$Z(3)$-symmetric effective theory for $SU(3)$ Yang-Mills theory at high temperature

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A three-dimensional effective theory for high temperature $SU(3)$ gauge theory, which maintains the $Z(3)$ center symmetry of the full theory, is constructed. Such a $Z(3)$ invariant effective theory should be applicable to a wider temperature range than the usual effective theory, known as EQCD, which fails to respect the center symmetry. This center-symmetric effective theory can reproduce domain wall and phase transition properties that are not accessible in EQCD. After identifying a convenient class of $Z(3)$ invariant effective theories, we constrain the coefficients of the various terms in the Lagrangian using leading-order matching to EQCD at high temperature, plus matching of domain wall properties in the full theory. We sketch the expected structure of the phase diagram of the effective theory and briefly discuss the prospects of numerical simulations and the addition of quarks.

I. INTRODUCTION

The thermodynamics of high-temperature QCD with various numbers of massless quarks has been widely studied using the method of dimensional reduction, in which one exploits the decoupling of the nonstatic degrees of freedom to build an effective three-dimensional theory known as electrostatic QCD (EQCD) [1]. This approach has been used to derive the perturbative expansion of the QCD pressure through order $g^6 \ln g$, and to compute various gluonic and mesonic correlation lengths [2–4]. It has also led to improved understanding of the nonperturbative dynamics on the "ultrasoft" $g^2 T$ energy scale [5]. This approach is applicable at asymptotically high temperatures where the running coupling $g^2(T)$ is small. But closer to the confinement/deconfinement transition, at temperatures of a few times T_c, even nonperturbative simulations of EQCD have failed to produce satisfactory results [6,7]. There have been attempts to build effective theories for the Wilson line near the phase transition [8–11], but none of these theories (which often come with very complicated Lagrangians) provide any connection to the perturbative regime at $T \gg T_c$.

Our goal is to formulate a simple three-dimensional effective theory for hot QCD that can reproduce equilibrium properties at temperatures ranging from asymptotically large down to T_c. For simplicity, most of our discussion will focus on the case of pure Yang-Mills theory, although the addition of dynamical quarks will be briefly discussed in the last section. The construction of our theory relies on a separation between the inverse correlation length and the lowest nonzero Matsubara frequency, $2\pi T$. In $SU(3)$ Yang-Mills theory, there is a parametrically large separation between these scales at very high temperatures, and a modest separation near T_c.[1]

In pure $SU(3)$ Yang-Mills theory, the confinement/deconfinement phase transition is a symmetry-breaking phase transition. The theory has a global $Z(3)$ symmetry corresponding to invariance of the action under gauge transformations which are periodic in the temporal direction only up to a twist belonging to the center of the gauge group. This $Z(3)$ symmetry is spontaneously broken in the deconfined phase where the Wilson line operator (which is an order parameter for this symmetry) acquires a nonzero expectation value. Close to the phase transition region, fluctuations in the Wilson line become increasingly impor-

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[1]It is natural to distinguish the correlation lengths of time-reflection even and odd operators. The Debye screening length m_D^{-1} is the longest correlation length of time-reflection odd operators [12], such as the imaginary part of the trace of the Wilson line. The longest correlation length of time-reflection even operators, which we will denote by μ^{-1}, is longer than the Debye length. In $SU(3)$ Yang-Mills theory at asymptotically high temperatures, $m_D/(2\pi T)$ approaches $g(T)/(2\pi)$ while $\mu/(2\pi T)$ is approximately as $0.1g(T)^2$ [13]. (This is an estimate based on the corresponding $SU(2)$ result, plus the fact that μ should scale nearly linearly with N_c.) Correlators of purely magnetic operators fall off like $e^{-\mu|\mathbf{x}|}$ at large distance, while correlators of the real part of the trace of the Wilson line initially decrease like $e^{-2m_D|\mathbf{x}|}$ but eventually fall more slowly as $e^{-\mu|\mathbf{x}|}$. The correlator of the imaginary part of the trace of the Wilson line initially decreases like $e^{-3m_D|\mathbf{x}|}$ but switches to $e^{-m_D|\mathbf{x}|}$ at large distance. As the temperature decreases, the ratio $\mu/(2\pi T)$, as deduced from the Wilson line correlator, rises to about 0.5 at $T \sim 2T_c$ [14,15] and then falls to about 0.1 just above T_c [15]. The ratio $m_D/(2\pi T)$ on the other hand rises to near unity at $T \sim 2T_c$ [14]. Because the confinement/deconfinement phase transition in $SU(3)$ Yang-Mills theory is only weakly first order [16], this ratio should decrease, similar to the behavior of $\mu/(2\pi T)$, closer to the phase transition, but we are unaware of any numerical simulations confirming this. Near T_c, correlations of purely magnetic operators appear to fall exponentially with a length scale near $(6T)^{-1}$ but should eventually switch to the longer scale μ^{-1} seen in the Wilson line correlator [14].

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tant and the $Z(3)$ invariance of the Wilson line probability distribution is an essential feature of the theory. Even in full QCD with dynamical quarks it has been argued that the $Z(3)$ symmetry, although explicitly broken by the presence of light quarks, may nevertheless play an important role in the dynamics near T_c [8].

The standard high-temperature effective theory, EQCD, cannot provide a smooth interpolation between asymptotia and T_c. This is evident even without considering numerical evidence, as the construction of EQCD violates one of the central tenets of effective field theory—it does not respect all the symmetries of the underlying theory. EQCD breaks the $Z(3)$ center symmetry explicitly, even in the absence of quarks. Its leading-order Lagrangian is obtained by expanding the one-loop effective potential of the Wilson line [17] around one of its three degenerate minima. Effects of the explicit $Z(3)$ breaking are clearly visible in the phase diagram of the effective theory [6].[2]

An effective theory that preserves the $Z(3)$ symmetry can capture the dynamics of thermal fluctuations (above T_c) which create bubbles sampling all three degenerate equilibrium states, and thus should have a phase structure similar to that of the full four-dimensional theory. In particular, its phase diagram should contain a quadruple point, where all three deconfined phases coexist with the confining phase, thereby extending the description of the theory into a temperature range inaccessible to EQCD.

Our guide during the process of constructing a better effective theory for $N_f = 0$ QCD is the requirement that the new theory provide a smooth link between asymptotically high temperatures, where perturbation theory is valid, and T_c, where a $Z(3)$ symmetry restoring phase transition occurs. At high temperatures, its predictions must reproduce those of EQCD (within its domain of applicability). But a $Z(3)$ invariant effective theory should also reproduce the thermodynamics of domain walls which interpolate between different equilibrium states related by the $Z(3)$ symmetry [18].

We begin in Sec. II with a brief review of the relevant aspects of the thermodynamics of $SU(3)$ Yang-Mills theory, including dimensional reduction and the role of the Wilson line as an order parameter for the confinement/deconfinement phase transition. Section III presents the general form of our effective theory. We perform lowest-order matching to ordinary $N_f = 0$ EQCD at high temperatures in Sec. IV, and also compute the semiclassical domain wall profile in our effective theory. The expected phase diagram of our theory is discussed in Sec. V, and compared to that of EQCD. Possible future directions,

including the addition of quarks, are briefly discussed in the concluding Sec. VI. Some details of the analysis of Sec. III are relegated to the appendices.

II. THERMODYNAMICS OF $SU(3)$ YANG-MILLS

A. $Z(3)$ center symmetry

The functional integral representation for the partition function of $SU(3)$ Yang-Mills theory involves an integral over all gauge fields which are periodic in (Euclidean) time with period β. In addition to invariance under gauge transformations which are periodic in time, the action and the space of field configurations are invariant under gauge transformations g which are only periodic in time up to a "twist" belonging to the $Z(3)$ center of the gauge group,

$$A_\mu(x) \rightarrow s(x)(A_\mu(x) + i\partial_\mu)s(x)^\dagger, \quad s(x) \in SU(3), \quad (2.1a)$$
$$s(x + \beta\hat{e}_t) = zs(x), \qquad z \in Z(3). \quad (2.1b)$$

Under such a transformation, the trace of the Wilson line (or Polyakov loop)

$$\text{tr } \Omega(\mathbf{x}) \equiv \text{tr}\left\{ \mathcal{P} \exp\left[i \int_0^\beta d\tau A_0(\tau, \mathbf{x}) \right] \right\}, \quad (2.2)$$

transforms in the fundamental representation of $Z(3)$,

$$\text{tr } \Omega(\mathbf{x}) \rightarrow z \,\text{tr } \Omega(\mathbf{x}). \quad (2.3)$$

The magnitude of the expectation value of $\text{tr } \Omega(\mathbf{x})$ may be interpreted physically as the exponential of the change in free energy due to the addition of an infinitely heavy fundamental representation test quark at position $\mathbf{x}$,

$$|\langle \text{tr}\Omega(\mathbf{x})\rangle| = e^{-\beta\Delta F_q(\mathbf{x})}. \quad (2.4)$$

This expectation value vanishes in the confining low-temperature phase, where the free energy cost to introduce a single test quark is infinite, but the expectation value is nonzero in the deconfined high-temperature phase, where the free energy cost of a test quark is finite. Hence, the $Z(3)$ center symmetry is unbroken in the confining phase, but is spontaneously broken in the deconfined phase, and the expectation value of $\text{tr } \Omega(\mathbf{x})$ is an order parameter for the $Z(3)$ center symmetry.[3]

Symmetry considerations [19], as well as lattice simulations [16], show that four-dimensional $SU(3)$ Yang-Mills theory belongs to the same universality class as the three-dimensional three-state Potts model, and undergoes a weak first order phase transition at the critical temperature. At the phase transition, the theory has a quadruple point with four coexisting equilibrium phases (the three spontaneously broken deconfined states, related by $Z(3)$ transformations, plus the confining phase). No correlation lengths diverge as $T \rightarrow T_c$.

