

Neutrino masses in quartification schemesAlison Demaria,^{*} Catherine I. Low,[†] and Raymond R. Volkas[‡]*School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria 3010, Australia*

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The idea of quark-lepton universality at high energies has recently been explored in unified theories based upon the quartification gauge group $SU(3)^4$. These schemes encompass a quark-lepton exchange symmetry that results upon the introduction of leptonic color. It has been demonstrated that, in models in which the quartification gauge symmetry is broken down to the standard model gauge group, gauge coupling constant unification can be achieved, and there is no unique scenario. The same is also true when the leptonic color gauge group is only partially broken, leaving a remnant $SU(2)_\ell$ symmetry at the standard model level. Here we perform an analysis of the neutrino mass spectrum of such models. We show that these models do not naturally generate small Majorana neutrino masses, thus correcting an error in our earlier quartification paper, but with the addition of one singlet neutral fermion per family there is a realization of seesaw suppressed masses for the neutrinos. We also show that these schemes are consistent with proton decay.

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I. INTRODUCTION

Quark-lepton universality at high energies has been advocated as one of the possible extensions to the standard model (SM). Within this class of extensions, a quark-lepton discrete symmetry is imposed after endowing the leptons with new *leptonic color* degrees of freedom described by the separate gauge symmetry $SU(3)_\ell$. This idea was first explored by Foot and Lew [1] with quarks and leptons being indistinguishable above an energy scale that could be as low as a few TeV. Within this context, the quark-lepton discrete symmetry does not require the unification of the gauge coupling constants. It is interesting that quark-lepton quantum number unification can be divorced from coupling constant unification; however, it is desirable to achieve the latter. To accommodate such unification, the gauge symmetry may be enhanced to the ‘‘quartification’’ group $SU(3)_q \otimes SU(3)_L \otimes SU(3)_\ell \otimes SU(3)_R$ [2,3], augmented by a discrete symmetry relating the $SU(3)$ factors.

This gauge group was independently explored by Volkas and Joshi [2] and Babu, Ma, and Willenbrock [3]. The former model achieved only the partial unification of the gauge coupling constants with two independent gauge coupling constants at the unification scale. The latter found a way to achieve full unification through enforcing additional discrete symmetries and introducing nonrenormalizable terms. This model also achieved suppressed nonzero neutrino masses with the addition of a heavy singlet per fermionic family. Both of these models had a residual $SU(2)_\ell$ symmetry at the SM level, a result of the leptonic color symmetry being only partially broken. The motivation for our recent paper [4] was to see if full coupling

constant quartification could be achieved without the introduction of additional symmetries and higher-dimension operators. The basic change from Ref. [3] was to admit the possibility of intermediate symmetry breaking scales.

We performed a systematic analysis of the renormalization-group equations (RGEs) for all possible symmetry breaking routes from the quartification gauge group down to the standard model gauge group G_{SM} . We demonstrated that full unification could be achieved using a number of these routes without the need for the additional discrete symmetry or the higher-dimension operators of [3]. This was true both for models that break the symmetry down to G_{SM} [5,6] or only to $G_{\text{SM}} \otimes SU(2)_\ell$ [1–3]. In the case where there was no residual $SU(2)_\ell$ symmetry, there were four independent choices that gave rise to unification of the gauge couplings, all of which predicted interesting, though distinct, phenomenology.

The purpose of the present paper is to examine the neutrino sector of quartification models in more detail, and, in the process, to correct an error in our previous paper [4].

The mass analysis of the leptonic sector for the models where $SU(2)_\ell$ was broken indicated that there were ten neutral leptonic components (per family) which, it was claimed, gained Majorana masses [4]. Nine of the resulting mass eigenvalues were of order of the grand unified theory (GUT) scale, and the tenth was of order $M_{\text{EW}}^2/M_{\text{GUT}}$ for a generic M_{GUT} and a single symmetry breaking event. This was the mass scale that would result from a regular seesaw mechanism [7] and this particle also displayed the correct weak coupling with the electron, suggesting that it was indeed a neutrino. It was claimed in this case that, for a general model, seesaw suppressed masses for the neutrinos arose naturally; however, it was anticipated that the introduction of intermediate scales, which would separate the values of the M_{GUT} entries, would increase the value of this small eigenvalue.

^{*}Electronic address: a.demaria@physics.unimelb.edu.au[†]Electronic address: c.low@physics.unimelb.edu.au[‡]Electronic address: r.volkas@physics.unimelb.edu.au

A more precise analysis of this neutral lepton sector was performed which sought to confirm seesaw suppressed masses for the neutrinos in the individual symmetry breaking cascades that achieved unification of the gauge coupling constants. In the course of this analysis, we discovered that two entries were incorrectly omitted from the Majorana mass matrix in the model with no residual $SU(2)_\ell$ symmetry of Sec. III B of Ref. [4]. The inclusion of these terms alters the general conclusion of Ref. [4] with regards to the neutrino masses. The results of the RGE analysis, however, remain unaffected with unification still possible.

Here we present a detailed analysis of the mass spectrum of the leptonic sector of these quartification models. For the general case where there is only one symmetry breaking event and no residual $SU(2)_\ell$ symmetry, the mass eigenvalues reveal that there are two light (though not ultralight) eigenvalues, disagreeing with our previous claim. Here we propose a simple extension to ensure that only one ultralight (i.e., seesaw suppressed) active Majorana neutrino per family is generated [7]. It turns out that the introduction of a singlet fermion S with bare mass M_S per family works well. Although the addition of these singlets slightly complicates the models, they are one way by which to generate seesaw suppression [8]. In this paper we show that this is all that is required to generate the desired neutrino mass spectrum in the unified schemes of [4], with the additional discrete symmetries and higher-dimensional operators of [3] still absent.

A summary of the neutral sector of the quartification model with the gauge symmetry fully broken down to the standard model gauge group is given in Sec. II. This contains details of the Yukawa sector and lists the four schemes which allow for the unification of the gauge coupling constants. In Sec. III we outline the mass analysis of the leptonic sector with and without the singlet present, and extend the analysis to consider the details of the four unified models. This analysis is repeated in the following section for the case where there is a residual $SU(2)_\ell$ symmetry. In Sec. V we estimate the lifetime of the proton, and we conclude in Sec. VI.

