Effective action for QED_3 in a region with borders

C. D. Fosco[1] and F. D. Mazzitelli[2]

[1]*Centro Atómico Bariloche and Instituto Balseiro, Comisión Nacional de Energía Atómica-R8402AGP Bariloche, Argentina*
[2]*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I-1428 Buenos Aires, Argentina*

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We study quantum effects due to a Dirac field in $2 + 1$ dimensions, confined to a spatial region with a nontrivial boundary, and minimally coupled to an Abelian gauge field. To that end, we apply a path-integral representation, which is applied to the evaluation of the Casimir energy and to the study of the contribution of the boundary modes to the effective action when an external gauge field is present. We also implement a large-mass expansion, deriving results which are, in principle, valid for any geometry. We compare them with their counterparts obtained from the large-mass ''bosonized'' effective theory.

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INTRODUCTION

The presence of borders drastically modifies the energy spectrum of a quantum field, by producing a vacuum energy with a nontrivial dependence on the geometry of the borders and the detailed form of the boundary conditions. The resulting ''Casimir energy'' has many interesting physical consequences, ranging from the existence of forces between uncharged metallic surfaces to potentially relevant effects in some cosmological scenarios [1].

On the other hand, quantum theories in the presence of background fields naturally arise in many different physical situations, like when considering the effects of classical gravitational or electromagnetic background fields on the vacuum persistence amplitude. Besides, the consideration of ''classical'' backgrounds is sometimes an important intermediate step in the context of the functional quantization approach, whereby one considers (trivial or nontrivial) classical backgrounds whose configurations may afterwards be allowed to fluctuate; usually this is done without modifying either the topology or the boundary conditions of the classical background.

Quantum fields coupled to background fields and models defined on spaces with nontrivial borders do share some important properties. Indeed, the latter can sometimes be regarded as a special limit of the former. Background fields do of course also modify the energy spectrum in a nontrivial way. As a result of this, the vacuum persistence amplitude, obtained by integrating out the quantum fields becomes a (usually) complicated functional of the background field.

QED in $2 + 1$ dimensions is an interesting arena for the analysis of the combined effect of boundary conditions and background fields on the quantum vacuum. The Casimir energy for massless and massive spinor fields in $2 + 1$ dimensions has been discussed at length, using the zeta function approach [2]. The effect of boundary conditions in the presence of external fields have also received some attention, in particular, in the case of fermions satisfying MIT boundary conditions on a circle in the presence of a magnetic flux string [3].

In this paper, we shall consider a path-integral approach to the computation of the effective action in the presence of nontrivial boundaries and external fields. This approach, introduced in [4], has been adapted to the case of the electromagnetic field satisfying perfect conductor boundary conditions on the borders [5], and successfully applied to the calculation of Casimir forces in different geometries [6]. The main idea is to implement the boundary conditions as delta functions in the functional integral, and to write them in terms of auxiliary fields living on the boundaries. Here we will apply a similar idea to the case of a Dirac field in $2 + 1$ dimensions. We will assume that the field is confined into a static spacetime region, and that it is minimally coupled to an Abelian gauge field. We shall obtain a general formula for the effective action in terms of a nonlocal kernel evaluated on the boundary. We will then analyze some of its formal properties, applying it next to the calculation of the Casimir energy and of the contribution of the borders to the effective action for the gauge field.

The paper is organized as follows. In Sec. II, we adapt the method of [4] to the present case. That approach is also used to understand the issue of gauge invariance, and to calculate the fermion propagator in the same system.

After studying some general properties of those functional representations, we apply them, in the following sections, to calculate the effective action under different approximation schemes and simplifying assumptions. In Sec. III, we consider the Casimir energy for massless Dirac fermions, which is derived from the effective action with a vanishing gauge field, for the special geometry of two parallel plates.

In Sec. IV, we evaluate the effective action in a large-m approximation, for the case on an arbitrary external gauge field. This yields a contribution coming from the boundary modes, which is local when the mass tends to infinity. In this section, we also discuss the same system from a different point of view: we start from the "dual" or bosonized version of the Dirac field in the large-mass limit, which is a Chern-Simons action. This action is then constrained to satisfy the corresponding boundary condition, which now is a kind of "perfect conductor" boundary condition for the Chern-Simons gauge field. We obtain the resulting functional integral for the boundary modes, and compare with the previous result.

In Sec. V, we consider the dependence of the effective action on the external gauge field, for the particular case of a linear wall.

II. THE EFFECTIVE ACTION

A. The model

We want to derive a general expression for the effective action due to a massive Dirac field in the presence of an external Abelian gauge field, in a spatial region $\mathcal{U}$ with a nontrivial (static) spatial boundary $\mathcal{C}$. We shall assume $\mathcal{C}$ to correspond to a simple closed plane curve $\mathcal{C}$ (Fig. 1).

The physical system, Dirac fermions in a background Abelian gauge field, may be conveniently defined by its Euclidean action S_f which, in our conventions, is given by:

$$S_f(\bar{\psi}, \psi, A) = \int_{\mathcal{U}} d^3x \bar{\psi}(x)(\not{D} + m)\psi(x) \tag{1}$$

where $\not{D} \equiv \gamma_\alpha D_\alpha$ and $D_\alpha \equiv \partial_\alpha + ieA_\alpha(x)$, γ_α are Dirac's matrices and A_α denotes an external Abelian gauge field. We shall adopt the prescription that indices from the beginning of the Greek alphabet $(\alpha, \beta, \ldots)$ can take the values 0, 1 and 2, those from the middle $(\mu, \nu, \ldots)$ run from 0 to 1, while Roman indices $(i, j, \ldots)$ can take the "spatial" values 1 or 2. Dirac's matrices are chosen according to the convention: $\gamma_0 \equiv \sigma_1$, $\gamma_1 \equiv \sigma_2$ and $\gamma_2 \equiv \sigma_3$ (σ_1, σ_2 and σ_3: Pauli's matrices) unless explicitly stated otherwise.

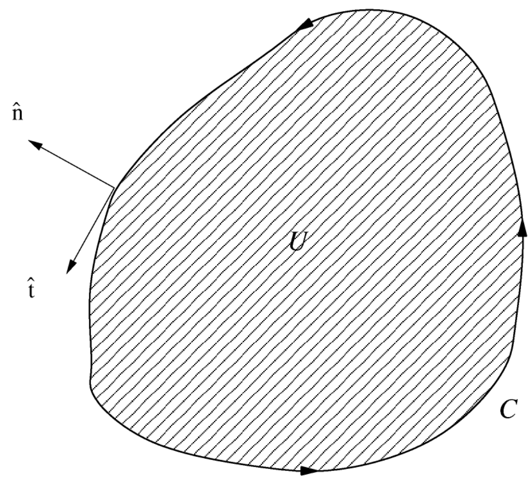

FIG. 1. The spatial region U, bounded by $\mathcal{C}$. $\hat{\mathbf{t}}$ and $\hat{\mathbf{n}}$ denote the unit tangent and normal vectors, respectively.