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[2]Even though it has been argued that a "partial dynamical restoration" of the $Z(3)$ symmetry takes place in EQCD [6], this theory does not have three distinct, degenerate, and physically equivalent equilibrium states which would signal a spontaneously broken $Z(3)$ symmetry. These aspects will be discussed in more detail in Sec. V.

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[3]Unlike more typical symmetry-breaking phase transitions, the low-temperature (confining) phase is the disordered phase, while the high-temperature (deconfined) phase is the ordered phase.

The transition from the deconfined plasma above T_c to the confining glueball phase below T_c may occur through two mechanisms: bubble nucleation or complete wetting. In the former, bubbles of the confining phase are formed via thermal fluctuations inside the (supercooled) deconfined matter just below $T = T_c$, and as the temperature is lowered these bubbles grow, eventually filling all space. Alternatively, complete wetting takes place if one begins with domain walls separating deconfined phases with differing $Z(3)$ "magnetization" (i.e., differing phases for $\langle \text{tr } \Omega \rangle$). Such a domain wall may be viewed as a thin film of the confining phase residing between the two deconfined phases. As the temperature is lowered below T_c, this film widens and splits the "deconfined-deconfined" domain wall into two "confined-deconfined" domain walls which repel, leading to the expansion of the confining phase in the direction perpendicular to the original domain wall.

Domain walls separating $Z(3)$ rotated deconfined phases are topologically stable, nonperturbative objects which play an important role in the dynamics of quarkless QCD at temperatures just above T_c. At sufficiently high temperatures, their properties may be computed using semiclassical techniques [18]. In hot QCD, the effective potential for the Wilson line only arises at one-loop order [17]. As a result, the Debye mass m_D (which is determined by the curvature of this potential at its minimum) is of order $g(T)T$, while the energy density at the top of the barrier separating $Z(3)$ minima is $O(T^4)$ (not order $T^4/g(T)^2$, as one might have expected for a semiclassical object). The width of a domain wall is determined by the Debye screening length. So high-temperature domain walls have an $O(T^4)$ energy density over a distance of order m_D^{-1}, yielding a tension which scales as $T^3/g(T)$. This parametric dependence must be reproduced by any effective theory that claims to capture the $Z(3)$ symmetry-breaking physics of hot Yang-Mills theory.

B. Dimensional reduction and EQCD

Because of the compact (Euclidean) temporal direction in a finite-temperature field theory, all four-dimensional fields may be decomposed into Fourier sums running over Matsubara frequencies. Each Fourier component acts as a three-dimensional field, with a mass (for bosonic fields) given by an integer multiple of $2\pi T$. At sufficiently high temperatures, all nonstatic modes act like heavy fields and decouple from dynamics on length scales large compared to $1/T$. The remaining static modes are the relevant degrees of freedom for physics on longer distance scales, yielding an effective three-dimensional theory. To lowest order, the form of the effective theory may be found by integrating out the heavy fields explicitly [17]. At higher orders, it is more efficient to start with the most general form of the Lagrangian and adjust its parameters by demanding that the effective theory reproduce a minimal set

of physical quantities which are computable in both the effective and underlying theories [1,20].

In the case of $SU(3)$ Yang-Mills theory at high temperature, the degrees of freedom of the effective theory correspond to the static components of the gluon field A_μ. Integrating out nonstatic fluctuations generates an order gT mass for the temporal component A_0, due to Debye screening. In addition, self-interaction terms are generated for the A_0 field. The resulting Lagrangian of $N_f = 0$ EQCD reads[4]

$$\mathcal{L}_{\text{EQCD}} = g_3^{-2}\{\tfrac{1}{2}\text{tr } F_{ij}^2 + \text{tr}[(D_i A_0)^2] + m_{\text{E}}^2 \text{tr}(A_0^2)$$
$$+ \lambda_{\text{E}} \text{tr}(A_0^4)\} + \delta\mathcal{L}_{\text{E}}, \qquad (2.5)$$

with $g_3 \equiv g(T)\sqrt{T}$ the three-dimensional gauge coupling and $D_i \equiv \partial_i - i[A_i, \cdot]$ an adjoint representation covariant derivative. The final term $\delta\mathcal{L}_{\text{E}}$ stands for higher dimension operators whose effects, on length scales large compared to T^{-1}, are subleading and only affect the thermodynamics at order g^7. The operators in $\delta\mathcal{L}_{\text{E}}$ will be irrelevant for our purposes.

A straightforward way to obtain the form of the above Lagrangian, plus the leading-order values of its parameters, is to consider the one-loop effective potential for the Wilson line $\Omega(\mathbf{x}) \equiv \mathcal{P}e^{i\int_0^\beta d\tau A_0(\tau,\mathbf{x})}$ in the full theory. The resulting potential [17] has three degenerate minima related by transformations belonging to the $Z(3)$ center of $SU(3)$. Choosing to work around the minimum where the Wilson line expectation value is real and positive (and subtracting from the potential an A_0^3 contribution originating from fluctuations in the static gauge field, which are not to be integrated out in the present case), one obtains Eq. (2.5) as the resulting effective potential for A_0 together with the lowest-order values for the effective theory parameters,

$$m_{\text{E}}^2 = g(T)^2 T^2, \qquad \lambda_{\text{E}} = \tfrac{3}{4}g(T)^2/\pi^2. \qquad (2.6)$$

To lowest order, the mass parameter m_{E} is the same as the physical Debye mass m_D.

This derivation also reveals the major shortcoming of the effective theory: when constructing the EQCD Lagrangian, one has chosen to work in the neighborhood of $A_0 \approx 0$, corresponding to a real, positive Wilson line, and thus loses the $Z(3)$ invariance of the original theory. The other $Z(3)$ minima correspond to eigenvalues of A_0 near $\pm\frac{2}{3}\pi T$, and are outside the domain of validity of the effective theory. At asymptotically high temperatures this is of little consequence for most physical quantities, as the probability of thermal fluctuations crossing the barriers separating different minima is exponentially small [with an exponent scal-

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[4]This form of the effective theory is valid for $SU(2)$ and $SU(3)$. With four or more colors, there is an additional $(\text{tr } A_0^2)^2$ term. But for both $SU(2)$ and $SU(3)$, the two fourth-order terms are not independent, as $(\text{tr } A_0^2)^2 = 2 \text{tr}(A_0^4)$.

ing as $1/g(T)$]. But fluctuations between the different minima become crucial for the dynamics closer to T_c, and hence a non-$Z(3)$-invariant effective theory cannot properly describe physics near the confinement/deconfinement phase transition.

III. $Z(3)$ INVARIANT EFFECTIVE THEORY

A. Degrees of freedom

To build a three-dimensional $Z(3)$ invariant effective theory, the minimal degrees of freedom are the spatial gauge field $\mathbf{A}(\mathbf{x})$ and the Wilson line $\Omega(\mathbf{x})$. But because $\Omega(\mathbf{x})$ is a unitary matrix, a theory with polynomial interactions for the Wilson line will not be perturbatively renormalizable. For an effective theory which is only intended to be valid below a UV cutoff, this is not a fundamental problem. However, it is a serious practical nuisance. It complicates perturbative matching to the underlying fundamental theory, as well as matching between lattice and continuum regulated versions of the effective theory.

Therefore, we take as our goal the construction of an effective theory whose short distance behavior is as benign as possible. To do so, we will replace the unitary matrix $\Omega(\mathbf{x})$ by an unconstrained 3×3 complex matrix $Z(\mathbf{x})$. This should be viewed as analogous to the relation between nonlinear and linear sigma models (or between an Ising model and a double-well ϕ^4 field theory). One may view the complex field $Z(\mathbf{x})$ as the result of applying a block-spin renormalization group transformation which averages $\Omega(\mathbf{x})$ over a small spatial region, producing a result which is no longer unitary.

