

Semileptonic decay of a polarized top quark in the noncommutative standard model

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In this paper we study the noncommutative effects to the lepton spectrum from the decay of a polarized top quark. It is shown that the lowest contribution comes from the quadratic terms of the noncommutative parameter. The deviations from the standard model are significant for small values of the noncommutative characteristic scale. However, the charged lepton spin correlation coefficient has a remarkable deviation from the standard model from very low values of the noncommutative characteristic scale to 1 TeV.

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I. INTRODUCTION

The standard model of the particles has been found to be in agreement with experiment in many of its aspects. However, in the framework of the standard model the top is the only quark which has a mass in the same order as the electroweak symmetry breaking scale, $v \sim 246$ GeV, whereas all other observed fermions have masses which are a tiny fraction of this scale. This huge mass might be a hint that the top quark plays an essential role in the electroweak symmetry breaking. On the other hand, the reported experimental data from Tevatron on the top quark properties are still limited, and no significant deviations from the standard model predictions have been seen [1]. The number of observed top quark events in the Tevatron experiment is increasing and now reaching to the order of a few hundred. Several properties of the top quark have been already examined. They consist of studies of the $t\bar{t}$ production cross section, the top quark mass measurement, the measurement of W helicity in the top decay, the search for flavor-changing neutral-current interactions (FCNC) and many other studies [1]. However, it is expected that top quark properties can be examined with high precision at the LHC due to very large statistics [2]. Since the dominant top quark decay mode is into a W boson and a bottom quark, the tWb coupling can be investigated accurately. Within the standard model, the top quark decays via electroweak interaction before hadronization. This important property is one the consequences of its large mass. Hence, the spin information of the top quark is transferred to its decay daughters. Thus, the top quark spin can be used as a powerful means for investigation of any possible new physics.

There are many studies for testing the top quark decay properties at hadron colliders. For instance, the nonstandard effects on the full top width have been investigated in the minimal supersymmetric standard model and in the technicolor model [3]. Some studies have been performed on the effects of anomalous tWb couplings on the top

width and some constraints have been applied on the anomalous couplings [4]. There have been some studies on the top quark rare decays. The two-body decay of the top quark, $t \rightarrow Wb$, has been considered within the noncommutative standard model in [5].

In this paper we study the semileptonic decay of the top quark, $t \rightarrow l\nu b$, in the noncommutative standard model. The noncommutativity in space-time is a possible generalization of the usual quantum mechanics and quantum field theory to describe the physics at very short distances of the order of the Planck length, since the nature of the space-time changes at these distances. The noncommutative spaces can be realized as spaces where coordinate operators \hat{x}_μ satisfy the commutation relations:

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}, \quad (1)$$

where $\theta_{\mu\nu}$ is a real antisymmetric tensor with the dimension of $[L]^2$. We note that a space-time noncommutativity, $\theta_{0i} \neq 0$, might lead to some problems with unitarity and causality [6]. A noncommutative version of an ordinary field theory can be obtained by replacing all ordinary products with Moyal \star product defined as [7]:

$$\begin{aligned} (f \star g)(x) &= \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^y\partial_\nu^z\right)f(y)g(z) \Big|_{y=z=x} \\ &= f(x)g(x) + \frac{i}{2}\theta^{\mu\nu}(\partial_\mu f(x))(\partial_\nu g(x)) + \mathcal{O}(\theta^2). \end{aligned} \quad (2)$$

The approach to the noncommutative field theory based on the Moyal product and Seiberg-Witten maps allows the generalization of the standard model to the case of noncommutative space-time, keeping the original gauge group and particle content [8,9]. Seiberg-Witten maps relate the noncommutative gauge fields and ordinary fields in commutative theory via a power series expansion in θ . Indeed the noncommutative version of the standard model is a Lorentz violating theory, but the Seiberg-Witten map shows that the zeroth order of the theory is the Lorentz invariant standard model. The effects of the noncommutative space-time on some rare and collider processes have

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been considered in [10]. The leptonic decay of the W and Z bosons on the noncommutative space-time have been studied in [11].

