \sqrt{\frac{1}{4}\frac{(d_f-m_{f2})(m_{f3}-d_f)}{m_f^2}} & -\sqrt{\frac{m_{f2}-d_f}{m_{f3}-m_{f2}}} & \sqrt{\frac{d_f-m_{f2}}{m_{f3}-m_{f2}}} \end{pmatrix}. \tag{3.5}$$

Case (e): For $m_f = m_{f1} < d_f < |m_{f2}| < m_{f3}$ (quasidegenerate 2), we have

$$O_f \simeq \begin{pmatrix} \sqrt{\frac{1}{2}\frac{d_f-m_{f1}}{m_{f3}-m_{f1}}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}\frac{m_{f3}-d_f}{m_{f3}-m_{f1}}} \\ -\sqrt{\frac{1}{2}\frac{d_f-m_{f1}}{m_{f3}-m_{f1}}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}\frac{m_{f3}-d_f}{m_{f3}-m_{f1}}} \\ \sqrt{\frac{m_{f3}-d_f}{m_{f3}-m_{f1}}} & -\sqrt{\frac{1}{4}\frac{(m_{f3}-d_f)(d_f-m_{f1})}{m_f^2}} & \sqrt{\frac{d_f-m_{f1}}{m_{f3}-m_{f1}}} \end{pmatrix}. \tag{3.6}$$

The inverse hierarchy and the quasidegenerate scenarios are unfavorable in our model.

IV. CKM QUARK MIXING MATRIX

Let us discuss the quark sector. The mass matrices M_u and M_d for the u- and d-quarks are, respectively, given by

$$M_u = P_u^\dagger \hat{M}_u P_u, \tag{4.1}$$

$$M_d = P_d^\dagger \hat{M}_d P_d, \tag{4.2}$$

where P_u and P_d are diagonal phase matrices and $\hat{M}_u$ and $\hat{M}_d$ are given by Eq. (1.4). The mass matrix M_f ($f = u$ and d) is diagonalized as

$$U_{Lf}^\dagger M_f U_{Lf} = \mathrm{diag}(-|m_{f1}|, m_{f2}, m_{f3}). \tag{4.3}$$

The unitary matrix U_{Lf} is described as

$$U_{Lf} = P_f^\dagger O_f. \tag{4.4}$$

Therefore the CKM quark mixing matrix U_{CKM} of the model is given by

$$U_{\mathrm{CKM}} = U_{Lu}^\dagger U_{Ld} = O_u^T P O_d, \tag{4.5}$$

where $P \equiv P_u P_d^\dagger$ is a diagonal phase matrix given by

$$\begin{aligned} P &= \mathrm{diag}(e^{i(\alpha_{u1}-\alpha_{d1})}, e^{i(\alpha_{u2}-\alpha_{d2})}, e^{i(\alpha_{u3}-\alpha_{d3})}) \\ &\equiv \mathrm{diag}(1, e^{i\alpha_2}, e^{i\alpha_3}). \end{aligned} \tag{4.6}$$

Here we take $\alpha_{d1} = \alpha_{u1} = 0$ without any loss of generality.

By using the expressions of O_d and O_u in Eq. (2.3), the explicit (i, j) elements of U_{CKM} are obtained as

$$\begin{aligned} (U_{\mathrm{CKM}})_{12} = {} & \sqrt{\frac{m_c m_t(d_u - m_u)}{R_{u1}d_u}}\sqrt{\frac{m_b m_d(d_d - m_s)}{R_{d2}d_d}} - e^{i\alpha_2}\sqrt{-\frac{m_u(d_u - m_u)}{R_{u1}}}\sqrt{-\frac{m_s(d_d - m_s)}{R_{d2}}} \\ & - e^{i\alpha_3}\sqrt{\frac{m_u(d_u - m_c)(d_u - m_t)}{R_{u1}d_u}}\sqrt{\frac{m_s(d_d - m_b)(d_d - m_d)}{R_{d2}d_d}}, \end{aligned} \tag{4.7}$$

$$(U_{\rm CKM})_{13} = \sqrt{\frac{m_c m_t(d_u - m_u)}{R_{u1} d_u}}\sqrt{\frac{m_d m_s(d_d - m_b)}{R_{d3} d_d}} - e^{i\alpha_2}\sqrt{-\frac{m_u(d_u - m_u)}{R_{u1}}}\sqrt{-\frac{m_b(d_d - m_b)}{R_{d3}}} - e^{i\alpha_3}\sqrt{\frac{m_u(d_u - m_c)(d_u - m_t)}{R_{u1} d_u}}\sqrt{\frac{m_b(d_d - m_d)(d_d - m_s)}{R_{d3} d_d}}, \tag{4.8}$$

$$(U_{\rm CKM})_{23} = \sqrt{\frac{m_t m_u(d_u - m_c)}{R_{u2} d_u}}\sqrt{\frac{m_d m_s(d_d - m_b)}{R_{d3} d_d}} - e^{i\alpha_2}\sqrt{-\frac{m_c(d_u - m_c)}{R_{u2}}}\sqrt{-\frac{m_b(d_d - m_b)}{R_{d3}}} - e^{i\alpha_3}\sqrt{\frac{m_c(d_u - m_t)(d_u - m_u)}{R_{u2} d_u}}\sqrt{\frac{m_b(d_d - m_d)(d_d - m_s)}{R_{d3} d_d}}, \tag{4.9}$$

where R_{ui} and R_{di} ($i = 1$, 2, and 3) are given by

$$R_{u1} = (m_u - m_c)(m_u - m_t), \tag{4.10}$$

$$R_{u2} = (m_c - m_t)(m_c - m_u), \tag{4.11}$$

$$R_{u3} = (m_t - m_u)(m_t - m_c). \tag{4.12}$$

$$R_{d1} = (m_d - m_s)(m_d - m_b), \tag{4.13}$$

$$R_{d2} = (m_s - m_b)(m_s - m_d), \tag{4.14}$$

$$R_{d3} = (m_b - m_d)(m_b - m_s). \tag{4.15}$$

Here, we denoted m_{ui} and m_{di} ($i = 1, 2, 3$) as (m_u, m_c, m_t) and (m_d, m_s, m_b) which are the masses of up and down quarks, respectively.

