

Canonical constraints on leptonic CP violation using ultrahigh energy cosmic ray neutrino fluxesK. R. S. Balaji,^{1,2,3,*} Gilles Couture,^{2,†} and Cherif Hamzaoui^{2,‡}¹*Department of Physics, McGill University, Montréal, Quebec, Canada H3A 2T8*²*Département des Sciences de la Terre et de l'Atmosphère, Université du Québec à Montréal, CP 8888, succ. centre-ville, Montréal, Quebec, Canada H3C 3P8*³*Physique des Particules, Université de Montréal, CP 6128, succ. centre-ville, Montréal, Quebec, Canada H3C 3J7*

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It is shown that one can in principle constrain the CP -violating parameter δ from measurements of four independent $|V_{ij}|^2$'s, or three $|V_{ij}|^2$'s and a ratio of two of them, in the leptonic sector. To quantify our approach, using unitarity, we derive simple expressions in terms of four independent $|V_{ij}|^2$'s for $\cos\delta$, and an expression for $\sin^2\delta$ from J^2 . Thus, depending on the values for $|V_{ij}|$ and their accuracy, we can set meaningful limits on $|\delta|$. To illustrate numerically, if $|V_{\mu 1}|^2$ is close to 0.1 with a 10% precision, and if $|V_{e 3}|^2$ is larger than 0.005 and for values of $|V_{e 2}|^2$ and $|V_{\mu 3}|^2$ that stay within ± 0.1 of the current experimental data leads to a bound: $\pi/2 \leq |\delta| \leq \pi$. Alternatively, a certain combination of parameters with values of $|V_{e 3}|^2$ larger than 0.01 leads to a closed bound of $73 \leq |\delta| \leq 103$. In general, we find that it is better to use $|V_{\mu 1}|^2$ or $|V_{\tau 1}|^2$ as the fourth independent $|V_{ij}|^2$ and that, over most of the parameter space, δ is least sensitive to $|V_{e 3}|^2$. With just three independent measurements (solar, atmospheric, and reactor), it is impossible to set limits on the CP phase. In this respect, we study the use of ultrahigh energy cosmic ray neutrino fluxes as the additional fourth information. We find that within the SM, neutrino fluxes of all three flavors will be very similar but that pushing current neutrino data to their extreme values still allowed, ratios of cosmic neutrino fluxes can differ by up to 20%; such large discrepancies could imply negligibly small CP violation. We also study a nonradiative neutrino decay model and find that the neutrino fluxes can differ by a factor of up to 3 within this model and that an accuracy of 10% on the neutrino fluxes is sufficient to set interesting limits on δ .

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I. INTRODUCTION

One of the significant achievements of particle physics has been our better understanding of the leptonic flavor mixings. Following some very detailed and painstaking measurements, the neutrino anomaly has now been confirmed; both in the solar and atmospheric sector [1]. An immediate conclusion is the existence of new physics beyond standard model and to introduce a small neutrino mass. Such an extension leads to the notion of neutrino oscillation [2] similar in spirit to quark sector. A genuine three flavor analysis allows for a solution space consisting of two mass-squared differences and three angles. The solution amounts to finding the allowed parameter space for the solar mixing angle, and the atmospheric mixing angle along with a strongly constrained reactor angle. Phenomenologically, best fit values and the allowed range for these three mixing parameters dictate a pattern that is almost being negligible (reactor) to moderate (solar) to maximal (atmospheric). For a review on the analysis, we refer to [3].

Similar to the quark sector, one can have CP violation in the leptonic sector due to massive neutrinos. Unlike the quark sector, however, the CP violation in the leptonic sector can come due to both Dirac and Majorana phases

in the mixing matrix, depending on the nature of the neutrinos [4]. Clearly, assessing the Dirac or Majorana nature of the neutrinos will be an important goal in the future. In this paper, we consider only CP violation *à la Dirac* and do not address at all Majorana phases. As a pertinent question, how to measure CP violation in neutrino oscillations has attracted a lot of attention in the past [5,6], and no doubt the search for leptonic CP violation will be one of the main goals of experimental particle physics in the years to come. Given the strong reactor constraints [7], it is very difficult for current neutrino oscillation experiments to measure CP violation. Nonetheless, as a proposal, a measurement is possible by one searching for differences between neutrino and antineutrino survival in a long-baseline experiment and measuring the spectrum [8].

In the present analysis, we want to explore the possibility of extracting information on CP violation in the leptonic sector without performing a direct CP violation experiment. Clearly, such an experiment will have to be done eventually, but in the near future it might be easier to measure individual lepton mixing matrix elements (V_{ij}). Thus, it becomes interesting to see how information about these can be translated into information about CP violation in the leptonic sector and what level of precision will be required on the $|V_{ij}|^2$'s to set interesting constraints on δ .

In the next section, we will contrast two parametrizations that represent leptonic mixing; the usual parametrization in terms of mixing angles and phases [9] and another

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one constructed purely as moduli elements, where CP violation is not explicit but still present. Following this, we will derive expressions that allow one to extract information on CP violation from measurements of four $|V_{ij}|$'s or combinations of them. These relations are used to calculate the precision on the measurements of the $|V_{ij}|^2$'s required in order to set constraints on δ . We will then consider the use of ultrahigh energy cosmic ray (UHCR) neutrino fluxes to obtain the fourth $|V_{ij}|^2$ needed to set constraints on δ [10]. We also derive general relations about their fluxes and estimate the precision required in order to set interesting limits on δ . To conclude, we will also consider a specific model that digresses from the SM and will strongly affect the neutrino fluxes that reach the Earth and see how useful this can be in our extraction of the phase δ .

II. LINKING FOUR $|V_{ij}|^2$ AND δ

A. Rephasing invariant parametrization

We assume that we have three neutrino flavors and consider only CP violation *à la Dirac*. We also assume unitarity in the mixing matrix even though some popular models of neutrino mass generation lead to violation of unitarity.

The elements of the lepton mixing matrix are defined as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (2.1)$$

It has been shown before [11] that all the information about mixing and CP violation in the quark sector can be parametrized in terms of four independent moduli of (2.1). These moduli are rephasing invariant and basis independent. A set of four independent parameters is not unique and any set of four parameters where we do not have three on the same row or column is acceptable. Thus, there are nine such ensembles which are allowed. For the present analysis, our choice is $|V_{e2}|$, $|V_{e3}|$, $|V_{\mu1}|$, and $|V_{\mu3}|$. This choice seems appropriate since we have experimental information on three of the four parameters.

The exact expressions between the moduli of the elements of the mixing matrix and the mixing angles are obtained to be

$$|V_{e3}|^2 = \sin^2\theta_{13}, \quad (2.2)$$

$$|V_{e2}|^2 = \sin^2\theta_{12}\cos^2\theta_{13}, \quad (2.3)$$

$$|V_{\mu3}|^2 = \sin^2\theta_{23}\cos^2\theta_{13}. \quad (2.4)$$

Our current knowledge on the magnitude of the elements of the leptonic mixing matrix comes from experiments on neutrino oscillation and can be summarized as follows [3]:

$$\begin{aligned} \sin^2\theta_{12} &= 0.3 \pm 0.08 & \sin^2\theta_{23} &= 0.5 \pm 0.18 \\ \sin^2\theta_{13} &\leq 0.05 \text{ at } 3\sigma \end{aligned} \quad (2.5)$$

B. Unitarity and $\cos(\delta)$

At the present time, there is no experimental information about $|V_{\mu1}|$ and one can only limit its modulus through unitarity. In the notation of (2.1), we can write

$$\begin{aligned} V_{\mu1} &= -\frac{1}{1-|V_{e3}|^2}(|V_{e2}||V_{\tau3}| + |V_{e1}||V_{\mu3}||V_{e3}|e^{i\delta}), \\ |V_{e1}| &= \sqrt{1-|V_{e2}|^2-|V_{e3}|^2}, \\ |V_{\tau3}| &= \sqrt{1-|V_{\mu3}|^2-|V_{e3}|^2}. \end{aligned} \quad (2.6)$$