This paper is organized as follows: In Sec. II, a short introduction for the noncommutative standard model (NCSM) is given. Section III is dedicated to present the noncommutative effects on the $t(\uparrow) \rightarrow l^+ + \nu_l + b$ decay. Finally, Sec. IV concludes the paper.

II. NONCOMMUTATIVE STANDARD MODEL

The action of the NCSM can be obtained by replacing the ordinary products in the action of the classical SM by the Moyal products and then matter and gauge fields are replaced by the appropriate Seiberg-Witten expansions. The action of the NCSM can be written as

$$S_{\text{NCSM}} = S_{\text{fermions}} + S_{\text{gauge}} + S_{\text{Higgs}} + S_{\text{Yukawa}}, \quad (3)$$

This action has the same structure group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the same fields number of coupling parameters as the ordinary SM. The approach, which has been used in [9] to build the NCSM, is the only known approach that allows one to build models of the electro-weak sector directly based on the structure group $SU(2)_L \times U(1)_Y$ in a noncommutative background. The NCSM is an effective, anomaly free, noncommutative field theory [12].

We just consider the fermions (quarks and leptons). The fermionic matter part in a very compact way is

$$S_{\text{fermions}} = \int d^4x \sum_{i=1}^3 (\bar{\Psi}_L^{(i)} \star (i\hat{D}_\mu \gamma^\mu \hat{\Psi}_L^{(i)})) + \int d^4x \sum_{i=1}^3 (\bar{\Psi}_R^{(i)} \star (i\hat{D}_\mu \gamma^\mu \hat{\Psi}_R^{(i)})), \quad (4)$$

where i is the generation index and $\Psi_{L,R}^i$ are

$$\Psi_L^{(i)} = \begin{pmatrix} L_L^i \\ Q_L^i \end{pmatrix}, \quad \Psi_R^{(i)} = \begin{pmatrix} e_R^i \\ u_R^i \\ d_R^i \end{pmatrix} \quad (5)$$

where L_L^i and Q_L^i are the well-known lepton and quark doublets, respectively. The Seiberg-Witten maps for the noncommutative fermion and vector fields yield

$$\hat{\psi}[V] = \psi - \frac{1}{2}\theta^{\mu\nu} V_\mu \partial_\nu \psi + \frac{i}{8}\theta^{\mu\nu} [V_\mu, V_\nu] \psi + \mathcal{O}(\theta^2),$$

$$\hat{V}_\alpha[V] = V_\alpha + \frac{1}{4}\theta^{\mu\nu} \{\partial_\mu V_\alpha + F_{\mu\alpha}, V_\nu\} + \mathcal{O}(\theta^2), \quad (6)$$

where ψ and V_μ are ordinary fermion and gauge fields, respectively. Noncommutative fields are denoted by a hat symbol.

For a full description and review of the NCSM, see [9]. The $t(p_1) \rightarrow W(q) + b(p_2)$ vertex in the NCSM up to the order of θ^2 can be written as [5,9]:

$$\Gamma_{\mu,\text{NC}} = \frac{gV_{tb}}{\sqrt{2}} \left[\gamma_\mu + \frac{1}{2}(\theta_{\mu\nu}\gamma_\alpha + \theta_{\alpha\mu}\gamma_\nu + \theta_{\nu\alpha}\gamma_\mu) q^\nu p_1^\alpha - \frac{i}{8}(\theta_{\mu\nu}\gamma_\alpha + \theta_{\alpha\mu}\gamma_\nu + \theta_{\nu\alpha}\gamma_\mu) \times (q\theta p_1) q^\alpha p_1^\nu \right] P_L, \quad (7)$$

where $P_L = \frac{1-\gamma_5}{2}$ and $q\theta p_1 \equiv q^\mu \theta_{\mu\nu} p_1^\nu$. This vertex is similar to the vertex of W decays into a lepton and anti-neutrino [9,11].