In order to introduce the boundary conditions, we shall assume that the curve $\mathcal{C}$ has been parametrized: $\zeta \longrightarrow \mathbf{r}(\zeta)$, where $\mathbf{r}(\zeta) = (r_1(\zeta), r_2(\zeta))$, and that the parameter ζ belongs to some interval I. Besides, for every point of $\mathcal{C}$, we introduce the unit vectors $\hat{\mathbf{t}}$ and $\hat{\mathbf{n}}$, tangent and (outer) normal to $\mathcal{C}$, respectively, (see Fig. 1).

An explicit expression for $\hat{\mathbf{t}}$ and $\hat{\mathbf{n}}$ may be written as follows:

$$t_i(\zeta) = \frac{\dot{r}_i(\zeta)}{|\dot{\mathbf{r}}(\zeta)|}, \qquad n_i(\zeta) = \varepsilon_{ij} t_j(\zeta), \tag{2}$$

where $\dot{r}_i(\zeta) \equiv \frac{dr_i(\zeta)}{d\zeta}$.

Besides, when considering the large-mass limit, we shall also need to invoke an alternative description for the curve $\mathcal{C}$, obtained by introducing u_1 and u_2, two orthogonal curvilinear coordinates for the plane, in such a way that $\mathcal{C}$ corresponds to $u_2 = 0$. Since they are orthogonal coordinates, the square of $d\mathbf{x}$ can be written as follows:

$$|d\mathbf{x}|^2 = h_1^2 du_1^2 + h_2^2 du_2^2, \tag{3}$$

where h_1 and h_2 may depend on u_1 and u_2. A further simplification we shall adopt is that we will fix u_1 to coincide with the arc length for the points on the curve $\mathcal{C}$ (of course, when $u_2 = 0$), namely,

$$u_2 = 0, \qquad du_2 = 0 \rightarrow |d\mathbf{x}|^2 = du_1^2 = d\xi_1^2. \tag{4}$$

We shall not need to construct that system of coordinates explicitly; rather, we note that, in a neighborhood of $u_2 = 0$, one can construct u_2-constant coordinate lines by dragging $\mathcal{C}$ along the direction of $\hat{\mathbf{n}}$. On the other hand, the u_1-constant lines are obtained by using the property that $\hat{\mathbf{n}}$ is tangent to them (at every point on the curve).

Equipped with the previous definitions, we introduce baglike boundary conditions on $\mathcal{C}$ for the fields ψ and $\bar{\psi}$, as follows:

$$\begin{aligned} &\mathcal{P}_L(\zeta)\psi(x_0, \mathbf{r}(\zeta)) = 0, \\ &\bar{\psi}(x_0, \mathbf{r}(\zeta))\mathcal{P}_R(\zeta) = 0, \quad \forall\ \zeta \in I, \end{aligned} \tag{5}$$

where $\mathcal{P}_L$ and $\mathcal{P}_R$ are the projectors:

$$\mathcal{P}_L(\zeta) = \frac{1 + \gamma \cdot \hat{\mathbf{n}}(\zeta)}{2}, \qquad \mathcal{P}_R(\zeta) = \frac{1 - \gamma \cdot \hat{\mathbf{n}}(\zeta)}{2}, \tag{6}$$

where the dot denotes the scalar product between (spatial) 2-component vectors: $\mathbf{a} \cdot \mathbf{b} \equiv a_i b_i = a_1 b_1 + a_2 b_2$. The conditions (5) ensure the vanishing, at all the points of $\mathcal{C}$, of j_n, the normal component of the induced fermion current:

$$j_n(x_0, \mathbf{r}(\zeta)) \equiv ie\langle \bar{\psi}(x_0, \mathbf{r}(\zeta))\gamma \cdot \hat{\mathbf{n}}(\zeta)\psi(x_0, \mathbf{r}(\zeta))\rangle = 0, \quad \forall\ \zeta \in I. \tag{7}$$

Here, the vacuum average $\langle \ldots \rangle$ is defined by:

$$\langle \ldots \rangle \equiv \frac{\int_{\mathcal{U}} \mathcal{D}\psi \mathcal{D}\bar{\psi} \ldots e^{-S_f(\bar{\psi},\psi;A)}}{\int_{\mathcal{U}} \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_f(\bar{\psi},\psi;A)}} \tag{8}$$

where $\int_{\mathcal{U}}$ means that the integration is constrained to verify the proper boundary conditions, we shall see how to implement them by the use of Lagrange multipliers (see below).

B. Functional representation for the effective action

Following the idea of the approach presented in [4], we introduce $\mathcal{Z}(A)$, the partition function, and $\Gamma(A)$, the effective action corresponding to the fluctuating Dirac field subject to the conditions (5), by means of the functional integral

$$\mathcal{Z}(A) = e^{-\Gamma(A)} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\chi_R \mathcal{D}\bar{\chi}_R e^{-S_f(\bar{\psi},\psi,A)} e^{i\int dx_0 \int d\zeta[\bar{\chi}_R(x_0,\zeta)\mathcal{P}_L(\zeta)\psi(x_0,\mathbf{r}(\zeta)) + \bar{\psi}(x_0,\mathbf{r}(\zeta))\mathcal{P}_R(\zeta)\chi_R(x_0,\zeta)]}, \tag{9}$$

where we introduced auxiliary chiral Grasmmann fields χ_R, $\bar{\chi}_R$ to exponentiate the δ functions. They are two-component fields living in $1 + 1$ dimensions, and we find it convenient to use a more symmetric notation for their arguments: $\chi_R = \chi_R(\xi_0, \xi_1)$, $\bar{\chi}_R = \bar{\chi}_R(\xi_0, \xi_1)$, where $\xi_0 \equiv x_0$ and $\xi_1 \equiv \zeta$. These chiral fields may, of course, be thought of as chiral projections of Dirac fiels:

$$\begin{aligned} \chi_R(\xi_0, \xi_1) &= \mathcal{P}_R(\xi_1)\chi(\xi_0, \xi_1) \\ \bar{\chi}_R(\xi_0, \xi_1) &= \bar{\chi}(\xi_0, \xi_1)\mathcal{P}_L(\xi_1). \end{aligned} \tag{10}$$

We note that the auxiliary fields functional integration measure is:

$$\mathcal{D}\chi_R \mathcal{D}\bar{\chi}_R = \prod_{-\infty<\xi_0<\infty} \prod_{\xi_1 \in I} [d\chi_R(\xi_0, \xi_1) d\bar{\chi}_R(\xi_0, \xi_1)]. \tag{11}$$

We see in (9) that the auxiliary fields will have a nontrivial dynamics as a result of the Dirac field fluctuations. Indeed, performing the (Gaussian) integral over the Dirac fields ψ, $\bar{\psi}$:

$$\mathcal{Z}(A) = \det(\not{D} + m) \times \int \mathcal{D}\chi_R \mathcal{D}\bar{\chi}_R e^{-\int d^2\xi \int d^2\xi' \bar{\chi}_R(\xi)\mathcal{K}_{\mathcal{C}}(\xi,\xi')\chi_R(\xi')} \tag{12}$$

where we introduced:

$$\mathcal{K}_{\mathcal{C}}(\xi, \xi') = \mathcal{P}_L(\xi_1)\langle \xi_0, \mathbf{r}(\xi_1)|(\not{D} + m)^{-1}|\xi_0', \mathbf{r}(\xi_1')\rangle \times \mathcal{P}_R(\xi_1'), \tag{13}$$

which is a kernel that induces a nonlocal action for the auxiliary fields. Here, and for the rest of this article, we use a "Dirac bracket" notation in order to simplify and clarify the formulae involving operator kernels.