An arbitrary complex matrix may always be decomposed into a product of unitary and Hermitian matrices, so we may write $Z(\mathbf{x}) = \Omega(\mathbf{x})H(\mathbf{x})$ with $H(\mathbf{x}) = H(\mathbf{x})^\dagger$. One may regard the matrix $H(\mathbf{x})$ as containing unphysical degrees of freedom (not present in the underlying Yang-Mills theory) which we have chosen to "integrate in" in order to construct an effective theory with good short distance behavior. We will arrange for these extra degrees of freedom to be "heavy"—to have masses large compared to the inverse correlation length.

We require that the effective theory Lagrangian be invariant under three-dimensional $SU(3)$ gauge transformations,

$$Z(\mathbf{x}) \rightarrow s(\mathbf{x})Z(\mathbf{x})s(\mathbf{x})^\dagger,$$
$$\mathbf{A}(\mathbf{x}) \rightarrow s(\mathbf{x})(\mathbf{A}(\mathbf{x}) + i\nabla)s(\mathbf{x})^\dagger, \qquad (3.1)$$

for $s(\mathbf{x}) \in SU(3)$, as well as global $Z(3)$ phase rotations,

$$Z(\mathbf{x}) \rightarrow e^{2\pi in/3}Z(\mathbf{x}), \qquad (3.2)$$

for integer n.

Instead of using the polar decomposition of $Z(\mathbf{x})$ mentioned above, it will be more convenient for our purposes to decompose this field into a traceless part and its trace

(times a unit matrix). Therefore, we define

$$L(\mathbf{x}) \equiv \mathrm{tr}\, Z(\mathbf{x}), \qquad (3.3)$$

and

$$M(\mathbf{x}) \equiv Z(\mathbf{x}) - \tfrac{1}{3}\mathrm{tr}\, Z(\mathbf{x})\mathbb{1}, \qquad (3.4)$$

so that

$$Z(\mathbf{x}) = M(\mathbf{x}) + \tfrac{1}{3}L(\mathbf{x})\mathbb{1}. \qquad (3.5)$$

B. Effective Lagrangian

The Lagrange density for the effective theory will have standard derivative terms plus a potential for the complex scalar Z,

$$\mathcal{L} = g_3^{-2}\{\tfrac{1}{2}\,\mathrm{tr}\, F_{ij}^2 + \mathrm{tr}(D_i Z^\dagger D_i Z) + V(Z)\}, \qquad (3.6)$$

with $D_i \equiv \partial_i - i[A_i, \cdot]$ and $F_{ij} \equiv \partial_i A_j - \partial_j A_i - [A_i, A_j]$.[5] The $Z(3)$ invariant potential $V(Z)$ will consist of two pieces,

$$V(Z) = V_0(Z) + g_3^2 V_1(Z), \qquad (3.7)$$

with

$$V_0(Z) = c_1\, \mathrm{tr}[Z^\dagger Z] + c_2(\det[Z] + \det[Z^\dagger]) + c_3\, \mathrm{tr}[(Z^\dagger Z)^2], \qquad (3.8)$$

and

$$V_1(Z) = \tilde{c}_1\, \mathrm{tr}[M^\dagger M] + \tilde{c}_2(\mathrm{tr}[M^3] + \mathrm{tr}[(M^\dagger)^3]) + \tilde{c}_3\, \mathrm{tr}[(M^\dagger M)^2]. \qquad (3.9)$$

We will refer to $V_0(Z)$ as the "hard potential" and $V_1(Z)$ as the "soft potential." This terminology is appropriate at high temperatures where g_3^2/T is small, in which case the effective masses arising from $V_0(Z)$ will be large compared to those arising from $g_3^2 V_1(Z)$.

The hard potential $V_0(Z)$ is composed of the three lowest-dimension operators which (in addition to being $Z(3)$ invariant) are invariant under the $SU(3) \times SU(3)$ transformation

$$Z(\mathbf{x}) \rightarrow AZ(\mathbf{x})B, \qquad (3.10)$$

for arbitrary $A, B \in SU(3)$. Note that the free $\mathrm{tr}(\partial_i Z^\dagger \partial_i Z)$ kinetic term is also invariant under this transformation. A short exercise, presented in Appendix A, shows that $V_0(Z)$ has extrema when $Z = \lambda\Omega$ for Ω any element of $SU(3)$ and λ a real root of $2c_3\lambda^3 + c_2\lambda^2 + c_1\lambda = 0$. The coefficient c_3 must be positive for stability. We will require that $c_2 < 0$ and $c_2^2 > 9c_1c_3$. These conditions ensure that the

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[5]The freedom to make multiplicative rescalings of Z is used to fix the coefficient of $\mathrm{tr}(D_i Z^\dagger D_i Z)$ in the Lagrangian (3.6). One could also add a $|\nabla(\mathrm{tr}\, Z)|^2$ term, as this preserves all the required symmetries. But this additional term may be eliminated with a $Z \rightarrow Z + \alpha(\mathrm{tr}\, Z)\mathbb{1}$ field redefinition.

global minimum corresponds to the largest positive real root which we will write, for later convenience, as $\lambda = \frac{1}{3}\upsilon$ with

$$\upsilon \equiv \frac{3}{4}\left(\frac{-c_2 + \sqrt{c_2^2 - 8c_1c_3}}{c_3}\right). \qquad (3.11)$$

Therefore, $V_0(Z)$ is minimized when Z equals an arbitrary $SU(3)$ matrix times $\upsilon/3$.[6]

The coupling constants in the soft potential $V_1(Z)$ will be required to satisfy the inequalities $\tilde{c}_3 > 0$ and $\tilde{c}_1\tilde{c}_3 > \tilde{c}_2^2$. As shown in Appendix A, these conditions ensure that $V_1(Z)$ is strictly positive for nonzero M. Hence, the soft potential $V_1(Z)$ is minimized when $M = 0$, or equivalently when Z is proportional to the identity matrix. The overall factor of g_3^2 multiplying the soft potential is designed to mimic the fact, noted earlier, that the effective potential for the Wilson line in high-temperature Yang-Mills theory only arises at one-loop order.

We have only included in the soft potential the three lowest-dimension $Z(3)$ invariant terms which can be built just from the traceless field M. There are more $Z(3)$ invariant terms, with the same dimensions, which we could have added—for example, terms including factors of the trace L. However, if Z is restricted to the manifold of minima of the hard potential V_0, and so is proportional to an $SU(3)$ matrix, then there are only three independent operators which are gauge invariant, $Z(3)$ invariant, and at most fourth order in the field. The three terms we have included in the potential (3.9) are a convenient choice for these three independent terms. As shown in the next section, these terms will be sufficient for matching with the EQCD Lagrangian.[7]

The soft potential $V_1(Z)$ lifts the continuous degeneracy at the minimum of $V_0(Z)$, leaving three discrete minima related by the $Z(3)$ symmetry. If Z equals $\frac{\upsilon}{3}$ times an element of the center of $SU(3)$, then it will simultaneously minimize both the hard and soft potentials, and hence will trivially minimize the combined potential $V(Z) = V_0(Z) + g_3^2 V_1(Z)$.

The theory defined by Eqs. (3.6), (3.7), (3.8), and (3.9) is a superrenormalizable three-dimensional field theory. The only renormalizations required (in addition to vacuum energy subtraction) are one- and two-loop adjustments of the coefficients c_1 and $\tilde{c}_1$ of the quadratic terms.

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[6]Note that sending the coefficients c_i to infinity, while holding their ratios fixed, would cause $V_0(Z)$ to act like a delta-function constraint forcing the matrix Z to equal an $SU(3)$ element (up to a scale factor). The resulting effective theory would be a gauged nonlinear sigma model for the Wilson line $\Omega(\mathbf{x})$. But this limit would return us to a nonrenormalizable effective theory, which we wish to avoid.

[7]Had we chosen to work with a gauge group $SU(4)$ or higher, we would need to include a quartic double trace term in Eq. (3.9) in order to reproduce the EQCD Lagrangian. This is unnecessary for $SU(3)$, due to the identity mentioned in footnote 4.

To carry out perturbative matching, as discussed in the next section, we will need the expansion of the Lagrangian (3.6) about the minima of the potential. In the vicinity of the minimum at $\frac{\upsilon}{3}e^{2\pi in/3}$, we may decompose Z as

$$Z = e^{2\pi in/3}\left\{\frac{1}{3}\upsilon\mathbb{1} + g_3\left[\frac{1}{\sqrt{6}}(\phi + i\chi)\mathbb{1} + (h + ia)\right]\right\}, \qquad (3.12)$$

where ϕ and χ are real numbers, while h and a are traceless Hermitian matrices. We will also rescale the three-dimensional gauge field $\mathbf{A} \to g_3\mathbf{A}$ to switch to conventional perturbative normalization of the gauge field. Note that the fluctuation fields ϕ, χ, h, and a, as well as the gauge field $\mathbf{A}$, will now have canonical dimension $1/2$, as appropriate for a three-dimensional theory. Expressed in terms of these fluctuation fields, the effective theory Lagrangian has the form

$$\mathcal{L} = V_{\min} + \frac{1}{2}\text{tr}\,F_{ij}^2 + \frac{1}{2}[(\partial_i\phi)^2 + m_\phi^2\phi^2]$$
$$+ \frac{1}{2}[(\partial_i\chi)^2 + m_\chi^2\chi^2] + \text{tr}[(D_ih)^2 + m_h^2h^2]$$
$$+ \text{tr}[(D_ia)^2] + V_{\text{int}}(\phi, \chi, h, a), \qquad (3.13)$$

where $D_i \equiv \partial_i - ig_3[A_i, \cdot]$ and $V_{\min}$ is the value of $V_0(Z)/g_3^2$ at its minimum.

