

**Large hierarchies from attractor vacua**

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We discuss a mechanism through which the multivacua theories, such as string theory, could solve the hierarchy problem, without any UV-regulating physics at low energies. Because of symmetry the number density of vacua with a certain hierarchically small Higgs mass diverges, and is an attractor on the vacuum landscape. The hierarchy problem is solved in two steps. It is first promoted to a problem of the superselection rule among the infinite number of vacua (analogous to  $\theta$  vacua in QCD) that are finely scanned by the Higgs mass. This rule is lifted by heavy branes, which effectively convert the Higgs mass into a dynamical variable. The key point is that a discrete “brane-charge-conjugation” symmetry guarantees that the fineness of the vacuum scanning is set by the Higgs mass itself. On a resulting landscape, in all but a measure-zero set of vacua, the Higgs mass has a common hierarchically small value. In minimal models this value is controlled by the QCD scale and is of the right magnitude. Although in each particular vacuum there is no visible UV-regulating low energy physics, the realistic models are predictive. For example, we show that in the minimal case the “charge-conjugation” symmetry is automatically a family symmetry, and imposes severe restrictions on quark Yukawa matrices.

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**I. INTRODUCTION**

It is usually assumed that the solution to the hierarchy problem requires the existence of some new physics around the TeV scale, in order to regulate quadratic divergences in the Higgs mass. In the present article, we shall attempt to provide a counterexample to this statement. Our approach will be based on generalization of [1], which we shall closely follow. Another inspiration is provided by the recent progress [2–5] in understanding the distribution of vacua on string theory moduli spaces. We will comment on the connection with the latter work below.

In [1] it was shown that in simple “string inspired” extensions of the standard model, postulating the coupling to the heavy branes and the three-form fields, the number of vacua with a hierarchically small value of the Higgs vacuum expectation value (VEV) *diverges* due to symmetry reasons. Such a value of the Higgs VEV was called an “attractor” point in the space of vacua. The crucial point is that by symmetry the attractor point is stable against the quantum corrections. The situation is rather peculiar. The theory in question has an infinite number of vacuum states separated by the large potential barriers. The Higgs mass (and the VEV) takes different values in different vacua, but the vacua with large values of the Higgs VEV are rare and gradually increase in number density towards the smaller Higgs VEV. An infinite number density of vacua cluster around a certain hierarchically small value of the Higgs VEV, which marks the attractor point. In other words, the scanning of the vacuum landscape by the Higgs mass becomes *hyperfine* near the attractor value.

In each particular vacuum within the neighborhood of the attractor point, there is no UV-regulating new physics

around the scale of the Higgs mass. So a naive observer, living inside of any such vacua and suspecting nothing about the multiplicity of the similar vacuum states, would attribute the smallness of the weak interaction scale to a mysterious fine-tuning. Nevertheless, for an “outside” observer, knowing that the number density of such vacua is divergent, the “fine-tuning” becomes perfectly natural. The hierarchy problem is solved since the prior probability distribution is singular, and probability is sharply peaked around the vacua with the hierarchically small Higgs mass.

These ideas indicate that multivacua theories, such as string theory, have the potential for solving the hierarchy problem without invoking any UV-stabilizing new physics around the weak scale. Hence, in such theories our ideas about naturalness must be reconsidered. This view is supported by the recent progress in the field.

It is becoming evident [2–10] that the string theory “landscape” [8], which reveals enormous complexity of the vacuum states, could play an important role in selection of the observed vacuum. So it is conceivable that the attractor solution of the hierarchy problem could find a natural implementation in string theory. Putting aside the question of the Higgs mass and the strength of the divergence in the vacuum number density, the attractor of [1] can be viewed as some sort of “holographic dual” to one class of vacua considered by Giddings, Kachru, and Polchinski [2], which arise at conifold points in Calabi-Yau moduli space.<sup>1</sup> The recent interesting studies [3–5] indicate that “attractive” conifolds with flux can lead to

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clustering of a high number of vacua. Although the clustering of the vacua in the case of conifolds is weaker than in the case of [1], it will be shown below that this feature is rather parameter dependent. This connection deserves further study [11]. Some related ideas can also be found in [10].

We should stress, however, that the vacuum structure of the theory considered in the present work is different from the conventionally discussed string landscapes, in the sense that in our case the attractor points accumulate literally an infinite number of vacua. Although we do not see any *a priori* fundamental reason that would make such sharp attractors impossible in string theory, we are not aware of any example.

Because of the above, in the present paper we shall limit ourselves to the effective field theory analysis of the attractor idea, although some string theory concepts will play a crucial role in this analysis. We discuss how the attractor behavior arises in effective field theory which involves some of the key ingredients of string compactifications, such as the three-form fields, axions (two-forms), and effective two-branes.

We should also emphasize that the two-forms that we shall employ should be exactly massless, unlike the standard QCD or the string theory axions, which typically acquire mass at low energies. They are demanded by the gauge invariance of the three-form-to-brane couplings, which in our case should depend on the expectation value of the Higgs field. They can only couple to the standard model particles via the heavy branes.

We shall show that, although the attractor solution does not require any UV-regulating new physics, the realistic models nevertheless have a predictive power and are potentially testable. For example, in the most minimal case, without any low energy extension of the standard model, we show that the attractor-stabilizing symmetry is automatically a *family* symmetry of standard model quarks, and implies restrictions on the structure of quark Yukawa couplings.

Among the possible approaches to the hierarchy problem, the attractor solution is probably the closest possible analog of the axion solution of the strong *CP* problem. Indeed, from the beginning of our treatment the hierarchy problem, from being a problem of the UV *stability* of the Higgs mass ( $m_\phi$ ), gets promoted to the problem of a *superselection rule* among the infinite vacua scanned by  $m_\phi$ . These are analogous to  $\theta$  vacua in QCD that are scanned by the  $\theta$  parameter [12]. The advantage of dealing with a superselection problem, as opposed to the one of UV stability, is the following. Unlike the latter problem, the new physics that solves the former can be arbitrarily weakly coupled to the standard model sector. The famous example of such new physics in the case of the Peccei-Quinn (PQ) solution [13] of the strong *CP* problem is the axion [14,15], which can be arbitrarily weakly coupled and

practically invisible [16,17]. As we shall see, in our case too, the new physics that solves the hierarchy problem via the attractor mechanism can be practically decoupled from the standard model sector. Of course, along with the analogies, there are fundamental differences with the axion mechanism. In both cases the parameter of interest is promoted to a dynamical variable. In the PQ case this is the  $\theta$  angle, and in our case the Higgs mass  $m_\phi$ . In both cases, the vacua with small values of the parameters are selected, but the selection mechanisms are different. In the case of the  $\theta$  angle the selection is via the vacuum relaxation mechanism, since small  $\theta$  corresponds to the true ground state of the system [18]. In our case, the selection happens through the enormous multiplicity of states. Because of the attractor, in all but a measure-zero number of vacua, the Higgs mass is small.

Finally, we wish to say that in the present paper we are addressing the statistics of the Higgs vacua, without putting them into a concrete cosmological scenario. That is, we show that the vacua with large hierarchy are *statistically most probable*, or to be more precise, are maximally probable. That is, if one had to choose randomly, with the equal weight, one would end up in such vacua with unit probability. This is all we can claim. Of course, in a concrete real time cosmology, the probabilities may be altered, for example, by the initial conditions of the Universe. We shall not attempt to estimate them here. One possible cosmological scenario was discussed in [1], in the context of the eternal inflation, and this answers some of the general questions. For instance, because inflation is eternal, the Universe realizes all the possibilities, and the vacuum statistics actually becomes a real statistics (it is analogous to throwing a biased die an infinite number of times). Over the infinite space-time range most of the volume of the Universe will be in the vacuum that is statistically the most probable one. Of course, there will be an infinite number of causally disconnected patches of such a vacuum, but they will all share the property that in all of them the electroweak scale will be low. This eliminates the need of anthropic considerations.

Of course, as in any other inflationary scenario in theories with different vacua (or branes), any patch that today is within our cosmological horizon should have inflated after it chooses its final vacuum, or else we would see the domain walls or branes in our observable Universe. This is not specific to our mechanism, and is common to any theory with domain walls or branes. In order not to have a cosmological problem, we need the inflationary scale to be lower than the brane (or wall) tension. When this is satisfied, the inflation solves the cosmological problems, as usual.

Notice that the fact that the last transition *within the observable patch* happens before the end of inflation does not jeopardize the statistical solution, but is exactly the opposite. This is precisely how inflation convert the vacuum statistics into the statistics of the causally discon-

nected patches of the Universe. Again, this is explained in detail in [1].

Note that the massless two-form field can only be produced together with the branes, and this is a tremendously suppressed process. Finally, because the large transition within our patch happened before the end of inflation, there are no immediate observable consequences for either dark matter or the cosmic microwave background (CMB) spectrum.

The paper is organized as follows. In Sec. II, we briefly summarize our criteria of naturalness and the essence of the attractor vacuum. In Sec. III, we gradually build the attractor vacuum, by putting all essential ingredients together. At the end of this section we derive an effective equation for finding the vacua on the landscape, and give their number counting. In Sec. IV, we give a general discussion of the shift of the attractor point to the realistic values of the Higgs VEV. The precise mechanisms are discussed in Secs. V and VI. In Sec. V, we study the effect of world-volume terms, such as brane-localized mass terms, on the attractor dynamics, and show that depending on the parameters they can either lead to sharpening of the attractor, or to regulating it (that is, cutting off the divergence in vacuum number densities). In Sec. VI, we discuss the realistic model building and present the two versions of the complete models. We show that the minimal one, which requires no enlargement of the electroweak sector, imposes severe restrictions on the structure of Yukawa matrices. The next to minimal case, which is less constraining, is discussed at the end of Sec. VI. In Sec. VII, we show that, when extended to grand unified theories, the attractor solution automatically solves the problem of doublet-triplet splitting. In Appendix A, we show how the theories with the attractor can be obtained from local gauge-invariant theories after integrating out the Stückelberg field, and in Appendix B we discuss some related potential issues. In Appendix C, we discuss some exact solutions. In Appendix D, we show, following [1], that the branes charged under the three-form fields can be the axionic domain walls. Finally, in Appendix E, we study how the axionic walls with field-dependent charges could be obtained without the Stückelberg method, by integrating out some intermediate scalars.

In the next article [19], we shall apply a very similar idea to the strong  $CP$  problem.

## II. NATURALNESS AND THE ESSENCE OF THE ATTRACTOR PHENOMENON

In this section, we wish to briefly formulate our criteria of naturalness, and summarize the essence of the attractor solution. The hierarchy problem is the problem of UV sensitivity of the Higgs mass ( $m_\phi$ ) and consequently of the Higgs VEV ( $\langle\phi\rangle$ ). The excellent formulation of the

problem can be found in [20]. The attractor mechanism solves this problem by selecting the vacuum with the hierarchically small Higgs mass. The selected value is UV *insensitive* due to the symmetry reasons. In this respect, the attractor solution of the hierarchy problem is natural in the same sense as the technicolor [20,21] or the low energy supersymmetry, but has some advantages over the latter. For example, as we shall show, the attractor mechanism also automatically solves the problem of doublet-triplet splitting in grand unified theories, which supersymmetry alone fails to solve.