Note that only one "chirality" of the auxiliary fields is actually coupled, but the decomposition between the would-be "left" and "right" components is point-dependent. This means, in particular, that $\bar{\chi}_R(\xi)\chi_R(\xi')$ does not necessarily vanish when $\xi \neq \xi'$. This fact prevents the introduction of one-component Weyl fermions as auxiliary fields, since their local (point dependent) definitions would render the apparent simplification illusory. We shall however, in some special situations, use Weyl fermions: that will be the case when the normal vector $\hat{n}$ is piecewise constant, like in the calculation of the Casimir effect for parallel "plates" (lines).

The determinant factor on the rhs of (12) agrees with the would-be $\mathcal{Z}(A)$ when the borders are sent to infinity (i.e., when there are no borders). Since we are interested precisely in the effects due to the presence of borders, we shall factor out that contribution, considering instead:

$$\begin{aligned} \mathcal{Z}_{\mathcal{C}}(A) &\equiv \frac{\mathcal{Z}(A)}{\det(\not{D} + m)} \equiv e^{-\Gamma_{\mathcal{C}}(A)} \\ &= \int \mathcal{D}\chi_R \mathcal{D}\bar{\chi}_R e^{-\int d^2\xi \int d^2\xi' \bar{\chi}_R(\xi)\mathcal{K}_{\mathcal{C}}(\xi,\xi')\chi_R(\xi')} \\ &= \det \mathcal{K}_{\mathcal{C}}. \end{aligned} \tag{14}$$

Thus, the effective action corresponding to this functional is given by

$$\Gamma_{\mathcal{C}}(A) = -\mathrm{Tr}\ln \mathcal{K}_{\mathcal{C}}. \tag{15}$$

At this point, it is useful to disentangle from $\Gamma_{\mathcal{C}}(A)$ the purely Casimir energy contribution from the part due to the external field:

$$\Gamma_{\mathcal{C}}(A) = \Gamma_{\mathcal{C}}(0) + \tilde{\Gamma}_{\mathcal{C}}(A) \tag{16}$$

where $\Gamma_{\mathcal{C}}(0)$ is proportional to the Casimir energy density $\mathcal{E}$, while $\tilde{\Gamma}_{\mathcal{C}}(A)$, which vanishes when $A = 0$, is a measure of the effect of the borders on the response of the system to the external field.

We shall use a $\mathcal{Z}$ functional corresponding to each of these terms; they will be denoted by $\mathcal{Z}_{\mathcal{C}}(0)$ and $\tilde{\mathcal{Z}}_{\mathcal{C}}(A)$ (in an obvious notation).

C. Gauge invariance of $\Gamma(A)$

Being a functional of A, the study of gauge invariance for $\Gamma(A)$, reduces to an analysis of its behavior under gauge transformations for the gauge field, namely:

$$\delta_\omega \Gamma(A) = \Gamma(A + \partial\omega) - \Gamma(A) \tag{17}$$

where ω is a smooth function of (all of) the spacetime coordinates. In order to understand the effect of those transformations, it is convenient to recall representation (9), in order to see that:

$$e^{-\Gamma(A+\partial\omega)} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\chi_R\mathcal{D}\bar{\chi}_R e^{-S_f(\bar{\psi},\psi,A+\partial\omega)} \times \exp\Big\{i\int d^2\xi[\bar{\chi}(\xi)\mathcal{P}_L(\xi_1)\psi(\xi_0,\mathbf{r}(\xi_1)) + \bar{\psi}(\xi_0,\mathbf{r}(\xi_1))\mathcal{P}_R(\xi_1)\chi(\xi)]\Big\}. \tag{18}$$

We then compensate the change in S_f due to the transformation of A, by means of a gauge transformation in the Dirac fields:

$$\psi(x)\to e^{-ie\omega(x)}\psi(x), \qquad \bar{\psi}(x)\to\bar{\psi}(x)e^{ie\omega(x)}, \tag{19}$$

which is, of course, non anomalous. The only source of noninvariance under the transformations we have just performed is in the coupling to the Lagrange multiplier fields, which is concentrated on the boundary $\mathcal{C}$:

$$e^{-\Gamma(A+\partial\omega)} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\chi_R\mathcal{D}\bar{\chi}_R e^{-S_f(\bar{\psi},\psi,A)} \times \exp\Big\{i\int d^2\xi[\bar{\chi}_R(\xi)\mathcal{P}_L(\xi_1) \times e^{-ie\omega(\xi_0,\mathbf{r}(\xi_1))}\psi(\xi_0,\mathbf{r}(\xi_1)) + \bar{\psi}(\xi_0,\mathbf{r}(\xi_1))e^{ie\omega(\xi_0,\mathbf{r}(\xi_1))}\mathcal{P}_R(\xi_1)\chi_R(\xi)]\Big\}. \tag{20}$$

At this point, we realize that all the dependence in ω can be erased by transforming the Lagrange multipliers:

$$\chi_R(\xi)\to e^{-ie\omega(\xi_0,\mathbf{r}(\xi_1))}\chi_R(\xi), \quad \bar{\chi}_R(\xi)\to\bar{\chi}_R(\xi)e^{ie\omega(\xi_0,\mathbf{r}(\xi_1))}. \tag{21}$$

Since they are chiral fields, there arises a nontrivial Jacobian $\mathcal{J}(\omega, A)$ from their integration measure:

$$\mathcal{D}\chi_R\mathcal{D}\bar{\chi}_R\to\mathcal{D}\chi_R\mathcal{D}\bar{\chi}_R\mathcal{J}(\omega,A). \tag{22}$$

To the first order in ω

$$\mathcal{J}(\omega,A)\simeq\exp\Big[ie\int d^2\xi\,\omega(\xi_0,\mathbf{r}(\xi_1))\mathcal{F}(A;\xi_0,\xi_1)\Big] \tag{23}$$

where $\mathcal{F}(A;\xi_0,\xi_1)$ is the anomaly a functional of A and a function of the parameters of the worldsheet corresponding to the border. We have assumed that $\xi_1\equiv u_1$, so that the $\mathcal{C}$ coincides with $u_2=0$.

