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Vacuum accumulation solution to the strong *CP* problem

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We suggest a solution to the strong *CP* problem in which there are no axions involved. The superselection rule of the θ vacua is dynamically lifted in such a way that an infinite number of vacua are accumulated within the phenomenologically acceptable range of $\theta < 10^{-9}$, whereas only a measure-zero set of vacua remains outside of this interval. The general prediction is the existence of membranes to which the standard model gauge fields are coupled. These branes may be light enough to be produced at the particle accelerators in the form of the resonances with a characteristic membrane spectrum.

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I. INTRODUCTION

The strong *CP* problem [1] can be expressed as an inexplicable smallness of the *CP*-violating θ parameter in the QCD Lagrangian,

$$\theta\,\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ and $\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\rho\kappa}G_{\rho\kappa}$ are the QCD gauge field strength and its dual, respectively. A nonzero θ implies a nonzero expectation value of the dual gauge field strength, $\langle \mathrm{Tr}G\tilde{G}\rangle \neq 0$, which would lead to the observable *CP* violation, unless θ is tiny, $\theta < 10^{-9}$ [2].

The strong *CP* problem is the problem of the vacuum superselection. The parameter θ scans a continuum of the vacuum states that have different expectation values of the operator $\mathrm{Tr}G\tilde{G}$. These vacua satisfy the superselection rule, which forbids any transition between the vacua with different values of θ. Because of this superselection rule, there is no *a priori* reason to give any preference to the vacua with small θ (small $\mathrm{Tr}G\tilde{G}$).

In this paper we shall propose a new approach to this problem, in which the θ vacua become rearranged in such a way that a vacuum with an acceptably small θ becomes "infinitely preferred" relative to the other vacua. In our treatment, the superselection rule gets partially lifted, but in such a way that the infinitely many vacua accumulate inside the region $\theta \ll 10^{-9}$, whereas only a measure-zero set of vacua remain outside this interval.

Our solution is based on the following key facts. First, because a nonzero θ implies a nonzero vacuum expectation value $\langle \mathrm{Tr}G\tilde{G}\rangle$, the explanation of the small θ reduces to the explanation of the small value of $\mathrm{Tr}G\tilde{G}$. Note that the latter gauge invariant can be rewritten as a dual four-form field strength of a Chern-Simons three-form in the following way (below everywhere we shall work in units of the QCD scale):

$$\frac{g^2}{8\pi^2}\mathrm{Tr}G\tilde{G} = F \equiv F_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta}, \tag{2}$$

where

$$F_{\alpha\beta\gamma\delta} = \partial_{[\alpha}C_{\beta\gamma\delta]}. \tag{3}$$

$C_{\alpha\beta\gamma}$ is a Chern-Simons three-form, which in terms of the gluon fields can be written as

$$C_{\alpha\beta\gamma} = \frac{g^2}{8\pi^2}\mathrm{Tr}\Big(A_{[\alpha}A_{\beta}A_{\gamma]} - \frac{3}{2}A_{[\alpha}\partial_{\beta}A_{\gamma]}\Big). \tag{4}$$

Here g is the QCD gauge coupling, $A_\alpha = A^a_\alpha T^a$ is the gluon gauge field matrix, and T^a are the generators of the $SU(3)$ group.

The above parametrization is not just a formality, but has a physical meaning. It is known [3] that in low energy QCD the Chern-Simons three-form behaves as a *massless* gauge field, and the corresponding four-form field strength F can assume an arbitrary constant value. Hence, the strong *CP* problem can be simply understood as the problem of an *arbitrary* constant four-form electric field. Any dynamical solution that would explain why such a field must be unobservably small would also automatically solve the strong *CP* problem.

For example, as explained in [4], the celebrated Peccei-Quinn solution [5] solves the strong *CP* problem by putting the three-form gauge theory into the Higgs phase. This happens because the three-form gauge field becomes massive by "eating up" a would-be massless axion [6], and acquires a propagating longitudinal degree of freedom. This effect is analogous to an ordinary Higgs effect in which the photon acquires a longitudinal polarization by eating up a Goldstone boson. The generation of the axion mass from the QCD instantons can be reformulated in this language as a three-form Higgs effect. As a result of this effect, the four-form electric field is *screened* [7], and the vacuum is automatically *CP* conserving. Thus, the three-form language gives a very simple explanation to the fact [8] that the minimum of the axion potential is always at $\theta = 0$ [4].

In the present paper, we shall present an alternative solution to the strong *CP* problem. We shall attempt to explain why the QCD four-form electric field (and thus θ) is small by employing the vacuum accumulation mecha-

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nism of [9,10]. This mechanism is ready-made for the three-form gauge theories and provides a general framework in which the small value of any parameter, that is determined by a four-form electric field, can become an accumulation point of the infinite number of vacua.