Expanding up to order $|V_{e3}|^2$, we calculate $|V_{\mu1}|^2$ to be

$$\begin{aligned} |V_{\mu1}|^2 &= |V_{e2}|^2(1-|V_{\mu3}|^2) + |V_{e3}|^2(|V_{e2}|^2 \\ &\quad - 3|V_{e2}|^2|V_{\mu3}|^2 + |V_{\mu3}|^2) \\ &\quad + 2|V_{e3}||V_{e2}||V_{\mu3}|\cos(\delta) \\ &\quad \times \sqrt{1-|V_{\mu3}|^2}\sqrt{1-|V_{e2}|^2} \end{aligned} \quad (2.7)$$

With current experimental data (2.5) and adding the errors in quadrature, the limits on $|V_{\mu1}|^2$ translate to

$$\begin{aligned} |V_{\mu1}|^2 &= 0.15 \pm 0.07 + (0.35 \pm 0.15)|V_{e3}|^2 \\ &\quad \pm (0.46 \pm 0.03)|V_{e3}|. \end{aligned} \quad (2.8)$$

If we use the 3σ limit of 0.05 on $|V_{e3}|^2$, we obtain a lower limit of 0 and an upper limit of 0.35 on $|V_{\mu1}|^2$ while the 1σ limit of 0.012 on $|V_{e3}|^2$ leads to $0.029 \leq |V_{\mu1}|^2 \leq 0.28$. Therefore, we can arrive at closed bound

$$0.17 \leq |V_{\mu1}| \leq 0.52 \text{ at } 1\sigma. \quad (2.9)$$

We can use this expression to set a bound on $|V_{\mu1}|^2$ or we can invert it to get information on $\cos(\delta)$ once we have information on all four $|V_{ij}|^2$, the exact relation being

$$\begin{aligned} \cos\delta &= \frac{|V_{\mu1}|^2 - |V_{e2}|^2(1-|V_{\mu3}|^2) + B|V_{e3}|^2 + C|V_{e3}|^4}{X}, \\ X &= 2|V_{e2}||V_{e3}||V_{\mu3}|\sqrt{1-|V_{e2}|^2-|V_{e3}|^2} \\ &\quad \times \sqrt{1-|V_{e3}|^2-|V_{\mu3}|^2}, \\ B &= |V_{e2}|^2 - |V_{\mu3}|^2 - 2|V_{\mu1}|^2 + |V_{e2}|^2|V_{\mu3}|^2, \\ C &= |V_{\mu1}|^2 + |V_{\mu3}|^2. \end{aligned} \quad (2.10)$$

Clearly, without information on $|V_{\mu1}|^2$, we cannot say anything on δ .

C. CP violation and the $|V_{ij}|$'s

The measure of CP violation is expressed, in general, through the Jarlskog parameter defined as [12]

$$J = \text{Im}(V_{e2}V_{\mu3}V_{e3}^*V_{\mu2}^*). \quad (2.11)$$

Once we have chosen our four independent parameters of the mixing matrix, J^2 is not independent but an explicit function of these four parameters, namely [11]

$$J^2 = (1 - |V_{e3}|^2 - |V_{e2}|^2)|V_{e3}|^2|V_{\mu1}|^2|V_{\mu3}|^2 - \frac{1}{4}(|V_{e2}|^2 - |V_{\mu1}|^2 + |V_{\mu1}|^2|V_{e3}|^2 - |V_{\mu3}|^2|V_{e3}|^2 - |V_{\mu3}|^2|V_{e2}|^2)^2. \quad (2.12)$$

This is a general expression based on rephasing invariants and we can use it to set limits on $|V_{\mu1}|^2$: requiring positivity of J^2 leads to (2.7) with $\cos\delta$ replaced by ± 1 . We can also calculate what value of $|V_{\mu1}|^2$ will maximize J^2 such that $|V_{\mu1}|^2_{J^2=J^2_{\max}}$. Differentiating J^2 with respect to $|V_{\mu1}|^2$ and retaining only first order in $|V_{e3}|^2$, one then recovers the first two terms of (2.7). This is consistent since maximizing J^2 requires $\sin^2\delta = 1$. One observes here that a value of $|V_{\mu1}|^2$ larger than $|V_{\mu1}|^2_{J^2=J^2_{\max}}$ requires δ to be in the first or in the fourth quadrant. This simple point does not require particularly precise data and could already be useful information for model builders.

In the standard basis, J^2 is given by [13]

$$J^2 = \sin^2(\theta_{12})\cos^2(\theta_{12})\sin^2(\theta_{23})\cos^2(\theta_{23})\sin^2(\theta_{13}) \times \cos^2(\theta_{13})\cos^2(\theta_{13})\sin^2(\delta) \quad (2.13)$$

Clearly, whether in the V_{PMNS} or in our current basis, J^2 is the same physical observable and we can compare the two expressions for this parameter. Using the relations between the V_{ij} and $\sin(\theta_{ij})$, we express J^2 in terms of $\sin^2(\delta)$ and three of our $|V_{ij}|^2$:

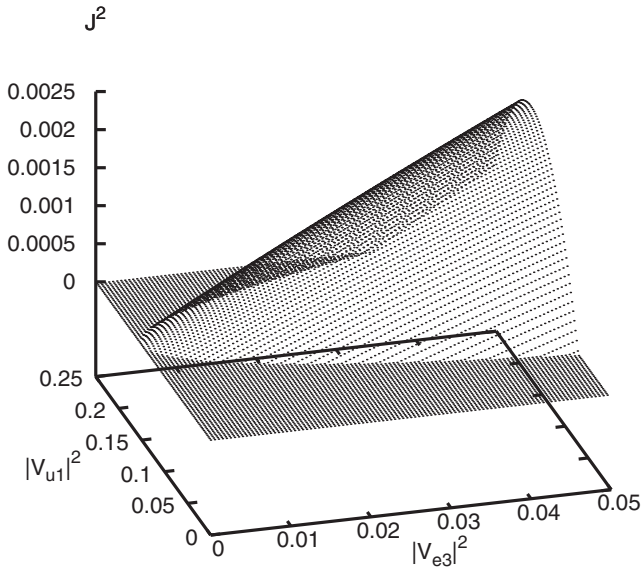


FIG. 1. J^2 as a function of $|V_{e3}|^2$ and $|V_{\mu1}|^2$ for the current central values of $|V_{e2}|^2$ (0.3) and $|V_{\mu3}|^2$ (0.5).

$$J^2 = \frac{1}{(1 - |V_{e3}|^2)^2} |V_{e2}|^2 |V_{\mu3}|^2 |V_{e3}|^2 (1 - |V_{e3}|^2 - |V_{e2}|^2) \times (1 - |V_{e3}|^2 - |V_{\mu3}|^2) \sin^2\delta. \quad (2.14)$$

Using (2.12) and (2.14) we now have a relation between $\sin^2\delta$ and our four $|V_{ij}|^2$. Therefore, we can relate δ to our set of four $|V_{ij}|^2$ through either $\cos\delta$ or $\sin^2\delta$. Since we want ultimately information on δ itself and there is more information through $\cos\delta$ than through $\sin^2(\delta)$, in what follows, we will concentrate on $\cos\delta$. We will be left with a twofold degeneracy as we will not be able to get the sign of δ .

In Fig. 1, we plot J^2 as a function of $|V_{\mu1}|^2$ and $|V_{e3}|^2$ for central values of $|V_{\mu3}|^2$ (0.5) and $|V_{e2}|^2$ (0.3). We see clearly that the vanishing of $|V_{e3}|^2$ leads to the vanishing of J^2 , as it must be. We also see that J^2 is a fairly linear function of $|V_{e3}|^2$; this is expected since the maximum value $|V_{e3}|^2$ can have is rather small compared to the other parameters and an expansion to first order in $|V_{e3}|^2$ would be adequate. For numerical purposes, we note that the largest value taken by J^2 on this figure is 2.42×10^{-3} ; it