II. THE QUARTIFICATION MODEL

The quartification gauge group is

$$G_4 = SU(3)_q \otimes SU(3)_L \otimes SU(3)_\ell \otimes SU(3)_R. \quad (2.1)$$

A Z_4 symmetry which cyclicly permutes the gauge groups in the order $q \rightarrow L \rightarrow \ell \rightarrow R \rightarrow q$ is imposed, ensuring a single gauge coupling constant. The multiplets which house the fermions are represented by 3×3 matrices:

$$\begin{aligned} q &\sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}) = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \\ q^c &\sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{3}) = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}, \\ \ell &\sim (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) = \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & N \end{pmatrix}, \\ \ell^c &\sim (\mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) = \begin{pmatrix} x_1^c & y_1^c & z_1^c \\ x_2^c & y_2^c & z_2^c \\ \nu^c & e^c & N^c \end{pmatrix}, \end{aligned} \quad (2.2)$$

where $q(\ell)$ contain the left-handed quarks (leptons) and $q^c(\ell^c)$ the left-handed antiquarks (antileptons). Both electrically neutral and charged exotic particles are needed to fill the representations. Of particular note, the exotic fermions x_i, y_i were required to be at the TeV scale in Ref. [3] to facilitate gauge coupling constant unification.

The Higgs fields are contained in two different $\mathbf{36}$'s of G_4 , and are labeled as per

$$\begin{aligned} \Phi_a &\sim (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}), & \Phi_b &\sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}), \\ \Phi_c &\sim (\mathbf{1}, \mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}), & \Phi_d &\sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}, \mathbf{1}), \\ \Phi_\ell &\sim (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}), & \Phi_{\ell^c} &\sim (\mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}), \\ \Phi_{q^c} &\sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{3}), & \Phi_q &\sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}). \end{aligned} \quad (2.3)$$

These fields are closed under the Z_4 symmetry and generate realistic fermion masses and mixings, with the coupling to the fermions described by the Z_4 -invariant Lagrangian¹

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & Y_1 \text{Tr}(\ell \ell^c \Phi_a + \ell^c q^c \Phi_b + q^c q \Phi_c + q \ell \Phi_d) \\ & + Y_2 \text{Tr}(\ell \ell^c \Phi_c^\dagger + \ell^c q^c \Phi_d^\dagger + q^c q \Phi_a^\dagger + q \ell \Phi_b^\dagger) \\ & + Y_L \epsilon^{jkm} \epsilon^{npr} (\ell^{jn} \ell^{kp} (\Phi_\ell)^{mr} + (\ell^c)^{jn} (\ell^c)^{kp} (\Phi_{\ell^c})^{mr} \\ & + (q^c)^{jn} (q^c)^{kp} (\Phi_{q^c})^{mr} + (q)^{jn} (q)^{kp} (\Phi_q)^{mr}) + \text{H.c.} \end{aligned} \quad (2.4)$$

The Higgs fields are sufficient to achieve the necessary symmetry breaking, with the pattern $G_4 \xrightarrow{V} SU(3)_q \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{u} SU(3)_q \otimes U(1)_Q$ accomplished by the fields obtaining vacuum expectation values (VEVs) of the form

¹The notation $\ell \ell^c \Phi_a$ means $\bar{\ell}_R \ell_L \Phi_a$, etc.

$$\langle \Phi_\ell \rangle = \begin{pmatrix} u_{\ell_1} & 0 & u_{\ell_2} \\ 0 & u_{\ell_3} & 0 \\ V_{\ell_1} & 0 & V_{\ell_2} \end{pmatrix}, \quad \langle \Phi_{\ell^c} \rangle = \begin{pmatrix} V_{\ell_1^c} & 0 & V_{\ell_2^c} \\ 0 & V_{\ell_3^c} & 0 \\ V_{\ell_4^c} & 0 & V_{\ell_5^c} \end{pmatrix},$$

$$\langle \Phi_a \rangle \sim \langle \Phi_c^\dagger \rangle = \begin{pmatrix} u_{a_1} & 0 & u_{a_2} \\ 0 & u_{a_3} & 0 \\ V_{a_1} & 0 & V_{a_2} \end{pmatrix}. \quad (2.5)$$

The u 's are of electroweak order and the V_i 's determine the higher energy breaking. Since Φ_a and Φ_c^\dagger have the same quantum numbers and hence VEV pattern, they can be treated together for the purposes of this paper.

The four independent symmetry breaking cascades which were shown to permit unification are

$$(1) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes U(1)_Y, \quad (2.6)$$

$$(2) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1) \\ \xrightarrow{y} SU(3)_q \otimes SU(2)_L \otimes U(1)_Y, \quad (2.7)$$

$$(3) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{y} SU(3)_q \otimes SU(2)_L \otimes U(1)_Y, \quad (2.8)$$

$$(4) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1) \\ \xrightarrow{y} SU(3)_q \otimes SU(2)_L \otimes U(1)_Y. \quad (2.9)$$

There are actually eight independent symmetry breaking routes from the quartification gauge group down to the standard model gauge group. Constraints on the unification of the gauge coupling constants revealed that four of these converged into the single possibility of Eq. (2.6), one disallowed unification entirely, and the other three are as listed in Eqs. (2.7), (2.8), and (2.9) [4].

With the introduction of these intermediate steps we now have five energy scales $v \geq w \geq x \geq y > u$; the VEV entries V_i of Eq. (2.5) now assuming the order y, x, w, v depending on the route. These are given by

$$(1) \quad V_{\ell_2, \ell_5^c, a_2} \sim v, \quad V_{\ell_1, \ell_1^c, \ell_2^c, \ell_3^c, \ell_4^c, a_1} \sim x, \quad (2.10)$$

$$(2) \quad V_{a_2} \sim v, \quad V_{\ell_2, \ell_5^c} \sim w, \quad V_{\ell_4^c, a_1} \sim x, \\ V_{\ell_1, \ell_1^c, 2, 3} \sim y, \quad (2.11)$$

$$(3) \quad V_{a_2} \sim v, \quad V_{\ell_2, \ell_5^c} \sim w, \quad V_{\ell_1, \ell_2^c} \sim x, \\ V_{\ell_1, 3, 4, a_1} \sim y, \quad (2.12)$$

$$(4) \quad V_{a_2} \sim v, \quad V_{a_1} \sim w, \quad V_{\ell_2, \ell_4^c, \ell_5^c} \sim x, \\ V_{\ell_1, \ell_1^c, 2, 3} \sim y. \quad (2.13)$$

As the V_i 's assume different energies across the four cascades, the mass spectrum resulting from the Yukawa couplings of Eq. (2.4) differs for each. This variation gives rise to distinct energy bounds on the intermediate scales when considering the RGEs [4]. The scales of all the breakings which allow unification are listed in Table I. The first cascade offered no flexibility, with the unification occurring at a fixed value of energy scales only. The other three cascades deliver unification within a range of energies, with the last stage of breaking able to occur at 10^6 GeV. Note that cascades 2 and 3 have identical energy thresholds despite being independent.