Applied to the strong *CP* problem, our strategy can be outlined as follows. We shall lift the superselection rule of the θ vacua by postulating the existence of two-branes that source the QCD Chern-Simons three-form C. These branes substitute the axion field a of the Peccei-Quinn solution in the following way. In the Peccei-Quinn scenario the QCD Chern-Simons three-form $C_{\alpha\beta\gamma}$ is sourced by the topological axionic current $\epsilon_{\alpha\beta\gamma\mu}\partial^{\mu}a$. This sourcing is the reason for screening the expectation value of F. In our treatment this topological current gets replaced by a world-volume history of a dynamical two-dimensional surface, a two-brane

$$\epsilon^{\alpha\beta\gamma\mu}\partial_{\mu}a(x) \rightarrow \int dY^{\alpha\beta\gamma}\delta^4(x-Y), \tag{5}$$

where Y^{α}'s are the target space coordinates of the brane. In this way the massless scalar that screens F gets replaced by a massive extended object that affects the expectation value of F in a different way. The precise origin of these branes is unimportant to us. The only nontrivial assumption is that the branes are *CP* odd. The transition within the subset of vacua then becomes possible quantum-mechanically via the two-brane nucleation. The branes in this picture play the role of the domain walls that separate vacua with different values of θ (and F). Although at low energies the real-time transitions are extremely rare, this does not concern us, since we are mostly preoccupied with the resulting vacuum statistics. This statistics turns out to be pretty profound, because of the *CP*-odd nature of branes.

By parity (P) and *CP* symmetries the charge of the branes with respect to $C_{\alpha\beta\gamma}$ must also be parity odd and is determined by the value of the parity-odd four-form electric field (3). Thus, in vacua with a smaller electric field F, the sourcing is correspondingly weaker, and entirely diminishes in the vacuum with $F=0$ ($\theta=0$). In this way, the brane charge is set by θ, and hence the step by which θ changes from vacuum to vacuum is set by θ too. This fact guarantees that the vacuum with $\theta=F=0$ is the maximally preferred one. That is, essentially all of the infinite number of (quantum-mechanically connected) vacua have arbitrarily small values of the θ parameter, and only a measure-zero fraction has an observably large *CP* violation.

Summarizing briefly, due to the lift of the superselection rule, the θ vacua get split in discrete sets of the vacuum states that we shall refer to as the *vacuum families*. Each family contains an infinite number of vacua. The defining property of a given family is that all its member vacua can be connected to one another by a quantum-mechanical tunneling, whereas the transition between the different families is forbidden. Within each family the number of vacua as a function of the θ parameter diverges for $\theta \rightarrow 0$ as

$$n_{\theta} \sim \theta^{-k} \quad \text{or} \quad n_{\theta} \sim \ln(\theta^{-1}), \tag{6}$$

where $k>0$. Thus, $\theta=0$ is the vacuum accumulation point. Such points in the space of vacua were called the *vacuum attractors* in [9,10].

This is the essence of the solution, which we shall discuss in detail below.

At the end, we should stress a couple of important points. First, the considered mechanism, due to the gauge invariance of the brane three-form coupling, requires us to postulate an exactly massless degree of freedom, the Stückelberg field. This field, however, is different from the standard axion, since it couples to the three-form only through the heavy branes. As explained in the paper, once such a field is introduced, the only way it could acquire a mass in perturbation theory is through the screening of the three-form field by the brane loops. Interestingly, such a screening (if it takes place in the first place) would by itself solve the strong *CP* problem, by default. Thus, massless or not, the above field does not jeopardize the solution of the strong *CP* problem. Instead, what happens is that the nature of the solution changes depending on the Stückelberg mass. In our work we shall concentrate on the case when the Stückelberg field is exactly massless, and, thus, the three-form electric field is not screened.

The second point is about cosmology. Our aim in this paper is purely field theoretical. We shall prove that in a given theory there is an infinite set of vacua, all of which, up to a measure-zero set, accumulate at small θ. In other words, we are concerned solely with the vacuum statistics, and we show that statistically the most probable vacua have small θ. We do not attempt to advocate any particular cosmological scenario, which would explicitly explain why the Universe ended up in a *statistically* most probable vacuum, although one could easily imagine such scenarios in the context of the eternal inflation. Obviously, it goes without saying that, since our theory postulates very heavy branes (in this respect we are not any different from any other theory with branes, solving the *CP* problem or not), some period of inflation should have happened after the comoving patch of today's observable Universe made its choice of the ''final'' θ vacuum. An interested reader is referred to [9] for the discussion of one particular possible cosmology, in the context when the similar ideas are applied to the hierarchy problem.

The general prediction of the above scenario is the existence of branes that source the Chern-Simons forms of the standard model gauge fields. The tension of these branes can be sufficiently low to be produced in particle collisions at CERN LHC in the form of the resonances with spacing and multiplicity characteristic of the brane spectrum.