III. THE NONCOMMUTATIVE EFFECTS ON THE $t(\uparrow) \rightarrow l^+ + \nu_l + b$ DECAY

The effective vertex in Eq. (7) contains γ_μ and the momenta of the involved particles. In order to simplify the calculations we ignore of the masses of the b quark and leptons. By using the Dirac equations, the vertices have simpler forms. For instance, Eq. (7) can be replaced by the following:

$$\bar{u}(p_2) \Gamma^\mu u(p_1) = \frac{gV_{tb}}{\sqrt{2}} \bar{u}(p_2) \left[\left(1 + \frac{1}{2}q\theta p_1 + \frac{i}{8}(q\theta p_1)^2 \right) \times \gamma^\mu P_L - m_t \left(\frac{1}{2} + \frac{i}{8}q\theta p_1 \right) \times \theta^{\mu\nu} p_{2,\nu} P_R \right] u(p_1), \quad (8)$$

According to Fig. 1, the event plane defines the $(x-z)$ plane. The z -axis is determined by the momentum of the lepton and θ_l is the angle between momentum of the lepton and the spin of the top quark in the rest frame of the top quark. Using the following identities:

$$u_\alpha \theta^{\alpha\beta} v_\beta = u\theta v = \vec{\theta} \cdot (\vec{u} \times \vec{v}),$$

$$u_\mu \theta^{\mu\nu} v_\nu^\alpha = |\vec{\theta}|^2 (\vec{u} \cdot \vec{v}) - (\vec{u} \cdot \vec{\theta})(\vec{v} \cdot \vec{\theta}), \quad (9)$$

where $\vec{\theta} = (\theta_{23}, \theta_{31}, \theta_{12})$, and if we assume that $\vec{\theta}$ is in the $(x-z)$ plane the squared matrix element for the reaction

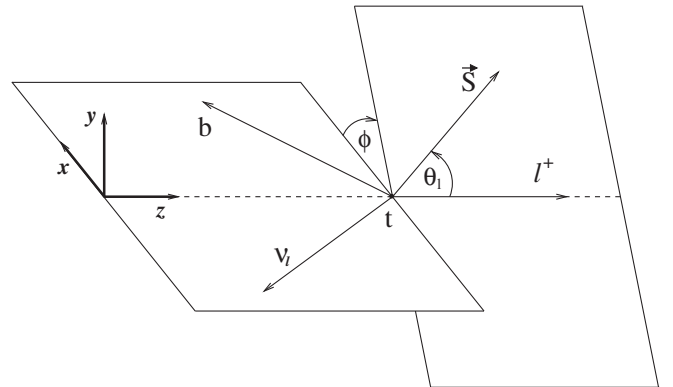


FIG. 1. The process $t(\uparrow) \rightarrow l^+ + \nu_l + b$ in the rest frame of the top quark.

$t(p_1) \rightarrow W(q) + b(p_2) \rightarrow l^+(k_1) + \nu_l(k_2) + b(p_2)$ has the following form:

$$\overline{|\mathcal{M}|^2} \propto \left((k_1 \cdot \hat{p}_1)(k_2 \cdot p_2) + \frac{m_t^2}{16} (|\vec{p}_2|^2 |\vec{\theta}|^2 - (\vec{p}_2 \cdot \vec{\theta})^2) \right. \\ \left. \times (k_1 \cdot k_2)(\hat{p}_1 \cdot p_2) \right), \quad (10)$$

where $\hat{p}_1^\mu = p_1^\mu - m_t s^\mu$ and $s^\mu = (0, \vec{s})$ is the polarization four-vector of the top quark.