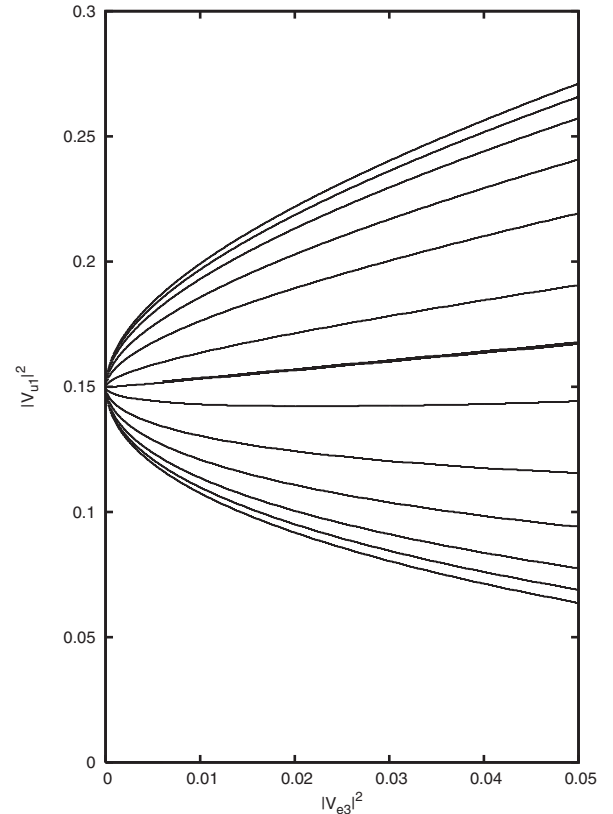


FIG. 2. Different values of $\sin^2(\delta)$ in the $|V_{e3}|^2 - |V_{\mu1}|^2$ plane. From the outside, inward, we plot $\sin^2(\delta) = 0, 0.1, 0.25, 0.5, 0.75, 0.95, 1$. The line in the middle corresponds to $\sin^2(\delta) = 1$ and is given by Eq. (2.7) with $\cos(\delta) = 0$; the most outward curves correspond to $\cos(\delta) = \pm 1$.

is clearly at the largest possible value of $|V_{e3}|^2$ and at $|V_{\mu 1}|^2 \approx 0.167$.

In Fig. 2, we plot curves of constant $\sin^2(\delta)$. When we fix $|V_{e2}|^2$, $|V_{\mu 3}|^2$, and $|V_{e3}|^2$, if we ask what value of $|V_{\mu 1}|^2$ will give 25% of the maximum value that J^2 can have, we are in fact setting $\sin^2(\delta) = 0.25$. The straight line in the middle is the curve $|V_{\mu 1}|^2_{J^2=J^2_{\max}}$ and is given by (2.7) with $\cos(\delta) = 0$.

III. CONSTRAINING δ FROM THE $|V_{ij}|^2$ OR THEIR RATIOS

In order to be able to set some bounds on δ , we need at least lower limits on $|V_{e3}|^2$ and $|V_{\mu 1}|^2$ besides the data that we have on $|V_{e2}|^2$ and $|V_{\mu 3}|^2$; as we can see in Fig. 2, just an upper limit cannot constrain δ . In what follows, we explore the full 4-dimensional parameter space: we allow $|V_{\mu 1}|^2$ to cover the whole range allowed by unitarity, and $|V_{e3}|^2$ varies from 0.001 to 0.03, since 0.03 is close to the current $3\text{-}\sigma$ limit and 0.001 makes CP violation extremely small at planned CP -violating experiments [14]. As for $|V_{e2}|^2$ and $|V_{\mu 3}|^2$, we will work with values that can differ

by ± 0.1 from their current central values of 0.3 and 0.5, respectively.

We assume that we have four $|V_{ij}|^2$'s with their experimental uncertainties. The experimental central values of the $|V_{ij}|^2$'s ($|V_{ij}|^2_c$) lead to the central value of $\cos\delta$. In order to estimate the error, or range that we should associate to this central value, we use a Monte Carlo technique and we cover, for each $|V_{ij}|^2$, the space $(|V_{ij}|^2_c - \text{experimental error}) - (|V_{ij}|^2_c + \text{experimental error})$. Within this 4-dimensional space, we are interested only in the largest and smallest values that $\cos\delta$ can have; these become the range that $\cos\delta$ covers with this particular combination of $|V_{ij}|^2$ and their associated errors.

As our fourth $|V_{ij}|^2$, we picked $|V_{\mu 1}|^2$. This choice is not unique and we could have chosen $|V_{\mu 2}|^2$, or $|V_{\tau 1}|^2$, or $|V_{\tau 2}|^2$. It is straightforward to rewrite all our equations in terms of these parameters through the following relations:

$$\begin{aligned} |V_{\mu 2}|^2 &= 1 - |V_{\mu 1}|^2 - |V_{\mu 3}|^2 \\ |V_{\tau 1}|^2 &= |V_{e2}|^2 + |V_{e3}|^2 - |V_{\mu 1}|^2 \\ |V_{\tau 2}|^2 &= |V_{\mu 3}|^2 + |V_{\mu 1}|^2 - |V_{e2}|^2. \end{aligned} \quad (3.1)$$

TABLE I. Values of $\cos(\delta)$ for different combinations of parameters: $\cos(\delta)_c$ is obtained from the central values of the parameters while $\cos(\delta)_-$ and $\cos(\delta)_+$ are the extremum values allowed by the uncertainties (shown in percent, in parentheses next to each parameter).

$ V_{e2} ^2$	$ V_{\mu 3} ^2$	$ V_{e3} ^2$	$\rho_1, \rho_2, V_{\mu 1} ^2$	$\cos(\delta)_-$	$\cos(\delta)_c$	$\cos(\delta)_+$
0.30 (10)	0.50 (10)	0.03 (10)	$\rho_1 = 1.0(10)$	-0.257	0.056	0.387
			$\rho_2 = 2.3(10)$	-0.300	0.111	0.457
			$ V_{\mu 1} ^2 = 0.15(10)$	-0.708	-0.132	0.442
0.30 (5)	0.50 (5)	0.01 (5)	$\rho_1 = 1.0(5)$	-0.224	0.033	0.294
			$\rho_2 = 2.3(5)$	-0.285	0.042	0.329
			$ V_{\mu 1} ^2 = 0.15(5)$	-0.569	-0.076	0.414
0.30 (5)	0.50 (5)	0.01 (20)	$\rho_1 = 1.0(5)$	-0.247	0.033	0.310
			$\rho_2 = 2.3(5)$	-0.303	0.042	0.343
			$ V_{\mu 1} ^2 = 0.15(5)$	-0.607	-0.076	0.457
0.30 (5)	0.50 (5)	0.01 (5)	$\rho_1 = 1.0(10)$	-0.306	0.033	0.391
			$\rho_2 = 2.3(10)$	-0.404	0.042	0.440
			$ V_{\mu 1} ^2 = 0.15(10)$	-0.747	-0.076	0.599
0.30 (5)	0.50 (5)	0.015 (5)	$\rho_1 = 1.0(5)$	-0.170	0.040	0.255
			$\rho_2 = 2.3(5)$	-0.190	0.065	0.300
			$ V_{\mu 1} ^2 = 0.15(5)$	-0.493	-0.093	0.306
0.30 (5)	0.50 (5)	0.005 (5)	$\rho_1 = 1.0(5)$	-0.339	0.023	0.392
			$\rho_2 = 2.3(5)$	-0.429	0.007	0.412
			$ V_{\mu 1} ^2 = 0.15(5)$	-0.756	-0.054	0.642
0.30 (5)	0.50 (2)	0.001 (25)	$\rho_1 = 1.0(5)$	-0.529	0.010	0.555
			$\rho_2 = 2.3(5)$	-1.0	-0.081	0.885
			$ V_{\mu 1} ^2 = 0.15(5)$	-1.0	-0.024	1.0
0.30 (2)	0.50 (5)	0.001 (25)	$\rho_1 = 1.0(5)$	-0.879	0.010	0.901
			$\rho_2 = 2.3(5)$	-0.796	-0.081	0.552
			$ V_{\mu 1} ^2 = 0.15(5)$	-1.0	-0.024	1.0

We studied all four parameters and we can say that, when they (i.e. $|V_{\mu 1}|^2$, $|V_{\mu 2}|^2$, $|V_{\tau 1}|^2$, $|V_{\tau 2}|^2$) have the same experimental error (e.g. 5%) then:

- (i) if the known parameters stay close to their current central values, in general, $|V_{\mu 1}|^2$ leads to slightly better bounds than $|V_{\tau 1}|^2$
- (ii) if the known parameters digress substantially from their current experimental values, then $|V_{\tau 1}|^2$ leads to slightly better bounds than $|V_{\mu 1}|^2$
- (iii) $|V_{\mu 2}|^2$ and $|V_{\tau 2}|^2$ are not as good as the previous two: the limits obtained from these parameters are degraded by about a factor of 2 when compared to those obtained from the previous two parameters; $|V_{\mu 2}|^2$ is, in general, a little bit better than $|V_{\tau 2}|^2$
- (iv) things improve a bit for the last two parameters if we give all four parameters the same absolute uncertainty instead of the same relative uncertainty (e.g. we compare $|V_{\mu 1}|^2 = 0.150 \pm 0.005$ and $|V_{\tau 2}|^2 = 0.350 \pm 0.005$ instead of $\pm 5\%$ for all parameters). Even then, $|V_{\mu 1}|^2$ and $|V_{\tau 1}|^2$ remain better than the other two parameters, but by a factor of 1.5 instead of a factor of 2.