The interaction terms in V_{int} are suppressed by one or more powers of g_3, and come from the soft potential plus the parts of the hard potential which involve more than two fluctuation fields. These terms are displayed explicitly in Appendix B. The leading-order masses induced by the hard potential are

$$m_\phi^2 = -2c_1 - \frac{1}{3}\upsilon c_2, \qquad m_\chi^2 = -\upsilon c_2,$$
$$m_h^2 = -2c_1 - \frac{4}{3}\upsilon c_2, \qquad (3.14)$$

with υ given in Eq. (3.11). The field a stays massless at this lowest order. Note that $m_h^2 = m_\chi^2 + m_\phi^2$, so this mass is not independent of the other two. The relations between the couplings in $V_0(Z)$ and the tree-level masses (and υ) may be inverted to give

$$c_1 = \frac{1}{6}(m_\chi^2 - 3m_\phi^2), \qquad c_2 = -m_\chi^2/\upsilon,$$
$$c_3 = \frac{3}{4}(m_\chi^2 + 3m_\phi^2)/\upsilon^2. \qquad (3.15)$$

The tree-level heavy masses m_ϕ and m_χ, plus the value of υ, should be regarded as arbitrary parameters for now, subject to the condition that $m_\phi > \frac{1}{3}m_\chi$. This is equivalent to the inequality $c_2^2 > 9c_1c_3$ which ensures that the global minima of the hard potential $V_0(Z)$ lie at $\frac{1}{3}\upsilon e^{2\pi in/3}$. The value of the potential (divided by g_3^2) at these minima is $V_{\min} \equiv -\frac{\upsilon^2}{108}(9m_\phi^2 - m_\chi^2)/g_3^2$.

IV. PARAMETER MATCHING AT HIGH TEMPERATURE

A. Reduction to EQCD

We now want to deduce the constraints on the coefficients $\tilde{c}_i$ in the soft potential $V_1(\mathcal{Z})$ which must be satisfied if the effective theory is to reproduce the long distance dynamics of $SU(3)$ Yang-Mills theory at asymptotically high temperature. In this regime, fluctuations on the scale of the temperature T [as well as on the Debye scale $g(T)T$] are weakly coupled, so perturbative matching techniques may be used. The most efficient way to carry this out is to start with our $Z(3)$ invariant effective theory and integrate out the heavy fluctuation fields ϕ, χ, and h. This will produce a non-$Z(3)$ invariant effective theory only involving the light field a (and the three-dimensional gauge field), which may be directly compared to EQCD.

To lowest nontrivial order, the result of integrating out the heavy fields is a "light" effective theory with Lagrange density[8]

$$\mathcal{L}_{\text{light}} = \tfrac{1}{2}\operatorname{tr} F_{ij}^2 + \operatorname{tr}[(D_i a)^2 + m_a^2 a^2 + \tilde{\lambda} a^4]. \quad (4.1)$$

Except for the trivial rescaling $\mathbf{A} \to g_3 \mathbf{A}$ of the gauge field, and renaming $A_0 \to g_3 a$, this has the same form, and identical derivative terms, as the EQCD Lagrangian (2.5).

The effective light mass m_a^2, and the quartic coefficient $\tilde{\lambda}$, can have contributions arising solely from the hard potential $V_0(\mathcal{Z})$, as well as having contributions which involve the soft potential $V_1(\mathcal{Z})$. The lowest-order contributions generated by the hard potential are illustrated in Fig. 1. Using the coefficients of the various terms in the expansion of V_0 about its minima, as given in Eq. (B3), one may easily evaluate these diagrams explicitly. However, this is not necessary—the diagrams shown in Fig. 1, at zero external momentum, sum to zero. This is a consequence of the $SU(3) \times SU(3)$ invariance (3.10) which the effective theory would have in the absence of the soft potential $V_1(\mathcal{Z})$ and the static gauge field. An infinitesimal $SU(3) \times SU(3)$ transformation acts as a shift on the light field a. Integrating out the heavy fields, without the soft potential, must produce a result which respects this invariance, and thus can only involve derivative interactions for the light field a.

Consequently, the light mass m_a^2 and the quartic coupling $\tilde{\lambda}$ in the Lagrangian (4.1) can arise only from contributions involving the non-$SU(3) \times SU(3)$ invariant soft

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[8]We omitted the unit operator from the original effective theory Lagrangian (3.6), as well as from (4.1), for simplicity of presentation. The coefficient of the unit operator gives the short distance contribution to the free energy density (divided by T). To match this contribution to the free energy, the coefficient of the unit operator in our effective theory must differ from the corresponding coefficient in EQCD by $-V_{\text{min}} + \frac{1}{12\pi} \times (m_\phi^3 + m_\chi^3 + 8m_h^3)$, up to higher order corrections suppressed by powers of g_3^2 divided by a heavy mass.
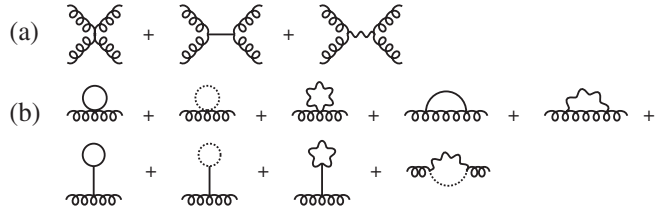

FIG. 1. (a) Tree-level diagrams originating from the hard potential V_0 which contribute to the quartic interaction vertex for the light field a. (b) One-loop diagrams produced by V_0 which contribute to the mass of the field a. Solid, dotted, wavy, and curly lines denote the propagators of the ϕ, χ, h, and a fields, respectively.

potential $V_1(\mathcal{Z})$. To lowest order, one may simply read off their values from the soft terms in the shifted potential (B3), giving

$$m_a^2 = \tilde{c}_1 g_3^2, \qquad \tilde{\lambda} = \tilde{c}_3 g_3^4. \quad (4.2)$$

Demanding that these coefficients coincide with the corresponding terms in the EQCD Lagrangian (2.5) (after identifying $A_0 \to g_3 a$) fixes

$$\tilde{c}_1 = T + \mathcal{O}(g_3^2), \quad (4.3)$$

and

$$\tilde{c}_3 = \frac{3}{4\pi^2 T} + \mathcal{O}\left(\frac{g_3^2}{T^2}\right), \quad (4.4)$$

while $\tilde{c}_2$ is left undetermined.

Integrating out the heavy fields beyond leading order will, of course, produce corrections to the coefficients (4.2), suppressed by additional powers of g_3^2 divided by a heavy mass, as well as induce additional higher dimension operators in the light effective Lagrangian (4.1). The first new operators, involving six powers of the light field a, will have a coefficient of order $(g_3^2/m_{\text{heavy}})^4$.

B. Domain wall properties

Having ensured that our theory reproduces the perturbative physics of $SU(3)$ Yang-Mills theory at very high temperature, we are left with two undetermined parameters, v and $\tilde{c}_2$ (in addition to the deliberately introduced heavy masses m_ϕ and m_χ). As we will show, these two parameters may be fixed by demanding that our effective theory reproduce the tension and width of domain walls separating different $Z(3)$ minima.

Consider a domain wall lying in the x-y plane, which interpolates between the state with $\langle \operatorname{tr} \mathcal{Z} \rangle = v$ at $z = -\infty$ and $\langle \operatorname{tr} \mathcal{Z} \rangle = e^{2\pi i/3} v$ at $z = +\infty$. To find the domain wall profile and evaluate its tension, we need to minimize the free energy for configurations of the scalar field $\mathcal{Z}(z)$ which depend on a single coordinate z and satisfy the boundary conditions

$$L(z = -\infty) = v, \qquad L(z = \infty) = e^{2\pi i/3}v, \qquad (4.5a)$$

$$M(z = -\infty) = 0, \qquad M(z = \infty) = 0. \qquad (4.5b)$$

To evaluate the (leading-order) domain wall tension in the high temperature or weak coupling limit it will be necessary to include:

(1) The contribution of the hard potential, $\frac{T}{g_3^2} \times \int dz V_0(\mathcal{Z}(z))$.