If we fix the quark masses (m_u, m_c, m_t) and (m_d, m_s, m_b) by the observed masses, two component parameters d_u and d_d and two phase parameters α_2 and α_3 are left as free parameters in the above expressions of $(U_{\rm CKM})_{ij}$. Using this feature of the model, we can reproduce the observed data for $(U_{\rm CKM})_{ij}$ as will be shown later. This model can be used for the improvement of the previous model [35] in which a rather small value for $|(U_{\rm CKM})_{13}|$ is predicted.

In the discussions of the CKM quark mixing matrix, we concentrate our attention on the case in which $|m_{f1}| \ll m_{f2} \ll d_f < m_{f3}$ (normal hierarchy 1). In this case, using two free parameters $x_u \equiv d_u/m_t$ and $x_d \equiv d_d/m_b$ instead of using d_u and d_d, we have

$$(U_{\rm CKM})_{12} \simeq \sqrt{\frac{|m_d|}{m_s}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}x_u x_d} - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}(1 - x_u)(1 - x_d)}, \tag{4.16}$$

$$(U_{\rm CKM})_{13} \simeq \sqrt{\frac{|m_d| m_s}{m_b^2}\frac{1 - x_d}{x_d}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}x_u(1 - x_d)} + e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}(1 - x_u)x_d}, \tag{4.17}$$

$$(U_{\rm CKM})_{23} \simeq \sqrt{\frac{|m_u|}{m_c}\frac{|m_d| m_s}{m_b^2}\frac{1 - x_d}{x_d}} + e^{i\alpha_2}\sqrt{x_u(1 - x_d)} - e^{i\alpha_3}\sqrt{(1 - x_u)x_d}. \tag{4.18}$$

By using the rephasing of the up and down quarks, Eq. (4.5) is changed to the standard representation of the CKM quark mixing matrix,

$$U^{\rm std}_{\rm CKM} = {\rm diag}(e^{i\zeta_1^u}, e^{i\zeta_2^u}, e^{i\zeta_2^u})U_{\rm CKM}{\rm diag}(e^{i\zeta_1^d}, e^{i\zeta_2^d}, e^{i\zeta_2^d}) = \begin{pmatrix} c_{13}^q c_{12}^q & c_{13}^q s_{12}^q & s_{13}^q e^{-i\delta_q} \\ -c_{23}^q s_{12}^q - s_{23}^q c_{12}^q s_{13}^q e^{i\delta_q} & c_{23}^q c_{12}^q - s_{23}^q s_{12}^q s_{13}^q e^{i\delta_q} & s_{23}^q c_{13}^q \\ s_{23}^q s_{12}^q - c_{23}^q c_{12}^q s_{13}^q e^{i\delta_q} & -s_{23}^q c_{12}^q - c_{23}^q s_{12}^q s_{13}^q e^{i\delta_q} & c_{23}^q c_{13}^q \end{pmatrix}. \tag{4.19}$$

Here ζ_i^q comes from the rephasing in the quark fields to make the choice of phase convention. By using the expressions of $U_{\rm CKM}$ in Eqs. (4.16), (4.17), and (4.18), the CP-violating phase δ_q in the quark mixing matrix is given by

$$\delta_q = \arg\left[\left(\frac{(U_{\rm CKM})_{12}(U_{\rm CKM})^*_{22}}{(U_{\rm CKM})_{13}(U_{\rm CKM})^*_{23}}\right) + \frac{|(U_{\rm CKM})_{12}|^2}{1 - |(U_{\rm CKM})_{13}|^2}\right] \tag{4.20}$$

$$\simeq \arg\left[\frac{(e^{i\alpha_3}\sqrt{(1 - x_u)(1 - x_d)} + e^{i\alpha_2}\sqrt{x_u x_d})^*}{(e^{i\alpha_3}\sqrt{(1 - x_u)x_d} - e^{i\alpha_2}\sqrt{x_u(1 - x_d)})(e^{i\alpha_2}\sqrt{x_u(1 - x_d)} - e^{i\alpha_3}\sqrt{(1 - x_u)x_d})^*}\right]. \tag{4.21}$$

Thus we have obtained the analytical expressions for $|(U_{\rm CKM})_{12}|$, $|(U_{\rm CKM})_{23}|$, $|(U_{\rm CKM})_{13}|$, and δ_q of the model which are given by Eqs. (4.16), (4.17), (4.18), and (4.21), respectively. They are functions of the four parameters x_u, x_d, α_2, and α_3. From the expressions of $|(U_{\rm CKM})_{13}|$ and $|(U_{\rm CKM})_{23}|$ in Eqs. (4.17) and (4.18), we obtain the following constraints in the parameters x_u and x_d, which hold irrespectively of the free phase parameters α_2 and α_3,

$$\frac{1}{1+\frac{\sqrt{\frac{|m_u|}{m_c}}|(U_{\rm CKM})_{23}|+|(U_{\rm CKM})_{13}|}{\frac{|m_d|m_s}{m_b^2}}} \lesssim x_d \lesssim \frac{1}{1+\frac{\sqrt{\frac{|m_u|}{m_c}}|(U_{\rm CKM})_{23}|-|(U_{\rm CKM})_{13}|}{\frac{|m_d|m_s}{m_b^2}}}, \tag{4.22}$$

$$\left|\sqrt{x_u(x_d-1)}-\sqrt{x_d(x_u-1)}\right| \lesssim |(U_{\rm CKM})_{23}| \lesssim \left|\sqrt{x_u(x_d-1)}+\sqrt{x_d(x_u-1)}\right|. \tag{4.23}$$