By simple choice of symmetry, the attractor solution gives the possibility of obtaining the Higgs VEV in terms of the QCD scale.

The essence of the attractor solution can be summarized as follows. The attractor is a special point on the energy landscape that contains an infinite number of discrete vacua. The vacua are scanned by a given parameter, which in the case of interest is the Higgs mass. What makes the attractor landscape special is the fact that, due to the symmetry reason, all but the measure-zero set of the vacuum states exhibit practically equal and hierarchically small values of the Higgs mass. This value is the attractor point.

The attractor vacuum is achieved in two steps. First, by coupling the Higgs to the field strength of an antisymmetric three-form, we promote the hierarchy problem into the problem of the superselection rule, analogous to the strong  $CP$  problem in QCD. The vacua are scanned by the Higgs mass  $m_\phi$ , which plays the role analogous to the  $\theta$  angle that scans the QCD vacua. The superselection rule is lifted by two-branes that effectively *source* the Higgs mass and allow the quantum transition between the vacua with different  $m_\phi$ . Thus, the Higgs mass gets promoted to a dynamical variable. This shares some analogy with the axion solution of the strong  $CP$  problem, in which the  $\theta$  angle gets promoted to a dynamical variable. However, the reason why the desired vacuum is selected in the attractor case is different from the axion scenario. In the latter case, because the dynamical  $\theta$  angle changes continuously, the selection is energetic. The selected vacuum is the true ground state of the theory, to which the system relaxes on a microscopic time scale. In our case, the transitions between the vacua with different  $m_\phi$  are discretized. The barrier between the vacua is very large, but the fineness of  $m_\phi$ -scanning changes throughout the landscape and becomes superfine around the attractor point. Because of the large potential barrier, each vacuum is extremely long-lived. In such a situation energetics plays no role in selecting the vacuum. Instead, what is important is the density of the vacua with the given values of  $m_\phi$ . We show that, due to symmetry reason, which triggers a profound backreaction on the brane charge, essentially all vacua cluster around a certain hierarchically small value of  $m_\phi$ , which is the attractor point.

### III. THE ATTRACTOR VACUUM

#### A. The superselection rule

Expanding the analysis of [1], we start with a detailed discussion of the attractor idea. String theory contains various antisymmetric form fields, which after compactification to four dimensions give rise to three-forms, two-forms (axions), and one-forms (vectors). The crucial role in our solution of the hierarchy problem is played by the three-form field  $C_{\alpha\beta\gamma}$ . For a free three-form field, the lowest order parity-invariant action has the following form,

$$\int_{3+1} \frac{1}{48} F_{\mu\alpha\beta\gamma} F^{\mu\alpha\beta\gamma} \quad (1)$$

where  $F_{\mu\alpha\beta\gamma} = d_{[\mu} C_{\alpha\beta\gamma]}$  is the four-form field strength. This action is invariant under the gauge transformation

$$C_{\alpha\beta\gamma} \rightarrow C_{\alpha\beta\gamma} + d_{[\alpha} \Omega_{\beta\gamma]}, \quad (2)$$

where  $\Omega$  is a two-form. Because of this gauge freedom in four dimensions  $C$  contains no propagating degrees of freedom, and its equation of motion

$$\partial^\mu F_{\mu\nu\alpha\beta} = 0 \quad (3)$$

is solved by

$$F_{\mu\nu\alpha\beta} = F_0 \epsilon_{\mu\nu\alpha\beta}, \quad (4)$$

where  $F_0$  is an arbitrary constant. Hence, in the absence of other interactions, the effect of the three-form is reduced to adding an arbitrary integration constant to the Lagrangian. This constant will contribute to the overall cosmological term. However, in the presence of interactions with the other fields, the integration constant  $F_0$  will also contribute to their effective masses and couplings. Consider, for instance, an interaction with a scalar field  $\phi$ . In what follows we shall treat  $\phi$  as the prototype of the standard model Higgs. The lowest order parity- and gauge-invariant Lagrangian describing a nontrivial interaction between  $\phi$  and  $C$  has the following form,

$$L = |\partial_\mu \phi|^2 - \frac{1}{48} F^2 + |\phi|^2 \left( m^2 + \frac{F^2}{48M^2} \right) - \frac{\lambda}{2} |\phi|^4 + \dots \quad (5)$$

Here  $\lambda$  is the quartic coupling, and  $m$  and  $M$  are the mass parameters that naturally are of the order of the UV cutoff, which we shall take to be around the Planck mass  $M_p$ . For definiteness, we shall assume  $m^2 > 0$ ,  $M^2 > 0$ . Note that the value of  $F$  determines the value of the effective mass  $2$  and consequently the VEV of the Higgs field. The latter is equal to

$$|\phi|^2 = \frac{1}{\lambda} (m^2 + F^2/48M^2). \quad (6)$$

The value of  $F$  is determined from the equation of motion

$$\partial^\mu ((1 - |\phi|^2/M^2) F_{\mu\nu\alpha\beta}) = 0, \quad (7)$$

which is solved by

$$F_{\mu\nu\alpha\beta} = \frac{F_0 \epsilon_{\mu\nu\alpha\beta}}{(1 - |\phi|^2/M^2)} \quad (8)$$

where  $F_0$  is an arbitrary integration constant. Plugging this solution into the equation for the Higgs field, we get the following effective equation determining the Higgs VEV,

$$\left( -m^2 + \frac{F_0^2}{2M^2(1 - |\phi|^2/M^2)^2} \right) \phi + \lambda |\phi|^2 \phi = 0. \quad (9)$$

It is obvious that the above theory has a continuum of the vacuum states, labeled by  $F_0$ . In many of these vacua the VEV and the mass of the Higgs are much smaller than the cutoff. These are the vacua with  $m^2 - F_0^2/2M^2 \ll M^2$ , in which  $\phi \ll M$ . For instance, there is a vacuum with  $F_0^2 = 2m^2M^2$  in which  $\phi = 0$ . In this vacuum  $\phi$  is exactly massless.

Although there exist vacua with a light scalar, the hierarchy problem is nevertheless *not* solved in the above theory. The reason is twofold. First, in the above theory the vacua are uniformly scanned by the integration constant  $F_0$ , and the light Higgs vacua are not special in any way. Second,  $F_0$  is not a dynamical quantity, and there is no transition between the different vacua. In other words, there is a superselection rule in  $F_0$ , no vacuum is preferred over any other, and any choice of  $F_0$  is good. In this respect  $F_0$  vacua are similar to theta vacua in QCD [12]. As it is well known, in QCD with no massless quarks there is a continuum of *physically distinct* vacuum states that can be parametrized by a periodic variable  $\theta$ . These  $\theta$  vacua obey the superselection rule; there is no transition between the states with different  $\theta$ . This situation gives rise to the celebrated strong  $CP$  problem, since the phenomenologically acceptable vacua, with  $\theta < 10^{-9}$ , are not particularly preferred by the system.

The situation in our case is analogous. The model (5) has a continuum of the vacuum states, scanned by  $F_0$  or equivalently by  $\langle \phi \rangle$ , and there is no transition between the different  $\langle \phi \rangle$  vacua. Thus, the hierarchy problem, from the problem of UV instability of the Higgs mass, is promoted to the superselection problem, analogous to the strong  $CP$  problem in QCD.

In order to solve the former, we shall try to follow, as closely as possible, the general strategy adopted by the axion solution of the strong  $CP$  problem. As it is well known, this solution is based on (a) first promoting  $\theta$  into a dynamical variable (axion), and (b) showing that  $\theta = 0$  is the true ground state of the system.

Thus, in order to solve the hierarchy problem we need to accomplish two more steps.

- (1) Promote  $F_0$  into a dynamical variable.
- (2) Find the symmetry reason that will ensure that the vacua are not uniformly distributed and that the vacua with the light Higgs are preferred over all the others.

As we shall see, the first step is achieved by the introduction of branes which permit the transition between the different  $\phi$  vacua. The difference with the axion case is that the transition is quantum mechanical and discrete, as opposed to being classical and continuous. This circumstance creates a profound difference between our solution of the hierarchy problem and the axion solution of the strong  $CP$  one. The impossibility of the continuous classical transition tells us that we cannot directly generalize the axion mechanism of  $\theta$  relaxation, but it also suggests a natural substitution. Indeed, because there is no classical transition, the energy difference is unimportant for selecting the vacuum state, and what matters is the multiplicity of vacua with a given value of  $\phi$ . We shall see that the hierarchy problem is solved, because an infinite number of vacua cluster around the small  $\phi$ -value, due to the symmetry reason.

In what follows we shall give a detailed discussion of this phenomenon.

## B. The role of the branes: Breaking the superselection

As said above, the superselection rule is lifted by introducing two-branes (membranes) that source our three-form field  $C_{\alpha\beta\gamma}$ . Ignoring the Higgs field for a moment, the effective action incorporating the interaction with branes can be written as

$$\frac{q}{6} \int_{2+1} d^3 \xi C_{\mu\nu\alpha} \left( \frac{\partial Y^\mu}{\partial \xi^a} \frac{\partial Y^\nu}{\partial \xi^b} \frac{\partial Y^\alpha}{\partial \xi^c} \right) \epsilon^{abc} - \int_{3+1} \frac{1}{48} F^2, \quad (10)$$

where the first explicitly written term describes the interaction between the brane and the three-form.  $q$  is the charge of the brane, and  $x^\mu = Y^\mu(\xi)$  specify a 2 + 1-dimensional history of the brane in 3 + 1 dimensions as a function of its world-volume coordinates  $\xi^a$  ( $a = 0, 1, 2$ ). This term can be rewritten in the form of the following four-dimensional integral,

$$\int d^4 x \frac{1}{6} J^{\alpha\beta\gamma} C_{\alpha\beta\gamma} \quad (11)$$

where  $J^{\alpha\beta\delta}$  is the brane current

$$J^{\alpha\beta\gamma}(x) = \int d^3 \xi \delta^4(x - Y(\xi)) q \left( \frac{\partial Y^\alpha}{\partial \xi^a} \frac{\partial Y^\beta}{\partial \xi^b} \frac{\partial Y^\gamma}{\partial \xi^c} \right) \epsilon^{abc}. \quad (12)$$

Obviously, the current  $J_{\alpha\beta\gamma}$  is conserved as long as  $q$  is a constant.

The brane self-action has the standard form

$$-T \int d^3 \xi \sqrt{-g}, \quad (13)$$

where  $T$  is the brane tension (a mass per unit surface), and  $g_{ab} = \partial_a Y^\mu \partial_b Y^\nu \eta_{\mu\nu}$  is the induced metric on the brane. Note that, since the bulk four-dimensional gravity plays no

essential role in our considerations, we have taken a flat Minkowskian four-dimensional metric  $\eta_{\mu\nu}$ . Despite this, the induced metric on the brane is not flat in general, due to the dynamical curving of the brane. With the brane source taken into the account, the equation of motion of the three-form now becomes

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q \int d^3 \xi \delta^4(x - Y(\xi)) \left( \frac{\partial Y^\nu}{\partial \xi^a} \frac{\partial Y^\alpha}{\partial \xi^b} \frac{\partial Y^\beta}{\partial \xi^c} \right) \epsilon^{abc}. \quad (14)$$

The brane can be taken to be flat and static,  $Y^\mu = \xi^\mu$  for  $\mu = 0, 1, 2$ , and  $Y^3 = 0$ . The equation of motion then simplifies to

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q \delta(z) \epsilon^{\nu\alpha\beta z} \quad (15)$$

where  $z = 0$  is the location of the brane. Both (14) and (15) show that the brane separates the two vacua in either of which  $F_0$  is constant, and the two values differ by  $|q|$ . Thus, the introduction of branes ensures that the transition between the vacua with different values of  $F_0$  is possible, as long as the value of  $F_0$  changes by the integer multiple of  $q$ . Hence the discrete quantum transition between the different vacua is possible via nucleation of closed branes (this fact was used in an interesting attempt [22] to explain the smallness of the cosmological term).