(2) The gradient energy, $\frac{T}{g_3^2} \int dz\,\mathrm{tr}|D_z \mathcal{Z}(z)|^2$.

(3) The contribution of the soft potential, $T \int dz V_1(\mathcal{Z}(z))$.

(4) The one-loop contribution from fluctuations in the gauge and scalar fields.

Our task is greatly simplified by the hierarchical structure of the potential. Because of their overall factors of g_3^{-2}, one might expect the first two contributions to be most important. However, a translationally noninvariant domain wall configuration can minimize $V_0(\mathcal{Z})$, everywhere in space, provided $\mathcal{Z}(z)$ equals $v/3$ times some (spatially varying) $SU(3)$ matrix. And the gradient energy will be comparable to the energy due to the soft potential, and not parametrically larger, if the width of the domain wall is inversely proportional to g_3. Because the soft potential is multiplied by an explicit factor of g_3^2 in the Lagrangian (3.6), its contribution is comparable to the effects due to fluctuations in the fields. Hence, the one-loop contribution cannot be neglected.[9]

Using gauge invariance, we may diagonalize $\mathcal{Z}(z)$ and write it as

$$\mathcal{Z}(z) = \mathrm{diag}(f_1(z), f_2(z), f_3(z)), \qquad (4.6)$$

with

$$f_1(z) \equiv \frac{v}{3} e^{(2\pi i/3)[\alpha(z)+3\beta(z)]}, \qquad (4.7a)$$

$$f_2(z) \equiv \frac{v}{3} e^{(2\pi i/3)[\alpha(z)-3\beta(z)]}, \qquad (4.7b)$$

$$f_3(z) \equiv \frac{v}{3} e^{(-4\pi i/3)\alpha(z)}. \qquad (4.7c)$$

Here α and β are real functions of z that satisfy the boundary conditions

$$\alpha(z = -\infty) = 0, \qquad \alpha(z = \infty) = 1, \qquad (4.8a)$$

$$\beta(z = -\infty) = 0, \qquad \beta(z = \infty) = 0. \qquad (4.8b)$$

Note that shifting (α, β) by any integer multiple of $(\frac{3}{2}, \pm\frac{1}{2})$ leaves $\mathcal{Z}(z)$ invariant (up to a permutation of its diagonal elements).

It will be convenient to introduce a dimensionless rescaled coordinate $\bar{z}$ and Wilson line expectation value $\bar{v}$ via

$$\bar{z} \equiv g_3\sqrt{T}z, \qquad \bar{v} \equiv \frac{v}{T}. \qquad (4.9)$$

The gradient and soft potential contributions to the domain wall free energy (per unit area) then become

$$F_{\mathrm{grad}} + F_{\mathrm{soft}} = g_3^{-1}(\pi\bar{v}T)^2 \left(\frac{2}{3}\sqrt{T}\right)^3 \int_{-\infty}^{\infty} d\bar{z}\{(\alpha')^2$$
$$+ 3(\beta')^2 + U_1(\alpha, \beta)\}, \qquad (4.10)$$

with $\alpha' \equiv \partial\alpha/\partial\bar{z}$, etc., and

$$U_1(\alpha, \beta) = \frac{1}{4\pi^2}\{3 - \cos(4\pi\beta) - 2\cos(2\pi\alpha)\cos(2\pi\beta)\}$$
$$+ \frac{\bar{v}\tilde{c}_2}{18\pi^2}\{6 + \cos(4\pi\alpha) - 3\cos(4\pi\beta)$$
$$+ 2\cos(2\pi\alpha)[\cos(6\pi\beta) - 3\cos(2\pi\beta)]\}$$
$$+ \left(\frac{\bar{v}}{12\pi^2}\right)^2\{9 + \cos(8\pi\beta) + 2\cos(4\pi\beta)$$
$$\times [\cos(4\pi\alpha) - 2] - 8\cos(2\pi\alpha)\cos(2\pi\beta)\}.$$
$$(4.11)$$

Fluctuations of the fields in the effective theory, at one-loop order, produce functional determinants depending on the background domain wall profile $\mathcal{Z}(z)$. The width of the domain wall will be parametrically large compared to $1/v$, and therefore one may use a gradient expansion in the evaluation of the resulting functional determinants. It is sufficient to keep just the first term, leading to[10]

$$\frac{F_{\mathrm{fluc}}}{T} = \ln\det(-\partial^2 + [\mathcal{Z}^\dagger, [\mathcal{Z}, \cdot]])/(\mathrm{Area})$$

$$= \sum_{ij} \int dz \int \frac{d^3k}{(2\pi)^3} \ln(k^2 + |f_i(z) - f_j(z)|^2)$$

$$= -\int \frac{dz}{3\pi}(|f_1 - f_2|^3 + |f_2 - f_3|^3 + |f_3 - f_1|^3)$$

$$= g_3^{-1}\pi^2\bar{v}^2 T\left(\frac{2}{3}\sqrt{T}\right)^3 \int_{-\infty}^{\infty} d\bar{z} U_2(\alpha, \beta), \qquad (4.12)$$

with

$$U_2(\alpha, \beta) = -\frac{\bar{v}}{3\pi^3}\{|\sin(2\pi\beta)|^3 + |\sin(\pi(\alpha - \beta))|^3$$
$$+ |\sin(\pi(\alpha + \beta))|^3\}. \qquad (4.13)$$

Combining the three contributions gives

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[9]This parallels the computation of the leading-order domain wall tension in the full theory [18].

[10]The evaluation of this one-loop contribution is similar to the calculation in Ref. [18]. One may neglect the soft potential $V_1(\mathcal{Z})$. Transverse fluctuations of **A** acquire an effective mass due to the background value of the scalar field, and yield the above functional determinant. If one uses the gauge fixing term $\mathrm{tr}(\partial_i A_i)^2$ and lets $\mathcal{Z}(x) = (\mathbb{1} + g_3\eta(x))\bar{\mathcal{Z}}$, with $\bar{\mathcal{Z}}$ the background value of the scalar field, then the longitudinal part of **A** mixes with the traceless anti-Hermitian part of η (which receives no tree-level mass from the hard potential V_0). The resulting functional determinant is independent of $\bar{\mathcal{Z}}$, and equals one with dimensional regularization. Fluctuations of the trace and traceless Hermitian parts of η are also independent of $\bar{\mathcal{Z}}$, and produce an irrelevant constant (given in footnote 8).
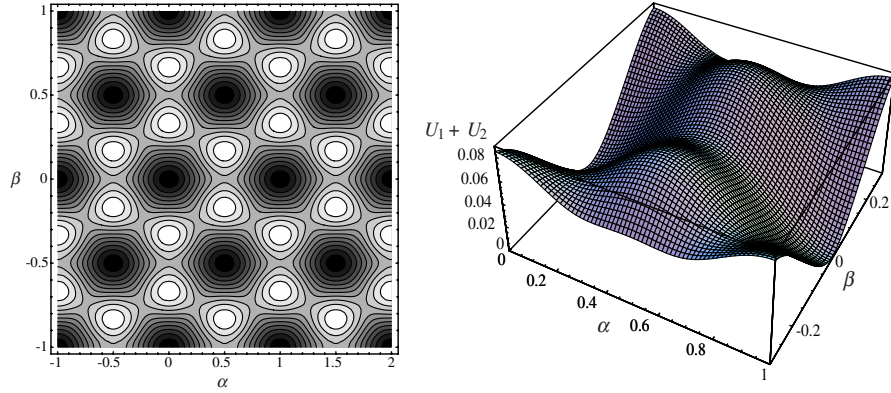
FIG. 2 (color online). The potential $U_1(\alpha, \beta) + U_2(\alpha, \beta)$ plotted for the optimal values of $\bar{v}$ and $\tilde{c}_2$ given in Eq. (4.23). In the contour plot on the left, minima correspond to the dark regions. A fundamental domain, representing all configurations of Z which are inequivalent under conjugation by $SU(3)$, is given by the triangle with vertices at $(\alpha, \beta) = (0, 0)$, $(1, 0)$ and $(1/2, 1/2)$—which represent the three distinct minima related by $Z(3)$ transformations. The $3d$ plot on the right shows the region between the two minima at $(\alpha, \beta) = (0, 0)$ and $(1, 0)$.

$$F_{\text{dw}}[\alpha, \beta] \equiv F_{\text{grad}} + F_{\text{soft}} + F_{\text{fluc}}$$
$$= g_3^{-1}(\pi\bar{v}T)^2\left(\frac{2}{3}\sqrt{T}\right)^3 \int_{-\infty}^{\infty} d\bar{z}[(\alpha')^2 + 3(\beta')^2$$
$$+ U_1(\alpha, \beta) + U_2(\alpha, \beta)]. \qquad (4.14)$$

This is the excess free energy density due to a domain wall (relative to the spatially constant equilibrium free energy density). Note that all three contributions are parametrically the same size, as anticipated. Minimizing this functional, subject to the boundary conditions (4.8), will yield the leading-order domain wall tension.