On the other hand, the numerical values of $|(U_{\rm CKM})_{12}|$, $|(U_{\rm CKM})_{23}|$, $|(U_{\rm CKM})_{13}|$, and δ_q at the unification scale $\mu = M_X$ are estimated from the experimental data observed at the electroweak scale $\mu = M_Z$ by using the renormalization group equation as [30]:

$$|(U_{\rm CKM})_{12}| = 0.2226\text{–}0.2259, \tag{4.24}$$

$$|(U_{\rm CKM})_{23}| = 0.0295\text{–}0.0387, \tag{4.25}$$

$$|(U_{\rm CKM})_{13}| = 0.0024\text{–}0.0038, \tag{4.26}$$

$$\delta_q = 46°\text{–}74°. \tag{4.27}$$

By using the above experimental constraints as inputs, we obtain the consistent solution for the parameter x_u, x_d, α_2, and α_3 of our model from our exact CKM matrix elements given by Eqs. (4.7), (4.8), (4.9), and (4.20). By doing parameter fitting, we find that the consistent CKM elements are realized only if (i) the parameter α_2 takes a value as $\alpha_2 \simeq \pi/2$ and (ii) the other three parameters α_3, x_u, and x_d take values in the allowed regions shown in Figs. 1–3. The best fit is realized for the following values of the parameters:

$$\alpha_2 = \pi/2, \tag{4.28}$$

$$\alpha_3 = 1.450, \tag{4.29}$$

$$x_u = 0.9560, \tag{4.30}$$

$$x_d = 0.9477. \tag{4.31}$$

For these best-fit-parameters of the model, we obtain

$$|(U_{\rm CKM})_{12}| = 0.2251, \tag{4.32}$$

$$|(U_{\rm CKM})_{23}| = 0.0340, \tag{4.33}$$

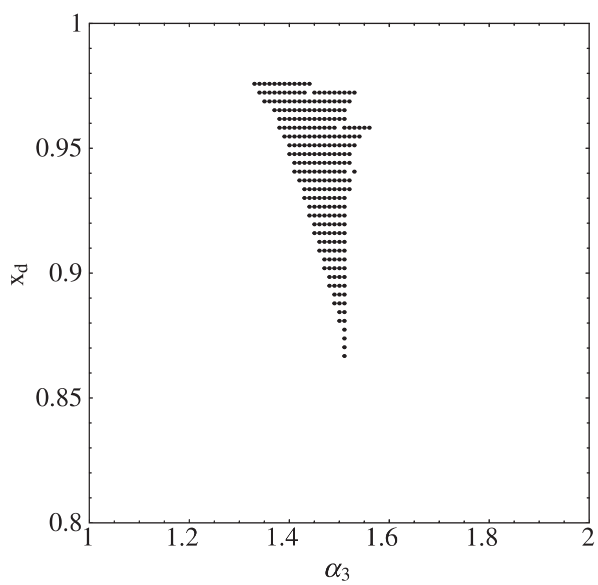

FIG. 1. The allowed region in the α_3-x_d parameter plane. Dotted regions are allowed from the experimental data for the CKM quark mixing matrix elements.

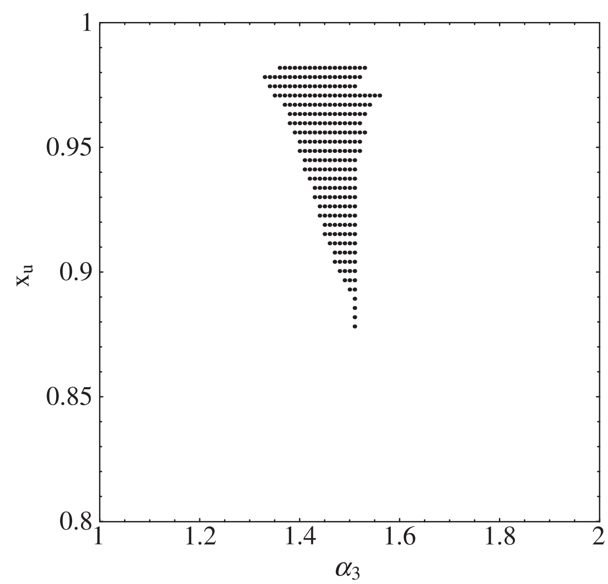

FIG. 2. The allowed region in the α_3-x_u parameter plane.

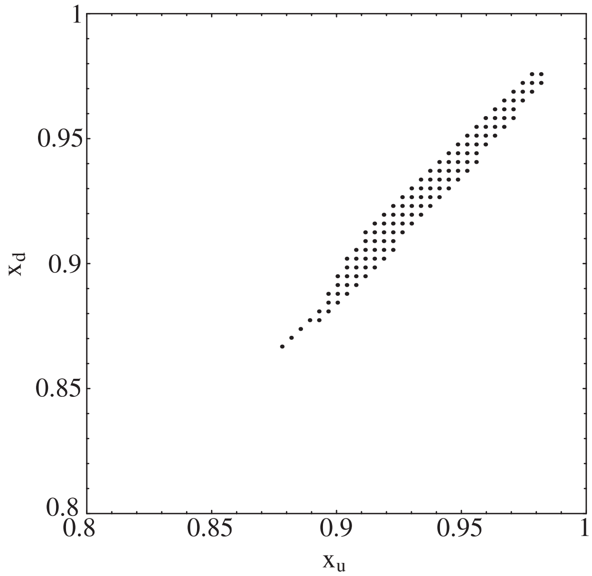

FIG. 3. The allowed region in the x_u-x_d parameter plane.