For details of the RGE analysis, including the exact VEV structure of Eq. (2.5) and the fermion and scalar

TABLE I. The range of energies for the symmetry breaking scales that will consistently give unification of the gauge coupling constants. Cascade 1 features the last stage of symmetry breaking necessarily occurring below a TeV. The other three choices allow for a range in the intermediate scales while still preserving unification. When y_{\max} is chosen for cascades 2, 3, and 4, they also become equivalent.

Cascade	y	x	w	v
1		7.1×10^2 GeV		1.3×10^{13} GeV
2 and 3	$y_{\min} \sim 1$ TeV	1 TeV	6.2×10^{12} GeV	$v_{\max} \sim 1.1 \times 10^{13}$ GeV
		4.2×10^7 GeV	4.2×10^7 GeV	$v_{\max} \sim 3.8 \times 10^{11}$ GeV
	$y_{\max} \sim 1.2 \times 10^6$ GeV	1.2×10^6 GeV	1.2×10^6 GeV	1.4×10^{11} GeV
4	$y_{\min} \sim 1$ TeV	8.8×10^3 GeV	3.6×10^{10} GeV	$v_{\min} \sim 3.6 \times 10^{10}$ GeV
		4.2×10^7 GeV	4.2×10^7 GeV	$v_{\max} \sim 3.8 \times 10^{11}$ GeV
	$y_{\max} \sim 1.2 \times 10^6$ GeV	1.2×10^6 GeV	1.2×10^6 GeV	1.4×10^{11} GeV

mass spectrum for each cascade, we refer the reader to Appendix B of Ref. [4].² It is sufficient for the purposes of this paper to be aware of the energy bounds on the intermediate and unification scales, and recognize that the internal structure of the scalar VEVs, and hence fermion mass matrices, will be unique for each cascade.

III. NEUTRINO MASSES WHEN $SU(2)_\ell$ IS BROKEN

Without restrictions imposed on the Higgs fields, the allowed Yukawa interaction terms in the leptonic sector are

$$Y_1 \text{Tr}[\ell \ell^c \Phi_a], \quad Y_L \epsilon^{jkm} \epsilon^{npr} \ell^{jn} \ell^{kp} (\Phi_\ell)^{mr}, \quad (3.1)$$

$$Y_L \epsilon^{jkm} \epsilon^{npr} (\ell^c)^{jn} (\ell^c)^{kp} (\Phi_{\ell^c})^{mr},$$

with the Y_2 term effectively subsumed into the Y_1 term. The leptons with a charge of +1 are the e^c , y_1^c , z_2 , and x_2 components of Eq. (2.2). They mix and form Dirac mass

$$(\chi^c)^T \begin{pmatrix} 0 & V_{a_2} & 0 & V_{a_1} & u_{\ell_3} & 0 & u_{\ell_1} & 0 & 0 & 0 \\ V_{a_2} & 0 & u_{a_2} & 0 & 0 & V_{\ell_3^c} & 0 & V_{\ell_1^c} & 0 & 0 \\ 0 & u_{a_2} & 0 & u_{a_1} & 0 & 0 & -V_{\ell_1} & 0 & -u_{\ell_3} & 0 \\ V_{a_1} & 0 & u_{a_1} & 0 & 0 & 0 & 0 & -V_{\ell_2^c} & 0 & -V_{\ell_3^c} \\ u_{\ell_3} & 0 & 0 & 0 & 0 & u_{a_1} & V_{\ell_2} & 0 & 0 & u_{a_2} \\ 0 & V_{\ell_3^c} & 0 & 0 & u_{a_1} & 0 & 0 & V_{\ell_5^c} & V_{a_1} & 0 \\ u_{\ell_1} & 0 & -V_{\ell_1} & 0 & V_{\ell_2} & 0 & 0 & u_{a_3} & -u_{\ell_2} & 0 \\ 0 & V_{\ell_1^c} & 0 & -V_{\ell_2^c} & 0 & V_{\ell_5^c} & u_{a_3} & 0 & 0 & -V_{\ell_4^c} \\ 0 & 0 & -u_{\ell_3} & 0 & 0 & V_{a_1} & -u_{\ell_2} & 0 & 0 & V_{a_2} \\ 0 & 0 & 0 & -V_{\ell_3^c} & u_{a_2} & 0 & 0 & -V_{\ell_4^c} & V_{a_2} & 0 \end{pmatrix} \chi, \quad (3.3)$$

where $\chi = (N \ N^c \ \nu \ \nu^c \ x_1 \ x_1^c \ y_2 \ y_2^c \ z_1 \ z_1^c)^T$. Equation (3.3) is the generic form of this mass matrix for all four cascades. Given that the V_i 's will have different orders depending upon the exact route, the strength of the mixings between particles will also vary across the four individual cascades, changing the internal structure of the matrix and hence eigenvalues. Although there are five energy scales within this matrix, the unification requirements give independence to only three of these. In the following analysis, the higher two scales w and v are constrained to be nontrivial functions of x and y . Unfortunately, diagonalization is not analytically tractable, with analytic approximations creating too much error in the energy regions of interest, so the relationship between the eigenvalues and the energy scales was ascertained numerically.

Cascade 1, with its fixed choice of energy scales, offers no choice of generating a small neutrino mass, revealing two light mass eigenvalues of $\mathcal{O}(u)$. These eigenvalues, although of the same order, are not identical, so they do not indicate a Dirac neutrino.