The differential rate for $t \rightarrow l^+ + \nu_l + b$ at the Born approximation can be written as

$$d\Gamma = \frac{1}{2m_t} \frac{64G_F^2}{(1-y/r)^2 + \gamma_W^2} \frac{dR_3}{(2\pi)^5} \left((k_1 \cdot \hat{p}_1)(k_2 \cdot p_2) \right. \\ \left. + \frac{m_t^2}{16} (|\vec{p}_2|^2 |\vec{\theta}|^2 - (\vec{p}_2 \cdot \vec{\theta})^2)(k_1 \cdot k_2)(\hat{p}_1 \cdot p_2) \right), \quad (11)$$

where $r = m_W^2/m_t^2$ and $\gamma_W = \Gamma_W/m_W$. The three-body phase space is parametrized as follows [13]:

$$dR_3 = \frac{1}{32} m_t^2 dx dy d(\cos\theta_l) d\alpha d\beta. \quad (12)$$

By defining $x = \frac{2p_1 \cdot k_1}{m_t^2}$ and $y = \frac{(k_1 + k_2)^2}{m_t^2}$ and from Fig. 1 one has

$$\begin{aligned} p_1 &= m_t(1; 0, 0, 0), \\ k_1 &= \frac{m_t}{2} x(1; 0, 0, 1), \\ k_{1 \cdot s} &= -\frac{xm_t}{2} \cos\theta_b, \\ p_2 &= \frac{m_t}{2} (1-y)(1; \sin\theta_b, 0, \cos\theta_b), \\ k_2 &= \frac{m_t}{2} (1-x+y)(1; -\sin\theta_\nu, 0, \cos\theta_\nu), \end{aligned} \quad (13)$$

where

$$\cos\theta_\nu = \frac{x(1+y-x) - 2y}{x(1-x+y)}, \quad \cos\theta_b = \frac{2y-x-xy}{x(1-y)}. \quad (14)$$

After replacing Eq. (13) in Eq. (11) and integrating over two Euler angles and in the W narrow width approximation, $\gamma_W \rightarrow 0$, the double differential $x - \theta_l$ distributions in the pointlike four fermion limit is

$$\frac{d^2\Gamma}{dx d\cos\theta_l} = \frac{G_F^2 m_t^3 m_W^3}{32\pi^2 \Gamma_W} |V_{tb}|^2 \left(F_0(x) + G_0(x) \cos\theta_l \right. \\ \left. + \frac{1}{\Lambda_{\text{NC}}^4} [F_1(x) + G_1(x) \cos\theta_l] \right), \quad (15)$$

where $\Lambda_{\text{NC}}^2 = \frac{1}{\sqrt{|\vec{\theta}|^2}}$ is the scale of the noncommutativity and $F_{0,1}(x)$ and $G_{0,1}(x)$ are defined in the following forms:

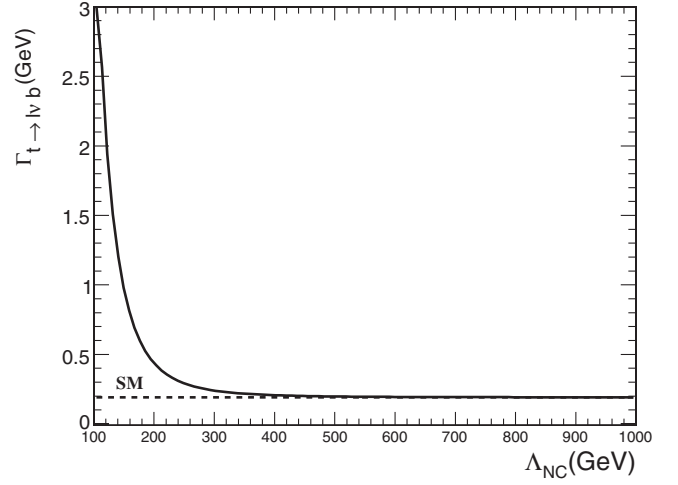


FIG. 2. The three-body decay rate as a function of noncommutativity scale (solid curve)

$$\begin{aligned} F_0(x) &= G_0(x) = x(1-x), \\ F_1(x) &= \frac{m_W^4 (1-x)(r-x)(1-r)}{16x^2}, \\ G_1(x) &= \frac{m_W^4 (1-x)(r-x)(2r-x-rx)}{16x^3}. \end{aligned} \quad (16)$$