$$|(U_{\rm CKM})_{13}| = 0.0032, \tag{4.34}$$

$$\delta_q = 58.86°. \tag{4.35}$$

Here we have used the best fit values of the following quark masses estimated [39] at the unification scale $\mu = M_X$,

$$\begin{aligned} |m_u(M_X)| &= 1.04^{+0.19}_{-0.20}\ \text{MeV}, \\ m_c(M_X) &= 302^{+25}_{-27}\ \text{MeV}, \\ m_t(M_X) &= 129^{+196}_{-40}\ \text{GeV}, \\ |m_d(M_X)| &= 1.33^{+0.17}_{-0.19}\ \text{MeV}, \\ m_s(M_X) &= 26.5^{+3.3}_{-3.7}\ \text{MeV}, \\ m_b(M_X) &= 1.00 \pm 0.04\ \text{GeV}. \end{aligned} \tag{4.36}$$

Finally let us mention the model in Ref. [35]. It corresponds to our present model with the parameter d_f fixed as $d_f = m_{f3} + m_{f1}$, namely $x_f = 1 - |m_{f1}|/m_{f3} \simeq 1$ and $1 - x_f = |m_{f1}|/m_{f3}$. In this case, the following CKM mixing matrix elements are derived as seen from Eqs. (4.16), (4.17), (4.18), and (4.21),

$$(U_{\rm CKM})_{12} \simeq \sqrt{\frac{|m_d|}{m_s}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}} - e^{i\alpha_3}\sqrt{\frac{m_u^2}{m_c m_t}\frac{|m_d|}{m_b}}, \tag{4.37}$$

$$(U_{\rm CKM})_{13} \simeq \sqrt{\frac{m_d^2 m_s}{m_b^3}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}\frac{|m_d|}{m_b}} + e^{i\alpha_3}\sqrt{\frac{m_u^2}{m_c m_t}}, \tag{4.38}$$

$$(U_{\rm CKM})_{23} \simeq \sqrt{\frac{|m_u|}{m_c}\frac{m_d^2 m_s}{m_b^3}} + e^{i\alpha_2}\sqrt{\frac{|m_d|}{m_b}} - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_t}}, \tag{4.39}$$

$$\begin{aligned} \delta_q &\simeq \arg\left[\frac{(e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_t}\frac{|m_d|}{m_b}} + e^{i\alpha_2})^*}{(e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_t}} - e^{i\alpha_2}\sqrt{\frac{|m_d|}{m_b}})(e^{i\alpha_2}\sqrt{\frac{|m_d|}{m_b}} - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_t}})^*}\right] \\ &\simeq \pi - \alpha_2. \end{aligned} \tag{4.40}$$

This model is more predictable for the CKM matrix elements than ours. However, this model predicts a rather smaller value for $|(U_{\rm CKM})_{13}|$ than the experimental data. This is seen from the fact that the values of the parameters $x_d = 1 - |m_d|/m_b = 0.998\,437$ and $x_u = 1 - |m_u|/m_t = 0.999\,987$ of this model are outside of the allowed regions shown in Figs. 1–3.

V. MNS LEPTON MIXING MATRIX

Let us discuss the lepton sector. The mass matrices M_ν and M_e for the Majorana neutrinos and the charged leptons are, respectively, given by

$$M_e = P_e^\dagger \hat{M}_e P_e, \tag{5.1}$$

$$M_\nu = P_\nu^\dagger \hat{M}_\nu P_\nu^*. \tag{5.2}$$

Here P_ν and P_e are diagonal phase matrices and $\hat{M}_\nu$ and $\hat{M}_e$ are given by Eq. (2.1). The charged-lepton mass matrix M_e is diagonalized as

$$U_{Le}^\dagger M_e U_{Le} = \text{diag}(-|m_e|, m_\mu, m_\tau), \tag{5.3}$$

where the unitary matrix U_e is described as

$$U_{Le} = P_e^\dagger O_e. \tag{5.4}$$

Since the mass matrix for the Majorana neutrinos is symmetric, M_ν is diagonalized as

$$U_\nu^\dagger M_\nu U_\nu^* = \text{diag}(|m_1|, m_2, m_3), \tag{5.5}$$

where $|m_1|$, m_2, and m_3 are real positive neutrino masses and the unitary matrix U_ν is described as

$$U_\nu = P_\nu^\dagger O_\nu Q_\nu. \tag{5.6}$$

Here, in order to make the neutrino masses for the first generation real positive, we introduce an additional diagonal phase matrix Q_ν defined by

$$Q_\nu \equiv \text{diag}(i, 1, 1). \tag{5.7}$$

In the following discussions, we consider the normal hierarchy 2 for the neutrino masses m_i, i.e. $|m_1| < m_2 \ll d_\nu < m_3$, and the normal hierarchy 1 for the charged-lepton masses, i.e. $|m_e| \ll m_\mu \ll d_e < m_\tau$. In this case, the orthogonal matrices O_e and O_ν are obtained from Eqs. (3.2) and (3.3) with $f = e$ and ν by replacing $|m_{f1}|$, m_{f2}, and m_{f3} with $|m_e|$, m_μ, and m_τ, and with $|m_1|$, m_2, and m_3, respectively. Therefore we have

$$O_\nu \simeq \begin{pmatrix} \sqrt{\frac{m_2}{|m_1|+m_2}} & \sqrt{\frac{|m_1|}{|m_1|+m_2}} & \sqrt{\frac{|m_1|m_2}{m_3^2}\frac{1-x_\nu}{x_\nu}} \\ -\sqrt{\frac{|m_1|}{|m_1|+m_2}x_\nu} & \sqrt{\frac{m_2}{|m_1|+m_2}x_\nu} & \sqrt{1-x_\nu} \\ \sqrt{\frac{|m_1|}{|m_1|+m_2}(1-x_\nu)} & -\sqrt{\frac{m_2}{|m_1|+m_2}(1-x_\nu)} & \sqrt{x_\nu} \end{pmatrix}, \tag{5.8}$$

$$O_e \simeq \begin{pmatrix} 1 & \sqrt{\frac{|m_e|}{m_\mu}} & \sqrt{\frac{|m_e|m_\mu}{m_\tau^2}\frac{1-x_e}{x_e}} \\ -\sqrt{\frac{|m_e|}{m_\mu}x_e} & \sqrt{x_e} & \sqrt{1-x_e} \\ \sqrt{\frac{|m_e|}{m_\mu}(1-x_e)} & -\sqrt{1-x_e} & \sqrt{x_e} \end{pmatrix}. \tag{5.9}$$