From (20), (22), and (23) we conclude that:

$$\partial_\mu\Big[\frac{\delta\Gamma(A)}{\delta A_\mu(x)}\Big] = ie\int d\xi_1\delta(\mathbf{x}-\mathbf{r}(\xi_1))\mathcal{F}(A;x_0,\xi_1), \tag{24}$$

which shows explicitly the fact that the gauge noninvariance will be concentrated on the boundary, although the actual form of the anomaly will, in principle, depend on the field A also at points slightly away from the boundary.

We see that (24) is relevant to the physical problem of imposing baglike boundary conditions. Indeed, we easily see that (24) implies:

$$\partial_\alpha j_\alpha(x) = -ie\int d\xi_1\delta(\mathbf{x}-\mathbf{r}(\xi_1))\mathcal{F}(A;x_0,\xi_1), \tag{25}$$

where $j_\alpha(x)$ is the induced vacuum current:

$$j_\alpha(x)\equiv ie\langle\bar{\psi}(x)\gamma_\alpha\psi(x)\rangle. \tag{26}$$

Integrating the anomalous divergence Eq. (25) on the world-volume generated by the (fixed) region $\mathcal{U}$ during a time interval $[0, T]$, we see that Gauss' theorem yields:

$$\int_{\mathcal{U}} d\mathbf{x}\,j_0(0,\mathbf{x}) - \int_{\mathcal{U}} d\mathbf{x}\,j_0(T,\mathbf{x}) = \int_{\mathcal{C}\times[0,T]} d^2\xi\,j_n(x_0,\mathbf{r}(\zeta)) + ie\int d^2\xi\,\mathcal{F}(A;\xi_0,\xi_1). \tag{27}$$

Then the existence of the anomaly implies that, under some circumstances, the bag condition will be violated. Indeed, assuming, for example, that the total charge of the 2 + 1 dimensional system is constant (insulated system), then the lhs of the previous equation vanishes, and we get a relation involving the integral of the anomaly and the flux of the current. If the former is not zero, the latter is necessarily different from zero. The explicit form for the anomaly is, in these coordinates ($\xi_1\equiv u_1$, $\xi_2\equiv u_2$):

$$\mathcal{F}(A;\xi_0,\xi_1) = -\frac{e}{2\pi}\varepsilon_{\mu\nu}\partial_\mu\tilde{A}_\nu(\xi), \tag{28}$$

where $\tilde{A}_\mu = A_\mu(\xi_0,\xi_1,0)$. Thus the nonvanishing of the anomalous contribution depends only on the circulation of $\tilde{A}_1$ (which is the tangential component of A on $\mathcal{C}$) at the times T and 0. This may also be put in terms of the *magnetic* flux through $\mathcal{U}$ at those times. Then:

$$\int_{\mathcal{C}\times[0,T]} d^2\xi\,j_n(x_0,\mathbf{r}(\zeta)) = \frac{e^2}{2\pi}\Big[\int d\mathbf{x}\,\varepsilon_{ij}\partial_iA_j(\mathbf{x},T) - \int d\mathbf{x}\,\varepsilon_{ij}\partial_iA_j(\mathbf{x},0)\Big]. \tag{29}$$

This anomalous current flux is of course just another manifestation of the fact that the effective theory shall contain a Chern-Simons like term, which introduces a gauge noninvariance on the boundary. Indeed, that is the usual set-up for the study of this phenomenon, which is dealt with in the context of the effective theory for the bulk, and the dynamics for the boundary modes is obtained therefrom [7].

Of course, the gauge noninvariance could be cured by adjusting the matter content, or by imposing conditions on the external gauge field, like the invariance of the total magnetic flux through $\mathcal{U}$.

D. Fermion propagator

Let us derive now an expression for the fermion propagator by using this representation. A simple way to do that is to introduce a generating functional containing linear couplings to two auxiliary Grassmann sources, denoted by $\bar{\eta}$ and η:

$$\mathcal{Z}(\bar{\eta},\eta)=\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\chi\mathcal{D}\bar{\chi}e^{-S_f(\bar{\psi},\psi,0)+\int d^3x(\bar{\eta}\psi+\bar{\psi}\eta)}e^{i\int dx_0\int d\xi[\bar{\chi}(x_0,\xi)\mathcal{P}_L(\xi)\psi(x_0,\mathbf{r}(\xi))+\bar{\psi}(x_0,\mathbf{r}(\xi))\mathcal{P}_R(\xi)\chi(x_0,\xi)]}, \tag{30}$$

whereby the fermion propagator $\langle\psi(x)\bar{\psi}(y)\rangle$ can be obtained as follows:

$$\langle\psi(x)\bar{\psi}(y)\rangle=\frac{1}{\mathcal{Z}(0,0)}\bigg[\frac{\delta^2}{\delta\eta(y)\delta\bar{\eta}(x)}\mathcal{Z}(\bar{\eta},\eta)\bigg]\bigg|_{\eta=0,\bar{\eta}=0}. \tag{31}$$

Performing the Gaussian integrations, and evaluating the derivatives, we obtain for the free fermion propagator the following expression:

$$\langle\psi(x)\bar{\psi}(y)\rangle=\langle x|(\partial+m)^{-1}|y\rangle-\int d^2\xi'\int d^2\xi''\langle x|(\not{\partial}+m)^{-1}|\xi'_0,\mathbf{r}(\xi'_1)\rangle\mathcal{P}_R(\xi'_1)\mathcal{K}_C^{-1}(\xi',\xi'')\mathcal{P}_L(\xi''_1)\langle\xi''_0,\mathbf{r}(\xi''_1)|(\not{\partial}+m)^{-1}|y\rangle. \tag{32}$$

It is evident, from the previous expression, that the propagator so obtained does verify the proper boundary conditions. Indeed,

$$\begin{aligned}\mathcal{P}_L(\xi_1)\langle\psi(\xi_0,\mathbf{r}(\xi_1))\bar{\psi}(y)\rangle &=\mathcal{P}_L(\xi_1)\langle\xi_0,\mathbf{r}(\xi_1)|(\not{\partial}+m)^{-1}|y\rangle-\int d^2\xi'\int d^2\xi''\mathcal{P}_L(\xi_1)\langle\xi_0,\mathbf{r}(\xi_1)|(\not{\partial}+m)^{-1}|\xi'_0,\mathbf{r}(\xi'_1)\rangle\\ &\quad\times\mathcal{P}_R(\xi'_1)\mathcal{K}_C^{-1}(\xi',\xi'')\mathcal{P}_L(\xi''_1)\langle\xi''_0,\mathbf{r}(\xi''_1)|(\not{\partial}+m)^{-1}|y\rangle=\mathcal{P}_L(\xi_1)\langle\xi_0,\mathbf{r}(\xi_1)|(\not{\partial}+m)^{-1}|y\rangle\\ &\quad-\int d^2\xi'\int d^2\xi''\mathcal{K}_C(\xi,\xi')\mathcal{K}_C^{-1}(\xi',\xi'')\mathcal{P}_L(\xi''_1)\langle\xi''_0,\mathbf{r}(\xi''_1)|(\not{\partial}+m)^{-1}|y\rangle\\ &=\mathcal{P}_L(\xi_1)\langle\xi_0,\mathbf{r}(\xi_1)|(\not{\partial}+m)^{-1}|y\rangle-\int d^2\xi''\delta(\xi_0-\xi''_0)\delta(\xi_1-\xi''_1)\\ &\quad\times\mathcal{P}_L(\xi''_1)\langle\xi''_0,\mathbf{r}(\xi''_1)|(\not{\partial}+m)^{-1}|y\rangle=0.\end{aligned} \tag{33}$$

In Sec. V, we will find an explicit expression for the free fermion propagator in the presence of a linear wall.