The combined potential $U_1(\alpha, \beta) + U_2(\alpha, \beta)$ is plotted in Fig. 2. From the general form of the potential it is easy to see that for positive $\tilde{c}_2$, and all values of α, there is a local minimum in the β-direction at $\beta = 0$. This suggests, and further inspection confirms, that one may simply set $\beta = 0$ in the search for the minimum. This reduces the computation to the minimization of

$$F_{\text{dw}}[\alpha, 0] = g_3^{-1}(\pi\bar{v}T)^2(\tfrac{2}{3}\sqrt{T})^3 \int_{-\infty}^{\infty} d\bar{z}[(\alpha')^2 + \tilde{U}(\alpha)], \qquad (4.15)$$

with

$$\tilde{U}(\alpha) \equiv \frac{1}{\pi^2}\sin^2(\pi\alpha) - \frac{2\bar{v}}{3\pi^3}\sin^3(\pi\alpha)$$
$$+ \frac{\bar{v}^2 + 4\pi^2\bar{v}\tilde{c}_2}{9\pi^4}\sin^4(\pi\alpha). \qquad (4.16)$$

Solutions to the equation of motion produced by varying the functional (4.15) conserve the "energy" $E \equiv (\partial_{\bar{z}}\alpha)^2 - \tilde{U}(\alpha)$, and the domain wall boundary conditions (4.8) imply that $E = 0$. The free energy functional (4.15) is invariant under the transformation $\alpha(\bar{z}) \rightarrow 1 - \alpha(-\bar{z})$, which is a combination of parity, charge conjugation, and a $Z(3)$ symmetry transformation. The minimal energy do-

main wall solution should have the same invariance, implying that $\alpha(\bar{z} = 0) = \frac{1}{2}$. Hence, the appropriate solution to the zero energy equation of motion $\partial_{\bar{z}}\alpha = \tilde{U}(\alpha)^{1/2}$ is given by

$$\int_{1/2}^{\alpha(\bar{z})} d\alpha\, \tilde{U}(\alpha)^{-1/2} = \bar{z}, \qquad (4.17)$$

while the resulting domain wall tension is

$$\sigma \equiv F_{\text{dw}}[\alpha, 0] = g_3^{-1}(4\pi\bar{v}T)^2\left(\frac{1}{3}\sqrt{T}\right)^3 \int_0^1 d\alpha\, \tilde{U}(\alpha)^{1/2}. \qquad (4.18)$$

We will define the width Δz of the domain wall as the ratio of the first moment of the domain wall free energy density divided by the tension,[11] or

$$\Delta z \equiv \frac{(4\pi\bar{v})^2(\frac{1}{3}T)^3}{g_3^2\sigma} \int_{-\infty}^{\infty} d\bar{z}|\bar{z}|\tilde{U}(\alpha(\bar{z}))$$
$$= \frac{2(4\pi\bar{v})^2(\frac{1}{3}T)^3}{g_3^2\sigma} \int_{1/2}^1 d\alpha_1 \tilde{U}(\alpha_1)^{1/2}$$
$$\times \int_{1/2}^{\alpha_1} d\alpha_2 \tilde{U}(\alpha_2)^{-1/2}. \qquad (4.19)$$

None of the integrals (4.17), (4.18), and (4.19) are analytically tractable but evaluating them numerically, or integrating the equation of motion $\partial_{\bar{z}}\alpha = \tilde{U}(\alpha)^{1/2}$, for particular values of $\tilde{c}_2$ and $\bar{v}$ is straightforward. The resulting tension and width may then be compared to the corresponding results in the underlying Yang-Mills theory. The (leading-order) domain wall free energy density in high-

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[11]Alternative definitions of the domain wall width could, of course, be chosen, but make little difference. For example, defining the width from the second moment of the domain wall free energy density changes the resulting optimal parameters (4.23) by about 0.1% for $\bar{v}$ and 2.6% for $\tilde{c}_2$.
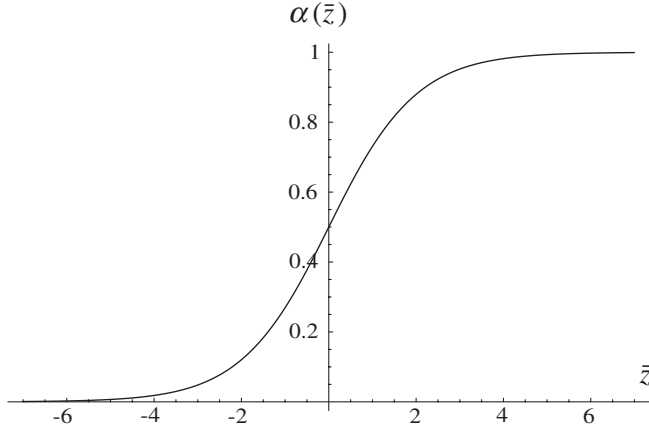
FIG. 3. The function $\alpha(\bar{z})$ plotted for the optimal values of $\bar{v}$ and $\tilde{c}_2$ given in Eq. (4.23).

temperature $SU(3)$ Yang-Mills theory is [18]

$$\mathcal{F}_{\text{YM}}(z) = \frac{\pi^2 T^4}{3\cosh^4(\bar{z}(z)/2)}. \qquad (4.20)$$

The resulting domain wall tension is

$$\sigma_{\text{YM}} = \int_{-\infty}^{\infty} dz \mathcal{F}_{\text{YM}}(z) = \frac{8\pi^2}{9} \frac{T^3}{g(T)}, \qquad (4.21)$$

and the (first moment) domain wall width equals

$$\Delta z_{\text{YM}} = \sigma_{\text{YM}}^{-1} \int_{-\infty}^{\infty} dz |z| \mathcal{F}_{\text{YM}}(z) = \frac{\ln(4) - \frac{1}{2}}{g(T)T}. \qquad (4.22)$$

A straightforward calculation shows that the effective theory reproduces the domain wall tension and width of the underlying Yang-Mills theory when

$$\bar{v} = 3.005\,868, \qquad \tilde{c}_2 = 0.118\,914. \qquad (4.23)$$

[Note that this value for $\bar{v}$, almost equal to 3, makes the minima of the potential lie almost exactly at T times an element of $Z(3)$.] In Fig. 3 we plot $\alpha(\bar{z})$ for these values of the parameters, and in Fig. 4 we compare the resulting domain wall free energy density in the effective theory, given by

$$\mathcal{F}_{\text{eff}}(z) = (4\pi\bar{v}T^2)^2(\tfrac{1}{3})^3 \tilde{U}(\alpha(\bar{z}(z))), \qquad (4.24)$$

with the full theory result (4.20). The agreement is rather remarkable.

C. Discussion

Perturbative matching with EQCD, plus comparison of domain wall properties, has determined (to leading order) all parameters of the effective theory [leading to Eqs. (4.3), (4.4), and (4.23)] except the two heavy masses m_ϕ and m_χ. These heavy masses are arbitrary, as long as they are parametrically of order T (or v), and satisfy $m_\phi > \frac{1}{3}m_\chi$.

It may seem surprising that domain wall properties can be used to fix parameters of the effective theory which

were left undetermined by the perturbative matching to EQCD. Correct matching with EQCD should imply that our effective theory reproduces all physics of hot Yang-Mills theory on energy scales small compared to T. Fields in the domain wall only vary on the Debye screening length scale $[g(T)T]^{-1}$ which, for asymptotically high temperatures, is parametrically large compared to T^{-1}. So why does matching to EQCD not yield an effective theory that automatically reproduces the correct domain wall structure, without any further tuning of parameters?

The resolution of this puzzle involves the realization that the inverse domain wall width is not the only relevant energy scale in a domain wall. As seen in the fluctuation contribution (4.12) to the tension, near the center of the domain wall off diagonal components of the static gauge field receive effective masses which are order v in the effective theory, and order T in the underlying Yang-Mills theory.[12] Hence, the $O(T)$ value of v, which determines the magnitude of the scalar field in our effective theory, is a physically relevant scale in the domain wall. Equivalently, the excess free energy density (4.24) near the center of the domain wall is $O(T^4)$, and thus comparable to the $O(T^4)$ equilibrium free energy density due to fluctuations on the scale of T.

The symmetry structure of the effective theory guarantees that it will have domain walls interpolating between different $Z(3)$ minima, and the matching to EQCD guarantees that these solutions will have the correct exponential decay, proportional to $e^{-m_D|z|}$, far from the domain wall. However, the precise value of the domain wall tension, and its width, are sensitive to physics on the scale of T. This is why reproducing these observables in the effective theory requires tuning of parameters beyond the perturbative matching to EQCD. If one were to demand that the effective theory exactly reproduce all moments of the domain wall energy density (4.20), then it would be necessary to add an infinite number of additional higher dimension operators to the effective theory, with appropriately tuned coefficients. But from Fig. 4, it is evident that near perfect agreement is achieved, without introducing any higher dimension operators, just by suitably adjusting those low energy parameters (v and $\tilde{c}_2$) of our superrenormalizable effective theory which were left undetermined by the matching to EQCD.