II. THREE-FORM OVERVIEW

For a massless three-form field the lowest order parity-invariant Lagrangian has the following form:

$$L = F_{\mu\alpha\beta\gamma}F^{\mu\alpha\beta\gamma} + C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}, \quad (7)$$

where $F_{\mu\alpha\beta\gamma} = \partial_{[\mu}C_{\alpha\beta\gamma]}$ is the four-form field strength, and $J^{\alpha\beta\gamma}$ is a conserved external current

$$\partial_\alpha J^{\alpha\beta\gamma} = 0. \quad (8)$$

The action (7) is then invariant under the gauge transformation

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + d_{[\alpha}\Omega_{\beta\gamma]}, \quad (9)$$

where Ω is a two-form. Because of this gauge freedom in four dimensions, C contains no propagating degrees of freedom. Despite the absence of propagating degrees of freedom, C nevertheless can create a "Coulomb"-type long-range electric field in the vacuum $F_{\mu\alpha\beta\gamma} = F_0\epsilon_{\mu\alpha\beta\gamma}$. In this respect, the $3+1$-dimensional three-form gauge theory is very similar to $1+1$ electrodynamics [11]. As is obvious from the equation of motion, in the absence of sources, the four-form electric field can assume an *arbitrary* constant value. Its equation of motion

$$\partial^\mu F_{\mu\nu\alpha\beta} = 0 \quad (10)$$

is solved by

$$F_{\mu\nu\alpha\beta} = F_0\epsilon_{\mu\nu\alpha\beta}, \quad (11)$$

where F_0 is an arbitrary constant.

Thus, the theory has a continuum of the vacuum states each labeled by an expectation value of a constant electric field F_0. These vacua obey the superselection rule. F_0 is not a dynamical quantity, and there is no transition between the different vacua. In other words, no F vacuum is preferred over any other, and any choice of F_0 is good. In this respect F_0 vacua are very similar to the θ vacua in QCD [1].

As said above, this connection between the F vacua and QCD θ vacua is deeper than one may naively think, and will play the central role in our approach to the strong *CP* problem. The key point in this connection is that the QCD θ vacua can be exactly reformulated in terms of the vacua with the four-form electric fields (3). We shall come back to this issue shortly.

Let us now briefly discuss how the superselection rule in the above example gets lifted in the presence of branes. In the presence of the external source $J^{\alpha\beta\gamma}$ the superselection rule gets partially lifted, permitting transitions between the vacua with certain discretized values of the electric field. As said above, the gauge invariance demands that the three-form be sourced by two-dimensional surfaces, two-branes, for which the conserved current takes the following form:

$$J^{\alpha\beta\gamma}(x) = \int d^3\xi\,\delta^4(x - Y(\xi))q\left(\frac{\partial Y^\alpha}{\partial\xi^a}\frac{\partial Y^\beta}{\partial\xi^b}\frac{\partial Y^\gamma}{\partial\xi^c}\right)\epsilon^{abc}, \quad (12)$$

where q is the charge of the brane, and $x^\mu = Y^\mu(\xi)$ specify a $2+1$-dimensional history of the brane in $3+1$ dimensions as a function of its world-volume coordinates ξ^a ($a = 0, 1, 2$). Obviously, the current $J_{\alpha\beta\gamma}$ is conserved as long as q is a constant.

The brane self-action has the standard form

$$-T\int d^3\xi\sqrt{-g}, \quad (13)$$

where T is the brane tension (a mass per unit surface), and $g_{ab} = \partial_a Y^\mu \partial_b Y^\nu \eta_{\mu\nu}$ is the induced metric on the brane. Note that, since the bulk four-dimensional gravity plays no essential role in our considerations, we have taken a flat Minkowskian four-dimensional metric $\eta_{\mu\nu}$. The brane can be taken to be flat and static, $Y^\mu = \xi^\mu$ for $\mu = 0, 1, 2$, and $Y^3 = 0$. The equation of motion then becomes

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q\delta(z)\epsilon^{\nu\alpha\beta z}, \quad (14)$$

where $z = 0$ is the location of the brane. Equation (14) shows that the brane separates the two vacua in either of which F_0 is constant, and the two values differ by $|q|$. Thus, the introduction of branes ensures that the transition between the vacua with different values of F_0 is possible, as long as the value of F_0 changes by an integer multiple of q. Hence the discrete quantum transitions between the different vacua are possible via nucleation of closed branes.

In other words, the theory given by the action (7) has a multiplicity of the discrete vacuum states. Among all possible vacua there are the subsets (the vacuum families) that are connected via quantum-mechanical tunneling. The different vacua within a given family can be labeled by an integer n. The value of the field strength in these vacua is

$$-\tfrac{1}{24}F_{\alpha\beta\gamma\mu}\epsilon^{\alpha\beta\gamma\mu} = F_0 = qn + f_0, \quad (15)$$

where f_0 is a constant, which is a fixed number for a given family, but changes from family to family. Thus, within a given family, the value of F is quantized in units of the brane charge.