III. CASIMIR ENERGY

Let us consider here the Casimir term $\Gamma_C(0)$, for the physically interesting case of $m=0$, evaluating it explicitly for a particular geometry.

We first write this object more explicitly, in terms of the corresponding functional integral over auxiliary fields:

$$\mathcal{Z}_C(0)=e^{-\Gamma_C(0)}=\int\mathcal{D}\chi\mathcal{D}\bar{\chi}e^{-\int d^2\xi\int d^2\xi'\bar{\chi}_R(\xi)\mathcal{P}_L(\xi_1)\langle\xi_0,\mathbf{r}(\xi_1)|\not{\partial}^{-1}|\xi_0,\mathbf{r}(\xi'_1)\rangle\mathcal{P}_R(\xi'_1)\chi_R(\xi')}. \tag{34}$$

The simplest nontrivial geometry is the one corresponding to the region: $\mathcal{U}=\{(x_1,x_2):0\leq x_2\leq l\}$, so that $\mathcal{C}$ is just the union of two lines: $\mathcal{C}_0$, corresponding to $x_2=0$ and $\mathcal{C}_l$, to $x_2=l$. Of course, in this case, the normal vectors shall be $-\hat{\mathbf{x}}_2$ and $\hat{\mathbf{x}}_2$, respectively. In order to parametrize the auxiliary fields, we find it convenient to use $x_1\in(-\infty,+\infty)$ as the (common) parameter, but using a label to distinguish the fields corresponding to the lower ($\chi^{(0)}(x_0,x_1)$), and upper ($\chi^{(l)}(x_0,x_1)$) borders.

Then,

$$e^{-\Gamma_C(0)}=\int\mathcal{D}\chi^{(0)}\mathcal{D}\bar{\chi}^{(0)}\mathcal{D}\chi^{(l)}\mathcal{D}\bar{\chi}^{(l)}e^{-S_C(\chi^{(0)},\bar{\chi}^{(0)};\chi^{(l)},\bar{\chi}^{(l)})} \tag{35}$$

where the "action" S_C is defined by:

$$S_{\mathcal{C}} = \int d^2x \int d^2x' [\bar{\chi}^{(0)}(x)\mathcal{P}_- \langle x_0, x_1, 0|\not\partial^{-1}|x_0', x_1', 0\rangle \mathcal{P}_+ \chi^{(0)}(x') + \bar{\chi}^{(l)}(x)\mathcal{P}_+ \langle x_0, x_1, l|\not\partial^{-1}|x_0', x_1', l\rangle \mathcal{P}_- \chi^{(l)}(x') + \bar{\chi}^{(0)}(x)\mathcal{P}_- \langle x_0, x_1, 0|\not\partial^{-1}|x_0', x_1', l\rangle \mathcal{P}_- \chi^{(l)}(x') + \bar{\chi}^{(l)}(x)\mathcal{P}_+ \langle x_0, x_1, l|\not\partial^{-1}|x_0', x_1', 0\rangle \mathcal{P}_+ \chi^{(0)}(x')] \tag{36}$$

where $\mathcal{P}_\pm = \frac{1\pm\gamma_2}{2}$. It should be clear now that, since these projectors are constant, the auxiliary fields $\chi^{(0,l)}$, when multiplied by those projectors, are trivial functions of (different) one-component Weyl fermions. Namely,

$$\mathcal{P}_+ \chi^{(0)}(x) = \begin{pmatrix} \eta^{(0)}(x) \\ 0 \end{pmatrix}, \qquad \mathcal{P}_- \chi^{(l)}(x) = \begin{pmatrix} 0 \\ \eta^{(l)}(x) \end{pmatrix}, \tag{37}$$

and

$$\bar{\chi}^{(0)}(x)\mathcal{P}_- = (0, \bar{\eta}^{(0)}(x)), \qquad \bar{\chi}^{(l)}(x)\mathcal{P}_+ = (\bar{\eta}^{(l)}(x), 0), \tag{38}$$

where $\eta^{(0)}$, $\eta^{(l)}$, and their adjoints, are one-component Weyl fields.

We may combine them into a two-component field χ:

$$\chi(x) \equiv \begin{pmatrix} \eta^{(0)}(x) \\ \eta^{(l)}(x) \end{pmatrix}, \qquad \bar{\chi}(x) \equiv (\bar{\eta}^{(0)}(x), \bar{\eta}^{(l)}(x)), \tag{39}$$

and write the action $S_{\mathcal{C}}$ as:

$$S_{\mathcal{C}} = \int d^2x \int d^2x' \bar{\chi}(x)\mathcal{D}(x, x')\chi(x'), \tag{40}$$

where

$$\mathcal{D}(x, x') = \begin{pmatrix} \langle x_0, x_1, 0|\frac{\partial^+}{\partial^2}|x_0', x_1', 0\rangle & \langle x_0, x_1, 0| -\frac{\partial_2}{\partial^2}|x_0', x_1', l\rangle \\ \langle x_0, x_1, l|\frac{\partial_2}{\partial^2}|x_0', x_1', 0\rangle & \langle x_0, x_1, l|\frac{\partial^-}{\partial^2}|x_0', x_1', l\rangle \end{pmatrix}, \tag{41}$$

where $\partial^+ \equiv \partial_0 + i\partial_1$ and $\partial^- \equiv \partial_0 - i\partial_1$. Then we have,

$$\Gamma_{\mathcal{C}}(0) = -\mathrm{Tr}\ln\mathcal{D} = -\frac{1}{2}\mathrm{Tr}\ln(\mathcal{D}^\dagger\mathcal{D}), \tag{42}$$

which is best evaluated by introducing a Fourier transformation with respect to the coordinates x_0 and x_1. We see that:

$$\tilde{\mathcal{D}}(k) = \begin{pmatrix} -i(k_0 + ik_1)/2k & e^{-lk}/2 \\ e^{-lk}/2 & -i(k_0 - ik_1)/2k \end{pmatrix}. \tag{43}$$

Then:

$$\Gamma_{\mathcal{C}}(0) = -\frac{1}{2}LT\int\frac{d^2k}{(2\pi)^2}\ln\left[\frac{1}{2}(1 + e^{-2lk})\right], \tag{44}$$

where L is the length of the plates, and T the extension of the (Euclidean) time interval. In this expression, there is a (divergent) l-independent contribution which we attribute to the self-energy or each plate, plus a Casimir energy (energy per unit length):

$$\mathcal{E} = -\frac{1}{2}\int\frac{d^2k}{(2\pi)^2}\ln(1 + e^{-2lk}), \tag{45}$$

which can be easily integrated:

$$\mathcal{E} = -\frac{3\zeta(3)}{64\pi l^2}. \tag{46}$$

An interesting feature of this result is that the Casimir energy is already given by an *integral* over the momenta which are parallel to the plates. Thus, the series over the discrete momenta along the normal direction to the plates has already been summed up.