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[12] An alternative way to see this is to consider the electric field contribution to the Yang-Mills action, $\text{tr}(D_0 A_i)^2 = \text{tr}(\partial_0 A_i - ig[A_0, A_i])^2$. A background value of A_0 effectively shifts the Matsubara frequencies from $2\pi nT$ to $2\pi nT$ minus the eigenvalues of gA_0 (in the adjoint representation). Near the center of a domain wall, the eigenvalues of A_0 (in a gauge where A_0 is static) are of order $\pi T/g$—since this corresponds to an order one change in the phase of the Wilson line $\Omega \sim e^{ig\beta A_0}$. Consequently, near the center of a domain wall there is no separation of scales between the effective frequencies of the static and nonstatic modes.
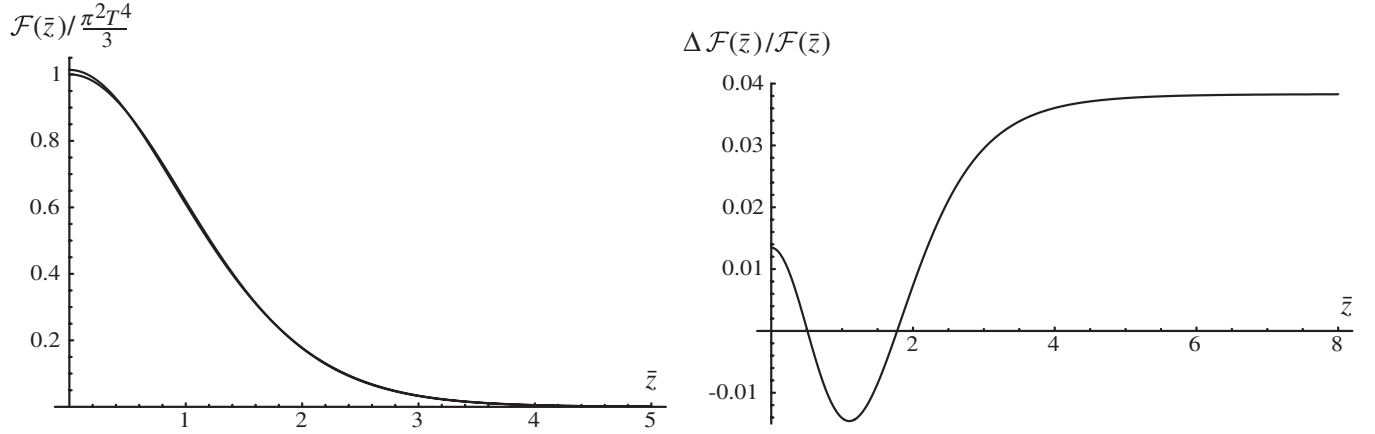
FIG. 4.   Left: The domain wall free energy densities $\mathcal{F}(\bar{z})$, divided by $\frac{1}{3}\pi^2 T^4$, in the effective theory and underlying Yang-Mills theory, plotted versus $\bar{z} \equiv g(T)Tz$. The two curves are nearly indistinguishable, differing by less than 1.5% at the center of the domain wall. Right: The relative difference of the two curves, $(\mathcal{F}_{\mathrm{eff}}(\bar{z}) - \mathcal{F}_{\mathrm{YM}}(\bar{z}))/\mathcal{F}_{\mathrm{YM}}(\bar{z})$.

To conclude this section, let us briefly review the different sources of corrections to the Lagrangian and parameters of the effective theory. These will become important if one aims at next-to-leading-order (NLO) accuracy in the determination of various physical quantities through the effective theory, such as bulk equilibrium thermodynamics or the $Z(3)$ domain wall tension and width.

Integrating out the heavy fields, with vertices originating from the hard potential $V_0(\mathcal{Z})$, will produce corrections to the scalar and gauge field kinetic terms. At first nontrivial order, the former involves determining the leading momentum dependence of the two-point graphs of Fig. 1(b), and will yield an order g^2 correction to the normalization of the scalar kinetic term in the Lagrangian of Eq. (4.1).[13] These corrections amount to multiplicative wave function renormalizations for the fields in question.

Integrating out heavy fields at NLO will also generate corrections to the values of the parameters in the light theory (4.1), whose matching to EQCD will require $\mathcal{O}(g^2)$ relative adjustments to the couplings in the soft potential $V_1(\mathcal{Z})$. For the parameters m_a and $\tilde{\lambda}$, one-loop contributions involving one soft vertex will produce $\mathcal{O}(g^4)$ and $\mathcal{O}(g^6)$ corrections, respectively. For bulk thermodynamics, the NLO correction to m_a^2 is more important than the leading-order value of $\tilde{\lambda}$.

Because the $SU(3) \times SU(3)$ invariance of V_0 is not respected by the soft potential V_1, or by the coupling of the scalar fields to the gauge field, integrating out the heavy fields may also produce entirely new operators involving **A** and a of dimension three or higher. The first such new

operators to appear include $\mathrm{tr}\, a^6$ and $\mathrm{tr}(a^2(D_i a)^2)$, and are suppressed by g_3^8.[14]

Ensuring that the domain wall properties of our theory agree with corresponding results in full Yang-Mills theory beyond leading order would, in addition to the above, require extending the derivative expansion of the one-loop fluctuation contribution F_{fluc} to next-to-leading order, as well as evaluating the two-loop effective potential for $\mathcal{Z}$. The analogous NLO calculation in the full Yang-Mills theory is discussed in more detail in Ref. [21]. The required analysis in the effective theory should be very similar.

V. PHASE DIAGRAMS

A. EQCD

The lack of $Z(3)$ center symmetry in EQCD has been a primary motivation for the work presented in this paper. The consequences of this explicit breaking of the $Z(3)$ symmetry are particularly apparent if one studies the phase diagram of EQCD for arbitrary values of its parameters, as done in Refs. [6,22]. One can form two dimensionless ratios from the parameters (g_3^2, m_E^2, and λ_E) of EQCD, conventionally chosen to be[15] $x \equiv \frac{1}{2}\lambda_E$ and $y \equiv m_E^2/g_3^4$. Parameter values which result from matching to hot $SU(3)$ Yang-Mills theory correspond to the curve

$$xy = \frac{3}{8\pi^2} + \frac{9}{16\pi^2}x + \mathcal{O}(x^2), \tag{5.1}$$

where the next-to-leading order correction, proportional to

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[13]Terms involving powers of the external momentum, which correspond to higher derivative terms in the Lagrangian, will be proportional to inverse powers of the heavy masses and are thus suppressed by additional factors of g when evaluated on momentum scales of order gT which are relevant in the effective theory.

[14]For $\mathrm{tr}\, a^6$, one factor of g_3^2 comes from the necessary inclusion of one soft vertex from V_1. For $\mathrm{tr}(a^2(D_i a)^2)$, the multiplicative coupling constant is of order g_3^6, but the covariant derivatives contribute additional factors of g_3—either explicitly or via spatial derivatives applied to gT scale physics. The same is true for other higher dimension operators involving derivatives.

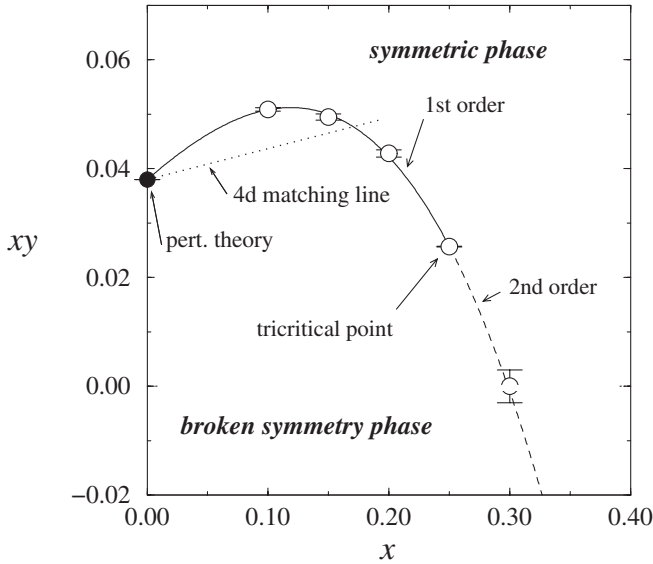[15]Our definition of λ_E differs from that of Ref. [6] by a factor of $2/g_3^2$.