We also take into consideration some ratios of $|V_{ij}|^2$ as the potential fourth parameter. The first one is $\rho_1 = |V_{\tau 1}|^2/|V_{\mu 1}|^2$ and has very similar properties to $|V_{\tau 2}|^2/|V_{\mu 2}|^2$; the second one is $\rho_2 = |V_{\tau 2}|^2/|V_{\tau 1}|^2$ and has very similar properties to $|V_{\mu 2}|^2/|V_{\mu 1}|^2$. We also studied $|V_{\tau 2}|^2/|V_{\mu 1}|^2$ but it turned out to be too sensitive to both $|V_{e 3}|^2$ and $|V_{\mu 3}|^2$ to be of any use; and similarly for

$|V_{\tau 1}|^2/|V_{\mu 2}|^2$. The same can be said of any ratio that involves $|V_{e 1}|^2$. So, we present in Tables I and II the limits that we can set on $\cos\delta$ from measurements on $|V_{\mu 1}|^2$, ρ_1 , and ρ_2 . These are representative of what can be achieved. After studying the parameter space described above, we can say that:

- (i) uncertainties of 10% on the four parameters can lead to very tight bounds on δ ; the uncertainty on $|V_{e 3}|^2$ can be much larger than this without affecting the bounds very much.
- (ii) an interesting constraint already occurs for the combination $(0.3, 0.5, P, |V_{e 3}|^2)$ with a 10% uncertainty on the parameters: the first two parameters ($|V_{e 2}|^2$ and $|V_{\mu 3}|^2$) are at their current central values while the third one (either $|V_{\mu 1}|^2 = 0.15$, $\rho_1 = 1.0$, or $\rho_2 = 2.3$) is close to the value that maximizes J^2 . We find that if $|V_{e 3}|^2$ turns out to be large (0.03), then $|\delta|$ has a range of 40–70 deg centered at about 90 deg, depending what P one uses; this range decreases as $|V_{e 3}|^2$ increases. If the uncertainties are reduced to 5%, the range becomes 30–60 deg and $|V_{e 3}|^2$ can be reduced to 0.01.
- (iii) for most combinations of $|V_{e 2}|^2$ and $|V_{\mu 3}|^2$, if $|V_{\mu 1}|^2$ turns out to be relatively small, about 0.1, then $|\delta|$ has to be between $\pi/2$ and π for any $|V_{e 3}|^2$ larger than 0.005.
- (iv) the limits are not very sensitive to the uncertainty on $|V_{e 3}|^2$; going from 5% to 20% does not change

TABLE II. Values of $\cos(\delta)$ for different combinations of parameters: $\cos(\delta)_c$ is obtained from the central values of the parameters while $\cos(\delta)_-$ and $\cos(\delta)_+$ are the extremum values allowed by the uncertainties (shown in percent, in parentheses next to each parameter).

$ V_{e 2} ^2$	$ V_{\mu 3} ^2$	$ V_{e 3} ^2$	$\rho_1, \rho_2, V_{\mu 1} ^2$	$\cos(\delta)_-$	$\cos(\delta)_c$	$\cos(\delta)_+$
0.30 (5)	0.40 (5)	0.01 (5)	$\rho_1 = 0.68(5)$	-0.204	0.025	0.247
			$\rho_2 = 2.2(5)$	-0.294	-0.034	0.195
			$ V_{\mu 1} ^2 = 0.18(5)$	-0.615	-0.076	0.458
0.30 (10)	0.40 (5)	0.01 (5)	$\rho_1 = 0.68(5)$	-0.213	0.025	0.330
			$\rho_2 = 2.2(5)$	-0.446	-0.034	0.330
			$ V_{\mu 1} ^2 = 0.18(5)$	-0.813	-0.076	0.665
0.30 (5)	0.40 (10)	0.01 (5)	$\rho_1 = 0.68(5)$	-0.349	0.025	0.383
			$\rho_2 = 2.2(5)$	-0.310	-0.034	0.199
			$ V_{\mu 1} ^2 = 0.18(5)$	-0.765	-0.076	0.582
0.30 (5)	0.40 (5)	0.01 (10)	$\rho_1 = 0.68(5)$	-0.212	0.025	0.250
			$\rho_2 = 2.2(5)$	-0.306	-0.034	0.196
			$ V_{\mu 1} ^2 = 0.18(5)$	-0.628	-0.076	0.471
0.25 (2)	0.55 (2)	0.01 (2)	$\rho_1 = 0.8(2)$	0.551	0.650	0.760
			$\rho_2 = 3.85(2)$	0.541	0.652	0.770
			$ V_{\mu 1} ^2 = 0.145(2)$	0.481	0.663	0.856
0.25 (2)	0.55 (2)	0.01 (25)	$\rho_1 = 0.8(2)$	0.553	0.650	0.854
			$\rho_2 = 3.85(2)$	0.543	0.652	0.852
			$ V_{\mu 1} ^2 = 0.145(2)$	0.482	0.663	0.993

the limits by much. If the other uncertainties are small (2%), the uncertainty on $|V_{e3}|^2$ can go up to 50% and it is still possible to get interesting bounds on δ

- (v) the limits are not very sensitive to $|V_{e3}|^2$ itself and going from 0.01 to 0.005 will not change the limits very much. When $|V_{e3}|^2$ becomes 0.001, however, the limits are somewhat degraded but still useful.
- (vi) as the value of $|V_{e3}|^2$ becomes smaller, the uncertainties on the other parameters must decrease in order to keep interesting bounds on δ ; note that the uncertainty on $|V_{e3}|^2$ can be rather large without affecting the bounds much
- (vii) in general, we find that the best parameter is ρ_1 , the second best is ρ_2 , and the third best is $|V_{\mu 1}|^2$.

The previous analysis is very general and describes which parameters must be measured and with what accuracy in order to set limits on δ . We now turn to some potential processes that could give us the fourth information that we need in order to be able to set limits on δ .

IV. ULTRAHIGH ENERGY COSMIC RAYS

Ultrahigh energy cosmic rays and their detectors have attracted a fair amount of attention in recent years [15–18]. In particular, the use of UHCRCs with the aim of observing leptonic CP violation has been considered in [19] An interesting consequence for the UHCRC neutrino spectra is due to maximal atmospheric mixings. Following maximal mixing, or $\tan\theta_A \approx 1$, leads to a unique prediction for UHCRC neutrino fluxes. It was shown that UHCRC neutrinos (which are expected to be sourced by cosmic objects such as AGNs) when measured by ground based detectors, the expected flavor ratio $\phi_e:\phi_\mu:\phi_\tau = 1:1:1$ [20]. This value is also known as the standard flavor ratio and by itself constitutes an independent confirmation of the neutrino mixing data from atmospheric sources. Since we propose to extract the Dirac CP phase present in the conventional PMNS mixing matrix, our discussion will be restricted to Dirac neutrinos. Before proceeding with the analysis, let us briefly allude to the significance of this proposal. It is well known that CP violation (due to oscillation) in the leptonic mixings (even if larger than the quark sector) will nonetheless remain a hard problem to resolve [14]. This arises from the strong constraints which reactor neutrinos set on the mixing element V_{e3} [7]. As a consequence of this, even if the CP phase is large, the smallness of V_{e3} leads to phase being insensitive to any CP violating measurements.

A. The basic formalism

In the same fashion as quarks mix, massive neutrinos mix (and also oscillate) between two eigenbases, the flavor (ν_α) and mass eigenbasis (ν_i). The two bases are related by a unitary matrix V such that $\nu_\alpha = V_{\alpha i}\nu_i$. Here we assume the summation over the mass eigenstates. Corresponding to

a particular mass eigenstates is a mass value, m_i , which in the limit of small mixings determines the mixing angles. Thus, in the limit of small mixings, θ_S mixes eigenstates ν_1 and ν_2 , while θ_A mixes eigenstates ν_2 and ν_3 and θ_R mixes the eigenstates ν_1 and ν_3 .