III. THE STRONG *CP* PROBLEM IN THREE-FORM LANGUAGE

As discussed above, the vacua with a constant four-form electric field F are very similar to θ vacua in CQD. We wish now to show that the real QCD θ vacua can be understood as the vacua with different values of an electric field of a composite QCD four-form (3). The detailed discussion of this connection can be found in [4]. Before going to real QCD let us formulate the θ-vacuum problem in a theory with a free fundamental three-form, with the simplest Lagrangian (7) and no sources. This theory is in the Coulomb phase, and this fact is the source of the generalized strong *CP* problem. As shown above, equations of motion are solved by an arbitrary constant electric field (11), where F_0 is arbitrary and plays the same role as the θ parameter in QCD. In particular, the constant electric field (11) is *CP* odd. Also, the F_0 vacua obey a superselection rule. Note that the expansion on a background with a constant electric field F_0 generates a direct analog of the θ term,

$$\theta F, \tag{16}$$

where $\theta = \frac{1}{24} F_0$. Hence, the θ parameter in a three-form gauge theory is equivalent to a constant four-form electric field in the vacuum. This is analogous to what happens in the free massless electrodynamics in two dimensions in which the θ parameter also appears as an electric field [11]. Thus, the strong *CP* problem reformulated in the language of a three-form gauge theory reduces to the following question. How can the four-form Coulomb electric field be made naturally small?

The strong *CP* problem in QCD can be reformulated in the above-presented three-form language. For this, consider a θ-term in $SU(N)$ gauge theory with a strong coupling scale Λ (which we shall set equal to 1) and no light fermion flavors,

$$L = \theta \frac{g^2}{32\pi^2} G^a \tilde{G}^a, \tag{17}$$

where g is the gauge coupling, and a is an $SU(N)$-adjoint index. As discussed in the Introduction, this term can be rewritten as a dual of the four-form field strength F of a composite three-form $C_{\alpha\beta\gamma}$ according to (2) and (4). Under the gauge transformation, $C_{\alpha\beta\gamma}$ shifts as (9) with

$$\Omega_{\alpha\beta} = A^a_{[\alpha} \partial_{\beta]} \omega^a, \tag{18}$$

where ω^a are the $SU(N)$ gauge transformation parameters. The four-form field strength $F_{\mu\alpha\beta\gamma} = \partial_{[\mu} C_{\alpha\beta\gamma]}$ is of course invariant under (9) and (18). Note that the $SU(N)$ Chern-Simons current K_μ can be written as

$$K_\mu = \epsilon_{\mu\alpha\beta\gamma} C^{\alpha\beta\gamma}. \tag{19}$$

It is known [3] that, at low energies, the three-form C becomes a massless *field* and creates a long-range Coulomb-type constant force. The easiest way to see that C mediates a long-range interaction is through the Kogut-Susskind pole [12]. The zero momentum limit of the following correlator,

$$\lim_{q\to 0} q^\mu q^\nu \int d^4x e^{iqx} \langle 0 | T K_\mu(x) K_\nu(0) | 0 \rangle, \tag{20}$$

is nonzero, as it is related to topological susceptibility of the vacuum, which is a nonzero number in pure gluodynamics. Hence, the correlator of the two Chern-Simons currents has a pole at zero momentum, and the same is true for the correlator of two three-forms. Thus, the three-form field develops a Coulomb propagator and mediates a long-range force. Because the probe sources for the three-form are two-dimensional surfaces (domain walls or the two-branes), the force in question is constant.

In other words, at low energies, the QCD Lagrangian contains a massless three-form field, and can be written as

$$L = \theta F + \mathbf{K}(F) + \cdots. \tag{21}$$

The exact form of the function $\mathbf{K}$ in QCD is unknown,[1] but it is unimportant for our purposes. It is obvious now that the θ problem in QCD is isomorphic to a problem of a constant four-form electric field, and that QCD θ vacua are simply vacua with different values of this electric field.

Notice that the axion solution of this problem is nothing but Higgsing the composite three-form given in (4). That is, the axion solves this problem by giving a gauge-invariant mass to the three-form field and screening its electric field in the vacuum.[2] Indeed, the axion solution is based on the idea of promoting θ into a dynamical pseudoscalar field a, which gives us the following Lagrangian:

$$L = \frac{f^2}{2}(\partial_\mu a)^2 - aF + \frac{1}{24}\mathbf{K}(F), \tag{22}$$

where f is the axion decay constant. The reader can easily check that the minimum of the axion potential in the above theory is always at $F = 0$, regardless of the form of the function $\mathbf{K}(F)$, and that the theory contains no massless correlators. In other words, the three-form gauge theory is in the Higgs phase, due to "eating up" the axion field.[3] The detailed discussion of this phenomenon can be found in [4].

We shall now choose a different path for solving the strong *CP* problem. We shall not introduce any massless axions. In our approach the four-form electric field will remain in the Coulomb phase, but we shall explain its smallness by altering the structure of the θ vacua. In our treatment the vacuum superselection rule will be lifted, but in such a way that the vacua will accumulate in a tiny-θ region.

IV. PROMOTING $\theta = 0$ INTO THE VACUUM ACCUMULATION POINT

We shall now discuss the dynamics that promotes $\theta = 0$ ($\mathrm{Tr} G\tilde{G} = 0$) to the *vacuum accumulation* point. As a result of this dynamics, all but a measure-zero set of vacua become piled up near $\theta = 0$.

[1] Some subleading terms were estimated in [13] using the large-N QCD expansion.

[2] Alternatively one could try screening θ by dramatically altering the topology of space [14].