In other words, the theory given by the action (10) has multiplicity of vacua that can be labeled by an integer  $n$ . The value of the field strength in this vacua is

$$-\frac{1}{24} F_{\alpha\beta\gamma\mu} \epsilon^{\alpha\beta\gamma\mu} = F_0 = qn + f_0, \quad (16)$$

where  $f_0$  is a constant, which we will set equal to zero. That is, the value of  $F$  is quantized in units of the brane charge. Restoring the coupling to the Higgs field, the equation (9) determining the Higgs VEV now becomes

$$\left( -m^2 + \frac{(nq)^2}{2M^2(1 - \frac{|\phi|^2}{M^2})^2} \right) \phi + \lambda |\phi|^2 \phi = 0. \quad (17)$$

The good news is that now the transition between the vacua with different  $n$  is possible; however, the hierarchy problem is still not solved. First, because both  $m$  and  $M$  are large, we need a very small  $q$  in order to ensure a fine enough scanning of the Higgs mass. Second, even for a small  $q$ , the vacua with small Higgs VEV are not preferred over the others. Both problems can be cured in one shot, by requiring a symmetry which will promote  $q$  into a continuous function of  $\phi$ . For example,

$$q \rightarrow q(\phi) \propto \phi^N. \quad (18)$$

Then, the zero of the function  $q(\phi)$  will become an attractor (accumulation of an infinite number of vacua) in the space of vacua. This will guarantee both the superfine scanning of the Higgs mass and the preference of the vacua with the small values of the Higgs VEV. Thus, the idea of

the attractor is that the multiplicity of the vacua with the small values of the Higgs VEV becomes divergent (or at least very sharply peaked), because of the symmetry reasons.

The essence of the attractor phenomenon can be summarized schematically in the following sequence (written in units of the fundamental scale):

$$(q = \phi^N) \rightarrow (\Delta F = q) \rightarrow \left(\frac{\Delta\phi}{\phi} \sim q\right) \rightarrow (\Delta q \sim q\phi^{N-2}). \quad (19)$$

The arrows indicate that a nonzero charge of the brane separating the two different vacua implies the change of  $F$  across the brane, which implies the change of  $\phi$ . The latter implies the change of the brane charge in a new vacuum, and closes the cycle. We shall give a detailed discussion of the dynamics of the above sequence throughout the paper.

### C. Charge conjugation: Creating an attractor

Thus, to achieve an attractor, we must promote the brane charge  $q$  into the function of  $\phi$ , such that  $q$  vanishes for small values of  $\phi$ . To guarantee this, we shall require a new discrete symmetry  $Z_{2N}$ ,

$$\phi \rightarrow e^{i(\pi/N)} \phi, \quad (20)$$

which acts on the brane as ‘‘charge conjugation.’’ That is, we require that  $q \rightarrow -q$  under  $Z_{2N}$ . The invariance under  $Z_{2N}$  then demands  $q$  to be an odd function of  $\phi^N$ . The simplest choice is

$$q = q_{\text{eff}}(\phi) = \frac{\mu}{2} \left(\frac{\phi}{M_P}\right)^N + \text{H.c.} \quad (21)$$

where  $\mu$  is some constant.

For successful implementation of the above idea, we need to address the following technical issue. For non-constant  $q$  the current (12) is not conserved, and hence the coupling (11) is not gauge invariant. In order to maintain the gauge invariance, following [1], we shall modify this coupling in the following way:

$$- \int d^4x \frac{1}{6} C_{\alpha\beta\gamma} J_{(T)}^{\alpha\beta\gamma} \quad (22)$$

where  $J_{(T)}$  is the transverse part of the current

$$J_{(T)}^{\alpha\beta\gamma} = \frac{1}{6} \Pi_{\mu}^{[\alpha} J^{\mu\beta\gamma]}. \quad (23)$$

Here  $\Pi_{\mu\nu} = \eta_{\mu\nu} - (\partial_{\mu}\partial_{\nu}/\partial^2)$  is the transverse projector.<sup>2</sup>

For constant  $q$ , we have  $\partial^{\alpha} J_{\alpha\beta\gamma} = 0$  and  $J_{(T)} = J$ . Thus, the coupling (22) reduces to (11). This fact accomplishes our goal. In each given vacuum the expectation value of the

<sup>2</sup> $J_{(T)}$  can also be written in the following form,  $J_{(T)}^{\alpha\beta\gamma} = \frac{1}{6} \epsilon^{\alpha\beta\gamma\mu} \Theta_{\mu}^{\nu} \epsilon_{\nu\tau\rho\omega} C^{\tau\rho\omega}$ , where  $\Theta_{\mu\nu} = \partial_{\mu}\partial_{\nu}/\partial^2$  is the longitudinal projector.

Higgs is fixed, and so  $q$  is constant. So in each vacuum with an unexcited Higgs field the brane couples to the three-form in a normal way. On the other hand, the change of  $q$  from vacuum to vacuum is permitted, because  $C_{\alpha\beta\gamma}$  only couples to the transverse part of  $J_{\alpha\beta\gamma}$ . The existence of the attractor point at  $\phi = 0$  is guaranteed by the fact that  $J_{(T)} \rightarrow 0$  when  $\phi \rightarrow 0$ .

The coupling (22) is the gauge-invariant generalization of (11) for the case of a nonconstant charge  $q(\phi)$ . For the constant  $\phi$  the above coupling is equivalent to (10) with  $q_{\text{eff}} = \mu \text{Re}(\phi/M_P)^N$ . Although the coupling (22) contains a projector, it actually can be obtained from a local underlying theory after integrating out certain degrees of freedom. This issue is discussed in the Appendix A. It is shown there how the coupling (22) can be obtained by integrating out the Goldstone-type degrees of freedom *à la* Stückelberg. In a very crude sense, the Stückelberg field plays the role analogous to the one of the axion in the solution of the strong  $CP$  problem. Some alternatives to the Stückelberg method will be discussed in [23].<sup>3</sup>

Putting all the ingredients together, let us now show that, with  $Z_{2N}$  symmetry, theory has an attractor point in the space of vacua at  $\phi = 0$ . For convenience, we write down the combined Lagrangian, which takes the following form:

$$L = |\partial_{\mu}\phi|^2 - \frac{1}{48} F^2 + |\phi|^2 \left( m^2 + \frac{F^2}{48M^2} \right) - \frac{\lambda}{2} |\phi|^4 - \frac{1}{6} C_{\alpha\beta\gamma} J_{(T)}^{\alpha\beta\gamma}. \quad (24)$$

The above theory admits the divergent number of vacua at small VEV of  $\phi$ , at least when<sup>4</sup>

$$m \gg \frac{\mu}{\sqrt{2}M} \left( \frac{m}{\sqrt{\lambda}M_P} \right)^N. \quad (25)$$

In order to see this, we shall integrate out the brane and the three-form field and write down the effective potential for  $\phi$ . Choosing the brane to be located at  $z = 0$ , the equations are

$$\partial^{\mu} \left( (1 - |\phi|^2/M^2) F_{\mu\alpha\beta\gamma} \right) = \mu \epsilon_{\alpha\beta\gamma\nu} \Theta^{\nu z} \left[ \text{Re} \left( \frac{\phi}{M_P} \right)^N \delta(z) \right], \quad (26)$$

$$\partial^2 \phi - \left( m^2 + \frac{F^2}{48M^2} \right) \phi + \lambda |\phi|^2 \phi + \frac{\mu N}{12} \frac{\phi^{*N-1}}{M_P^N} \delta(z) \Theta_{z\nu} \epsilon^{\nu\alpha\beta\gamma} C_{\alpha\beta\gamma} = 0. \quad (27)$$

<sup>3</sup>We thank Gregory Gabadadze for enlightening discussions on this and other issues.

<sup>4</sup>The meaning of this constraint is to avoid ‘‘overshooting’’ to the vacua with the large positive Higgs mass, in case we start in a vacuum with  $F = 0$ , in which the Higgs VEV is maximal, and so is the brane charge.

In integrating these equations, for simplicity, we will first make the following approximation which is well justified in the vacua with the small VEVs of  $\phi$  (a more rigorous derivation is discussed in Appendix C).

- (1) We shall ignore terms of order  $\phi^2/M^2$  in the gauge-kinetic function of  $F$  in the left-hand side (l.h.s.) of Eq. (26).
- (2) We shall ignore the terms proportional to the derivatives of  $q_{\text{eff}}$  in the right-hand side of (26).

The above is justified, because the change in  $q$  in each elementary transition between the small  $\phi$  vacua is very small. So the correction to  $\Delta F_0$  because of the change in  $q$  is of the higher order smallness in  $\phi/M_P$  and can be ignored.

Indeed, the change in  $\phi$  ( $\Delta\phi$ ), in each elementary step that connects the neighboring vacua, is due to change of  $F$ , which is  $\Delta F \simeq q_{\text{eff}}$ . Because the VEV of  $\phi$  in any vacuum is given by (6), the change in  $\phi$  is

$$\Delta(\phi^2) \simeq \frac{\Delta F_0 F_0}{\lambda M^2} \simeq q_{\text{eff}} \frac{F_0}{\lambda M^2}. \quad (28)$$

Thus, even if the attractor point happens to be for the maximal value  $F \sim M^2$ , the change  $\Delta\phi^2 \sim q_{\text{eff}}$ , and consequently

$$\Delta q_{\text{eff}} = \frac{N\mu}{2} \frac{\phi^{N-2}}{M_P^N} \Delta(\phi^2) \sim q_{\text{eff}} \frac{\mu\phi^{N-2}}{M_P^N}. \quad (29)$$

Thus, the change in the brane charge is higher order in  $\phi/M_P$ . So, around the attractor point ( $\phi \rightarrow 0$ ), this sub-leading correction can be safely ignored in each elementary step and  $q_{\text{eff}}$  can be regarded as constant. Only after many steps can the accumulated change in  $\phi$  become significant.

This fact simplifies the equation (26). Integrating it for the constant  $q_{\text{eff}}$  we get that  $F$  is given by (4) with

$$F_0 = nq_{\text{eff}}, \quad (30)$$

where  $n$  is an integer that labels the different vacua. Plugging this result into the equation for  $\phi$  we get the effective equation defining the VEV of  $\phi$ ,<sup>5</sup>

$$-\left(m^2 - n^2 \frac{\mu^2}{2M^2} \left(\frac{\phi^N}{M_P^N}\right)\right)\phi + \lambda\phi^3 = 0. \quad (31)$$

Thus,  $n$  labels different vacua. It is obvious that there are an infinite number of vacua close to  $\phi = 0$ . Thus, the vacuum with a vanishing VEV is an *attractor*.