We now discuss the MNS lepton mixing matrix $U_{\rm MNS}$ of the model, which is given by

$$U_{\rm MNS} = U_{Le}^\dagger U_\nu = O_e^T P_\ell O_\nu Q_\nu, \tag{5.10}$$

where $P_\ell \equiv P_e P_\nu^\dagger$ is a diagonal phase matrix and we take

$$P_\ell = {\rm diag}(1, e^{i\beta_2}, e^{i\beta_3}), \tag{5.11}$$

without any loss of generality. Thus we obtain

$$U_{\rm MNS} \simeq \begin{pmatrix} i\sqrt{\frac{m_2}{|m_1|+m_2}} & \sqrt{\frac{|m_1|}{|m_1|+m_2}} & \sqrt{\frac{|m_1|m_2}{m_3^2}\frac{1-x_\nu}{x_\nu}} + \xi_5\sqrt{\frac{|m_e|}{m_\mu}} \\ -i\xi_1\sqrt{\frac{|m_1|}{|m_1|+m_2}} & \xi_1\sqrt{\frac{m_2}{|m_1|+m_2}} & \xi_2 \\ i\xi_3\sqrt{\frac{|m_1|}{|m_1|+m_2}} & -\xi_3\sqrt{\frac{m_2}{|m_1|+m_2}} & \xi_4 \end{pmatrix}, \tag{5.12}$$

where ξ_i are complex quantities defined by

$$\xi_1 = \sqrt{x_\nu x_e}e^{i\beta_2} + \sqrt{(1-x_\nu)(1-x_e)}e^{i\beta_3}, \tag{5.13}$$

$$\xi_2 = \sqrt{(1-x_\nu)x_e}e^{i\beta_2} - \sqrt{x_\nu(1-x_e)}e^{i\beta_3}, \tag{5.14}$$

$$\xi_3 = -\sqrt{x_\nu(1-x_e)}e^{i\beta_2} + \sqrt{(1-x_\nu)x_e}e^{i\beta_3}, \tag{5.15}$$

$$\xi_4 = \sqrt{(1-x_\nu)(1-x_e)}e^{i\beta_2} + \sqrt{x_\nu x_e}e^{i\beta_3}, \tag{5.16}$$

$$\xi_5 = \sqrt{(1-x_\nu)x_e}e^{i\beta_2} + \sqrt{x_\nu(1-x_e)}e^{i\beta_3}. \tag{5.17}$$

Equation (5.12) is changed to the standard representation of the MNS lepton mixing matrix as well as the CKM quark mixing matrix,

$$U_{\rm MNS}^{\rm std} = {\rm diag}(e^{i\zeta_1^e}, e^{i\zeta_2^e}, e^{i\zeta_2^e})U_{\rm MNS} = \begin{pmatrix} c_{13}^l c_{12}^l & c_{13}^l s_{12}^l & s_{13}^l e^{-i\delta_l} \\ -c_{23}^l s_{12}^l - s_{23}^l c_{12}^l s_{13}^l e^{i\delta_l} & c_{23}^l c_{12}^l - s_{23}^l s_{12}^l s_{13}^l e^{i\delta_l} & s_{23}^l c_{13}^l \\ s_{23}^l s_{12}^l - c_{23}^l c_{12}^l s_{13}^l e^{i\delta_l} & -s_{23}^l c_{12}^l - c_{23}^l s_{12}^l s_{13}^l e^{i\delta_l} & c_{23}^l c_{13}^l \end{pmatrix} {\rm diag}(1, e^{i\phi_2}, e^{i\phi_3}). \tag{5.18}$$

Here ζ_i^e comes from the rephasing in the charged-lepton fields, δ_ν is the Dirac phase, and ϕ_i is the Majorana phase in the MNS lepton mixing matrix.

In order to realize the maximal lepton mixing angle between the second and third generations, we must choose the free parameters x_ν, x_e, β_2, and β_3 to satisfy the following condition:

$$|\xi_1| = |\xi_2| = |\xi_3| = |\xi_4| = \sqrt{\tfrac{1}{2}}. \tag{5.19}$$

In the present paper, we take the following choice:

$$x_\nu = 1/2 \quad \text{and} \quad x_e \simeq 1, \tag{5.20}$$

which satisfies the above condition irrespectively of the phases β_2 and β_3. Then, the explicit magnitudes of the components of $|(U_{\rm MNS})_{ij}|$ are obtained as

$$\begin{aligned}
|(U_{\rm MNS})_{11}| &\simeq \sqrt{\frac{m_2}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{12}| &\simeq \sqrt{\frac{|m_1|}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{13}| &\simeq \left|\sqrt{\frac{|m_1|m_2}{m_3^2}}+e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right|,\\
|(U_{\rm MNS})_{21}| &\simeq \frac{1}{\sqrt{2}}\sqrt{\frac{|m_1|}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{22}| &\simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{23}| &\simeq \frac{1}{\sqrt{2}},\\
|(U_{\rm MNS})_{31}| &\simeq \frac{1}{\sqrt{2}}\sqrt{\frac{|m_1|}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{32}| &\simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{m_2+|m_1|}},\\
|(U_{\rm MNS})_{33}| &\simeq \frac{1}{\sqrt{2}}.
\end{aligned} \tag{5.21}$$