²The cascades from Ref. [4] have been relabeled here for convenience. Thus, here cascade 1 corresponds to cascades 1, 2, 7, and 8 of Ref. [4]; two is four, three is five, and four represents six.

terms with the charge -1 lepton components e , y_1 , z_2^c , and x_2^c , in the manner

$$(e \ y_1 \ z_2^c \ x_2^c) \begin{pmatrix} u_{a_3} & 0 & -u_{\ell_1} & V_{\ell_1} \\ 0 & u_{a_3} & u_{\ell_2} & -V_{\ell_2} \\ -V_{\ell_1^c} & V_{\ell_4^c} & V_{a_2} & u_{a_2} \\ V_{\ell_2^c} & -V_{\ell_5^c} & V_{a_1} & u_{a_1} \end{pmatrix} \begin{pmatrix} e^c \\ y_1^c \\ z_2 \\ x_2 \end{pmatrix} + \text{H.c.} \quad (3.2)$$

where all the Yukawa coupling factors have been absorbed into the VEVs. There are three Dirac mass eigenvalues (per family) of the intermediate scales v , w , or x , and one eigenvalue (per family) of electroweak scale corresponding to the e , μ , and τ masses.

The leptonic components N , N^c , ν , ν^c , x_1 , x_1^c , y_2 , y_2^c , z_1 , and z_1^c are all neutral. These ten neutral leptons gain Majorana masses, as per

At first glance, the other three cascades seem more promising given that there is flexibility in the energy scales. However, the mass eigenvalues of cascades 2 and 3 adopt a similar pattern, with two light eigenvalues which are again not identical. These eigenvalues are of $\mathcal{O}(u)$ and they do not exhibit any significant fluctuation from this order within the energy bounds listed in Table I.

Cascade 4 is the only option which returns light eigenvalues which depart from the electroweak order. Again we have two similar, light eigenvalues with the discrepancies between their values more evident for low x , y . At this end of the symmetry breaking spectrum, we have a mass eigenvalue of no less than $\mathcal{O}(100)$ MeV, which increases to $\mathcal{O}(u)$ as x and y approach their maximum values. Thus, one cannot obtain a small enough Majorana mass for the neutrino here.

Additionally, in all of the above cases, the electron couples to two of the lightest eigenvalues, so there are two ‘‘neutrino-like’’ particles per family. Thus, we are unable to return nonzero suppressed neutrino masses in these models, and there are unwanted partners for the lightest eigenstates also.

For schemes which are not forthcoming of a natural seesaw mechanism for neutrinos, there are three main approaches by which to induce the correct relations which feature in model building. These are the employment of

discrete symmetries [9], the addition of a neutral fermion per family which transforms as a singlet under the gauge group [8], and the implementation of nonrenormalizable higher-dimensional operators [10]. These three were all invoked in the model of Babu *et al.* [3], although not all were introduced to resolve the issue of neutrino mass. The former was required to forbid some of the exotic leptons from gaining GUT scale masses, a result which would be incompatible with the unification of the gauge coupling constants. In this case, a small Majorana neutrino mass was obtained by the addition of a fermion singlet which was given a GUT scale bare mass.

Unfortunately, the minimal quartification models fail to automatically yield seesaw suppressed neutrinos. As the next best alternative, we try to construct the simplest satisfactory nonminimal class of models. The logical choice to consider is the addition of a singlet fermion S with Majorana mass M_S . This action does not impact the renormalization-group equation analysis of Ref. [4], with the unification constraints still as per Table I. We shall consider a range of values for the mass M_S . Being a fermion mass, naturalness does not automatically prejudice us to take its value to be at the highest scale in the theory. In

fact, a small mass term for this singlet has been extensively employed to generate suppressed neutrino masses [11], and this was specifically shown in quark-lepton symmetric models in Ref. [6]. In the latter model, the neutrino mass term (family structure suppressed) is of the form

$$(\bar{\nu}_L^c \quad \bar{\nu}_R \quad \bar{S}^c) \begin{pmatrix} 0 & u & 0 \\ u & 0 & v \\ 0 & v & M_S \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \\ S \end{pmatrix}. \quad (3.4)$$

If $M_S = 0$ then there is a massless Weyl neutrino and Dirac partner. For small, nonzero M_S this Weyl state transforms into a Majorana neutrino with a mass twice suppressed by the GUT scale, being of the order $u(u/v)(M_S/v)$. Motivated by this, we try adding a singlet to our more complicated scenario.

The singlet couples to the neutral leptons via the gauge invariant terms

$$\lambda_S (\text{Tr}[\ell \Phi_\ell^\dagger] + \text{Tr}[\ell^c \Phi_{\ell^c}^\dagger]) S. \quad (3.5)$$

The mass matrix for the neutral leptons is now simply Eq. (3.3) with the addition of a column and a row corresponding to S ,

$$(\chi'^c)^T \begin{pmatrix} 0 & V_{a_2} & 0 & V_{a_1} & u_{\ell_3} & 0 & u_{\ell_1} & 0 & 0 & 0 & \lambda_S V_{\ell_2} \\ V_{a_2} & 0 & u_{a_2} & 0 & 0 & V_{\ell_3^c} & 0 & V_{\ell_1^c} & 0 & 0 & \lambda_S V_{\ell_5^c} \\ 0 & u_{a_2} & 0 & u_{a_1} & 0 & 0 & -V_{\ell_1} & 0 & -u_{\ell_3} & 0 & \lambda_S u_{\ell_2} \\ V_{a_1} & 0 & u_{a_1} & 0 & 0 & 0 & 0 & -V_{\ell_2^c} & 0 & -V_{\ell_3^c} & \lambda_S V_{\ell_4^c} \\ u_{\ell_3} & 0 & 0 & 0 & 0 & u_{a_1} & V_{\ell_2} & 0 & 0 & u_{a_2} & \lambda_S u_{\ell_1} \\ 0 & V_{\ell_3^c} & 0 & 0 & u_{a_1} & 0 & 0 & V_{\ell_5^c} & V_{a_1} & 0 & \lambda_S V_{\ell_1^c} \\ u_{\ell_1} & 0 & -V_{\ell_1} & 0 & V_{\ell_2} & 0 & 0 & u_{a_3} & -u_{\ell_2} & 0 & \lambda_S u_{\ell_3} \\ 0 & V_{\ell_1^c} & 0 & -V_{\ell_2^c} & 0 & V_{\ell_5^c} & u_{a_3} & 0 & 0 & -V_{\ell_4^c} & \lambda_S V_{\ell_3^c} \\ 0 & 0 & -u_{\ell_3} & 0 & 0 & V_{a_1} & -u_{\ell_2} & 0 & 0 & V_{a_2} & \lambda_S V_{\ell_1} \\ 0 & 0 & 0 & -V_{\ell_3^c} & u_{a_2} & 0 & 0 & -V_{\ell_4^c} & V_{a_2} & 0 & \lambda_S V_{\ell_2} \\ \lambda_S V_{\ell_2} & \lambda_S V_{\ell_5^c} & \lambda_S u_{\ell_2} & \lambda_S V_{\ell_4^c} & \lambda_S u_{\ell_1} & \lambda_S V_{\ell_1^c} & \lambda_S u_{\ell_3} & \lambda_S V_{\ell_3^c} & \lambda_S V_{\ell_1} & \lambda_S V_{\ell_2} & M_S \end{pmatrix} \chi', \quad (3.6)$$

with

$$\chi' = \begin{pmatrix} \chi \\ S \end{pmatrix}.$$

The same methodology for the eigenvalue analysis is now applied to this new mass matrix; however, now our search space expands to allow $M_S \in [1 \text{ TeV}, v]$ as motivated above, and an arbitrary choice of the coupling λ_S .