We should stress that  $Z_{2N}$  symmetry guarantees the UV stability of the attractor point. Indeed, the attractor point is the vacuum in which the brane charge vanishes,  $q = 0$ . Because of  $Z_{2N}$  symmetry, this happens when  $\phi = 0$ . Thus, any renormalization of  $\phi$  implies the corresponding

<sup>5</sup>Without the loss of generality we can put the VEV of  $\phi$  to be real.

renormalization of the brane charge, such that  $\phi = 0$  remains an attractor point.

#### D. Counting the number of vacua

We shall now give a simple general rule for counting the number of vacua near the attractor point. For simplicity, we set the parameter  $\lambda \sim 1$ . We wish to estimate the number of vacua in which the expectation value of the Higgs field is of the order of a given value  $\phi_0 \ll M$ . The number of the vacua in which the Higgs VEV is  $\phi \sim \phi_0$ , we shall denote by  $n_{\phi_0}$ . There is a simple way to estimate this number. We start with a vacuum with a given VEV  $\phi = \phi_0$  and begin lowering  $\phi$  by jumping to new vacua via creating branes, until the VEV changes in first nonzero digit. The VEV of  $\phi$  in each vacuum is given by (6). The change of  $\phi$  in each elementary step is given by (28) and is small, because  $q$  is small. The number of vacua  $n_{\phi_0}$  is equal to the number of steps that will make the accumulated relative change of  $\phi$  of order one. That is,  $n_{\phi_0}$  is defined by the condition that the quantity

$$\frac{\Delta\phi}{\phi_0} \sim \frac{qn_{\phi_0}}{\phi_0^2} \quad (32)$$

should become of order one. Thus, the number of vacua is

$$n_{\phi_0} \sim \frac{\phi_0^2}{q} \sim \frac{M_P^N}{\mu\phi_0^{N-2}}, \quad (33)$$

which for  $N > 2$  diverges as  $\phi_0 \rightarrow 0$ .

The above expression should not create a false impression that there is no attractor for  $N = 2$  or less. The equation (33) indicates that the number of vacua in which  $\phi$  is of the order of a given value  $\phi_0$  diverges as  $\phi_0 \rightarrow 0$ , as long as  $N > 2$ . For smaller  $N$ , this number is either constant or decreases with  $\phi_0$ , but the total number of vacua accumulated around  $\phi = 0$  is always infinite.

In other words, the number of vacua with the Higgs VEV in an interval  $M_P > \phi > \phi_0$  is given by the sum

$$n_{(>\phi_0)} = \sum n_{\phi_0} \quad (34)$$

where the sum is taken over all the discrete vacua. As said above, for the condition (25) the vacua can be arranged into the groups labeled by  $\phi_0$ . In any vacuum belonging to the  $\phi_0$ th group the VEV of the Higgs is equal to  $\phi_0$  in the first nonzero digit, and the number in each group is given by  $n_{\phi_0}$ . Although for  $N = 2$  or less this number either stays constant or decreases with  $\phi_0$ , the density of the groups grows as  $1/\phi_0^2$  and overcompensates the decrease of  $n_{\phi_0}$  for any  $N > 0$ . To put it shortly, for the small  $\phi_0$  the sum can be approximated by the following integral,

$$n_{(>\phi_0)} \sim M_P \int_{\phi_0}^{M_P} n_{\phi} \frac{d\phi}{\phi^2} \quad (35)$$

which shows that even for  $N = 1$  the total number of vacua

diverges as  $\log(\phi_0)$ . The divergence of  $n_{(>\phi_0)}$  for  $\phi_0 \rightarrow 0$  for  $N > 0$  can also be seen from (31), which for  $N = 1$  gives the following expression for the VEV of  $\phi$ :

$$|\phi_0|^2 = \frac{m^2}{\lambda + n^2(\mu^2/2M^2M_p^2)}. \quad (36)$$

This VEV approaches  $\phi_0 = 0$  for  $n \rightarrow \infty$ .

#### IV. THE WEAK SCALE SHIFT OF THE ATTRACTOR

We have shown that a theory in which the brane charge  $q$  is set by the VEV of a scalar field has a divergent number of vacua with the values of the scalar field for which  $q \rightarrow 0$ . This is the essence of the attractor phenomenon. In the above example the charge conjugation  $Z_{2N}$  symmetry guaranteed that the attractor point was at  $\phi = 0$ . However, in order to solve the hierarchy problem in the standard model, we have to make sure that the attractor point is not at zero, but instead at the observed value  $\phi \sim 100$  GeV. As it was shown in [1], when the attractor theory is considered in the cosmological context of an eternally inflating universe, the attractor point can be shifted to the value of the Hubble parameter during inflation. The solution of the hierarchy problem then would require this parameter to be around the weak scale or so.

Despite the cosmological possibilities, it is important to have other mechanisms that could generate the small shift of the attractor point in a cosmology-independent way. One universal possibility is to shift the brane charge by a small  $\phi$ -independent amount

$$q_{\text{eff}} \rightarrow \frac{\mu}{2} \left( \frac{\phi^N}{M_p^N} - \xi \right) + \text{H.c.} \quad (37)$$

where  $\xi$  is a  $\phi$ -independent part. We should think of  $q_0$  as of ‘‘spurion,’’ the VEV that spontaneously (softly) breaks  $Z_{2N}$  symmetry. It is important to stress that even for constant  $\xi$ , because  $\xi = 0$  is an enhanced symmetry point, the value of  $\xi$  is *perturbatively stable*. The attractor point then will be shifted to

$$\phi = (\xi)^{1/N} M_p. \quad (38)$$

The origin of  $\xi$  is model dependent. We shall explore two possibilities. In Sec. V, we will show that  $\xi$  can come from the QCD condensate of the standard model quarks.<sup>6</sup> Another avenue, to be discussed in the next section, is to take into account the possible impact of the brane-localized mass term of the Higgs field. As we will show, such terms can have nontrivial effects on the attractor dynamics.

<sup>6</sup> $\xi$  can also be set by the expectation value of some hidden sector fermionic condensate, which has no direct interactions with standard model particles.

#### V. THE EFFECT OF THE BRANE-LOCALIZED POTENTIAL

The  $\phi$  field may have various potential terms on the brane world volume, compatible with symmetries. The most important of these is a brane-localized mass term, which can be introduced in the four-dimensional action in the following form,

$$- \int d^4x M_{\text{br}}(x)^2 |\phi|^2, \quad (39)$$

where

$$M_{\text{br}}^2(x) = \pm \int d\xi^3 \sqrt{-g} M_B \delta^4(x - Y). \quad (40)$$

In the above expression  $M_B$  is a positive mass parameter. The question is as follows: what is the effect of this brane-localized mass term on the attractor dynamics? We shall show that this effect depends on the sign in (66). For the positive sign the attractor at  $\phi = 0$  becomes sharper, meaning that the number of vacua diverges faster for  $\phi \rightarrow 0$ . In the case of the negative sign, the two subregimes are possible, depending on the parameters. One possibility is that the attractor point is shifted away from zero, but the divergence of vacua is kept intact. Another possibility is that the attractor becomes ‘‘softer,’’ meaning that the brane charge cannot decrease below a certain minimal value. In the latter case, the scanning of the Higgs VEV cannot get finer beyond the certain minimal step, and correspondingly the divergence in the number density of vacua gets cut off at some maximal value. We shall now give a more detailed discussion of the above two regimes.

##### A. A positive mass term: Sharpening the attractor

Consider the effect of the positive mass term first. Such a mass term ‘‘repels’’ the  $\phi$  field from the brane, and effectively diminishes its VEV at the brane location. Ignoring the effect of  $\sim q_{\text{eff}}$ -terms, the equation for  $\phi$  in the background of the brane located at  $z = 0$  is

$$\partial^2 \phi - (m_{\text{bulk}}^2 - \delta(z)M_B)\phi + \lambda\phi^3 = 0 \quad (41)$$

where  $m_{\text{bulk}}^2$  is the effective bulk mass term which includes the contribution coming from  $F$ ,

$$m_{\text{bulk}}^2 = m^2 + F^2/2M^2. \quad (42)$$

From (41) it is clear that the positive brane-localized mass term is seen by the field as a potential barrier, and for  $M_B \gg m_{\text{bulk}}$  the expectation value at the brane location  $\phi(0)$  is considerably smaller than its bulk counterpart  $\phi(\infty)$ .

$\phi(0)$  can be estimated by minimizing the following expression (we ignore the factors of order one):

$$E = M_B \phi(0)^2 + (\phi(0) - m_{\text{bulk}})^2 m_{\text{bulk}} + (\phi(0)^2 - m_{\text{bulk}}^2)^2 m_{\text{bulk}}^{-1}. \quad (43)$$



The first term in this expression comes from the brane mass term. The second and the third terms are the expenses in the gradient and the bulk potential energies. The full expression is minimized at

$$\phi(0) \sim \frac{m_{\text{bulk}}^2}{M_B}. \quad (44)$$

Thus, in any given vacuum, the brane expectation value of  $\phi$  is by the factor  $m_{\text{bulk}}/M_B$  smaller than its bulk counterpart  $\phi(\infty) \sim m_{\text{bulk}}$ . Thus, for a given value of the bulk mass term, the value of the brane charge  $q_{\text{eff}}$  is smaller by a factor of  $(m_{\text{bulk}}/M_B)^N$ , as compared to what it would be for  $M_B = 0$ . Correspondingly, according to the general rule of vacuum counting, the number of vacua with a given VEV of the Higgs field  $\phi \sim \phi_0$  becomes

$$n_{\phi_0} \sim \left( \frac{M_P^N}{\mu \phi_0^{N-2}} \right) \left( \frac{M_B}{\phi_0} \right)^N, \quad (45)$$

and is by a factor of  $(M_B/\phi_0)^N$  bigger than what it would be for  $M_B = 0$ . Thus, the positive sign brane-localized mass term makes the attractor stronger.

### B. A negative mass: Smoothing out the attractor

Now let us show that in the case of the negative sign the brane-localized mass term has an opposite effect, and may either shift the attractor or cut off the divergence of the number of vacua. This is because, for the negative brane mass term,  $\phi$  develops a nonzero expectation value on the brane even for  $m_{\text{bulk}}^2 = 0$  (that is, when the bulk VEV is zero). Thus, the brane continues to have a nonzero charge  $q$  even in  $\phi = 0$  vacua, and the step of change stays finite. As a result, the number of vacua gets cut off.

Let us show that, for the negative sign of the brane mass term,  $\phi$  indeed develops a nonzero value on the brane in the limit  $m_{\text{bulk}}^2 \rightarrow 0$ . The fact that  $\phi$  wants to condense on the brane can be seen by examining the linearized equation for small perturbations about the  $\phi = 0$  solution in the brane background. This equation has the following form:

$$(\partial^2 - \delta(z)M_B)\phi = 0. \quad (46)$$

It is obvious that there is a normalizable exponentially growing tachyonic mode, localized on the brane,

$$\phi = e^{(1/2)M_B t} e^{-(1/2)|z|M_B}. \quad (47)$$

This instability signals that  $\phi$  condenses on the brane and develops a nonzero expectation value there. This condensate is  $\phi(0) \sim M_B$ , since for  $m_{\text{bulk}} = 0$ ,  $M_B$  is the only mass scale in the problem. Hence, the expectation value on the brane is  $\sim M_B$  even though the bulk VEV vanishes.