From Eqs. (5.18) and (5.21), the neutrino oscillation angles and phases of the model are related to the lepton masses as follows:

$$\tan^2\theta_{\rm solar} = \frac{|(U_{\rm MNS})_{12}|^2}{|(U_{\rm MNS})_{11}|^2} \simeq \frac{|m_1|}{m_2}, \tag{5.22}$$

$$\sin^2 2\theta_{\rm atm} = 4|(U_{\rm MNS})_{23}|^2|(U_{\rm MNS})_{33}|^2 \simeq 1, \tag{5.23}$$

$$|(U_{\rm MNS})_{13}|^2 \simeq \left|\sqrt{\frac{|m_1|m_2}{m_3^2}}+e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right|^2, \tag{5.24}$$

$$\delta_\nu \simeq -\arg\left(\sqrt{\frac{|m_1|m_2}{m_3^2}}+e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right), \tag{5.25}$$

$$\phi_2 \simeq \phi_3 \simeq -\frac{\pi}{2}. \tag{5.26}$$

It should be noted that the present model leads to the same results for $\theta_{\rm solar}$ and $\theta_{\rm atm}$ as the model in Ref. [27], while a different feature for $|(U_{\rm MNS})_{13}|^2$ is derived.

On the other hand, we have [40] an experimental bound for $|(U_{\rm MNS})_{13}|^2_{\rm exp}$ from the CHOOZ [41], solar [42], and atmospheric neutrino experiments [1]. From the global analysis of the SNO solar neutrino experiment [40,42], we have Δm_{12}^2 and $\tan^2\theta_{12}$ for the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution [43]. From the atmospheric neutrino experiment [1,40], we also have Δm_{23}^2 and $\tan^2\theta_{23}$. These experimental data with 3σ range are given by

$$|U_{13}|^2_{\rm exp} < 0.054, \tag{5.27}$$

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\rm sol}^2 = (5.2\text{–}9.8)\times 10^{-5}\ {\rm eV}^2, \tag{5.28}$$

$$\tan^2\theta_{12} = \tan^2\theta_{\rm sol} = 0.29\text{–}0.64, \tag{5.29}$$

$$\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq \Delta m_{\rm atm}^2 = (1.4\text{–}3.4)\times 10^{-3}\ {\rm eV}^2, \tag{5.30}$$

$$\tan^2\theta_{23} \simeq \tan^2\theta_{\rm atm} = 0.49\text{–}2.2. \tag{5.31}$$

Hereafter, for simplicity, we take $\tan^2\theta_{\rm atm} \simeq 1$. Thus, by combining the present model with the mixing angle $\theta_{\rm sol}$, we have

$$\frac{m_1}{m_2} \simeq \tan^2\theta_{\rm sol} = 0.29\text{–}0.64. \tag{5.32}$$

Therefore we predict the neutrino masses as follows.

$$\begin{aligned}
m_1^2 &= (0.48\text{–}6.8)\times 10^{-5}\ {\rm eV}^2,\\
m_2^2 &= (5.7\text{–}16.6)\times 10^{-5}\ {\rm eV}^2,\\
m_3^2 &= (1.5\text{–}3.6)\times 10^{-3}\ {\rm eV}^2.
\end{aligned} \tag{5.33}$$

Let us mention a specific feature of the model. Our model predicts a rather large value for $|(U_{\rm MNS})_{13}|$ as

$$\begin{aligned}
|(U_{\rm MNS})_{13}|^2 &\simeq \left|\sqrt{\frac{|m_1|m_2}{m_3^2}}+e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right|^2 \simeq \frac{|m_1|m_2}{m_3^2}\\
&= (0.041\text{–}9.6)\times 10^{-2}.
\end{aligned} \tag{5.34}$$

The predicted value for $|(U_{\rm MNS})_{13}|$ in Eq. (5.34) is close to the present experimental constraints Eq. (5.27) in contrast to the previously proposed model [25,27]. Therefore our model will be checked in neutrino factories in the near future.

In the preset model, the neutrino mass matrix M_ν is given by

$$M_\nu \simeq P_\nu^\dagger \begin{pmatrix} 0 & \sqrt{2|m_1|m_2} & 0\\ \sqrt{2|m_1|m_2} & m_3/2 & m_3/2\\ 0 & m_3/2 & m_3/2 \end{pmatrix} P_\nu^*. \tag{5.35}$$

Now we discuss the requirements for the mass matrix elements of M_D and M_R to realize the above structure for M_ν. In our model we have assumed the seesaw mechanism $M_\nu = -M_D M_R^{-1} M_D^T$ and the following structure for M_D and M_R,

$$M_D = P_D^\dagger \begin{pmatrix} 0 & a_D & 0 \\ a_D & b_D & c_D \\ 0 & c_D & d_D \end{pmatrix} P_D, \quad M_R = \begin{pmatrix} 0 & a_R & 0 \\ a_R & b_R & c_R \\ 0 & c_R & d_R \end{pmatrix}, \tag{5.36}$$

where $P_D = \mathrm{diag}(e^{i\alpha_{D1}}, e^{i\alpha_{D2}}, e^{i\alpha_{D3}})$. Here we assume a real symmetric M_R for simplicity. In this case, we have

$$M_\nu = -M_D M_R^{-1} M_D^T \tag{5.37}$$

$$\simeq -P_\nu^\dagger \begin{pmatrix} 0 & \frac{a_D^2}{a_R} & 0 \\ \frac{a_D^2}{a_R} & \frac{c_D^2}{d_R} & \frac{c_D d_D}{d_R} \\ 0 & \frac{c_D d_D}{d_R} & \frac{d_D^2}{d_R} \end{pmatrix} P_\nu^* \qquad (\text{for } a_D \ll c_D, d_D), \tag{5.38}$$

where $P_\nu = \mathrm{diag}(e^{i(\alpha_{D3}-\alpha_{D2})}, e^{-i(\alpha_{D3}-\alpha_{D2})}, 1)$. Therefore, the following conditions should be satisfied in order to realize our M_ν in Eq. (5.35),

$$\frac{a_D^2}{a_R} \ll \frac{c_D^2}{d_R} \simeq \frac{c_D d_D}{d_R} \simeq \frac{d_D^2}{d_R}. \tag{5.39}$$