Again, cascade 1 can be immediately ruled out, the presence of the singlet not altering the pattern of the eigenvalues from the case above. The same is not true for the other three unification schemes. Consider the second cascade. With the unification constraints imposed, both v and w are dependent on the u , y , and x scales, giving us three variables. For low values of y and x , the inclusion of the singlet does not have much of an observable effect on the eigenvalues from the scenario above. However, for

larger values of x , and a small bare mass, we have one light mass eigenvalue. This occurs for $x \geq 10^6 \text{ GeV}$ and $M_S < 10^6 \text{ GeV}$, with the mass becoming lighter for larger x and smaller M_S . The most favorable outcome is when there exists a larger hierarchy between x and y . Within this domain, w and v decrease, with $w \rightarrow x$ and $v/x \rightarrow 10^4 \text{ GeV}$. For the range of energies that return a sole light eigenvalue, this particle displays the correct weak coupling with the electron and thus can be identified as the neutrino. Subsequently, if we consider a light singlet, we can generate seesaw suppressed neutrino masses for this choice of unification schemes.

Cascade 3, although similar to two in its unification characteristics, does not offer as attractive a mass spectrum. The mass of the singlet must always be less than 10^6 GeV to return a single light eigenvalue. However, although light, its value does not become ultralight unless

we consider the limit $y \sim y_{\min}$ and $x \sim 10^7$ GeV when this choice becomes equivalent to cascade 2. It is interesting to note that both cascades 2 and 3 require $M_S \lesssim \mathcal{O}(x)$ in order to return a ultralight eigenvalue. This suggests a nontrivial relationship between these two scales. This is not altogether surprising given that we have considered w and v to be functions of x , and so x is the order parameter governing the lightness of this Majorana mass term.

Cascade 4 also generates an ultralight neutrino mass. As long as $M_S \lesssim M_{\text{GUT}}$ we have two lighter Majorana eigenvalues. For larger values of x , we have a greater hierarchy in these two lightest eigenvalues for all values of $y \neq x$. Within this region, $x/w \rightarrow 1$ and the hierarchy between v and w increases. The lightest eigenvalue becomes more favorably light as the mass of the singlet is lowered, with an ultralight eigenvalue obtained for $M_S \lesssim 10^6$ GeV. This one light mass eigenvalue displays the correct weak coupling with the electron. In general, this eigenvalue is lighter for all values of the breaking scales than the previous cascades, giving us a lot more freedom. In particular, the last stage of symmetry breaking can occur at higher energies while still allowing for small neutrino mass. Additionally, as an ultralight eigenvalue can be generated for larger values of x , this pushes the unification scale to 10^{11} GeV, which, although still low, is more favorable than the lower unification bound of 10^{10} GeV.

In all the above cascades, the presence of the singlet only has a significant influence on the two lightest eigenvalues, the remaining spectrum of particle mass maintaining consistency with that of Ref. [4] and Table I. Without an analytic form of the eigenvalues, it is difficult to ascertain precisely how the presence of the singlet influences the behavior of the lightest eigenvalue and, in particular, why $M_S < M_{\text{GUT}}$ yields a nice spectrum. In particular, the location of the coupling of order x appears to, at least numerically, drive the order of this light eigenvalue. This arises as we have treated the scales undemocratically. Changing x alters the relative hierarchies of all scales, and thus provides an energy domain in which there is seesaw suppression. From this analysis, it appears that Eq. (3.6) is a (rather complicated) generalization of Eq. (3.4).

In particular instances, analytic approximations of this light eigenvalue were obtained. However, in the energy regions of interest, these approximations became too erroneous. Nevertheless, inclusion of the singlet generates a hierarchy between the two lightest neutral mass terms, with the heavier of the two pushed up to an energy $x > m_{\nu^c} > y$. This, however, has no impact on the RGE analysis of Ref. [4]. In the regions in which we have an ultralight eigenvalue for m_{ν} , the second lightest term, corresponding to m_{ν^c} , has a value that, in general, approaches but never reaches order x , and thus these values are consistent with the previous calculations. Even if this mass were to exceed order x , the resultant effect on the RGEs would be negli-

gible. This means also that any new physics due to the presence of this exotic singlet will be above a TeV.

So, adding a light singlet fermion to the quartification models of Ref. [4] gives rise to a small Majorana mass for the neutrinos for a subset of these schemes. The success of these scenarios depends nontrivially on the relative hierarchies between the intermediate scales. It is interesting to note that the introduction of intermediary symmetry breaking stages in quartification models yields both gauge coupling unification [4] and seesaw suppressed neutrino masses upon the addition of a fermion singlet. The presence of this singlet, while a complication, does preserve many of the attractive features of quartification. In particular, the previous RGE analysis is unaffected, giving us a unified scheme based on the quartification gauge group that can deliver the SM and predicts interesting new phenomenology at 1 TeV and above. As the addition of the singlet has not added any new phenomenology below a TeV here, the phenomenological account of Sec. V of Ref. [4] is still valid.