This fact has profound implications for the attractor dynamics, since now even in the vacua with  $m_{\text{bulk}} = 0$  the brane charge is nonzero and is given by

$$q_{\text{min}} \sim \mu(M_B/M_P)^N. \quad (48)$$

To see what these implications are, first note that the value of  $F_0$  in the vacuum with  $m_{\text{bulk}} = 0$  is  $F_0^2 = 2M^2 m^2$ . Then, if

$$M_B^2 \gg 4\sqrt{2} \frac{m}{M} q_{\text{min}}, \quad (49)$$

the attractor will be shifted to a positive value  $m_{\text{bulk}}^2 = M_B^2/4$ . In the opposite limit,

$$M_B^2 < 4\sqrt{2} \frac{m}{M} q_{\text{min}}, \quad (50)$$

the attractor will be regulated, and the divergence in the vacuum number density will be cut off at

$$n_{\text{max}} \sim \frac{mM}{q_{\text{min}}}. \quad (51)$$

The above two regimes can be understood from the fact that the equation

$$(\partial^2 - m_{\text{bulk}}^2 - \delta(z)M_B)\phi = 0 \quad (52)$$

has an exponentially growing normalizable tachyonic mode

$$\phi = e^{(1/2)t\sqrt{M_B^2 + 4m_{\text{bulk}}^2}} e^{-(1/2)|z|M_B}, \quad (53)$$

as long as<sup>7</sup>

$$M_B^2 + 4m_{\text{bulk}}^2 > 0. \quad (54)$$

Thus, only in this regime does  $\phi$  develop a nonzero VEV in the vicinity of the brane. If (50) holds in the  $m_{\text{bulk}} = 0$  vacuum, then the condition (54) will be violated within a single step, and the branes will become chargeless. Thus,  $q_{\text{min}}$  is the smallest possible nonzero brane charge. Because of this, the singularity in the number density of vacua will get smoothed out at this point, according to (51).

## VI. REALISTIC MODEL BUILDING AND PREDICTIONS

### A. The need for a second $SU(2) \times U(1)$ doublet

So far we have been discussing the attractor solution of the hierarchy problem on a toy example in which the prototype for the standard model Higgs was a complex singlet  $\phi$ . In order to implement this idea in the realistic model, we have to promote  $\phi$  into the doublet representation of the  $SU(2) \times U(1)$  group. This creates an issue of how to write down the gauge-invariant interaction with the brane. Since  $C_{\alpha\beta\gamma}$  carries no electroweak quantum numbers,  $q(\phi)$  must be an  $SU(2) \times U(1)$ -invariant, but  $Z_{2N}$ -odd function of  $\phi$ . This is, however, impossible to achieve by employing a single Higgs doublet, since the

<sup>7</sup>For  $M_B^2 + 4m_{\text{bulk}}^2 = 0$  there is a localized zero mode  $\phi = \beta e^{-(|z|M_B/2)}$  that can be given an arbitrary expectation value without costing any energy up to bilinear in  $\phi$ . In other words, for  $\lambda = 0$  there is a one-parameter class of the zero energy solutions with arbitrary  $\beta$ .

only possible gauge-invariant  $\phi^* \phi$  is also automatically a  $Z_{2N}$  invariant. This is unacceptable, since  $Z_{2N}$  oddness of the brane charge is what guarantees the UV stability of the attractor point. Fortunately, there are a number of ways for circumventing the above technicality. Here we shall discuss one of the simplest and most economical ones.

### B. Quark condensate as a second doublet

The QCD condensate of the light quarks in the standard model carries the quantum numbers identical to the Higgs doublet and contributes to the Higgsing of  $SU(2) \otimes U(1)$  symmetry. In fact, because of this quark condensate, the electroweak  $W$  and  $Z$  bosons would get masses even in the absence of the Higgs scalar [20]. We can use this condensate in convolution with the Higgs doublet for creating the gauge-invariant (but  $Z_{2N}$ -odd) brane charge. There are two lowest possible gauge invariants (per generation) that can be made out of the Higgs and the quark bilinears. These are [ $SU(2)$  indices are suppressed]

$$\phi \bar{Q}_L U_R \quad (55)$$

and

$$\phi^* \epsilon \bar{Q}_L D_R, \quad (56)$$

respectively. Here  $Q_L$  is the left-handed quark doublet, and  $U_R, D_R$  are the right-handed up and down quarks, respectively.  $\epsilon$  is an antisymmetric  $SU(2)$  tensor. Because of the QCD quark condensate, the expectation value of these invariants is nonzero as long as  $\phi \neq 0$ . Thus, any of these invariants can be used for creating the attractor at  $\phi = 0$ . For this we have to require that the given invariant, e.g., (55) (or any power of it) is odd under the brane conjugation symmetry. For example,

$$(\phi \bar{Q}_L U_R) \rightarrow e^{i(\pi/N)} (\phi \bar{Q}_L U_R). \quad (57)$$

Then the brane charge becomes the function of (55)

$$q = \frac{\mu}{6} \operatorname{Re} \left( \frac{\phi \bar{Q}_L U_R}{M_P^4} \right)^N. \quad (58)$$

Note that, since (55) transforms nontrivially under  $Z_{2N}$  conjugation, it cannot appear in the Lagrangian. Thus, the standard model Yukawa coupling that could give a diagonal mass to the given  $U$  quark is forbidden in this theory. This fact highlights a generic feature of the model: irrespective of which quark condensate creates a brane charge, appearance of certain zeros in the Yukawa matrix is the generic prediction. Precise structure depends on the  $Z_{2N}$  charge assignment, and will not be discussed here, but it is an important aspect for understanding the predictivity of the attractor solution.

### C. A complete model

Putting all the ingredients together, we can now write down a simple extension of the standard model which

solves the hierarchy problem via the attractor mechanism. The action is

$$S = \int_{3+1} |D_\mu \phi|^2 - \frac{1}{48} F^2 + |\phi|^2 \left( m^2 + \frac{F^2}{48M^2} \right) - \frac{\lambda}{2} |\phi|^4 + \frac{1}{6} C_{\alpha\beta\gamma} J_{(T)}^{\alpha\beta\gamma} - M_{\text{br}} |\phi|^2, \quad (59)$$

plus the usual action of the standard model. In (59)  $D_\mu$  is the covariant derivative.  $M_{\text{br}}$  is the brane-localized mass term given by (66), and we choose the sign to be positive. The current  $J_{(T)}^{\alpha\beta\gamma}$  is given by (12) with

$$q_{\text{eff}} = \frac{\mu}{12} \left[ \left( \frac{\phi \bar{Q}_L U_R}{M_P^4} \right)^N - \left( \frac{\bar{Q}_L U_R \bar{Q}_L D_R}{M_P^6} \right)^K \right] + \text{H.c.} \quad (60)$$

In this model we choose to use the quark condensate as a second Higgs doublet in order to construct a gauge-invariant brane charge. The attractor value is shifted away from  $\phi = 0$  by the second quark condensate. For the latter invariant to be nonzero the quarks must be taken from different generations. The integers  $N$  and  $K$  are determined by the transformation properties of the various fields under the discrete symmetry. These transformation properties also restrict the structure of Yukawa matrix elements. For instance, as said above for arbitrary  $N$ , the diagonal Yukawa coupling of the up quark is forbidden.

Taking all these terms into account, the attractor value for the bulk Higgs VEV is

$$\phi_{\text{attr}} \sim \sqrt{M_B M_P} \left( \frac{\Lambda_{\text{QCD}}}{M_P} \right)^{[3(K/N) - (3/2)]} \quad (61)$$

where  $\Lambda_{\text{QCD}} \sim \text{GeV}$  is the strong interaction scale. For  $M_B \sim M_P \sim 10^{19} \text{ GeV}$ , the correct attractor value is established around  $K/N \simeq 5/6$  or so. Since the brane charge-conjugation symmetry implies nontrivial restrictions on the matrix of Yukawa couplings, it would be interesting to classify predictions of fermion mass relations for various assignments that lead to the correct attractor value.

We wish to note that other standard model parameters, such as, for example, Yukawa coupling constants, can (and, in general, will) depend on the values of  $F$  and  $\phi$  through some high-dimensional  $M_P$ -suppressed operators. The question then is how the attractor influences the values of such parameters. To answer this question it is useful to classify parameters by their transformation properties under  $Z_{2N}$ . The general rule then is that the parameters that are  $Z_{2N}$  even do not change significantly near the attractor point, since in the zeroth order such parameters do not depend on the  $\phi$  VEV. Examples of  $Z_{2N}$ -even parameters are, for instance, all the Yukawa coupling constants of the couplings that are allowed by the  $Z_{2N}$  symmetry. The dependence of such a coupling constant on  $\phi$  and  $F$  can be parametrized in the form of the expansion in a series of invariants  $F^2/M_P^4$  and  $|\phi|^2/M_P^2$ ,

$$g = g_0 + g_1 F^2/M_P^4 + g_2 |\phi|^2/M_P^2 + \dots \quad (62)$$

where  $g_0, g_1, g_2$  are field-independent constants. It is obvious that, unless  $g_0$  is minuscule, the change of  $g$  near the attractor point is negligible, since both  $\Delta F$  and  $\Delta\phi$  vanish there. So, generically, at the attractor point the expectation values of  $Z_{2N}$ -even parameters will be set by some attractor-insensitive physics.

#### D. The heavy Higgs doublet

An alternative to the quark condensate for creating an  $SU(2) \times U(1)$ -invariant brane charge is to introduce a second Higgs doublet  $H$ . We shall assume that  $H$  has a positive mass square  $M_H^2$  of order  $M_P^2$ , and has no expectation value in the bulk vacuum. However,  $H$  is allowed by symmetries to have a large brane-localized mass term. We shall choose the sign of this mass term to be negative so that  $H$  develops a nonzero VEV on the brane. This is sufficient for forming the  $Z_{2N}$ -odd but  $SU(2) \times U(1)$ -invariant brane charge out of the two doublets.

To achieve this we shall require that the  $SU(2) \times U(1)$ -invariant product of the two doublets transforms under the brane ‘‘charge conjugation’’  $Z_{2N}$  symmetry

$$(H^* \phi) \rightarrow e^{i\pi/N} (H^* \phi). \quad (63)$$

The new  $H$ -dependent terms in the action are

$$S = \int_{3+1} -\frac{1}{6} C_{\alpha\beta\gamma} J_{(T)}^{\alpha\beta\gamma} + M_{\text{br}}(x)^2 |H|^2 + |D_\mu H|^2 - M_H^2 |H|^2 - \frac{1}{4} |H|^4 + \dots \quad (64)$$

The current  $J_{(T)}$  is given by (23) with

$$q_{\text{eff}} = \frac{\mu}{6} \text{Re} \left( \frac{H^* \phi}{M_P^2} \right)^N. \quad (65)$$

The second term is the brane-localized mass term

$$M_{\text{br}}^2(x) = \int d\xi^3 \sqrt{-g} M_{\text{BH}} \delta^4(x - Y), \quad (66)$$

where  $M_{\text{BH}} > 0$ . In addition, the total action will contain all possible  $SU(2) \times U(1) \times Z_{2N}$ -invariant couplings among  $H, \phi$ , and  $F$ . These are unessential and are not shown, for simplicity. The only requirement on such cross-interaction terms of the form  $|H|^2 |\phi|^2, |H|^2 F^2$  is that they do not make the effective bulk mass  $2$  of  $H$  negative. This is easy to arrange by choosing the signs of these interactions, and can be assumed to be the case without any loss of generality. Note that  $Z_{2N}$  symmetry forbids the appearance of the mixing term  $H^* \phi$  in the action.