[3] The Higgs effect can be clearly visualized by dualizing the axion to an antisymmetric two-form field [4,7].

We shall achieve this effect by replacing the *topological* axion current $\epsilon^{\alpha\beta\gamma\mu}\partial_\mu a$ in the second term of the Lagrangian (22) by the CP-odd two-brane current, according to (5). In other words, we trade a massless CP-odd scalar for a massive (P) CP-odd extended object, the two-brane.

(The meaning of a CP-odd brane requires some clarification. In fact, a topological domain wall formed by a P- and CP-even scalar χ represents a simplest field theoretic example of a P- and CP-odd two-brane. Indeed, let us assume that χ changes by $\Delta\chi$ through the wall. The topological current $J^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma\mu}\partial_\mu\chi$ then has the parity which is opposite to the one of $C_{\alpha\beta\gamma}$. Thus, in such a case, the coupling $C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}$ is not permitted by P and CP and must be accompanied by the odd powers of a P-odd electric field F. In the effective low energy description, in which the wall thickness is integrated out, the topological current plays the role of the two-brane current which sources $C_{\alpha\beta\gamma}$,

$$\epsilon^{\alpha\beta\gamma\mu}\partial_\mu\chi(x) \to \int dY^{\alpha\beta\gamma}\delta^4(x - Y). \tag{23}$$

In view of this, if we wished to limit ourselves to entirely field theoretic realization, with no fundamental extended object, we could have simply replaced a massless pseudoscalar axion field with a heavy scalar χ that forms topological domain walls. One can easily construct other more involved examples of the CP-odd walls. However, we wish to keep our treatment maximally general, without specifying any underlying nature of two-branes.)

Up to total derivatives, the Lagrangian then becomes

$$L = \tfrac{1}{24}\mathbf{K}(F) + C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}_{(T)}, \tag{24}$$

where $J^{\alpha\beta\gamma}_{(T)}$ is the transverse part of the brane current (12). The reason for transversality is as follows. From the assumption that the brane is CP odd it follows that the brane charge q can no longer be a constant, but should be an odd continuous function of F. For example,

$$q \to q(F) \propto F^{2n+1}, \tag{25}$$

where n is a positive integer. This is exactly what we need, since, according to the general mechanism of [10], the zero of the electric field F will become an accumulator in the space of vacua. We shall demonstrate this explicitly in a moment. But let us first note that, in order to maintain the gauge invariance in the case of a field-dependent q, we shall follow the prescription of [10] and couple $C_{\alpha\beta\gamma}$ and $J^{\alpha\beta\gamma}$ transversely. That is, we shall adopt the following coupling,

$$C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}_{(T)}, \tag{26}$$

where $J_{(T)}$ is the transverse part of the current

$$J^{\alpha\beta\gamma}_{(T)} = \Pi^{[\alpha}_{\mu}J^{\mu\beta\gamma]}. \tag{27}$$

Here $\Pi_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}$ is the transverse projector.

For constant q, we have $\partial^\alpha J_{\alpha\beta\gamma} = 0$ and $J_{(T)} = J$. Thus, the coupling (26) reduces to the one in (7). This fact accomplishes our goal. In each given vacuum the expectation value of F is fixed to a constant. On the other hand, the change of q from vacuum to vacuum is permitted, because $C_{\alpha\beta\gamma}$ only couples to the transverse part of $J_{\alpha\beta\gamma}$. The existence of the attractor point at $F = 0$ is guaranteed by the fact that $J_{(T)} \to 0$ when $F \to 0$.

The coupling (26) is the gauge-invariant generalization of (7) for the case of a nonconstant charge $q(F)$. Although the coupling (26) contains a projector, as shown in [10], it is local, and can be obtained from a local underlying theory after integrating out the Stückelberg field. For completeness, we repeat this derivation in the Appendix.

Putting all the ingredients together, let us now show that the theory has an attractor point in the space of vacua at $F = 0$. Since the existence of the attractor point at $F = 0$ is independent of the form of the function $\mathbf{K}(F)$, we shall take $\mathbf{K}(F) = F^2/2$ for simplicity,

$$L = \tfrac{1}{48}F^2 + C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}_{(T)}. \tag{28}$$

The equations of motion are

$$\epsilon^{\mu\alpha\beta\gamma}\partial_\mu\left(F + 24C_{\kappa\rho\tau}\frac{\partial J^{\kappa\rho\tau}_{(T)}}{\partial F}\right) = J^{\alpha\beta\gamma}_{(T)}. \tag{29}$$

The vacuum structure of a similar toy example of 1 + 1-dimensional electrodynamics was studied in [15]. The analysis in our case is pretty similar. In order to visualize the vacuum structure, we have to figure out how the constant four-form electric field F changes at the static brane. For definiteness, we shall place the latter at $x^3 = z = 0$. Then the only nonzero components of the current become $J^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma 3}\delta(z)$. We shall now look for the static z-dependent configuration $F = F(z)$ and $C^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma 3}C(z)$. Then the only nontrivial equation is

$$d_z\left(F(z) + 864\delta(z)C(z)\frac{\partial q(F)}{F}\right) = -q(F(z))\delta(z). \tag{30}$$

To find a jump in the value of F, we integrate the equation in a small interval near $z = 0$. This integration gives

$$\Delta F = -q(F(0)). \tag{31}$$

Because $q(F)$ is an odd function of F, Eq. (31) shows that jumps are becoming smaller and smaller as $F \to 0$, which proves that there are an infinite number of vacua within an arbitrarily small neighborhood of $F = 0$. Thus, $F = 0$ ($\theta = 0$) is a vacuum accumulator point.