Of course, both approaches are related, as can be easily seen by first noting that the eigenvalues of $\mathcal{D}^\dagger\mathcal{D}$ are identical to the squares of the eigenvalues of a Dirac *Hamiltonian* in 3 + 1 dimensions (with one of the spatial coordinates playing the role of the time). Those eigenvalues are known to be [8]:

$$\lambda_{n,k} = \sqrt{\omega_n^2 + k^2}, \tag{47}$$

where $\omega_n = \frac{(2n+1)\pi}{2l}$. Then we see that:

$$\ln(1 + e^{-2lk}) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\ln[(2l)^2(\omega_n^2 + k^2)], \tag{48}$$

where we have neglected l-independent terms. The sum on the rhs of (48) arises naturally when one evaluates the Casimir energy by finding the eigenvalues of the Dirac operator for the modes constrained to satisfy the bag boundary conditions.

One can also obtain an expression similar to (45) for the Casimir energy starting from its usual definition as the sum of the zero-point energies of the field modes, and using Cauchy's theorem to write the sum as a contour integral in the complex plane [9].

IV. THE LARGE-MASS LIMIT

We shall approach this limit by following two different strategies: first, we shall begin with a quantized Dirac field, implementing the approximations and simplifications that follow from the assumption that the fermionic mass is much larger than the other relevant dimensionful objects; i.e., the gauge field derivatives. Our second approach amounts to start from the effective "bosonized" theory that follows by taking the large-mass limit beforehand, and introducing the boundary condition afterwards.

A. Fermionic representation

In the large-mass limit, we can obtain some explicit results as a consequence of the fact that the kernel $\mathcal{K}_\mathcal{C}$ becomes *local*. We begin by noting that $\tilde{\mathcal{Z}}_\mathcal{C}(A)$ may be regarded as a regularized version (with the fermion mass m playing the role of an UV cutoff) of the determinant of a *local* operator. Indeed, taking into account the fact that $\mathcal{P}_L$ and $\mathcal{P}_R$ are orthogonal projectors at every point of $\mathcal{C}$, we may rewrite $\mathcal{K}_\mathcal{C}$ as:

$$\mathcal{K}_\mathcal{C}(\xi,\xi') = \mathcal{P}_L(\xi_1)\langle \xi_0, \mathbf{r}(\xi_1) \left| \frac{-\not\partial}{-\not\partial^2 + m^2} \right| \xi'_0, \mathbf{r}(\xi'_1)\rangle \times \mathcal{P}_R(\xi'_1) \quad (49)$$

or:

$$\mathcal{K}_\mathcal{C}(\xi,\xi') = \frac{1}{m^2}\mathcal{P}_L(\xi_1)\langle \xi_0, \mathbf{r}(\xi_1)| f\left(\frac{-\not\partial^2}{m^2}\right) \times (-\not\partial)|\xi'_0, \mathbf{r}(\xi'_1)\rangle \mathcal{P}_R(\xi'_1) \quad (50)$$

where $f(x) \equiv \frac{1}{1+x}$. Since $f(0) = 1$, and f and all its derivatives tend to zero when $x \to \infty$, it is evident that $\tilde{\mathcal{Z}}_\mathcal{C}(A)$ is a regularized version of another functional, which we denote by $\mathcal{Z}_{\rm loc}(A)$, defined as the result of taking the $f \to 1$ limit in $\tilde{\mathcal{Z}}_\mathcal{C}(A)$:

$$\mathcal{Z}_{\rm loc}(A) = [\tilde{\mathcal{Z}}_\mathcal{C}(A)]_{m\to\infty} = \int \mathcal{D}\chi\mathcal{D}\bar{\chi} e^{-S_{\rm loc}(\bar{\chi},\chi,A)}, \quad (51)$$

where

$$S_{\rm loc} = \int d^2\xi \bar{\chi}(\xi)\mathcal{K}_{\rm loc}(\xi,\xi')\chi(\xi), \quad (52)$$

and:

$$\mathcal{K}_{\rm loc}(\xi,\xi') = -\mathcal{P}_L(\xi_1)\langle \xi_0, \mathbf{r}(\xi_1)|\not\partial|\xi'_0, \mathbf{r}(\xi'_1)\rangle \mathcal{P}_R(\xi'_1). \quad (53)$$

$S_{\rm loc}$ is a local action, and we have neglected an infinite (A-independent) factor $\det(m^{-2})$. It is important at this point to remark that, since the auxiliary fields behave as $1+1$ dimensional Dirac fermions with a minimal gauge coupling, no infinity arises when removing the regulator ($m \to \infty$). Of course, this will not necessarily be the case in higher dimensions. Besides, the regulator only affects the real part of the effective action (namely, the modulus of the fermionic determinant). The imaginary part is of course still there, and requires its own regularization. Note, however, that the imaginary part is also determined by the local action, since the "regulator" affects only the modulus of the eigenvalues of the Dirac operator, and those are gauge invariant.

It should be obvious that, to make further progress, it is convenient to write (53) more explicitly, in terms of coordinates which are more adapted to the geometry of $\mathcal{C}$. To that end, we invoke the coordinates u_1 and u_2, introduced in the previous section, recalling that u_1 and ξ_1 actually coincide on $\mathcal{C}$, to see that the local action may be written as follows:

$$S_{\rm loc} = \int d^2\xi \bar{\chi}(\xi_0,\xi_1)\mathcal{P}_L(\xi_1)(\tilde{\gamma}_\mu d_\mu)\mathcal{P}_R(\xi_1)\chi(\xi_0,\xi_1) \quad (54)$$

where:

$$\mathcal{P}_L(\xi_1) = \frac{1+\tilde{\gamma}_2(\xi_1)}{2}, \qquad \mathcal{P}_R(\xi_1) = \frac{1-\tilde{\gamma}_2(\xi_1)}{2}$$
$$d_\mu \equiv \partial_\mu + ie\tilde{A}_\mu(\xi_0,\xi_1) \quad (55)$$
$$\tilde{A}_\mu(\xi_0,\xi_1) \equiv A_\mu(\xi_0, r_1(\xi_1), r_2(\xi_1))$$

and

$$\tilde{\gamma}_0 = \gamma_0, \qquad \tilde{\gamma}_1(\xi_1) = \gamma\cdot\hat{\mathbf{t}}(\xi_1), \qquad \tilde{\gamma}_2(\xi_1) = \gamma\cdot\hat{\mathbf{n}}(\xi_1). \quad (56)$$

Note that there is no coupling to the component of A that is normal to the curve.