As long as

$$M_{\text{BH}}^2 > 4M_H^2, \quad (67)$$

$H$  develops a nonzero VEV on the brane. This is because for (67) the linearized equation for  $H$  in the brane background (located at  $z = 0$ ),

$$(\partial^2 + M_H^2 - M_{\text{BH}} \delta(z)) H = 0, \quad (68)$$

has a localized exponentially growing tachyonic mode,

$$H = e^{(1/2)t\sqrt{M_{\text{BH}}^2 - 4M_H^2}} e^{-(1/2)M_{\text{BH}}|z|}. \quad (69)$$

Because  $H$  is nonzero on the brane, the effective brane charge (65) vanishes only for  $\phi \rightarrow 0$ , and the vacuum  $\phi = 0$  is an attractor.

#### VII. THE DOUBLET-TRIPLET SPLITTING

In grand unified theories (GUTs) the low energy supersymmetry alone cannot guarantee the smallness of the Higgs mass, due to the problem of doublet-triplet splitting. The problem can be illustrated on an example of a simplest  $SU(5)$  extension of the standard model. Because of  $SU(5)$  symmetry the Higgs doublet  $\phi$  acquires a color-triplet partner ( $T$ ), and the two together form the five-dimensional representation of the  $SU(5)$  group,  $5_{\text{Higgs}} = (\phi, T)$ . Thus, because of GUT symmetry both the weak doublet and the color triplet are forced to couple to quarks and leptons that transform as  $10 + \bar{5}$  dimensional representations per generation,

$$5_{\text{Higgs}} 10 10 + \bar{5}_{\text{Higgs}}^* 10 \bar{5}. \quad (70)$$

$SU(3) \times SU(2) \times U(1)$  reduction of the above coupling shows that the three-level exchange of the color-triplet Higgs violates the baryon number and would mediate an unacceptably fast proton decay, unless  $T$  acquires a very large mass due to GUT symmetry breaking. At the same time  $\phi$  should stay light, and this requires an additional fine-tuning not provided by supersymmetry alone. Let  $\Sigma_i^j$  be an  $SU(5)$ -adjoint Higgs that breaks the GUT symmetry [ $i, j = 1, 2, \dots, 5$  are  $SU(5)$  indices]. The doublet-triplet mass splitting is accomplished through the coupling of  $5_{\text{Higgs}}$  with  $\Sigma$ ,

$$5_{\text{Higgs}}^* (a \Sigma^2 + b \Sigma) 5_{\text{Higgs}} + (c \text{Tr} \Sigma^2 + m^2) 5_{\text{Higgs}}^* 5_{\text{Higgs}} \quad (71)$$

where  $a, b, c$  are some constants. After  $\Sigma$  develops the VEV  $\Sigma = \text{diag}(2, 2, 2, -3, -3)\sigma$ , the masses of  $\phi$  and  $T$  become split as

$$m_\phi^2 = (9a + 30c)\sigma^2 - 3b\sigma + m^2 \quad (72)$$

and

$$m_T^2 = (4a + 30c)\sigma^2 + 2b\sigma + m^2, \quad (73)$$

respectively. The additional fine-tuning amounts to setting (72) to  $\sim (100 \text{ GeV})^2$ .

We wish to point out now that the attractor solution of the hierarchy problem automatically solves the problem of doublet-triplet mass splitting in grand unified theories. Applying the attractor idea to GUTs simply amounts to promoting  $m^2$  in (72) and (73) into the function of the four-form field strength  $F$ . The  $SU(5)$ -symmetric generalization

of the brane charge (58) is straightforward,

$$q = \frac{\mu}{6} \operatorname{Re} \left( \frac{5_{\text{Higgs}} 10 10}{M_P^4} \right)^N, \quad (74)$$

where a standard  $SU(5)$ -invariant contraction of the indices  $(5_a 10_{bc} 10_{de} \epsilon^{abcde})$  is assumed. In the same way (65) gets generalized to

$$q_{\text{eff}} = \frac{\mu}{12} \left[ \left( \frac{5_{\text{Higgs}} 10 10}{M_P^4} \right)^N - \left( \frac{10 10 10 \bar{5}}{M_P^6} \right)^K \right] + \text{H.c.} \quad (75)$$

Then, just as in case of standard model,  $m_\phi^2$  is attracted to a small value, and at the same time  $m_T^2$  is attracted to a large one. Note that, depending of the parameters, because of  $SU(5)$  symmetry of the brane charge, there may be other attractor points, e.g., at  $m_T^2 = 0$ .

### VIII. DISCUSSIONS AND OUTLOOK

The attractor solution is probably as close as the axion-type dynamical relaxation mechanism could come to the solution of the hierarchy problem. Indeed, the first step of the attractor solution is exactly to bring the hierarchy problem on the same footing as the strong  $CP$  problem [12] in QCD. The latter, as we know, is not the problem of UV sensitivity but rather the problem of the superselection rule among the infinitely many  $\theta$  vacua. As we have shown in Sec. III A, this is precisely what happens to the hierarchy problem when we couple the Higgs to the three-form field: From the problem of UV sensitivity of the Higgs mass, it gets converted into a *superselection problem* of the latter. The continuum of the vacua scanned by  $m_\phi$  are all good, very much like QCD  $\theta$  vacua in QCD.

Having achieved this, we realize that there is a profound difference between the problem of UV sensitivity and the problem of superselection. The solution of the UV-sensitivity problem always requires a new strongly coupled physics at low energies, whereas the solution of the superselection problem does not. In the former case the new strongly coupled physics is required in order to regulate the quadratic divergency in the Higgs mass. An example for such a regulating physics is the low energy supersymmetry, whereas the new physics which solves the superselection problem can be *arbitrarily weakly* coupled. A good example of such new physics in the case of the strong  $CP$  problem is the axion, which can have an arbitrarily high scale and be practically invisible. Likewise, in our case the new physics that selected the attractor vacuum is arbitrarily decoupled, and can be practically unobservable at low energies. Nevertheless, as we have seen already the minimal realistic models can be predictive and potentially testable. The attractor is the mechanism through which the multivacua fundamental theory could make a sharp low energy prediction.

If the attractor solution of the hierarchy problem can be fully implemented in string theory, it will most likely have softer properties. For instance, the infinite number density of vacua will probably be regulated, as already suggested by the distribution of vacua in [4,5]. This should not be an obstacle for solving the hierarchy problem, provided the number of vacua is sharply peaked around the small Higgs mass. Moreover, as we saw in Sec. VB, the softened attractors can appear already in an effective field theory treatment.

In theories with attractor vacua, there are a number of open important questions, such as whether some version of the attractor mechanism could select the small cosmological constant. Some of these questions will be addressed in [23].

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### APPENDIX A: GAUGE INVARIANCE FROM GOLDSTONE FIELDS

In this appendix we shall discuss how the coupling (22) can be obtained by integration of Goldstone-type degrees of freedom in a local, gauge-invariant theory. Let us first illustrate the idea on an example of electrodynamics. Imagine that we wish to couple a photon  $A_\mu$  to a non-conserved current  $J_\mu$  ( $\partial_\mu J^\mu \neq 0$ ) in a gauge-invariant way. For example, such can be a 1 + 1-dimensional version of the current (12)

$$J^\mu(x) = \int d\xi \delta^2(x - Y(\xi)) q(Y) \left( \frac{\partial Y^\mu}{\partial \xi} \right) \quad (A1)$$

which, in general, is not conserved unless  $q$  is a constant. The conservation can be restored by introducing additional degrees of freedom that will compensate the divergence of (A1) for nonconstant  $q$ .

This can be accomplished by introducing a compensating Stückelberg field  $\theta$ . The coupling to the source then can be written in the following form:

$$(A_\mu - \partial_\mu \theta) J^\mu. \quad (A2)$$

This coupling will be gauge invariant if we demand that under the gauge transformation

$$A_\mu \rightarrow A_\mu + \partial_\mu \omega \quad (A3)$$

the compensating field shifts as

$$\theta \rightarrow \theta + \omega. \quad (\text{A4})$$

Hence,  $\theta$  is in fact a Goldstone field. Because of the gauge invariance, the Lagrangian can only depend on  $\theta$  through the combination  $(A_\mu - \partial_\mu \theta)$ . Since the current  $J_\mu$  is not conserved, to maintain the gauge invariance we have to impose an additional constraint,

$$\partial^2 \theta = \partial_\mu A^\mu. \quad (\text{A5})$$

Then, integrating out  $\theta$  we can write down an effective Lagrangian for  $A_\mu$ ,

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu \Pi_{\mu\nu} A^\nu. \quad (\text{A6})$$

We see that, after integrating out of the Stückelberg field, the photon only couples to the transverse part of  $J_\mu$ , as required by gauge invariance.

The generalization of the above construction to the case of the three-form field  $C_{\alpha\beta\gamma}$  is straightforward. Again, in order to achieve a gauge-invariant coupling of the three-form field to a nonconserved current  $J_{\alpha\beta\gamma}$ , we introduce a compensating two-form  $B_{\alpha\beta}$ , which under the gauge transformation (2) shifts in the following way:

$$B_{\alpha\beta} \rightarrow B_{\alpha\beta} + \Omega_{\alpha\beta}. \quad (\text{A7})$$

The gauge-invariant coupling to an arbitrary nonconserved source  $J_{\alpha\beta\gamma}$  is

$$(C_{\alpha\beta\gamma} - F_{\alpha\beta\gamma}^B)J^{\alpha\beta\gamma}, \quad (\text{A8})$$

where  $F^B$  is the field strength of  $B$ ,

$$F_{\alpha\beta\gamma}^B = d_{[\alpha} B_{\beta\gamma]}. \quad (\text{A9})$$

As in the photon example, we have to impose the following constraint on  $B$ :

$$\partial^\mu F_{\mu\alpha\beta}^B = \partial^\nu C_{\nu\alpha\beta}. \quad (\text{A10})$$

This constraint can be enforced by introducing an additional auxiliary two-form field  $X^{\beta\gamma}$  with the following coupling in the Lagrangian:

$$X^{\beta\gamma} \partial^\alpha (C_{\alpha\beta\gamma} - F_{\alpha\beta\gamma}^B). \quad (\text{A11})$$

The equation of motion of  $X^{\alpha\beta}$  then imposes the constraint (A10). Now integrating out the  $B_{\alpha\beta}$ -field we write down the effective Lagrangian for  $C$

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{6}J^{\mu\alpha\beta} \Pi_{[\mu}^\nu C_{\nu\alpha\beta]}. \quad (\text{A12})$$

The above effective coupling coincides with (22) up to a total derivative.