As it was shown in [10,15] the number of vacua with the electric field exceeding a certain value F_* diverges with $F_* \to 0$ as

$$n_{F_*} \sim \int_{F_*}^{1} \frac{dF}{q(F)}. \tag{32}$$

For $q(F) = F$, this gives

$$n_{F_*} \sim \ln\left(\frac{1}{F_*}\right). \tag{33}$$

Translated to the statistics of the θ vacua, this implies that the number of θ vacua within each family diverges as

$$n_\theta \sim \ln(\theta^{-1}) \tag{34}$$

for $\theta \to 0$.

V. THE ACCUMULATOR-SHIFT PROBLEM AND THE FIX

We shall now address a potential problem which could destabilize the above solution of the strong *CP* problem. The problem may arise from the perturbative gluon loops that may generate a constant part in $q(F)$ and therefore shift the accumulator point away from $F = 0$ ($\theta = 0$). Such a constant part may be generated because of the following reason. The symmetries that guarantee that $q(F)$ is an odd function of F are P and CP. However, both symmetries are broken in the electroweak sector of the standard model, and this breaking will result in a contribution to the θ-term, which we shall call $\theta_{\rm SM}$. We shall assume that $\theta_{\rm SM} \sim 1$. The breaking of CP by the $\theta_{\rm SM}$-term can (and, in general, will) be communicated to the brane via the gluon loops, and this will result in the shift of the function $q(F)$ by an F-independent constant proportional to the $\theta_{\rm SM}$-term,

$$q(F) \to q(F) + \theta_{\rm SM}\Lambda_c^4, \tag{35}$$

where Λ_c is a cutoff. Such a shift by a large constant would be a disaster for our solution, since the vacuum accumulation point now would be shifted to the $F \neq 0$ value that, in general, would result in $\theta_{\rm observable} \sim \theta_{\rm SM}$.

We shall now solve the above problem dynamically. For this we shall assume that *CP* is broken spontaneously at some scale M, by a nonzero vacuum expectation value (VEV) of some scalar field ϕ. In such a case all the *CP*-violating operators, and, in particular, the $\theta_{\rm MS}$-term will be generated from some effective high-dimensional operator of the form

$$\left(\frac{\phi}{M}\right)^N {\rm Tr} G\tilde{G} \to \theta_{\rm SM} F, \tag{36}$$

where N is some power, which depends on the details of model building and, in particular, on quantum numbers of the field ϕ under different gauge symmetries. After ϕ gets a VEV, the above coupling will translate into an effective $\theta_{\rm SM}$-term with $\theta_{\rm SM} = (\frac{\langle\phi\rangle}{M})^N$.

At this point we should make it clear that our approach should not be confused with the one by Nelson and Barr [16]. In fact, our strategy is exactly orthogonal. If we translate the strong *CP* problem in three-form language, the initial postulate in the Nelson-Barr type approach amounts to choosing the vacuum by the requirement of the *CP* invariance, that is, by the requirement that the electric field (2) (equivalently θ) should vanish in the vacuum.

Our philosophy is precisely to avoid this type of choice, because the electric field ($\langle G\tilde{G}\rangle \propto \theta$) is *not* a parameter in the fundamental action, but rather a *solution* of the equations of motion, which, as any other solution, does not have to respect the symmetry of the initial action. Therefore, we do not put any such restrictions on the vacuum states, and consider all of them. We then solve the strong *CP* problem, by explaining why the ones that conserve strong *CP* are *statistically more probable*.

The crucial difference is already obvious from the fact that we are *not* demanding any particular smallness of $\theta_{\rm SM}$. Because in the vacuum $\langle\phi\rangle \sim M$, the $\theta_{\rm SM}$ is naturally of order 1. So in the absence of our *CP*-odd branes the strong *CP* problem would be there, as usual. What we gain by promoting $\theta_{\rm SM}$ into a VEV of a field is that its expectation value can diminish in the vicinity of the brane due to the influence of the latter. This fact solves the accumulator-shift problem. The shift of the brane charge by the gluonic loops now will manifest itself through correcting $q(F)$ by the following local operator:

$$q(F) \to q(F) + \left(\frac{\phi}{M}\right)^N \Lambda_c^4. \tag{37}$$

The important point is that, in evaluating this shift, the value of ϕ has to be taken not in the vacuum, but at the location of the brane. Because of the influence of the brane, the latter value can be negligibly small, as we show in a moment. In this way the accumulator shift is avoided.