IV. NEUTRINO MASSES WHEN $SU(2)_\ell$ IS UNBROKEN

We now consider the case where a residual $SU(2)_\ell$ symmetry remains at the standard model level. The four independent routes for this symmetry breaking are

$$(1) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(3)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1)_Y, \quad (4.1)$$

$$(2) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(3)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1)_Y, \quad (4.2)$$

$$(3) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1)_Y, \quad (4.3)$$

$$(4) \quad G_4 \xrightarrow{v} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes SU(2)_R \otimes U(1) \\ \xrightarrow{w} SU(3)_q \otimes SU(2)_L \otimes SU(3)_\ell \otimes U(1) \\ \xrightarrow{x} SU(3)_q \otimes SU(2)_L \otimes SU(2)_\ell \otimes U(1)_Y, \quad (4.4)$$

all of which allow for unification of the gauge coupling constants. The VEV pattern which achieves this breaking is

TABLE II. Range of energy scales of symmetry breaking that yield unification of the gauge coupling constants when the quartification gauge group is broken to $G_{\text{SM}} \otimes SU(2)_\ell$. There is only one scenario which allows for a TeV level breaking scale. The others occur at higher energies, with much narrower ranges for the final stage of breaking.

Cascade	V 's	x	w	v
1	$V_{\ell_2^c} \sim v, V_{a_2, \ell} \sim w,$ $V_{a_1, \ell_1^c} \sim x$	$x_{\text{min}} \sim 1 \text{ TeV}$ $x_{\text{max}} \sim 6.4 \times 10^7 \text{ GeV}$	$2.7 \times 10^{12} \text{ GeV}$ $7.5 \times 10^{13} \text{ GeV}$	$1.2 \times 10^{17} \text{ GeV}$ $7.5 \times 10^{13} \text{ GeV}$
2	$V_\ell \sim v, V_{a_2, \ell_2^c} \sim w,$ $V_{a_1, \ell_1^c} \sim x$	$x_{\text{min}} \sim 6.5 \times 10^5 \text{ GeV}$ $x_{\text{max}} \sim 6.5 \times 10^7 \text{ GeV}$	$6.5 \times 10^5 \text{ GeV}$ $7.4 \times 10^{13} \text{ GeV}$	$3.9 \times 10^{19} \text{ GeV}$ $7.4 \times 10^{13} \text{ GeV}$
3	$V_{a_2} \sim v, V_{\ell, \ell_2^c} \sim w,$ $V_{a_1, \ell_1^c} \sim x$	$x_{\text{min}} \sim 6.3 \times 10^7 \text{ GeV}$ $x_{\text{max}} \sim 4.9 \times 10^{10} \text{ GeV}$	$7.7 \times 10^{13} \text{ GeV}$ $4.9 \times 10^{10} \text{ GeV}$	$7.7 \times 10^{13} \text{ GeV}$ $7 \times 10^{12} \text{ GeV}$
4	$V_{a_2} \sim v, V_{a_1} \sim w,$ $V_{\ell, \ell_2^c} \sim x$	$x_{\text{min}} \sim 6.2 \times 10^8 \text{ GeV}$ $x_{\text{max}} \sim 4.8 \times 10^{10} \text{ GeV}$	$1.7 \times 10^{12} \text{ GeV}$ $4.8 \times 10^{10} \text{ GeV}$	$1.7 \times 10^{12} \text{ GeV}$ $7 \times 10^{12} \text{ GeV}$

$$\begin{aligned}
\langle \Phi_\ell \rangle &= \begin{pmatrix} 0 & 0 & u_\ell \\ 0 & 0 & 0 \\ 0 & 0 & V_\ell \end{pmatrix}, & \langle \Phi_{\ell^c} \rangle &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_{\ell_1^c} & 0 & V_{\ell_2^c} \end{pmatrix}, & (x_1 \ z_1 \ y_2^c) & \begin{pmatrix} u_{a_1} & u_{a_2} & V_\ell \\ V_{a_1} & V_{a_2} & -u_\ell \\ V_{\ell_2^c} & -V_{\ell_1^c} & u_{a_3} \end{pmatrix} \begin{pmatrix} x_1^c \\ z_1^c \\ y_2 \end{pmatrix} \\
\langle \Phi_a \rangle = \langle \Phi_c^\dagger \rangle &= \begin{pmatrix} u_{a_1} & 0 & u_{a_2} \\ 0 & u_{a_3} & 0 \\ V_{a_1} & 0 & V_{a_2} \end{pmatrix}. & (4.5) & + (x_2 \ z_2 \ y_1^c) & \begin{pmatrix} u_{a_1} & u_{a_2} & -V_\ell \\ V_{a_1} & V_{a_2} & u_\ell \\ -V_{\ell_2^c} & V_{\ell_1^c} & u_{a_3} \end{pmatrix} \begin{pmatrix} x_2^c \\ z_2^c \\ y_1 \end{pmatrix} + u_{a_3} e e^c.
\end{aligned}
\tag{4.6}$$

The relationship between the V 's and the breaking scales for each cascade are listed in Table II along with the energy bounds of each symmetry breaking event which allows for the unification of the gauge couplings. For full details of the derivation of these scales, including the exact VEV structure and mass spectrum for each cascade, see Sec. IV A and Appendix A of Ref. [4].

Again, there is a range of possible energies at which unification and symmetry breaking are compatible, with the unification condition constraining v and w to be functions of x and u only. Only one choice allows for a TeV level breaking, the other three routes having a much narrower range for the final stage of breaking.

The $x_1, x_1^c, y_2, y_2^c, z_1,$ and z_1^c components are now electrically charged and decouple from the N, N^c, ν, ν^c components. The mixing between the charged sector is given by

There is one electroweak Dirac term (per family) corresponding to the $e, \mu,$ and τ masses, and the exotic charged leptons obtain Dirac masses of order V .

As was concluded in Ref. [4], this model does not admit a Majorana mass for the neutrinos. Adding the singlet, the Majorana mass matrix has the form

$$\begin{aligned}
& \begin{pmatrix} N & N^c & \nu & \nu^c & S \end{pmatrix} \\
& \times \begin{pmatrix} 0 & V_{a_2} & 0 & V_{a_1} & \lambda_s V_\ell \\ V_{a_2} & 0 & u_{a_2} & 0 & \lambda_s V_{\ell_2^c} \\ 0 & u_{a_2} & 0 & u_{a_1} & \lambda_s u_\ell \\ V_{a_1} & 0 & u_{a_1} & 0 & \lambda_s V_{\ell_1^c} \\ \lambda_s V_\ell & \lambda_s V_{\ell_2^c} & \lambda_s u_\ell & \lambda_s V_{\ell_1^c} & M_S \end{pmatrix} \begin{pmatrix} N \\ N^c \\ \nu \\ \nu^c \\ S \end{pmatrix}.
\end{aligned}
\tag{4.7}$$