Namely, it turns out that the large lepton mixing angle is realized through the seesaw mechanism by using the following M_D and M_R,

$$M_D = P_D^\dagger \begin{pmatrix} 0 & a_D & 0 \\ a_D & * & d_D \\ 0 & d_D & d_D \end{pmatrix} P_D, \quad M_R = \begin{pmatrix} 0 & a_R & 0 \\ a_R & * & * \\ 0 & * & d_R \end{pmatrix}, \tag{5.40}$$

with $c_D = d_D$ and a hierarchy condition

$$\left(\frac{a_D}{d_D}\right)^2 \ll \frac{a_R}{d_R}. \tag{5.41}$$

It should be noted that the components b_D in M_D, b_R, and c_R in M_R which are denoted as asterisks are not important for reproducing the large lepton mixing angle at all.

VI. CONCLUSION

We have reconsidered the mass matrix model with a universal and seesaw-invariant form of four-zero structure given by

$$M_f = P_f^\dagger \begin{pmatrix} 0 & a_f & 0 \\ a_f & b_f & c_f \\ 0 & c_f & d_f \end{pmatrix} P_f, \quad \text{for } f = u, d, \text{ and } e, \tag{6.1}$$

$$M_f = P_f^\dagger \begin{pmatrix} 0 & a_f & 0 \\ a_f & b_f & c_f \\ 0 & c_f & d_f \end{pmatrix} P_f^*, \quad \text{for } f = \nu. \tag{6.2}$$

The analytical expressions for the CKM quark mixing matrix are derived as functions of the four parameters x_u, x_d, α_2, and α_3. We do fine tuning of the parameters so as to reproduce the experimental data. It turns out that the CKM quark mixing matrix can be consistent with the data at the special value of the parameter given by $\alpha_2 \simeq \pi/2$ and in the allowed regions among α_3, x_u, and x_d as shown in Figs. 1–3.

We have also analyzed the MNS lepton mixing matrix analytically and shown that it is consistent with the observed large lepton mixings. The model predicts a relatively large (1, 3) element for the MNS lepton mixing matrix element:

$$|(U_{\mathrm{MNS}})_{13}|^2 \simeq \frac{|m_1|m_2}{m_3^2} \simeq (0.041\text{–}9.6) \times 10^{-2}, \tag{6.3}$$

which is close to the experimental upper bound at present. Therefore a determination of the finite value for $|(U_{\mathrm{MNS}})_{13}|^2$ in the near future experiment will be expectable in our model.

We have assumed the seesaw mechanism $M_\nu = -M_D M_R^{-1} M_D^T$ and the same four-zero structure for M_D and M_R. Within this framework, we have derived the conditions given by Eqs. (5.40) and (5.41) for the components of M_D and M_R to realize our structure for M_ν.

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[1] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1562 (1998); **86**, 5651 (2001); Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **87**, 071301 (2001); **89**, 011301 (2002); K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003).
[2] H. Fritzsch, Phys. Lett. B **73**, 317 (1978); **85**, 81 (1979); Nucl. Phys. **B155**, 189 (1979); L. F. Li, Phys. Lett. B **84**, 461 (1979); H. Georgi and C. V. Nanopoulos, Nucl. Phys. **B155**, 52 (1979); A. C. Rothman and K. Kang, Phys. Rev. Lett. **43**, 1548 (1979); A. Davidson, V. P. Nair, and K. C. Wali, Phys. Rev. D **29**, 1513 (1984); M. Shin, Phys. Lett. B **145**, 285 (1984); H. Georgi, A. Nelson, and M. Shin, Phys.