The suppression of the ϕ VEV on the brane can happen, because the ϕ field is allowed to have various potential terms on the brane world volume, compatible with symmetries. The most important of these is a brane-localized mass term, which can be introduced in the four-dimensional action in the following form:

$$-\tfrac{1}{2}\int dx^4 M_{\rm br}(x)^2\phi^2, \tag{38}$$

where

$$M_{\rm br}^2(x) = \pm\int d\xi^3\sqrt{-g}M_B\delta^4(x - Y). \tag{39}$$

In the above expression M_B is a positive mass parameter. As shown in [10] in the case of the positive sign, the brane-localized mass term has an effect of suppressing the ϕ VEV on the brane. The source for this suppression is easy to understand. The equation for ϕ in the background of the brane (located at $z = 0$) is

$$\partial^2\phi - \lambda^2(\phi^2 - m^2)\phi - \delta(z)M_B\phi = 0. \tag{40}$$

Here we have assumed that the bulk potential for the ϕ field is $V(\phi) = \frac{\lambda^2}{4}(\phi^2 - m^2)^2$ so that the bulk VEV is $\phi_{\text{bulk}} = m$. From (40) it is clear that the positive brane-localized mass term is seen by the field as a potential barrier, which for $M_B \gg \lambda m$ strongly suppresses the expectation value on the brane, $\phi_{\text{br}} \ll \phi_{\text{bulk}}$.

ϕ_{br} can be estimated by minimizing the following expression (we ignore the factors of order 1):

$$E = M_B\phi_{\text{br}}^2 + (\phi_{\text{br}} - m)^2 m + \lambda^2(\phi_{\text{br}}^2 - m^2)^2 m^{-1}. \quad (41)$$

The first term in this expression comes from the brane mass term. The second and the third terms are the expenses in the gradient and the bulk potential energies. The full expression is minimized at

$$\phi_{br} \sim \phi_{\text{bulk}}\frac{m_\phi}{M_B}, \quad (42)$$

where $m_\phi \equiv \lambda m$ is the bulk mass. Thus, in any given vacuum, the brane expectation value of ϕ is by the factor m_ϕ/M_B smaller relative to its bulk counterpart.

This statement can be checked exactly. Equation (40) has an explicit solution

$$\phi(z) = m\,\text{th}\left[\frac{m_\phi}{\sqrt{2}}\left(|z| + \frac{1}{\sqrt{2}m_\phi}\text{arcsh}\left(\frac{2\sqrt{2}m_\phi}{M_B}\right)\right)\right], \quad (43)$$

which confirms the above estimate, since $\phi_{\text{br}} = \phi(0) \simeq \sqrt{2}mm_\phi/M_B$.

To summarize, by decreasing the bulk mass of ϕ relative to its brane mass, the expectation value at the brane gets diminished. The brane surrounds itself by a "halo" of a restored CP region. The size of this halo is $\sim m_\phi^{-1}$. Inside this region, the only source of the CP violation is the expectation value of F that is sourced by the CP-odd brane.

Because of the above effect, the accumulator gets shifted to the point

$$q(F) \sim \left(\frac{m_\phi}{M_B}\right)^N. \quad (44)$$

This value can be naturally extremely small. To get a rough feeling about the possible smallness, consider an extreme case, when the brane has a Planck scale tension. Then, we can take $M_B \sim M_P$. If ϕ is a modulus that couples to the standard model fields via the M_P suppressed interaction, the natural lower value for m_ϕ is somewhere around 10^{-3} eV. This will be the case if the supersymmetry breaking scale is around TeV. Then taking $q(F) = F$ the value of the observable θ-term at the accumulator point will be $\theta_{\text{observable}} \sim 10^{-31N}$, which is practically unobservable. A phenomenologically more interesting situation occurs for the lower values of the brane tensions, in which case the predicted $\theta_{\text{observable}}$ may be close to the phenomenological lower bound, and be potentially observable.

VI. EXPERIMENTAL SIGNATURES

We wish to briefly discuss some possible experimental signatures of the presented scenario. The crucial role is played by the branes to which the standard model gauge fields are coupled. These branes can be introduced either as some field theoretic domain walls (see [10]), or as the fundamental objects. The latter possibility is phenomenologically the most interesting one, since branes can be produced at particle colliders.

There is no obvious particle physics upper bound on the tension of these branes (although the specific cosmological considerations may place one). In this respect, the role played by the brane tension in our scenario is analogous to the one played by the axion decay constant in the Peccei-Quinn solution. The latter can be arbitrarily high, allowing the axion to be arbitrarily weakly coupled and practically unobservable [17].

However, if the brane tension is at the TeV scale or lower, the branes may be observed at LHC in the form of the resonances with a specific spectrum. One can think of a number of possible production channels for these resonances. Because branes are predicted to be coupled to all possible CP-odd combinations of the standard model Chern-Simons forms and dual field strengths, they can be produced in gauge boson scattering. For instance, they can be produced in gluon-gluon collisions accompanied by the production of the additional gauge fields. A typical process would be

$$2g \to \text{brane resonance} + 2\gamma, \quad (45)$$

in which two gluons produce a brane resonance and two photons. The brane resonances can then decay into a combination of the standard model gauge fields [for instance, into four photons (gluons, weak bosons)]. They can also decay into the CP-violating scalar ϕ, which then (if mass allows) can decay into the standard model fermion-antifermion pair and a Higgs.