## APPENDIX B: THE CHARGE SCREENING

We shall now discuss a potential effect, which may lead to the screening of the three-form charge by the brane loops. The effect is somewhat similar in spirit to the charge screening by fermion loops in the massless Schwinger

model [24], except that this issue in our case is more subtle, as we shall now discuss. For simplicity, let us consider the analogous question in electrodynamics first. Consider the action given in (A6) where  $J_\mu$  is some generic current. Since the photon couples only to its transverse part, the theory is automatically gauge invariant regardless of whether the current  $J_\mu$  is conserved or not. Consider now a correlator of the two currents

$$\langle J_\mu(x) J_\nu(x') \rangle = P \Pi_{\mu\nu} + R \Theta_{\mu\nu} \quad (\text{B1})$$

where  $P$  and  $R$  are some scalar functions of the cutoff and the momentum. If the transverse part of the correlator is nonzero ( $P \neq 0$ ), it will generate the following effective operator in the photon action:

$$A^\mu P \Pi_{\mu\nu} A^\nu. \quad (\text{B2})$$

Depending on the structure of the operator  $P$ , this can be interpreted as the correction either to the photon kinetic terms, to the mass term, or to both. If the propagator  $\frac{1}{\partial^2 + P}$  has a physical pole, (B2) generates the mass for the photon, and charges will be screened. This is, in particular, the case for  $P = \text{constant}$ . The result depends on the underlying structure of the theory. For instance, for the conserved fermionic current  $J^\mu$ , (A6) is equivalent to the usual massless electrodynamics, which gives different answers in different numbers of dimensions. In  $3 + 1$  dimensions, for the conserved fermionic current  $P \propto \partial^2$ , and no photon mass is generated. On the other hand, in the  $1 + 1$  Schwinger model the answer depends on the fermion mass [24]. In the case of massless fermions, the photon mass is generated and the charges are screened. For massive fermions, however, the screening is only partial.

Let us now show that the analogous question can be posed in our case. Consider the coupling (22). The correlator of the two currents  $J_\mu$  can again be parametrized in the form (B1). If  $R \neq 0$  this will generate the following operator,

$$C^{\mu\alpha\beta} R \Pi_{[\mu}^\nu C_{\nu\alpha\beta]} \quad (\text{B3})$$

which would signal the generation of mass, if  $\frac{1}{(1+R)}$  had a physical pole. Because, in general,  $R$  is expected to be a function of  $\partial^2/M^2$ , the requirement of the absence of the physical poles below the cutoff scale reduces to the requirement of the absence of the constant part in  $R$ . In our case,  $R$  depends on the loops of the superheavy branes, and we will not attempt to calculate it.

## APPENDIX C: EXACT SOLUTIONS

The effective equation, Eq. (31), was derived in the approximation of constant  $\phi$  per unit step. In reality, there will be a small backreaction on  $\phi$  in each individual step, which will lead to the readjustment of the brane charge, and subsequent readjustment of the VEVs in the bulk. It is obvious that, in the attractor neighborhood, this backreac-

tion is negligible, but it is instructive to take it into account for completeness.

So we shall now derive the effective bulk equation determining the VEV of  $\phi$  without ignoring the variation of  $\phi$  in an elementary step. For this, we shall solve the equations (26) and (27) without the simplifying approximations listed below them. Substituting the form (4), Eq. (26) now becomes

$$\partial^\nu((1 - |\phi|^2/M^2)F_0) = -\mu\Theta^{\nu z}\left[\text{Re}\left(\frac{\phi}{M_P}\right)^N\delta(z)\right] \quad (\text{C1})$$

or equivalently

$$\partial^2((1 - |\phi|^2/M^2)F_0) = -\mu\partial_z\left[\text{Re}\left(\frac{\phi}{M_P}\right)^N\delta(z)\right]. \quad (\text{C2})$$

The solution with the correct boundary conditions is

$$F_0(x_j, z) = \frac{\mu}{2M_P^N(1 - \frac{|\phi(x_j, z)|^2}{M^2})}\left[\text{Re}(\phi^N(z=0, f_i(x_j, z)))\theta(z) - \text{Re}(\phi^N(z=0, f_i(x_j, -z)))\theta(-z) + f_0(M_P^N/\mu)\right] \quad (\text{C3})$$

where  $\theta(z)$  is the step function, and  $x_i$ ,  $i = 0, 1, 2$  are the three remaining space-time coordinates parallel to the brane. For clarity, we have indicated the coordinate dependence, in order to stress how the values of the  $\phi$  field at different locations determine the bulk value of  $F_0$ . The functions  $f_i(z)$  are such that  $f_i(x_j, z)|_{z=0} = x_i$ ,

$$\partial^2\phi^N(z=0, f_i(x_j, z)) = 0. \quad (\text{C4})$$

Note that the function  $\phi^N(z=0, x_i)$  is the value of  $\phi^N$  at the brane location.  $f_0$  is the integration constant. Thus, we see that for small  $\phi$  the value of  $F_0(z)$  in the bulk vacuum  $z \neq 0$  is essentially determined by the value of the  $\phi^N$  function *at the brane location*  $z = 0$ . For  $z \neq 0$  the dependence of  $F_0(z)$  on local values of  $\phi(z)$  is rather mild and has an additional suppression factor of  $\phi^2/M_P^2$ .

For instance, for the background values that are functions of  $z$  and  $t$  only, the solution is

$$F_0(z, t) = \frac{\mu}{2M_P^N(1 - \frac{|\phi(z, t)|^2}{M^2})}\left[\text{Re}(\phi^N(0, (t-z)))\theta(z) - \text{Re}(\phi^N(0, (t+z)))\theta(-z) + f_0(M_P^N/\mu)\right]. \quad (\text{C5})$$

From here it is clear that the change of the expectation value of  $\phi$  on the brane triggers the corresponding change in  $\phi$ , which propagates away from the brane at the speed of light. Existence of such waves indicates the presence of some ‘‘hidden’’ massless degrees of freedom. This is not surprising, since the degree in question is the Stückelberg field  $B_{\mu\nu}$ , which we have integrated out. In some sense, this degree of freedom in our case plays the role analogous to the one played by the invisible axion in the solution of the strong  $CP$  problem in QCD.

Substituting the solution (C5) into Eq. (27), we get the following bulk equation for  $\phi$ :

$$\partial^2\phi - \left(m^2 - \frac{\mu^2}{8M^2M_P^{2N}\left(1 - \frac{|\phi(x_j, z)|^2}{M^2}\right)^2}\right) \times [(\text{Re}(\phi^N(0, (t-z)))\theta(z) - \text{Re}(\phi^N(0, (t+z)))\theta(-z) + f_0(M_P^N/\mu))^2]\phi + \lambda|\phi|^2\phi = 0. \quad (\text{C6})$$

Since in the vacua  $F_0$  takes discrete values that between the two neighboring vacua are spaced by  $\sim\phi^N$ , for small  $\phi$  the scanning becomes almost continuous. So the dynamics of the small- $\phi$  vacua can be studied by considering the appropriate values of the integration constant  $f_0$ . In small- $\phi$  vacua this constant takes the value

$$f_0^2 \simeq 8m^2M^2. \quad (\text{C7})$$

Expanding (C6) about such a vacuum, and noticing that the value of the  $\phi^N$  determining  $F_0$  is taken at the brane location, we see that self-interactions are weak in the attractor vacuum.

#### APPENDIX D: RESOLVING THE BRANE

We shall now resolve the structure of the brane, and show that it can arise in the effective low energy theory in the form of an ‘‘axionic’’ domain wall.<sup>8</sup> Such a possibility was suggested in [1,25], but we shall review it here in more detail for completeness. For this we shall introduce an axion field  $a$ , defined modulo  $2\pi$ , with the decay constant  $f_a$ . First, let us show that, in the presence of the coupling between the axion and the three-form field  $C_{\alpha\beta\gamma}$ , the axionic domain walls become branes charged under  $C$ .<sup>9</sup>

For demonstrating this, the possible coupling to the Higgs field  $\phi$  plays no role and we shall ignore the latter for simplicity. Consider then the following Lagrangian:

$$L = \frac{f_a^2}{2}(\partial_\mu a)^2 - V(a) - \frac{q}{12\pi}\partial_\alpha a C_{\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta} - \frac{1}{48}F^2, \quad (\text{D1})$$

where  $V(a)$  is the axion potential, which is required to be periodic under

$$a \rightarrow a + 2\pi n. \quad (\text{D2})$$

The equations of motion are

$$\partial^\mu F_{\mu\nu\alpha\beta} = \frac{q}{2\pi}\partial^\mu a \epsilon_{\mu\nu\alpha\beta}, \quad (\text{D3})$$

<sup>8</sup>The ‘‘axion’’ should not be confused with the one solving the strong  $CP$  problem. The role of the former can be played by one of the axions appearing in string compactifications.

<sup>9</sup>At the level of the present discussion we shall treat both  $a$  and  $C$  as ‘‘elementary’’ objects, without specifying their possible origin from the fundamental underlying theory [11].

$$f_a^2 \partial^2 a + V_a - \frac{q}{12\pi} \frac{F^{\mu\nu\alpha\beta}}{24} \epsilon_{\mu\nu\alpha\beta} = 0. \quad (\text{D4})$$

Integrating the first equation, we get

$$F_{\mu\nu\alpha\beta} = \frac{q}{2\pi} (a + 2\pi k) \epsilon_{\mu\nu\alpha\beta} \quad (\text{D5})$$

where  $k$  is an integer integration constant. Under shift symmetry (D2)  $k$  changes as

$$k \rightarrow k + n. \quad (\text{D6})$$

The relation (D5) implies that any axionic domain wall, through which  $a$  changes by  $\Delta a$ , effectively acquires a three-form charge

$$q_{\text{eff}} = \frac{q\Delta a}{2\pi}. \quad (\text{D7})$$

The value of  $\Delta a$  through the elementary wall can be found by substituting the solution (D5) into Eq. (D4). This gives the following effective equation for the axion:

$$f_a^2 \partial^2 a + V_a + \frac{q^2}{24\pi^2} (a + 2\pi k) = 0. \quad (\text{D8})$$

The value of the  $\Delta a$  through the elementary wall is determined by the distance between the neighboring minima of the effective potential:

$$V_{\text{eff}} = V(a) + \frac{q^2}{48\pi^2} (a + 2\pi k)^2. \quad (\text{D9})$$

For the small  $q$  and fixed  $k$ , the number of local minima is roughly  $\sim V(a)_{\text{max}}/q^2$ , where  $V_{\text{max}}(a)$  is the maximal value of  $V(a)$ . The elementary step is  $\Delta a \simeq 2\pi$ . Thus, the branes acquire an effective three-form charge  $q_{\text{eff}} = q$ . The above counting of vacua is not surprising, since the model (D1) can be viewed as the simplest four-dimensional generalization of the massive Schwinger model, in which there is an analogous counting of states [26].