Then, in the infinite mass limit, the effective action due to the presence of the boundary reduces to the one of a chiral fermion determinant:

$$\mathcal{Z}_{\rm loc}(A) = e^{-\Gamma_{\rm loc}(A)}, \qquad \Gamma_{\rm loc}(A) = -{\rm Tr}\ln[\tilde{\gamma}_\mu d_\mu \mathcal{P}_R]. \quad (57)$$

By our comment above on the imaginary part, it is clear that:

$$\text{Im}\,\Gamma_\mathcal{C}(A) = \text{Im}\Gamma_{\rm loc}(A), \quad (58)$$

where, of course, a regularization procedure has to be invoked (as it has to be also in a local theory).

B. Bosonic representation

To describe a Dirac field coupled to an external gauge field A_α we may, in the limit when the fermion mass is large (in comparison with the momenta of the external fields) use an approximate bosonization procedure [10–13]. The fermion $\leftrightarrow$ boson mapping leads to a bosonic action, $S^{(b)}$, whose leading form in a large-mass expansion is given by:

$$S^{(b)}(a,A) = S_{\rm CS}(a) + i\int d^3x \varepsilon_{\alpha\beta\gamma}\partial_\beta a_\gamma A_\alpha, \quad (59)$$

where a_α is a new gauge field, introduced to implement the duality, and $S_{\rm CS}$ is the Chern-Simons action:

$$S_{\rm CS}(a) = i\frac{\kappa}{2}\int d^3x \varepsilon_{\alpha\beta\gamma} a_\alpha \partial_\beta a_\gamma, \tag{60}$$

where κ is a constant.

The second term in (59) corresponds to the standard coupling between current and external gauge field, since a_α is related to the average value of the fermionic current, j_α, by:

$$j_\alpha = i\varepsilon_{\alpha\beta\gamma}\partial_\beta a_\gamma, \tag{61}$$

a relation which is exact, i.e, independent of the approximation used to obtain the bosonized action.

The bosonized partition function *in the absence of boundaries*, $\mathcal{Z}^{(b)}(A)$, can be defined as follows:

$$\mathcal{Z}^{(b)} = \int \mathcal{D}a e^{-S^{(b)}(a,A)}. \tag{62}$$

To take into account the boundary conditions corresponding to the fermionic theory in this setting, we note that the mapping between the fermionic and bosonic representations for the current implies that we should impose:

$$(\partial_0 a_l - \partial_l a_0)t_l = 0 \quad \text{on } \mathcal{C} \tag{63}$$

i.e., the wall must behave like a "perfect conductor" for the a_α gauge field, since there is no tangential component for its electric field on the border. Note that this boundary condition is independent of the large-mass expansion, since it only relies upon the exact mapping (61) between j_α and a_α.

Introducing now a new field $\varphi(\xi_0, \xi_1)$, a Lagrange multiplier field for the previous condition, we are lead to $\tilde{\mathcal{Z}}^{(b)}_{\mathcal{C}}$, the bosonized form of the partition function for the contribution due to the modes localized on the borders:

$$\tilde{\mathcal{Z}}^{(b)}_{\mathcal{C}}(A) = e^{-\tilde{\Gamma}^{(b)}_{\mathcal{C}}(A)} = \frac{\mathcal{Z}^{(b)}_{\mathcal{C}}(A)}{\mathcal{Z}^{(b)}_{\mathcal{C}}(0)} \tag{64}$$

where now

$$\mathcal{Z}^{(b)}_{\mathcal{C}}(A) = \frac{1}{\mathcal{Z}^{(b)}(A)}\int \mathcal{D}a\mathcal{D}\varphi e^{-S_{\rm CS}(a) - i\int d^3x\varepsilon_{\alpha\beta\gamma}A_\alpha\partial_\beta a_\gamma} \times e^{i\int d^2\xi\varphi(\xi_0,\xi_1)f_{0l}(\xi_0,\mathbf{r}(\xi_1))t_l(\xi_1)} \tag{65}$$

where $f_{\alpha\beta} \equiv \partial_\alpha a_\beta - \partial_\beta a_\alpha$.

When evaluating the Gaussian integral, an important point arises as a consequence of the existence of a boundary term coming from an integration by parts. Indeed, to perform the Gaussian integral we need to rewrite the term that couples the bosonized current to the external field A_α. After performing an integration by parts and applying Gauss' theorem, we see that:

$$\int d^3x\varepsilon_{\alpha\beta\gamma}\partial_\beta a_\gamma A_\alpha = \int d^3x\varepsilon_{\alpha\beta\gamma}\partial_\beta A_\gamma a_\alpha + \int d^3x a_\alpha(x)R_\alpha(x) \tag{66}$$

where

$$\begin{aligned} R_0(x) &= -\int d\xi_1\delta(\mathbf{x} - \mathbf{r}(\xi_1))A_l(x)t_l(\xi_1) \\ R_k(x) &= \int d\xi_1\delta(\mathbf{x} - \mathbf{r}(\xi_1))A_0(x)t_k(\xi_1). \end{aligned} \tag{67}$$

To keep this boundary term amounts to reproducing the proper result, in particular, for the anomalous behavior of the effective action under gauge transformations.

The Gaussian integral over a_α can now be performed, what yields an action $S_{\mathcal{C}}$ for the Lagrange multiplier field:

$$\mathcal{Z}^{(b)}_{\mathcal{C}}(A) = \int \mathcal{D}\varphi e^{-S_{\mathcal{C}}(\varphi,A)}, \tag{68}$$

where a "bulk" Chern-Simons term has been cancelled out, and:

$$\begin{aligned} S_{\mathcal{C}}(\varphi, A) = \frac{1}{2}\int d^2\xi\int d^2\xi'(\partial_0\varphi(\xi) - A_0(\xi_0,\mathbf{r}(\xi_1))t_j(\xi_1)M_{jk}(\xi,\xi')t_k(\xi'_1)(\partial'_0\varphi(\xi') - A_0(\xi'_0,\mathbf{r}(\xi'_1)) \\ + \int d^2\xi\int d^2\xi'\varphi(\xi)t_l(\xi_1)\partial_l M_j(\xi,\xi')t_j(\xi'_1)(\partial'_0\varphi(\xi') - A_0(\xi'_0,\mathbf{r}(\xi'_1)) \\ + \int d^2\xi\int d^2\xi' t_l(\xi_1)A_l(\xi_0,\mathbf{r}(\xi_1))M_j(\xi,\xi')t_j(\xi'_1)(\partial'_0\varphi(\xi') - A_0(\xi'_0,\mathbf{r}(\xi'_1)) \\ - \frac{i}{\kappa}\int d^2\xi\varphi(\xi)t_l(\xi_1)F_{0l}(\xi_0,\mathbf{r}(\xi_1)) \end{aligned} \tag{69}$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, and

$$\begin{aligned} M_{jk}(\xi,\xi') &= -\frac{i}{\kappa}\varepsilon_{jk}\left\langle \xi_0,\mathbf{r}(\xi_1)\left|\frac{1}{-\partial^2}\right|\xi'_0,\mathbf{r}(\xi'_1)\right\rangle \\ M_j(\xi,\xi') &= \frac{i}{\kappa}\varepsilon_{jk}\left\langle \xi_0,\mathbf{r}(\xi_1)\left|\frac{\partial_k}{-\partial^2}\right|\xi'_0,\mathbf{r}(\xi'_1)\right\rangle. \end{aligned} \tag{70}$$