In the context of UHCRC neutrinos which travel astronomical distances, the coherence between the various mass eigenstates is averaged out. In other words, after production these neutrinos essentially travel as individual mass eigenstates. At the point of detection, they are in the flavor states. Therefore, the detection probability in a given flavor eigenstate is

$$\begin{aligned}\phi_e &= 1 + 2x(2c_A^2 - 1); \\ x &= (s_S c_S)^2, \\ \phi_\mu &= 2xc_A^2 + 2(c_A^4(1 - 2x) + s_A^4), \\ \phi_\tau &= s_A^2 + 2xs_A^2(1 - c_A^2),\end{aligned}\tag{4.1}$$

where s and c denote sine and cosine, respectively. From the above expressions and the experimental fact $\theta_A \approx \pi/4$, it follows that all neutrino flavors must be detected with the same weight factor. In deriving this result, we have disregarded the mixing corresponding to reactor experiments, which is consistent with zero [7]. This result is also independent of the solar angle θ_S .

B. Neutrino fluxes and $|V_{ij}|^2$

Let us revise in more detail the arguments from the previous section. Consider the probability of oscillating from flavor α to β

$$\begin{aligned}P_{\alpha\beta} &= \delta_{\alpha\beta} - 4 \sum_{i>j=1}^3 \text{Re}[K_{\alpha\beta,ij}] \sin^2(\Delta_{ij}) \\ &\quad + 4 \sum_{i>j=1}^3 \text{Im}[K_{\alpha\beta,ij}] \sin(\Delta_{ij}) \cos(\Delta_{ij}),\end{aligned}\tag{4.2}$$

$$K_{\alpha\beta,ij} = V_{\alpha i} V_{\beta i}^* V_{\alpha j}^* V_{\beta j}; \quad \Delta_{ij} = \frac{m_i^2 - m_j^2}{L/4E}.$$

We can express the real and imaginary parts of $K_{\alpha\beta,ij}$ in terms of moduli as follows:

$$\begin{aligned}2 \text{Re}[K_{\alpha\beta,ij}] &= |V_{\alpha i}|^2 |V_{\beta j}|^2 + |V_{\beta i}|^2 |V_{\alpha j}|^2 \\ &\quad - \sum_{\gamma,k} c_{\alpha\beta\gamma} c_{ijk} |V_{\gamma k}|^2\end{aligned}\tag{4.3}$$

and

$$\text{Im}[K_{\alpha\beta,ij}] = J \varepsilon_{\alpha\beta} \varepsilon_{ij},\tag{4.4}$$

where we have defined

$$\begin{aligned}c_{ijk} &= 1 \quad \text{if } i \neq j, j \neq k, k \neq i \\ &= 0 \quad \text{otherwise}\end{aligned}\tag{4.5}$$

and ε as the antisymmetric tensor

$$\varepsilon = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}. \quad (4.6)$$

We are interested here only in the probabilities for $\nu_\mu \rightarrow \nu_\tau$, $\nu_\mu \rightarrow \nu_e$, and $\nu_\tau \rightarrow \nu_e$. After combining two terms ($\Delta_{13} = \Delta_{23}$ to high accuracy), expressing the third term as a combination of $|V_{ij}|^2$ and averaging over the oscillating terms, we obtain

$$\begin{aligned} P_{\mu\tau} &= 2|V_{\mu 3}|^2|V_{\tau 3}|^2 - (|V_{e1}|^2|V_{e2}|^2 - |V_{\mu 1}|^2|V_{\mu 2}|^2 \\ &\quad - |V_{\tau 1}|^2|V_{\tau 2}|^2), \\ P_{e\tau} &= 2|V_{e3}|^2|V_{\tau 3}|^2 - (|V_{\mu 1}|^2|V_{\mu 2}|^2 - |V_{e1}|^2|V_{e2}|^2 \\ &\quad - |V_{\tau 1}|^2|V_{\tau 2}|^2), \\ P_{e\mu} &= 2|V_{\mu 3}|^2|V_{e3}|^2 - (|V_{\tau 1}|^2|V_{\tau 2}|^2 - |V_{\mu 1}|^2|V_{\mu 2}|^2 \\ &\quad - |V_{e1}|^2|V_{e2}|^2). \end{aligned} \quad (4.7)$$

One then uses the relations

$$\begin{aligned} |V_{e1}|^2 &= 1 - |V_{e2}|^2 - |V_{e3}|^2, \\ |V_{\mu 2}|^2 &= 1 - |V_{\mu 1}|^2 - |V_{\mu 3}|^2, \\ |V_{\tau 1}|^2 &= |V_{e2}|^2 + |V_{e3}|^2 - |V_{\mu 1}|^2, \\ |V_{\tau 2}|^2 &= |V_{\mu 1}|^2 + |V_{\mu 3}|^2 - |V_{e2}|^2, \end{aligned} \quad (4.8)$$

to express these probabilities in terms of our four $|V_{ij}|^2$.

Note that when averaging over the oscillating terms, the third term of (4.2) vanishes; therefore, one will not be able to observe directly CP violation with UHCR's.

If we assume that the main production mode of these UHCR's is [21]

$$p + \gamma \rightarrow \pi^\pm + X, \quad (4.9)$$

with subsequent decay of the π^\pm into muons, electrons, and neutrinos, we conclude that there will be two ν_μ (or $\bar{\nu}_\mu$) for every ν_e (or $\bar{\nu}_e$) and virtually no ν_τ (or $\bar{\nu}_\tau$). The initial fluxes are then $\phi_e^0, \phi_\mu^0 = 2\phi_e^0$, and $\phi_\tau^0 = 0$. In order to calculate a given neutrino flux that reaches the Earth, we take into account the probability that this neutrino will oscillate into other types of neutrinos and the probabilities that other neutrinos will oscillate into this type of neutrino. Up to a common geometrical factor, the observed terrestrial fluxes are

$$\begin{aligned} \phi_e^t &= \phi_e^0(1 + P_{e\mu} - P_{e\tau}), \\ \phi_\mu^t &= \phi_e^0(2 - P_{e\mu} - 2P_{\mu\tau}), \quad \phi_\tau^t = \phi_e^0(2P_{\mu\tau} + P_{e\tau}). \end{aligned} \quad (4.10)$$

In terms of matrix elements, we get

$$\begin{aligned} \phi_e^t &= \phi_e^0(1 - 2|V_{e2}|^2(|V_{\mu 3}|^2 - |V_{e2}|^2) \\ &\quad + 2|V_{\mu 1}|^2(1 - 2|V_{e2}|^2) - 2|V_{e3}|^2(1 + |V_{\mu 1}|^2 \\ &\quad - |V_{e2}|^2 - |V_{\mu 3}|^2) + 2|V_{e3}|^4), \\ \phi_\mu^t &= \phi_e^0(2 + (1 - |V_{\mu 3}|^2)(|V_{e2}|^2 - 4|V_{\mu 3}|^2) \\ &\quad - |V_{\mu 1}|^2(3 + 2|V_{e2}|^2 - 4|V_{\mu 3}|^2 - 4|V_{\mu 1}|^2) \\ &\quad + |V_{e3}|^2(|V_{\mu 3}|^2 - |V_{\mu 1}|^2)), \\ \phi_\tau^t &= \phi_e^0(4|V_{\mu 3}|^2(1 - |V_{\mu 3}|^2) - |V_{e2}|^2(1 - |V_{e2}|^2) \\ &\quad + 3|V_{e2}|^2(|V_{\mu 3}|^2 - |V_{e2}|^2) + |V_{e3}|^2(2 + 3|V_{\mu 1}|^2 \\ &\quad - 3|V_{\mu 3}|^2 - 2|V_{e2}|^2 - 2|V_{e3}|^2) \\ &\quad + |V_{\mu 1}|^2(1 - 4|V_{\mu 1}|^2 - 4|V_{\mu 3}|^2 + 6|V_{e2}|^2)). \end{aligned} \quad (4.11)$$

Clearly, we do not know the original neutrino fluxes at the source. Therefore, it is more meaningful to try to measure ratios of neutrino fluxes. We will consider the ratios $R_{e\mu} = \phi_e^t/\phi_\mu^t$ and $R_{e\tau} = \phi_e^t/\phi_\tau^t$ and calculate how they vary when we cover the parameter space available. Our results are summarized in Table III. One can see that:

- (i) by numerical accident, $|V_{\mu 3}|^2$ close to 0.5 has a very strong influence on $R_{e\mu}$ and $R_{e\tau}$ and tends to keep them close to 1
- (ii) if $|V_{\mu 3}|^2$ stays close 0.5 to within 10%, $R_{e\mu}$ and $R_{e\tau}$ remain close to 1, for any value of $|V_{e2}|^2$ and $|V_{e3}|^2$ that are within the experimentally allowed ranges

TABLE III. Different values of $R_{e\mu}$ and $R_{e\tau}$ for different combinations of 4 independent $|V_{ij}|^2$. $|V_{\mu 1}|^2$, and $|V_{\mu 1}|^2_\pm$ span the range allowed by unitarity and positivity of $|J|^2$; these extremum values of $|V_{\mu 1}|^2$ correspond to vanishingly small CP violation.