Note that the brane tension may be below the W, Z masses, in which case the decay in these particles is excluded. Also note that the mass of ϕ in the extreme case can naturally be as small as 10^{-3} eV. This will be the case if the supersymmetry breaking scale is TeV, and ϕ couples via Planck scale suppressed operators to the standard model fields. In such a case ϕ can only decay into the photons, and the lifetime will be too long for being observed within the detector. In such a case, ϕ can also manifest itself through a new gravity-competing force at submillimeter scales.

Brane resonances can also be produced in quark-antiquark annihilation together with the Higgs in the final state, via an intermediate ϕ scalar.

The generic distinctive feature of brane resonances will be their characteristic spectrum. For instance, it is known [18] that the radial excitations of a spherical membrane have the spectrum of a radial Schrödinger equation with a quartic potential (for simplicity we work in units of the brane tension),

$$\left(-\frac{d^2}{dr^2} + r^4\right)\psi(r) = m^2\psi(r), \tag{46}$$

which has a $m^2 \sim n^{4/3}$ scaling for large n.

VII. CONCLUSIONS

The source of the strong *CP* problem is that the QCD has a continuum of the vacuum states labeled by the parameter θ, which obey the superselection rule and most of which are excluded observationally. The phenomenologically acceptable values $\theta < 10^{-9}$ are not preferred by any statistics or dynamical considerations.

What we have achieved in our treatment is that we have lifted the superselection rule in such a way that the transition within an infinite subset of vacua, called a *vacuum family*, is now permitted by nucleation of branes. A given family of vacua can be constructed by randomly choosing the value of the parameter θ and adding all possible vacua that can be created from it by nucleation of the arbitrary number of branes. The set of vacua created in this way will by default be isolated from the other sets by the superselection rule. Thus, the transition among the families is still forbidden. *A priori*, the number of families may be large or even infinite, but within each family the accumulation of the infinite number of vacua happens within the phenomenologically permitted region $\theta < 10^{-9}$.

In this way, in the full theory, the phenomenologically unacceptable vacua form a measure-zero set.

In this work we were mainly concerned with the vacuum statistics, and we did not discuss an explicit cosmological scenario that in the real-time picture would drive the Universe towards the vacuum accumulation point. However, it is intuitively clear that, in any early universe scenario in which the statistics of vacua matters, the highest probability will be given to the phenomenologically acceptable ones, due to their enormous number.

We also wish to remark that, although fundamentally different, the presented mechanism shares some spiritual connection with the irrational axion idea [19].

Finally, there are possible experimental signatures of the above theory. The general prediction is the existence of the two-branes to which the standard model gauge fields are coupled. The tension of these branes may be around the TeV scale or lower. In such a case, they can be produced in particle collisions in the form of the resonances with a characteristic spectrum.

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APPENDIX

In this appendix, we briefly repeat the method of Ref. [10] for obtaining the couplings (27) from the local gauge-invariant theory. This is achieved by introducing the following couplings in the Lagrangian:

$$(C_{\alpha\beta\gamma} - \partial_{[\alpha}B_{\beta\gamma]})J^{\alpha\beta\gamma} + X^{\beta\gamma}\partial^{\alpha}(C_{\alpha\beta\gamma} - \partial_{[\alpha}B_{\beta\gamma]}). \tag{A1}$$

Here, two-form $B_{\alpha\beta}$ is a compensating Stückelberg field, which under the gauge transformation (9) shifts in the following way:

$$B_{\alpha\beta} \rightarrow B_{\alpha\beta} + \Omega_{\alpha\beta}. \tag{A2}$$

$X^{\beta\gamma}$ is a two-form Lagrange multiplier that, through its equation of motion, imposes the transversality constraint

$$\partial^{\mu}\partial_{[\mu}B_{\alpha\beta]} = \partial^{\nu}C_{\nu\alpha\beta}. \tag{A3}$$

Integrating out the X and B fields, we arrive at the effective coupling (27).

Let us briefly comment on a potential effect, discussed in [10], which is a possible screening of the electric field F by the virtual brane loops. The effect is somewhat analogous in spirit to the charge screening by fermion loops in the massless Schwinger model [20], except that this issue in the brane case is more subtle. The point is that the loops of the branes could, in principle, generate operators of the form

$$C^{\mu\alpha\beta}R\Pi^{\nu}_{[\mu}C_{\nu\alpha\beta]} \tag{A4}$$

where R is some function that can be expanded in a series of the positive powers of ∂^2/M^2, where M is a cutoff. If this expansion contains a constant ∂^2-independent term, the propagator of the three-form would acquire a physical pole. This would screen the electric field F. In this case the strong *CP* problem would be solved automatically, just as in the axion case, and there would be no need to allude to the vacuum accumulation effects.

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