In Fig. 2, the three-body top decay width is presented as a function of the noncommutativity scale. This figure shows that the noncommutative effect is negligible for $\Lambda_{\text{NC}} \geq 500$ GeV and is not observable. Figure 3 shows the energy spectrum for the charged lepton in the top quark decay at the Born approximation for the ordinary SM (dashed curve) and for when $\Lambda_{\text{NC}} = 200$ GeV in the NCSM (solid curve). It shows that the probability of having

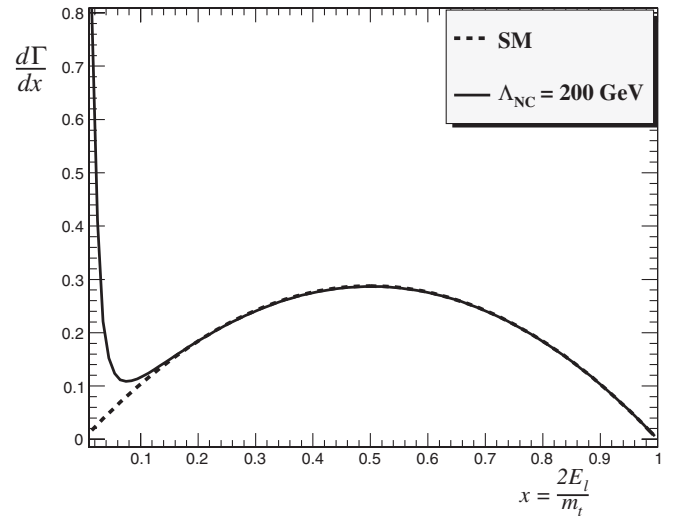


FIG. 3. The energy spectrum (Born approximation) for the charged lepton in the top quark decay for the ordinary SM (dashed curve) and for when $\Lambda_{\text{NC}} = 200$ GeV in the NCSM (solid curve).

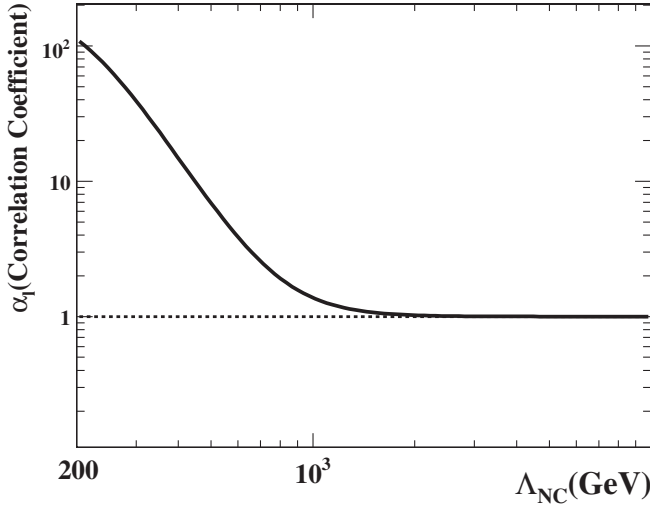


FIG. 4. The correlation coefficient α_l as a function of non-commutativity scale in the logarithmic scale.