$ V_{e2} ^2$	$ V_{\mu 3} ^2$	$ V_{e3} ^2$	$\frac{ V_{\mu 1} ^2_-}{ V_{\mu 1} ^2_+}$	$R_{e\mu}$	$R_{e\tau}$
0.3	0.5	0.01	0.11	0.94	0.95
			0.20	1.04	1.06
0.3	0.5	0.005	0.12	0.96	0.96
			0.18	1.03	1.03
0.35	0.5	0.01	0.13	0.94	0.96
			0.23	1.03	1.05
0.35	0.55	0.01	0.12	0.88	0.91
			0.21	0.95	0.99
0.35	0.4	0.01	0.17	1.05	1.14
			0.26	1.14	1.22
0.35	0.6	0.01	0.10	0.80	0.86
			0.19	0.86	0.95
0.25	0.6	0.01	0.063	0.81	0.86
			0.145	0.90	1.00
0.3	0.6	0.01	0.081	0.81	0.86
			0.17	0.88	0.98

- (iii) when one varies $|V_{\mu 3}|^2$ by 20%, either up or down, it is possible for $R_{e\mu}$ and $R_{e\tau}$ to differ from 1 by as much as 20%. Unfortunately, these large discrepancies require values of $|V_{\mu 1}|^2$ such that $\delta \simeq 0$ or π , which means very little CP violation
- (iv) $|V_{e3}|^2$ has very little impact and varying it from 0.01 to 0.005 will change $R_{e\mu}$ and $R_{e\tau}$, typically by 1% or so.

Therefore, we can say that, even within the standard production mechanism for the UHCR, deviations of $R_{e\mu}$ and $R_{e\tau}$ by 20% from their expectation value of 1 are possible with the current experimental data but require in general pushing these data to their limits and CP violation to be vanishingly small.

C. Neutrino fluxes and δ

In order to study the bounds one could set on δ if one were to measure $R_{e\mu}$ with a given accuracy, we express $|V_{\mu 1}|^2$ in terms of $R_{e\mu}$ and use that result directly into our expression of $\cos(\delta)$ (2.10). Again, we use a Monte Carlo technique to scan the allowed parameter space. As a technical detail, we note that we have a quadratic equation to solve with two possible values for $|V_{\mu 1}|^2$ but only one value respects unitarity and leads to a positive value of J^2 ; events that do not respect unitarity are excluded from

our Monte Carlo. In Table IV, we summarize our results. We see that:

- (i) the most important parameters are $|V_{\mu 3}|^2$ and $R_{e\mu}$, where the uncertainties must be rather small, while $|V_{e2}|^2$ and $|V_{e3}|^2$ can tolerate much larger errors without degrading the limits much
- (ii) in general, a precision of 2% on all 4 parameters leads to tight constraints on δ ; this can be relaxed to 10% for $|V_{e2}|^2$ and up to 25% for $|V_{e3}|^2$ without loosening much on the bounds on δ
- (iii) large or small values of $R_{e\mu}$ (1.13 or 0.84, for example) require higher precision in the four parameters in order to keep $\cos(\delta)$ away from $+1$ or -1 ; the precision required on $|V_{e2}|^2$ and $|V_{e3}|^2$ is still lower than that on the other two parameters. Recall that $\cos(\delta) = \pm 1$ means no CP violation.
- (iv) as $|V_{e3}|^2$ gets smaller, a higher precision on the parameters is necessary in order to keep the same tight range on δ
- (v) with some combinations of parameters, it is possible to exclude $\delta = \pi$ or 0 with a precision of a few percent on the parameters
- (vi) an uncertainty of 5% on $R_{e\mu}$ can limit δ to a 90-degree range or smaller with certain combination of parameters

TABLE IV. Values of $\cos(\delta)$ for different combinations of parameters: $\cos(\delta)_c$ is obtained from the central values of the parameters while $\cos(\delta)_-$ and $\cos(\delta)_+$ are the extremum values allowed by the uncertainties (shown in percent, in parentheses next to each parameter).

$ V_{e2} ^2$	$ V_{\mu 3} ^2$	$ V_{e3} ^2$	$R_{e\mu}$	$\cos(\delta)_-$	$\cos(\delta)_c$	$\cos(\delta)_+$
0.30 (1)	0.50 (1)	0.01 (1)	1.00 (1)	-0.183	0.118	0.463
0.30 (10)	0.5 (1)	0.01 (25)	1.00 (1)	-0.265	0.118	0.567
0.30 (2)	0.50 (2)	0.01 (2)	1.00 (2)	-0.463	0.118	0.891
0.30 (1)	0.50 (1)	0.005 (1)	1.00 (1)	-0.332	0.882	1.0
0.30 (2)	0.50 (2)	0.005 (2)	1.00 (2)	-0.718	0.082	1.0
0.30 (5)	0.50 (1)	0.005 (10)	1.00 (1)	-0.373	0.082	0.600
0.30 (1)	0.60 (1)	0.01 (1)	0.84 (1)	-0.605	-0.211	0.257
0.30 (2)	0.60 (2)	0.01 (2)	0.84 (2)	-0.949	-0.211	0.867
0.30 (5)	0.60 (1)	0.01 (10)	0.84 (1)	-0.653	-0.211	0.348
0.35 (1)	0.45 (1)	0.01 (1)	1.03 (1)	-0.769	-0.485	-0.186
0.35 (2)	0.45 (1)	0.01 (2)	1.03 (1)	-0.792	-0.485	-0.181
0.35 (10)	0.45 (1)	0.01 (10)	1.03 (1)	-0.931	-0.485	-0.121
0.35 (10)	0.45 (2)	0.01 (10)	1.03 (2)	-0.936	-0.485	+0.160
0.24 (2)	0.40 (2)	0.01 (2)	1.06 (2)	-0.476	-0.141	0.205
0.24 (10)	0.40 (2)	0.01 (25)	1.06 (2)	-0.679	-0.141	0.290
0.24 (2)	0.40 (2)	0.01 (2)	1.13 (2)	0.446	0.845	1.0
0.24 (10)	0.40 (1)	0.01 (25)	1.13 (1)	0.579	0.845	1.0
0.24 (2)	0.40 (2)	0.01 (2)	1.13 (5)	-0.076	0.845	1.0
0.25 (1)	0.55 (1)	0.01 (1)	0.94 (1)	-0.165	0.135	0.475
0.25 (10)	0.55 (1)	0.01 (25)	0.94 (1)	-0.244	0.135	0.627
0.25 (2)	0.55 (2)	0.01 (2)	0.94 (2)	-0.440	0.135	0.874
0.25 (5)	0.55 (2)	0.01 (10)	0.94 (2)	-0.46	0.135	0.950

The precision required in order to get interesting constraints on δ is daunting but this was expected since we saw that $|V_{\mu 1}|^2$ was the least sensitive of our three parameters to set limits on δ .

V. NEW PHYSICS AND δ

This general behavior of cosmic neutrinos fluxes has been noted before [20] and comes directly from the fact that $|V_{\mu 3}|^2$ is close to 0.5. As $|V_{\mu 3}|^2$ goes away from 0.5 (current data allow a deviation of up to 20%), the ratios can differ substantially from 1:1:1, as we just saw. This brings the question of how one would interpret ratios that would differ much more than 8%, say 30% or more. It would be difficult to explain these anomalies within the SM and such discrepancies might suggest some new physics. This new physics could bring its own CP violation, which could be *à la Dirac*; we would then deal with an effective CP phase such that $\delta_{\text{effective}} = \delta + \delta_{\text{new}}$.