For a generic symmetry breaking, the light eigenvalue can be approximated to be

$$m_\nu \approx \frac{(u_{a_1} V_{a_2} - u_{a_2} V_{a_1}) [M_S (u_{a_1} V_{a_2} - u_{a_2} V_{a_1}) + 2\lambda_s^2 (-u_\ell V_{a_2} V_{\ell_1^c} + u_{a_2} V_\ell V_{\ell_1^c} + u_\ell V_{a_1} V_{\ell_2^c} - u_{a_1} V_\ell V_{\ell_2^c})]}{\lambda_s^2 (V_{a_2} V_{\ell_1^c} - V_{a_1} V_{\ell_2^c})^2}.
\tag{4.8}$$

If there were only one symmetry breaking event to $G_{\text{SM}} \otimes SU(2)_\ell$, then $V_i \rightarrow v$ and the light eigenvalue approaches u^2/v in the limit $\lambda_s \rightarrow 1$. However, we know from the unification conditions of Table II that our intermediate scales are different and these influence the value of the light eigenvalue. In fact, it appears that the hierarchy between the intermediate scales aids in generating the

lightness of this mass term, and Eq. (4.8) remains valid only for certain subsets of the individual cascades upon identification of the V 's.

For the first cascade, the singlet does not conspire to give favorable results, with two light eigenvalues of order the electroweak scale for all possible values allowed by unification.

We turn now to cascade 2 where the value of the last stage of symmetry breaking has a much more narrow range. The analytic approximation of the light eigenvalue is valid only for $x \rightarrow x_{\max}$. In this limit, we obtain two light eigenvalues, both of electroweak order. If $M_S \ll v$ then one of these becomes lighter and the other heavier; however, we still cannot generate an ultralight mass. Thus we are concerned with the lower bound of the last stage of symmetry breaking. Fixing M_S at the GUT scale, there is one ultralight mass eigenvalue for $x \lesssim 10^6$ GeV. This light eigenvalue appears to be independent of M_S , remaining at approximately the same order for all choices of the singlet mass, and it displays the correct electroweak coupling. In this limit, however, we have two additional light eigenvalues of electroweak order. The values of these two eigenvalues are significantly influenced only by x , not M_S , and they become larger than a TeV in the region in which we no longer have an ultralight eigenvalue. In this case, the presence of the singlet alters the spectrum of our particles, with there being a light exotic singlet in addition to the right-handed neutrino and the SM neutrino. This does not impact the results of the RGE analysis of Ref. [4], as these exotic singlets can be taken to be the N^c and ν^c components which carry hypercharge $Y = 0$. So, one must choose whether the generation of seesaw suppressed masses in this scenario warrants the presence of exotic singlets with masses under a TeV.

Cascade 3, conversely, exhibits seesaw suppressed neutrino masses for values of x closer to the upper energy bound. For smaller values of x , the presence of the singlet, and the variation of its mass, does not prove fruitful. The Majorana mass matrix has three eigenvalues of order v and two electroweak order eigenvalues which in the limit $w \rightarrow v$ are approximately $[uv(v-x)]/v^2$. Again, considering $M_S \sim v$, increasing x affects three of the five eigenvalues. An order v mass term becomes slightly lighter, approaching order w , while the hierarchy between the two lightest eigenvalues becomes more pronounced. As $x \rightarrow x_{\max}$, the second lightest tends towards 10^6 GeV, and the lightest becomes of order $\sim \{u^2[M_S - 2\lambda_s^2 w(x)]\}/\lambda_s^2 w(x)^2$ for smaller M_S , as given by Eq. (4.8). Lowering the singlet mass lowers the value of x at which the hierarchy between the two lightest Majorana terms becomes prominent, and lowers the third eigenvalue closer to w . At $x = x_{\max}$ we can generate an ultralight eigenvalue, the particle displaying the correct coupling with the electron. The second lightest eigenvalue approaches order x but never attains it, and the limiting value of the heavier eigenvalue is always larger than order w . This preserves the spectrum of masses considered for the RGE analysis in Ref. [4] and, in particular, there are no exotic singlets below a TeV.

The final cascade is the only choice which returns the two lightest Majorana masses at distinct scales for all values of Table II irrespective of the scale of the singlet mass. In the limit $x \rightarrow w$, Eq. (4.8) reduces to $[u^2 M_S (v -$

$w)^2]/\lambda_s^2 w^4$, this mass becoming lighter for larger x , w . An ultralight eigenvalue can be obtained in this high x limit, with the variation of the singlet mass not affecting this order much. This particle has the correct coupling with the electron. The other eigenvalues are influenced by the presence of the singlet; however, as in cascade 3, the ordering of the particle spectrum is consistent with the RGE analysis, and the linear combination corresponding to ν^c gains a mass larger than a TeV.

So, when there is a residual $SU(2)_\ell$ gauge symmetry, we still have three scenarios which achieve unification of the gauge coupling constants and generate neutrino masses that would result from the regular seesaw mechanism. One of these choices is less desirable, having exotic singlets at electroweak order. Again, this appears to be a generalization of the case of Ref. [6]. Furthermore, the range of energies in which this results demands that $M_{\text{GUT}} > 10^{12}$ GeV, meaning that Higgs induced proton decay has a greater suppression. Although this result necessitates the addition of a fermion singlet per family, this is still an improvement on the model of Ref. [3], as unification is still possible without the need to invoke a further discrete symmetry and nonrenormalizable operators. In particular, this analysis cements the fact that unification in this case is not unique. Additionally, for the latter two cascades, all exotic singlets lie above a TeV; thus, in these scenarios there is no new phenomenology at a TeV other than that previously commented [4].

V. PROTON DECAY

We finish with a brief comment on the proton decay in those models that return light Majorana masses for the neutrinos. Although in quartification baryon number is conserved by the gauge interactions, proton decay may still proceed, mediated by Higgs boson exchange. In the absence of S , baryon number conservation can be imposed on the model. With S present, this is no longer the case, so we need to examine the finite proton lifetime induced by the S Yukawa coupling terms. Proton decay occurs through the mode $p \rightarrow \pi^+ \nu$. In addition to the terms in Eq. (3.5), the singlet must also couple to the quark multiplets as per

$$\lambda_s (\text{Tr}[q\Phi_q^\dagger] + \text{Tr}[q^c\Phi_{q^c}^\dagger])S \quad (5.1)$$

in order to be closed under the Z_4 symmetry. However, while the Yukawa Lagrangian of Eq. (2.4) assigns Φ_q and Φ_{q^c} the baryon numbers $-\frac{2}{3}$ and $\frac{2}{3}$, respectively, the simultaneous presence of the interaction terms of Eq. (5.1) leads to baryon number violation by one unit. Proton decay is mediated by a tree-level graph involving a virtual colored Higgs field. The decay products must be $\pi^+ \nu$ because the field S must be involved.