very low energy charged leptons, $x < 0.1$ or $E_l < 9$ GeV, is high because of the noncommutative effects. However, from the experimental point of view this effect is not observable since we are restricted by the detector resolution. The detected leptons with transverse momentum less than 15 GeV are usually not reliable and rejected in the experiments [1]. It is noticeable that this effect is present only for low values of noncommutativity scale. The lepton energy spectrum is matched with the SM case for $\Lambda_{\text{NC}} \geq 500$ GeV. It is more useful to express Eq. (15) in the following way:

$$\frac{d^2\Gamma}{dx d\cos\theta_l} = \frac{d\Gamma}{dx} \times \frac{1}{2} (1 + \alpha_l(x) \cos\theta_l), \quad (17)$$

where $\alpha_l(x)$, *correlation coefficient* or *spin analyzing power*, is

$$\alpha_l(x) = \frac{G_0(x) + G_1(x)/\Lambda_{\text{NC}}^4}{F_0(x) + F_1(x)/\Lambda_{\text{NC}}^4}. \quad (18)$$

In the ordinary standard model and in the limit of vanishing the lepton masses, α_l is independent of x and is equal to one [14]. As a result of Eq. (17), in the NCSM the angular distribution of a polarized top decay has the same form as the ordinary SM:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_l} = \frac{1}{2} (1 + \alpha_l \vec{\sigma} \cdot \hat{\mathbf{p}}_l), \quad (19)$$

where $\hat{\mathbf{p}}_l$ describes the direction of the flight of lepton in the rest frame of the top quark and σ_i denotes the Pauli matrices. However, according to Fig. 4 in the NCSM, the spin analyzing power α_l depends on the noncommutativity

scale. Figure 4 reveals a significant deviation from the SM even for the case that Λ_{NC} is around 1 TeV.

It is well known that top quarks produced in hadron colliders are scarcely polarized. As a result, the correlations between top spin and anti-top spin in the $t\bar{t}$ are considered [2,15]. In order to illustrate we consider the dileptonic decay of the $t\bar{t}$ events in hadron colliders:

$$P P, P\bar{P} \rightarrow t\bar{t} + X \rightarrow l^+ l'^- + X, \quad (20)$$

The double differential angular distribution of the leptons coming from the top and anti-top in the ordinary SM is [2,15]

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_{l^+} d\cos\theta_{l^-}} = \frac{1}{4} (1 + \kappa \cos\theta_{l^+} \cos\theta_{l^-}),$$

$$\kappa = \alpha_{l^+} \alpha_{l^-} \times \frac{N_{\parallel} - N_{\times}}{N_{\parallel} + N_{\times}}. \quad (21)$$

where $\theta_{l^+} (\theta_{l^-})$ is the angle between the direction of the $l^+ (l^-)$ in the rest frame of the $t(\bar{t})$ and the $t(\bar{t})$ direction in the $t\bar{t}$ center of mass. N_{\parallel} is the number of top pair events where both quarks have spin up or spin down and N_{\times} is the number of top pair events where one quark is spin up and the other is spin down. Equation (21) shows the strong dependence of the experimental observable κ to the spin analyzing power (α_l). For the $t\bar{t}$ production at the Tevatron, the SM predicts $(N_{\parallel} - N_{\times})/(N_{\parallel} + N_{\times}) = 0.88$. The D0 measurement for the quantity κ is 2.3 ± 2.5 [16], which is not very good due to low statistics. Having a good measurement for the κ and knowing the dependency of N_{\parallel} and N_{\times} on the noncommutativity scale might provide valuable information concerning the noncommutativity scale. Just as an example, using this rough measurement from D0 and ignoring of any dependency of N_{\parallel} and N_{\times} on the noncommutativity scale yields $\Lambda_{\text{NC}} \sim 900$ GeV.

IV. CONCLUSION

The noncommutative effects only show up for the low values of the noncommutativity scale in the lepton energy spectrum from the decay of top quarks. The effects are not visible for low values of the noncommutativity scale because of the experimental restrictions. However, the charged lepton spin correlation coefficient is sensitive to the noncommutativity scale from very low values to 1 TeV. It is a powerful tool for investigation of non-SM effects, particularly in $t\bar{t}$ events.

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