We will consider new physics beyond massive neutrinos and for illustration purposes, we choose nonradiative neutrino decays.

A. Nonradiative neutrino decays and δ eff

The notion of nonradiative neutrino decay as a source of deviations from the standard ratio was first discussed in [22] and we refer to this paper for details. Essentially, in this model, the observable flavor ratio is modified to be

$$\phi_e : \phi_\mu : \phi_\tau = |V_{e1}|^2 : |V_{\mu 1}|^2 : |V_{\tau 1}|^2, \quad (5.1)$$

where we assume normal mass hierarchy.

Contrary to the SM flux ratios, the ratio $R_{\tau\mu} = \phi_\tau / \phi_\mu$ is a direct measure of

$$\rho_1 = \frac{|V_{\tau 1}|^2}{|V_{\mu 1}|^2}, \quad (5.2)$$

which proved to be the best variable to set constraints on δ .

Table V shows what kind of limits one could obtain with such a scenario. We note that:

- (i) uncertainties in the 5%–10% range can lead to interesting constraints on δ_{eff}
- (ii) the limits are insensitive to the uncertainty on $|V_{e3}|^2$
- (iii) in this model, flux ratios of up to 3 are allowed with current data. This limit depends on $|V_{e3}|^2$ and as $|V_{e3}|^2$ decreases, so does the upper limit of the ratio.

As we just saw, if $R_{e\mu}$ or $R_{e\tau}$ is close to 1, there is no need for new physics and a precision of 1%–2% is required to set bounds on δ . Such a precision appears extremely difficult at planned detectors such as IceCube, Antares, Nestor, Anita, or Baikal [23]. An optimistic view would be the detection of a few tens of events per year at IceCube, for example. If we assume that all these detectors will have comparable performance after a few years of running, then we could hope for about 100 events per year. This will not be sufficient to reach a precision of 1% in a reasonable time frame but a 10% precision appears within reach. Therefore, the better scenario, when considering the experimental precision, is to have very different neutrino flux ratios; in that case, a 10% precision leads to interesting constraints on $\delta_{\text{effective}}$.

VI. CONCLUSIONS

In this paper we have shown how it is possible to get information about CP violation in the leptonic sector without performing any direct experiment on CP violation. In the usual parametrization of the leptonic mixing, the mixings are described by three mixing angles and the fourth parameter is the CP -violating phase, δ . All the information about mixing and CP violation can also be described completely by four mixing elements, four $|V_{ij}|^2$. We have shown how one can extract information on $\sin^2(\delta)$ and $\cos(\delta)$ from four $|V_{ij}|^2$ or three $|V_{ij}|^2$'s and a ratio of

TABLE V. Values of $\cos(\delta_{\text{eff}})$ for different combinations of parameters in a nonradiative decay scenario: $\cos(\delta_{\text{eff}})_c$ is obtained from the central values of the parameters while $\cos(\delta_{\text{eff}})_-$ and $\cos(\delta_{\text{eff}})_+$ are the extremum values allowed by the uncertainties (shown in percent, in parentheses next to each parameter).

$ V_{e2} ^2$	$ V_{\mu 3} ^2$	$ V_{e3} ^2$	$\rho_1 = \phi_\tau^t / \phi_\mu^t$	$\cos(\delta_{\text{eff}})_-$	$\cos(\delta_{\text{eff}})_c$	$\cos(\delta_{\text{eff}})_+$
0.30 (5)	0.50 (5)	0.01 (5)	0.7 (5)	0.377	0.628	0.920
0.30 (5)	0.50 (5)	0.01 (10)	0.7 (5)	0.378	0.628	0.940
0.30 (5)	0.50 (5)	0.01 (5)	0.7 (10)	0.303	0.628	1.0
0.30 (5)	0.50 (5)	0.01 (25)	0.7 (5)	0.380	0.628	1.0
0.30 (5)	0.50 (5)	0.01 (5)	1.3 (5)	−0.685	−0.407	−0.161
0.30 (5)	0.50 (5)	0.01 (5)	1.3 (10)	−0.765	−0.407	−0.072
0.30 (5)	0.50 (5)	0.01 (5)	1.8 (10)	−1.0	−0.931	−0.598
0.25 (5)	0.60 (5)	0.01 (5)	1.8 (5)	−0.522	−0.263	−0.013
0.25 (5)	0.60 (5)	0.01 (5)	3.0 (10)	−1.0	−0.920	−0.624
0.25 (5)	0.60 (5)	0.005 (5)	2.5 (10)	−1.0	−0.971	−0.544

two $|V_{ij}|^2$'s. We used two such ratios: $\rho_1 = |V_{\tau 1}|^2/|V_{\mu 1}|^2$ and $\rho_2 = |V_{\tau 2}|^2/|V_{\tau 1}|^2$.

We find that if one uses $|V_{e2}|^2$, $|V_{\mu 3}|^2$, and $|V_{e3}|^2$, then it is better to use $|V_{\mu 1}|^2$ or $|V_{\tau 1}|^2$ as the fourth independent $|V_{ij}|^2$. When the first two parameters remain close to their current central experimental values, $|V_{\mu 1}|^2$ is slightly better than $|V_{\tau 1}|^2$ while $|V_{\tau 1}|^2$ is slightly better when the first two parameters digress from their current central experimental values.

As a first constraint that does not require particularly precise data, we find that if $|V_{\mu 1}|^2$ were larger than $|V_{\mu 1}|^2_{j^2=j^2_{\max}}$, then δ would have to be in the first or fourth quadrant.

In general, when the four parameters have uncertainties of 10%, it is possible to get interesting constraints on δ on some of the parameter space. For example, assuming 10% errors, if $|V_{e2}|^2$ and $|V_{\mu 3}|^2$ keep their current experimental central values and $|V_{\mu 1}|^2$ turns out to be about 0.15, then $|\delta|$ has a 70 deg range centered at about 90 deg if $|V_{e3}|^2$ turns out to be 0.03; this range decreases to about 40 deg if one measures ρ_1 and it decreases as $|V_{e3}|^2$ increases. If the uncertainties are reduced to 5%, then the ranges on δ become 30–60 deg and can tolerate a value of 0.01 for $|V_{e3}|^2$. We also found that, for most combinations of $|V_{e2}|^2$ and $|V_{\mu 3}|^2$, a small value of $|V_{\mu 1}|^2$ (0.1) implies that $|\delta|$ is between $\pi/2$ and π . It turns out that $\cos(\delta)$ is not particularly sensitive to $|V_{e3}|^2$ over most of the parameter space. This implies that, once the errors are reduced to the 2% level, even if the error on $|V_{e3}|^2$ stayed at 10%, it would not degrade the constraints on δ by much. Of the three variables investigated here, the best one turned out to be ρ_1 .

In order to get the fourth piece of information needed to set bounds on δ , we explored the use of ultrahigh energy cosmic neutrinos. Generally, due to a numerical accident

stemming mostly from $|V_{\mu 3}|^2 \simeq 0.5$, the cosmic neutrino fluxes are very similar once they reach the Earth. However, current data allow cosmic neutrino fluxes to differ from their central, expected value of 1 by up to 20%: this requires pushing the parameters to the acceptable limits and implies that CP violation in the leptonic sector is vanishingly small. Unfortunately, the uncertainties on the data would have to be at the few percent level on the cosmic neutrino fluxes and on $|V_{e2}|^2$ in order to set interesting limits on δ . Finally, we studied a nonradiative decay model of neutrinos that could lead to flux ratios very different from 1. Indeed, in this model, the current data allow for neutrino fluxes to differ by a factor of up to three. Fortunately, in this model, one ratio of neutrino fluxes is ρ_1 and leads to rather interesting constraints on δ_{eff} with experimental uncertainties of 10% on ratios of cosmic neutrino fluxes.

It appears that 1%–2% precision on extremely high energy neutrino fluxes will be out of reach for the planned detectors but that 10% precision will be within reach. It would then be difficult to constrain δ within the SM context using these cosmic rays and one would have to rely on other means to measure $|V_{\mu 1}|^2$ or $|V_{\tau 1}|^2$ or ρ_1 or ρ_2 . On the other hand, if the neutrino fluxes are very different, then one is likely outside the SM and a 10% precision on the neutrino flux ratios is sufficient in some models to set interesting bounds on $\delta_{\text{effective}}$.