Lett. B **150**, 306 (1985); T. P. Cheng and L. F. Li, Phys. Rev. Lett. **55**, 2249 (1985).
[3] B. Stech, Phys. Lett. B **130**, 189 (1983); G. Ecker, Z. Phys. C **24**, 353 (1984).
[4] M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985); R. Johnson, J. Schechter, and M. Gronau, Phys. Rev. D **33**, 2641 (1986).
[5] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[6] K. Kang and M. Shin, Phys. Lett. B **165**, 383 (1985); **185**, 163 (1987).
[7] A. Bottino, C. W. Kim, H. Nishiura, and W. K. Sze, Phys. Rev. D **34**, 862 (1986).
[8] R. Johnson, S. Ranfone, and J. Schechter, Phys. Lett. B **179**, 355 (1986); Phys. Rev. D **35**, 282 (1987).
[9] G. C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D **39**, 3443 (1989).
[10] P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993).
[11] D. Du and Z. Z. Xing, Phys. Rev. D **48**, 2349 (1993); H. Fritzsch and Z. Z. Xing, Phys. Lett. B **353**, 114 (1995).
[12] K. Kang and S. K. Kang, Phys. Rev. D **56**, 1511 (1997); H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D **60**, 013006 (1999); K. Matsuda, T. Fukuyama, and H. Nishiura, Phys. Rev. D **61**, 053001 (2000); K. Kang, S. K. Kang, C. S. Kim, and S. M. Kim, Mod. Phys. Lett. A **16**, 2169 (2001); H. Fritzsch and Z. Z. Xing, Phys. Rev. D **61**, 073016 (2000); C. Giunti and M. Tanimoto, Phys. Rev. D **66**, 113006 (2002); M. Frigerio and A. Yu. Smirnov, Nucl. Phys. **B640**, 233 (2002); Phys. Rev. D **67**, 013007 (2003); P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002); **594**, 324 (2004); E. Ma, Mod. Phys. Lett. A **17**, 2361 (2002); Phys. Rev. D **66**, 117301 (2002); K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003); Z. Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004); M. Bando, S. Kaneko, M. Obara, and M. Tanimoto, Phys. Lett. B **580**, 229 (2004); O. L. G. Peres and A. Yu. Smirnov, Nucl. Phys. **B680**, 479 (2004); C. H. Albright, Phys. Lett. B **599**, 285 (2004); J. Ferrandis and S. Pakvasa, Phys. Lett. B **603**, 184 (2004); S. T. Petcov and W. Rodejohann, Phys. Rev. D **71**, 073002 (2005); S. S. Masood, S. Nasri, and J. Schechter, Phys. Rev. D **71**, 093005 (2005); R. Dermíšek and S. Raby, Phys. Lett. B **622**, 327 (2005); F. Plentinger and W. Rodejohann, Phys. Lett. B **625**, 264 (2005).
[13] T. Fukuyama and H. Nishiura, hep-ph/9702253; in *Proceedings of the International Workshop on Masses and Mixings of Quarks and Leptons, Shizuoka, Japan, 1997*, edited by Y. Koide (World Scientific, Singapore, 1998), p. 252.
[14] R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999).
[15] E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001).
[16] C. S. Lam, Phys. Lett. B **507**, 214 (2001); Phys. Rev. D **71**, 093001 (2005).
[17] K. R. S. Balaji, W. Grimus, and T. Schwetz, Phys. Lett. B **508**, 301 (2001).
[18] W. Grimus and L. Lavoura, Acta Phys. Polon. B **32**, 3719 (2001); J. High Energy Phys. 07 (2001) 045; Eur. Phys. J. C **28**, 123 (2003); Phys. Lett. B **572**, 189 (2003); J. Phys. G **30**, 1073 (2004); Phys. Lett. B **579**, 113 (2004); J. High Energy Phys. 08 (2005) 013; W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, and M. Tanimoto, J. High Energy Phys. 07 (2004) 078; W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, Nucl. Phys. **B713**, 151 (2005); W. Grimus, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, J. High Energy Phys. 01 (2006) 110.
[19] T. Kitabayashi and M. Yasue, Phys. Lett. B **524**, 308 (2002); Int. J. Mod. Phys. A **17**, 2519 (2002); Phys. Rev. D **67**, 015006 (2003); I. Aizawa, M. Ishiguro, T. Kitabayashi, and M. Yasue, Phys. Rev. D **70**, 015011 (2004); I. Aizawa, T. Kitabayashi, and M. Yasue, Phys. Rev. D **71**, 075011 (2005); **72**, 055014 (2005); Nucl. Phys. **B728**, 220 (2005); T. Kitabayashi and M. Yasue, Phys. Lett. B **607**, 267 (2005); **621**, 133 (2005); I. Aizawa and M. Yasue, Phys. Rev. D **73**, 015002 (2006); K. Fuki and M. Yasue, Phys. Rev. D **73**, 055014 (2006).
[20] S. Kaneko, H. Sawanaka, and M. Tanimoto, J. High Energy Phys. 08 (2005) 073.
[21] N. Haba and W. Rodejohann, hep-ph/0603206.
[22] R. N. Mohapatra, J. High Energy Phys. 10 (2004) 027; R. N. Mohapatra and S. Nasri, Phys. Rev. D **71**, 033001 (2005); R. N. Mohapatra, S. Nasri, and Hai-bo Yu, Phys. Lett. B **615**, 231 (2005); R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005); R. N. Mohapatra and H. Yu, Phys. Lett. B **636**, 114 (2006).
[23] W. Rodejohann and M. A. Schmidt, hep-ph/0507300.
[24] H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D **65**, 097301 (2002).
[25] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D **66**, 093006 (2002).
[26] K. Matsuda and H. Nishiura, Phys. Rev. D **69**, 053005 (2004).
[27] K. Matsuda and H. Nishiura, Phys. Rev. D **69**, 117302 (2004).
[28] A. Ghosal, Mod. Phys. Lett. A **19**, 2579 (2004).
[29] Y. Koide, Phys. Rev. D **69**, 093001 (2004).
[30] K. Matsuda and H. Nishiura, Phys. Rev. D **71**, 073001 (2005); **72**, 033011 (2005).
[31] K. Matsuda and H. Nishiura, Phys. Rev. D **73**, 013008 (2006).
[32] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); **25**, 774 (1982).
[33] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[34] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].
[35] H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D **60**, 013006 (1999).
[36] R. G. Roberts, A. Romanino, Graham G. Ross, and L. Velasco-Sevilla, Nucl. Phys. **B615**, 358 (2001).

[37] H. D. Kim, S. Raby, and L. Schradin, Phys. Rev. D **69**, 092002 (2004).
[38] G. C. Branco, D. Emmanuel-Costa, and R. Gonzalez Felipe, Phys. Lett. B **477**, 147 (2000); **483**, 87 (2000); G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Lett. B **597**, 155 (2004).
[39] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[40] See, for a recent review, M. C. Gonzalez-Garcia, NOON2004 (http://www-sk.icrr.u-tokyo.ac.jp/noon2004/).
[41] M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999).
[42] Q. R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002); **89**, 011302 (2002).
[43] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979); S. P. Mikheev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento Soc. Ital. Fis. C **9**, 17 (1986).