The current experimental lower bound on the lifetime of this decay mode is $\sim 10^{32}$ years [12], with the partial width for this channel approximately

$$\Gamma \sim \frac{1}{32\pi} (\lambda_s Y_L \sin\theta)^2 \frac{m_p^5}{m_\Phi^4}. \quad (5.2)$$

m_p and m_Φ are the masses of the proton and colored Higgs fields, respectively, Y_L is the Yukawa coupling defined in Eq. (2.4), and $\sin\theta$ describes the admixture of S within the light neutrino eigenstate. It turns out that this tiny mixing angle provides the necessary suppression.

Given that both λ_s and θ are not independent of the Majorana neutrino masses, it is not possible to make a generic estimation of the proton decay rate. Rather, one must look at the cascades above individually. When the leptonic color symmetry is broken entirely, there are only two choices which return realistic neutrino masses. Looking first at cascade 2, choosing the singlet coupling strength $\lambda_s \sim 10^{-3}$, an ultralight Majorana mass term results when the unification scale is $\sim 10^{11}$ GeV. The admixture of the singlet within this light neutrino eigenstate is $\theta \sim 10^{-4}$, giving an estimation of the lifetime to be $\tau \sim \frac{1}{Y_L} 10^{28}$ years. (In general, for this case, over the range of allowable scales that give unification, the mixing angle is $\theta \gtrsim 10^{-4}$). If the Yukawa coupling strength is $Y_L \lesssim 10^{-2}$, then the decay lifetime is above experimental bounds.

Cascade 4 yields a more favorable estimate. For the same singlet coupling strength, the neutrino eigenstate has a mixing angle $\theta \sim 10^{-6}$, and the unification scale is again of order 10^{11} GeV. This gives the lifetime of this particular mode to be $\tau \sim \frac{1}{Y_L} 10^{32}$ years.

When the $SU(2)_\ell$ symmetry remains unbroken, the unification scales are of larger orders and so generate a greater suppression of the decay rate (independent of the mixing angle suppression) than the previous case. Cascade 4 returns the lowest of the unification scales and thus returns the shortest estimation of the lifetime. For symmetry breaking scales of $v \sim 10^{12}$, $w \sim 10^{11}$, $x \sim 10^9$ GeV, and $\lambda_s = 10^{-2}$, the mixing angle is of order $\theta \sim 10^{-7}$. This gives an estimate of the proton lifetime to be $\tau \sim \frac{1}{Y_L} 4 \times$

10^{36} years. So, unless the Yukawa coupling $Y_L > 10^2$, then this lifetime is well above current experimental bounds.

VI. CONCLUSION

Models based upon a quark-lepton symmetry offer alternative unification schemes. Gauge coupling constant unification can result in several of these schemes which consider different symmetry breaking routes from the quartification gauge group down to the standard model gauge group [4]. These schemes considered a minimal quartification scenario in which there are no additional symmetries or higher-dimension operators. This paper investigated the neutrino sector of these successful unified schemes. We corrected an error in Ref. [4], and as a consequence our claim in that paper that light Majorana masses for the neutrinos can be generated in these minimal schemes was shown to be false. To achieve a satisfactory neutrino mass outcome, nonminimal models must therefore be considered. We chose to add a fermion singlet per family. The addition of these singlets provided the necessary dynamics to return seesaw suppressed Majorana masses for the neutrinos in a subset of the unification schemes. When there was no residual leptonic color symmetry, there were four unique cascades consistent with the gauge coupling unification. Of these, two returned both favorable neutrino masses and realistic bounds on proton stability. In the models that had a remnant $SU(2)_\ell$ symmetry, of the four possible choices, there were three which displayed consistency with both neutrino masses and proton decay.

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- [1] R. Foot and H. Lew, Phys. Rev. D **41**, 3502 (1990); Nuovo Cimento Soc. Ital. Fis. A **104**, 167 (1991); R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 1531 (1991).
- [2] G. C. Joshi and R. R. Volkas, Phys. Rev. D **45**, 1711 (1992).
- [3] K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. D **69**, 051301 (2004).
- [4] A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **72**, 075007 (2005).
- [5] R. R. Volkas, Phys. Rev. D **50**, 4625 (1994).
- [6] R. Foot and R. R. Volkas, Phys. Lett. B **358**, 318 (1995).
- [7] P. Minkowski, Phys. Lett. **67B**, 421 (1977); S. L. Glashow, "Quarks and Leptons," in Proceedings of the Advanced Study Institute, 1979 (unpublished); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. X. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979) (KEK Report No. 79-18, 1979); see also R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [8] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986); A. S. Joshipura and A. Y. Smirnov, Phys. Lett. B **439**, 103 (1998).
- [9] G. C. Branco and C. Q. Geng, Phys. Rev. Lett. **58**, 969 (1987); B. A. Campbell, J. R. Ellis, K. Enqvist, M. K.

- Gaillard, and D. V. Nanopoulos, *Int. J. Mod. Phys. A* **2**, 831 (1987); D. London, G. Belanger, and J.N. Ng, *Phys. Rev. D* **34**, 2867 (1986); A. S. Joshipura and U. Sarkar, *Phys. Rev. Lett.* **57**, 33 (1986); M. Frank, I. Turan, and M. Sher, *Phys. Rev. D* **71**, 113001 (2005).
- [10] S. Nandi and U. Sarkar, *Phys. Rev. Lett.* **56**, 564 (1986); M. Cvetič and P. Langacker, *Phys. Rev. D* **46**, 2759 (1992).
- [11] D. Wyler and L. Wolfenstein, *Nucl. Phys.* **B218**, 205 (1983); R. N. Mohapatra and J. W. F. Valle, *Phys. Lett. B* **177**, 47 (1986); R. N. Mohapatra, hep-ph/0306016.
- [12] S. Eidelman *et al.* (Particle Data Group), *Phys. Lett. B* **592**, 1 (2004).