Restoration of the  $\phi$  dependence of the brane charge can now be done in a straightforward way, by promoting  $q$  into the function of  $\phi$  given by (21). The gauge-invariant Lagrangian has the following form:

$$\begin{aligned} L = & \frac{f_a^2}{2} (\partial_\mu a)^2 - V(a) - \frac{q_{\text{eff}}(\phi)}{12\pi} \partial_\alpha a \Theta_\mu^\alpha C_{\beta\gamma\delta} \epsilon^{\mu\beta\gamma\delta} \\ & - \frac{1}{48} F^2 + |\partial_\mu \phi|^2 + |\phi|^2 \left( m^2 + \frac{F^2}{48M^2} \right) \\ & - \frac{\lambda}{2} |\phi|^4 + \dots \end{aligned} \quad (\text{D10})$$

The equations of motions are

$$\begin{aligned} f_a^2 \partial^2 a + V_a - \frac{q_{\text{eff}}(\phi)}{12\pi} \frac{F^{\mu\nu\alpha\beta}}{24} \epsilon_{\mu\nu\alpha\beta} \\ + (q_{\text{eff}} - \text{derivatives}) = 0, \end{aligned} \quad (\text{D11})$$

$$\begin{aligned} \partial^\mu ((1 + |\phi|^2/M^2) F_{\mu\nu\alpha\beta}) - \frac{q_{\text{eff}}(\phi)}{2\pi} \Theta^{\gamma\mu} \partial_\gamma a \epsilon_{\mu\nu\alpha\beta} \\ + (q_{\text{eff}} - \text{derivatives}) = 0, \end{aligned} \quad (\text{D12})$$

$$\begin{aligned} \partial^2 \phi + \left( -m^2 - \frac{F^2}{48M^2} \right) \phi + \lambda |\phi|^2 \phi \\ + \frac{\mu N}{24\pi} \frac{\phi^{*N-1}}{M_p^N} \partial_\alpha a \Theta_\mu^\alpha C_{\beta\gamma\delta} \epsilon^{\mu\beta\gamma\delta} = 0, \end{aligned} \quad (\text{D13})$$

and again ignoring the derivatives of  $q_{\text{eff}}$  and the terms of order  $\phi^2/M^2$  in the l.h.s. of Eq. (D12), and integrating the equation for  $C$ , we get the following effective equation for  $\phi$  vacua:

$$-(m^2 - (a - 2\pi k)^2) \frac{\mu^2}{4\pi^2 M^2} (\text{Re}(\phi^N/M_p^N)^2) \phi + \lambda \phi^3 = 0, \quad (\text{D14})$$

where  $a$  is defined by the equation

$$V_a - \frac{q_{\text{eff}}^2}{24\pi^2} (a + k) = 0 \quad (\text{D15})$$

as discussed above; for small  $q_{\text{eff}}$  the minima of the  $\phi$  potential are at  $a = 2\pi n$ , which means that near the attractor point the  $\phi$  vacua are defined by Eq. (31).

## APPENDIX E: AXION-THREE-FORM COUPLINGS FROM MASSLESS SCALARS

We shall now discuss how the axionic domain walls with Higgs-dependent three-form charges can be generated by integrating out some intermediate scalar fields, and show that this is only possible if the scalars in question are exactly massless. The idea is to start with the local coupling

$$q_{\text{eff}} \partial_\mu a J^\mu, \quad (\text{E1})$$

where  $J^\mu$  is the gauge-invariant current, with the divergence

$$\partial_\mu J^\mu = cF + \dots \quad (\text{E2})$$

where  $c$  is some constant. Then ‘‘integrating out’’  $J^\mu$  we write the effective coupling between  $a$  and  $C$ . In order to fulfill this program, we shall consider a toy 1 + 1-dimensional example first. This is a 1 + 1-dimensional electrodynamics coupled to two pseudoscalars, an axion  $a$  and an additional pseudoscalar  $\chi$ . The Lagrangian is

$$\begin{aligned} L = & \frac{1}{2} (\partial_\mu a)^2 + \frac{1}{2} (\partial_\mu \chi)^2 + q_{\text{eff}} \partial_\mu a \partial^\mu \chi - V(a) - V(\chi) \\ & - \tau \partial_\alpha \chi C_\beta \epsilon^{\alpha\beta} - \frac{1}{4} F^2. \end{aligned} \quad (\text{E3})$$

Here  $C_\alpha$  is the electromagnetic vector potential, and  $\tau$  is some constant.  $q_{\text{eff}}$ , which is understood to be the function of the Higgs VEV, will be treated as a constant for a moment. The reason for us to consider the above example is that the electric field in 1 + 1 dimensions shares some

properties with a three-form in 3 + 1 dimensions. Just like the latter, the noninteracting 1 + 1 electric field has no propagating degrees of freedom, but its value can change in the presence of charges, which play the role analogous to two-branes in four dimensions.

Our aim is to start from the local Lagrangian (E14), which for arbitrary  $q_{\text{eff}}$  is invariant under the axionic shift symmetry (D2) as well as under the gauge symmetry

$$C_\alpha \rightarrow C_\alpha + \partial_\alpha \Omega, \quad (\text{E4})$$

which is a 1 + 1-dimensional analog of (2). However,  $a$  and  $C$  do not couple directly, but through the “intermediate” field  $\chi$ . In fact,  $\partial_\mu a$  couples to the current  $\partial_\mu \chi$  whose divergence is set by  $F$ . So we expect that after integrating out  $\chi$  we arrive at (the 1 + 1-dimensional analog of) the coupling (D1). This coupling should guarantee that axionic “walls,” which in 1 + 1 dimensions are just particles, acquire electric charges controlled by  $q_{\text{eff}}$ . We shall now check if this is indeed the case. The equations of motion are

$$\partial^2 a + q_{\text{eff}} \partial^2 \chi + V_a = 0, \quad (\text{E5})$$

$$\partial^2 \chi + q_{\text{eff}} \partial^2 a + V_\chi - \frac{\tau}{2} F_{\mu\nu} \epsilon^{\mu\nu} = 0, \quad (\text{E6})$$

$$\partial^\mu F_{\mu\nu} = \tau \partial^\alpha \epsilon_{\alpha\nu} \chi. \quad (\text{E7})$$

The last equation is solved by

$$F_{\mu\nu} = \epsilon_{\mu\nu} \tau (\chi + \chi_0) \quad (\text{E8})$$

where  $\chi_0$  is some constant. Substituting this result in Eq. (E9), we get the following effective equation for  $\chi$ :

$$\partial^2 \chi + q_{\text{eff}} \partial^2 a + V_\chi + \tau^2 \chi + \tau^2 \chi_0 = 0. \quad (\text{E9})$$

For the special choice of

$$V_\chi = -\tau^2 \chi - \tau^2 \chi_0 \quad (\text{E10})$$

the system of equations is solved by

$$\chi = -q_{\text{eff}} a \quad (\text{E11})$$

where  $a$  satisfies the equation

$$\partial^2 a + \frac{1}{1 - q_{\text{eff}}^2} V_a = 0. \quad (\text{E12})$$

Because of the  $2\pi$  periodicity of  $V(a)$ , the latter equation always has an axionic domain wall solution, through which  $a$  changes by  $2\pi$ . According to (E11) the corresponding change in  $\chi$  is  $\Delta\chi = -q_{\text{eff}} 2\pi$  and according to (E9) the change in  $F$  is  $\Delta F = -q_{\text{eff}} \tau 2\pi$ . Thus, axionic walls indeed acquire an electric charge. However, such a behavior is a peculiarity of the choice (E10). In fact, for no other choice of  $V_\chi$  may axionic walls have a charge proportional to  $q_{\text{eff}}$ . Indeed, the fields at infinity on both sides of the wall must assume the constant values satisfying

$$V_a = 0 \quad (\text{E13})$$

and Eq. (E10). Unless the latter is identically zero, the change of  $\chi$  through the wall will be determined by the neighboring minima of (E10), which contains no reference to  $q_{\text{eff}}$ . Hence, the change of  $F$  will not be set by  $q_{\text{eff}}$  either.

The reason why the choice (E10) is the only possible one is easy to understand from the symmetry point of view. For such a choice the Lagrangian can be rewritten as

$$L = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu \chi)^2 + q_{\text{eff}} \partial_\mu a \partial^\mu \chi - V(a) - \frac{1}{4}(\tau \chi \epsilon_{\mu\nu} - F_{\mu\nu})^2 \quad (\text{E14})$$

which has an exact shift symmetry

$$C_\mu \rightarrow C_\mu + b \epsilon_{\mu\nu} x^\nu, \quad \chi \rightarrow \chi - b/\tau. \quad (\text{E15})$$

Because of this symmetry, after integrating out  $F$ , the action for  $\chi$  cannot be anything other than the one of a scalar field with a vanishing potential.

The fact that in the model (E14) the existence of the branes with electric charge  $\propto q_{\text{eff}}$  requires a very special choice of  $V(\chi)$  can also be understood in the following way. In the limit  $V(a) = V(\chi) = 0$  there are two continuous shift symmetries,

$$a \rightarrow a + \text{constant} \quad (\text{E16})$$

and

$$\chi \rightarrow \chi + \text{constant}. \quad (\text{E17})$$

The corresponding currents are

$$J_a^\mu = \partial^\mu a + q_{\text{eff}} \partial^\mu \chi \quad (\text{E18})$$

and

$$J_\chi^\mu = \partial^\mu \chi + q_{\text{eff}} \partial^\mu a, \quad (\text{E19})$$

respectively. Only the second shift symmetry is “anomalous” and the corresponding current is not conserved,

$$\partial^\mu J_\chi^\mu = \frac{\tau}{2} F_{\mu\nu} \epsilon^{\mu\nu}. \quad (\text{E20})$$

Hence,  $\chi$  is the only pseudo-Goldstone boson whose shift is tied directly to  $F$ . But the shift of  $\chi$  is determined by  $V(\chi)$  which, in general, carries no information about  $q_{\text{eff}}$ . Only in the limit (E10), when the “bare” mass of  $\chi$  is exactly canceled by the “anomaly” contribution, the shift symmetry (E17) remains exact and the shift in  $\chi$  adjusts to the minimal step of  $a$  suppressed by the small charge  $q_{\text{eff}}$ .

It is useful to reformulate the above effect in fermionic language. Indeed, for  $V(\chi) = m^2 \cos^2 \sqrt{\pi} \chi$ , the Lagrangian (E14) is a bosonized version of the following fermionic Lagrangian:

$$L = \frac{1}{2}(\partial_\mu a)^2 - V(a) + q_{\text{eff}} \partial^\mu a \epsilon_{\mu\nu} \bar{\psi} \gamma^\mu \psi + i \bar{\psi} \gamma_\mu D^\mu \psi - m \bar{\psi} \psi - \frac{1}{4} F^2. \quad (\text{E21})$$

In the limit  $m \rightarrow 0$ ,  $V(a) \rightarrow 0$  there are two continuous axial symmetries (E16) and



$$\psi \rightarrow e^{i\gamma_5\theta}\psi. \quad (\text{E22})$$

But only the second is anomalous, and the corresponding pseudo-Goldstone boson is the fermionic composite, and not  $a$ . The composite pseudo-Goldstone boson can be related to an elementary scalar  $\chi$  via standard bosonization [27],

$$\bar{\psi}\gamma_\mu\psi = \epsilon_{\mu\nu}\partial^\nu\frac{\chi}{\sqrt{\pi}} \quad \text{and} \quad m\bar{\psi}\psi = m\kappa\cos^2\sqrt{\pi}\chi \quad (\text{E23})$$

where  $\kappa$  is a charge-related constant. The reason why the axial  $U(1)$  symmetry cannot “see”  $a$  as its pseudo-Goldstone boson has to do with the peculiarities of  $1+1$  dimensions, and, in particular, the fact that the axial and vector currents are related through

$$\epsilon_{\mu\nu}\bar{\psi}\gamma^\nu\psi = \bar{\psi}\gamma_\mu\gamma^5\psi. \quad (\text{E24})$$

Generalization of the model (E14) to four dimensions is straightforward and the results are essentially unchanged.

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