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- [1] Y. Fukuda (Super-Kamiokande Collaboration), *Phys. Rev. Lett.* **81**, 1562 (1998); W. Hampel *et al.* (GALLEX Collaboration), *Phys. Lett. B* **447**, 127 (1999); J.N. Abdurashitov *et al.* (SAGE Collaboration), *Phys. Rev. C* **60**, 055801 (1999); B.T. Cleveland *et al.* (Homestake Collaboration), *Astrophys. J.* **496**, 505 (1998); M. Altmann *et al.* (GNO Collaboration), *Phys. Lett. B* **490**, 16 (2000); Q.R. Ahmad *et al.* (SNO Collaboration), *Phys. Rev. Lett.* **87**, 071301 (2001); S. Fukuda *et al.* (Super-Kamiokande Collaboration), *Phys. Lett. B* **539**, 179 (2002); S.N. Ahmed *et al.* (SNO Collaboration), *Phys. Rev. Lett.* **92**, 181301 (2004); Y. Ashie *et al.* (Super-Kamiokande Collaboration), *Phys. Rev. Lett.* **93**, 101801 (2004).
- [2] B. Pontecarvo, *J. Exp. Theor. Phys.* **33**, 549 (1957) [*Sov. Phys. JETP* **6**, 429 (1958)]; *J. Exp. Theor. Phys.* **34**, 247 (1957) [*Sov. Phys. JETP* **7**, 172 (1958)].
- [3] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, *New J. Phys.* **6**, 122 (2004); G.L. Fogli, E. Lisi, A. Marrone, and A. Palazzo, *Prog. Part. Nucl. Phys.* **57**, 742 (2006).
- [4] J. Schechter and J. W. F. Valle, *Phys. Rev. D* **21**, 309 (1980); **22**, 2227 (1980).
- [5] J. Arafune and J. Sato, *Phys. Rev. D* **55**, 1653 (1997); J. Arafune, M. Koide, and J. Sato, *Phys. Rev. D* **56**, 3093 (1997); **60**, 119905(E) (1999); H. Minakata and H. Nunokawa, *Phys. Lett. B* **413**, 369 (1997); H. Minakata and H. Sugiyama, *Phys. Lett. B* **580**, 216 (2004); K. Hiraide *et al.*, *Phys. Rev. D* **73**, 093008 (2006); K. Kimura, A. Takamura, and T. Yoshikawa, hep-ph-0603141; F.R. Klinkhamer, *Phys. Rev. D* **73**, 057301 (2006); H. Minakata, hep-ph-0604088.

- [6] K. Dick, M. Freund, M. Lindner, and A. Romanino, Nucl. Phys. **B562**, 29 (1999); M. Freund, P. Huber, and M. Lindner, Nucl. Phys. **B585**, 105 (2000); **B615**, 331 (2001); P. Huber, M. Lindner, and W. Winter, Nucl. Phys. **B654**, 3 (2003); **B645**, 3 (2002); S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Nucl. Phys. **B674**, 401 (2003); P. Huber, M. Lindner, M. Rolinec, T. Schwetz, and W. Winter, Phys. Rev. D **70**, 073014 (2004); P. Huber, M. Lindner, and W. Winter, J. High Energy Phys. 05 (2005) 020.
- [7] M. Apollonio *et al.* (CHOOZ Collaboration), Phys. Lett. B **466**, 415 (1999); F. Boehm *et al.*, Phys. Rev. Lett. **84**, 3764 (2000). Phenomenological implications for CHOOZ data on active three flavor oscillations was first examined by Mohan Narayan, G. Rajasekaran, and S. Uma Sankar, Phys. Rev. D **58**, 031301 (1998).
- [8] See, for example, M. Diwal *et al.*, hep-ex-0211001.
- [9] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [10] We could also use supernova neutrino data as additional observable, but, given the possibility of detecting UHCR neutrinos more frequently than a supernova burst, we shall not consider this option.
- [11] C. Hamzaoui, Phys. Rev. Lett. **61**, 35 (1988); C. Hamzaoui, *Geometrical Parametrization of the Quark Mixing Matrix*, in Rare Decay Symposium, Proceedings, Vancouver, Canada, 1988, edited by D. Bryman, J. Ng, R. Numao, and J.-M. Poutissou (World Scientific, Singapore, 1989), p. 469; A. Campa, C. Hamzaoui, and V. Rahal, Phys. Rev. D **39**, 3435 (1989).
- [12] M. Gronau and J. Schechter, Phys. Rev. Lett. **54**, 385 (1985); **54**, 1209(E) (1985); C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); I. Dunietz, O. W. Greenberg, and D.-D. Wu, Phys. Rev. Lett. **55**, 2935 (1985).
- [13] See, for example, R. H. Mohapatra *et al.*, hep-ph-0510210; M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. **75**, 345 (2003).
- [14] S. Petcov, in *Neutrino 2004*, edited by J. Dumarchez, Th. Patzi, and F. Vannucci (Elsevier, New York, 2005); Nucl. Phys. B, Proc. Suppl. **143**, 159 (2005); O. Mena, S. Palomares-Ruiz, and S. Pascoli, Phys. Rev. D **73**, 073007 (2006); O. Mena, Mod. Phys. Lett. A **20**, 1 (2005).
- [15] S. Pakvasa, Mod. Phys. Lett. A **19**, 1163 (2004); Yad. Fiz. **67**, 1179 (2004); J. F. Beacom, N. F. Bell, D. Hooper, J. G. Learned, S. Pakvasa, and T. J. Weiler, Phys. Rev. Lett. **92**, 011101 (2004); P. Keranen, J. Maalampi, M. Myyrylainen, and J. Riittinen, Phys. Lett. B **574**, 162 (2003).
- [16] J. Ahrens *et al.* (IceCube Collaboration), Astropart. Phys. **20**, 507 (2004); Teresa Montaruli *et al.* (ANTARES Collaboration), Proceedings of 28th International Cosmic Ray Conferences (ICRC 2003), Tsukuba, Japan, 2003, physics/0306057; S. E. Tzamarias (NESTOR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **502**, 150 (2003); M. Takeda *et al.*, Astrophys. J. **522**, 225 (1999); D. J. Bird *et al.* (HIRES Collaboration), Astrophys. J. **424**, 491 (1994).
- [17] A. Osipowicz *et al.* (KATRIN Collaboration), hep-ex/0109033; H. Paes and Thomas J. Weiler, Phys. Rev. D **63**, 113015 (2001).
- [18] J. Jones, I. Mocioiu, I. Sarcevic, and M. H. Reno, Int. J. Mod. Phys. A **20**, 1204 (2005); P. D. Serpico, Phys. Rev. D **73**, 047301 (2006); W. Winter, hep-ph-0604191.
- [19] Y. Farzan and Yu. Smirnov, Phys. Rev. D **65**, 113001 (2002); P. D. Serpico and Kachelriess, Phys. Rev. Lett. **94**, 211102 (2005).
- [20] J. G. Learned and S. Pakvasa, Astropart. Phys. **3**, 267 (1995); H. Athar, M. Jezabek, and O. Yasuda, Phys. Rev. D **62**, 103007 (2000); Luis Bento, Petteri Keranen, and Jukka Maalampi, Phys. Lett. B **476**, 205 (2000).
- [21] Other production mechanisms can affect the fluxes seen on earth. See, for example, J. Jones, I. Mocioiu, M. H. Reno, and I. Sarcevic, Phys. Rev. D **69**, 033004 (2004).
- [22] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. Lett. **90**, 181301 (2003); Phys. Rev. D **69**, 017303 (2004).
- [23] I. Mocioiu, Int. J. Mod. Phys. A **20**, 7079 (2005); J. G. Learned and K. Mannheim, Annu. Rev. Nucl. Part. Sci. **50**, 679 (2000); S. W. Barwick *et al.* (ANITA Collaboration), Phys. Rev. Lett. **96**, 171101 (2006); R. Wischniewski *et al.* (Baikal Collaboration), Int. J. Mod. Phys. A **20**, 6932 (2005); A. Achterberg *et al.* (IceCube Collaboration), astro-ph/0604450; F. Halzen, astro-ph/0602132; F. Halzen and D. Hooper, astro-ph/0605103; see also <http://antares.in2p3.fr/> and <http://www.nestor.org.gr>.