It is straightforward to see that (69) has the same transformation properties as its fermionic equivalent. Indeed, all the terms in $S_{\mathcal{C}}$ except for the last one are invariant under gauge transformations restricted to the border:

$$A_\alpha(\xi_0,\mathbf{r}(\xi_1)) \to A_\alpha(\xi_0,\mathbf{r}(\xi_1)) + \partial_\alpha\omega(\xi_0,\mathbf{r}(\xi_1)) \tag{71}$$

if the scalar field is also transformed:

$$\varphi(\xi_0, \xi_1) \to \varphi(\xi_0, \xi_1) + \omega(\xi_0, \mathbf{r}(\xi_1)). \tag{72}$$

It is clear that the last term in $S_{\mathcal{C}}$ does reproduce the chiral anomaly, since under the previous gauge transformation:

$$\delta_\omega S_{\mathcal{C}}(\varphi, A) = -\frac{i}{\kappa} \int d^2\xi \omega(\xi_0, \mathbf{r}(\xi_1)) t_l(\xi_1) F_{0l}(\xi_0, \mathbf{r}(\xi_1)), \tag{73}$$

which is, of course, consistent with the result obtained from the fermionic representation. The results agree when $\kappa = \frac{4\pi}{e^2}$, which is the proper value for the bosonized theory with "minimal" regularization.

V. LINEAR WALL

In this section we calculate the free fermion propagator and the effective action for the particularly simple case of a linear boundary, which we assume to be at $x_2 = 0$, with $\mathcal{U} = \{(x_1, x_2): x_2 \geq 0\}$.

Let us first consider the free fermion propagator. Using x_0 and x_1 as coordinates we write the free kernel $\mathcal{K}^{(0)}_{\text{linear}}$ as

$$\mathcal{K}^{(0)}_{\text{linear}}(x_0, x_1; x'_0, x'_1) = \left\langle x_0, x_1, 0 \left| \frac{-\gamma_\mu \partial_\mu}{-\partial_\mu\partial_\mu + m^2} \mathcal{P}_R \right| x'_0, x'_1, 0 \right\rangle = \left\langle x_0, x_1 \left| \frac{-\gamma_\nu \partial_\nu}{2\sqrt{-\partial_\mu\partial_\mu + m^2}} \mathcal{P}_R \right| x'_0, x'_1 \right\rangle, \tag{74}$$

where $\mathcal{P}_R = \frac{1+\gamma_2}{2}$. Inserting this expression into (32), after some algebra we obtain:

$$\langle \psi(x)\bar{\psi}(y) \rangle = \langle x|(\not{\partial} + m)^{-1}|y\rangle - 2\int d^2x' \int d^2x'' \langle x|(\not{\partial} + m)^{-1}|x'_0, x'_1, 0\rangle \mathcal{P}_+ \langle x'_0, x'_1|V|x''_0, x''_1\rangle \mathcal{P}_- \langle x''_0, x''_1, 0|(\not{\partial} + m)^{-1}|y\rangle. \tag{75}$$

where $\mathcal{P}_\pm \equiv \frac{1\pm\gamma_2}{2}$, and

$$V = \frac{\gamma_\mu \partial_\mu + m}{\sqrt{-\partial_\nu\partial_\nu + m^2}}. \tag{76}$$

Now we consider the evaluation of the effective action $\tilde{\Gamma}_f$, which is given by

$$\tilde{\Gamma}_f = -\mathrm{Tr}\ln \mathcal{K}_{\text{linear}} \tag{77}$$

with

$$\mathcal{K}_{\text{linear}}(x_0, x_1; x'_0, x'_1) = \left\langle x_0, x_1, 0 \left| \frac{-\gamma_\mu D_\mu}{-(\not{D})^2 + m^2} \mathcal{P}_R \right| x'_0, x'_1, 0 \right\rangle. \tag{78}$$

Note that, in the previous expression, the Dirac operator in the denominator will in general depend on A_2, which does not commute with ∂_2. Thus, in general, no simpler expression may be written for (78) unless some simplifying assumptions are introduced.

Assuming that the A_α's are smooth functions of x_2 in the region around $x_2 = 0$, the leading term in a ∂_2 derivative expansion is

$$\mathcal{K}_{\text{linear}}(x_0, x_1; x'_0, x'_1) \simeq \left\langle x_0, x_1, 0 \left| \frac{-\gamma_\mu D_\mu}{-(\gamma_\nu D_\nu)^2 - \partial_2^2 + e^2 A_2^2(x_0 . x_1, 0) + m^2} \mathcal{P}_R \right| x'_0, x'_1, 0 \right\rangle \tag{79}$$

or,

$$\mathcal{K}_{\text{linear}}(x_0, x_1; x'_0, x'_1) = \left\langle x_0, x_1 \left| \frac{-\gamma_\nu D_\nu}{2\sqrt{-(\gamma_\mu D_\mu)^2 + e^2 A_2^2(x_0 . x_1, 0) + m^2}} \mathcal{P}_R \right| x'_0, x'_1 \right\rangle, \tag{80}$$

which is a sort of dimensional reduction of the original problem, although at the expense of having to deal with a nonlocal theory. This nonlocal kernel may properly be called the effective Dirac operator for the boundary modes, in a *microscopic* representation. It clearly shows the well-known fact that the corresponding Dirac determinant contains gapless excitations (as it should be [7]) and also captures part of the non locality which would have been lost if the $m \to \infty$ had been taken beforehand.

The imaginary part of the effective action, on the other hand, can be borrowed from the known result about the chiral fermion determinant in $1 + 1$ dimensions:

$$\mathrm{Im}\,\tilde{\Gamma}_f = \frac{e^2}{2\pi}\int d^2x \frac{\partial\cdot A}{\partial^2}\epsilon_{\mu\nu}\partial_\mu A_\nu, \tag{81}$$

while for the real part we have:

$$\mathrm{Re}\,\tilde{\Gamma}_f(A) = -\frac{1}{2}\,\mathrm{Tr}\ln\left[\frac{-\gamma_\nu D_\nu}{2(-(\gamma_\mu D_\mu)^2 + e^2 A_2^2(x_0, x_1, 0) + m^2)}\right]. \tag{82}$$

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