FIG. 5. The phase diagram of EQCD (from Ref. [6]) parametrized by $x \equiv \frac{1}{2}\lambda_E$ and $y \equiv m_E^2/g_3^4$. The phase with spontaneous symmetry breaking of the $A_0 \rightarrow -A_0$ symmetry lies in the lower left region of the diagram; the upper region is the unbroken symmetry phase. The dotted "4d matching line" is the slice of the phase diagram which satisfies relation (5.1) giving the result of next-to-leading order matching with hot SU(3) Yang-Mills theory.

x, has been included. Increasing temperature corresponds to decreasing values of x (proportional to $g(T)^2$).

For sufficiently small values of m_E^2 (or y) one finds that the $A_0 \rightarrow -A_0$ symmetry, which is a remnant of time-reflection symmetry in the original 4-d theory, is spontaneously broken. Within the broken phase, there are two degenerate equilibrium states, with opposite (and nonzero) expectation values for $\mathrm{tr}\,A_0^3$. The phase transition line separating the symmetric phase (with vanishing $\langle\mathrm{tr}\,A_0^3\rangle$) from this symmetry-broken phase is first order for small values of x and second order for large values, with a tricritical point at $x \approx 0.26$. This is illustrated in Fig. 5. On the first order part of the transition line there are three coexisting extremal equilibrium states, distinguished by positive, negative, and zero values of the order parameter $\langle\mathrm{tr}\,A_0^3\rangle$.

Near the first order transition line, these three equilibrium states with differing values of $\langle\mathrm{tr}\,A_0^3\rangle$ may be regarded as deformed remnants of the three equilibrium states of hot SU(3) gauge theory (which are related by the center symmetry). But in EQCD, there is no threefold symmetry relating these states, only a Z(2) symmetry interchanging the two symmetry-broken states. The states with nonzero $\langle\mathrm{tr}\,A_0^3\rangle$ have, for example, energy densities and correlation lengths which differ from the symmetric $\langle\mathrm{tr}\,A_0^3\rangle=0$ state [6].

Only the symmetric phase corresponds to the physics of the underlying hot Yang-Mills theory. However, as indicated in Fig. 5, the dimensional reduction line (5.1) lies inside the broken phase. This line intersects the phase

transition curve at $x = 0$ (or $T = \infty$), but for any nonzero value of x (or finite temperature T), matching with hot Yang-Mills theory requires studying the symmetric phase on the wrong side of the phase transition—that is, studying a metastable "supercooled" phase. For sufficiently small x, this is feasible. The lifetime of the metastable phase is so long that transitions to the stable symmetry-broken phase are never seen in practical numerical simulations. But this is surely a less-than-satisfying feature of EQCD.

As one increases x (or lowers the temperature) in EQCD, there is no region where a new "confining" phase appears, and no quadruple point where four equilibrium states coexist, analogous to the transition temperature T_c in real SU(3) Yang-Mills theory, where the three spontaneously broken Z(3) deconfined states and the Z(3) symmetric confining state all coexist.

B. Z(3) invariant effective theory

Viewed as a superrenormalizable three-dimensional field theory with no constraints on its coupling constants (besides stability), our Z(3) invariant effective theory (3.6), (3.7), (3.8), and (3.9) depends on a total of seven parameters, which may be taken to be one overall scale v, the ratios of the heavy masses to this scale (m_ϕ^2/v^2 and m_χ^2/v^2), a dimensionless gauge coupling g_3^2/v, and three dimensionless "soft" couplings ($\tilde{c}_1/v$, $\tilde{c}_2$, and $\tilde{c}_3 v$). Exploring the resulting phase diagram via numerical simulations, as all six dimensionless ratios are varied, is obviously impractical. However, it is inevitable that the parameter space of this theory contains (i) a region in which the Z(3) symmetry is spontaneously broken, and (ii) a region in which the Z(3) symmetry is unbroken.[16]

There are two ways in which one can easily imagine changing the parameters of the theory so as to move from the symmetry-broken (deconfined, high-temperature) phase to the unbroken (confining, low-temperature) phase. If the heavy masses are varied and m_ϕ becomes less than $\frac{1}{3}m_\chi$, then the global minimum of the potential $V_0(Z)$ switches from $Z = v\mathbb{1}$ (times a cube root of unity) to the symmetric point $Z = 0$.[17] If the theory is kept weakly

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[16]Gauge symmetries are, of course, never truly spontaneously broken. In addition to the global Z(3) symmetry, our effective theory is also invariant under $Z \rightarrow Z^*$ (for real values of the couplings). Hence, the global symmetry group is really Z(3) $\times$ Z(2). The Z(3) broken phase is a phase in which this Z(3) $\times$ Z(2) global symmetry spontaneously breaks to a Z(2) subgroup. For real values of the couplings, we do not believe that any other symmetry-breaking pattern (such as Z(3) $\times$ Z(2) $\rightarrow$ nothing) is possible.

[17]Within a mean field approximation, m_ϕ and m_χ are the inverse correlation lengths for the real and imaginary parts of the trace of the Wilson line, respectively. A ratio of three for m_χ/m_ϕ at the point where the tree-level potential undergoes symmetry restoration is in accord with the mean field analysis of Dumitru and Pisarski [23] for a Z(3) symmetric model involving only the trace of the Wilson line (with up to quartic interactions).

coupled, so that $g_3^2/v \ll 1$, then this tree-level (or mean field) analysis is reliable, and the resulting transition will generically be a strong first order transition.

Alternatively, if m_ϕ remains greater than $\frac{1}{3}m_\chi$ but the theory becomes sufficiently strongly coupled, so that g_3^2/v is no longer small, then this should drive a fluctuation induced phase transition in which the entropy-driven effects of fluctuations overwhelm the symmetry-breaking influence of the potential. This is analogous to symmetry restoration in nonlinear sigma models.

The confining/deconfining phase transition in $SU(3)$ Yang-Mills theory is known to be a relatively weak first order transition, and near the transition fluctuations of the Wilson line, or the static gauge field, are not weakly coupled. Consequently, we expect that the second regime of fluctuation induced phase transitions is the more appropriate model for $SU(3)$ gauge theory near its confinement transition.

An interesting two-dimensional slice of the parameter space of our effective theory which would probe both of the above regimes, as well as the portion of parameter space which corresponds to asymptotically high-temperature Yang-Mills theory, is given by varying g_3^2/v and m_ϕ^2/v^2 while fixing the soft couplings at the values determined from the high-temperature matching in the previous section, namely

$$\tilde{c}_1/v = 0.332683, \quad \tilde{c}_2 = 0.118914, \quad \tilde{c}_3 v = 0.228419,$$
(5.2)

and choosing m_χ^2/v^2 to equal, say, unity. Within this two-dimensional slice, there will be a region of unbroken $Z(3)$ symmetry, a region of spontaneously broken $Z(3)$ symmetry, and a phase transition line separating these regions. The entire phase transition line is expected to be first order (for the same reasons discussed in Ref. [19]), and everywhere on the transition line there will be four coexisting equilibrium states—just as in the full $SU(3)$ gauge theory.

Exploring this two-dimensional parameter space via numerical simulations is surely feasible. As a warm-up, one could focus on a single line with a fixed value m_ϕ^2/v^2, and only vary g_3^2/v. It would be very interesting to see if the behavior of physical observables (such as the ratios of correlation lengths in different symmetry channels) along such a line closely mimics the behavior in hot $SU(3)$ gauge theory as the temperature varies from below T_c to far above.

VI. CONCLUSION

We have formulated a three-dimensional $Z(3)$-invariant effective theory which can reproduce properties of hot $SU(3)$ Yang-Mills theory both at asymptotically high temperatures, and near the confinement/deconfinement transition. Lowest-order perturbative matching plus comparison of domain wall properties was used to fix the parameters of the effective theory at high temperatures. The form of the

effective theory ensures that as one increases the dimensionless gauge coupling g_3^2/v (which increases the size of fluctuations), or decreases the heavy mass m_ϕ (which changes the shape of the tree-level potential), the theory will undergo a first order phase transition from a deconfined phase with spontaneously broken $Z(3)$ symmetry to a confining phase with unbroken $Z(3)$ symmetry. This reproduces, by design, the qualitative behavior of hot $SU(3)$ Yang-Mills theory near T_c. Numerical simulations of the effective theory will be needed to determine the tuning of parameters that makes the effective theory, near its phase transition, most closely reproduce quantitative properties of hot Yang-Mills theory near the confinement transition. As discussed in the previous section, just varying g_3^2/v while holding the other parameters fixed at the values determined by high-temperature matching will give a theory that reproduces the correct high-temperature behavior at small gauge coupling, together with a fluctuation induced phase transition at a critical value of g_3^2/v. This, we hope, will be an accurate effective theory for quarkless QCD for temperatures ranging from near T_c to asymptotically large. We look forward to numerical simulations testing this hypothesis.[18]

In the case of a positive result, it will be interesting to investigate generalizations of our effective theory to other values of N_c, or to QCD with dynamical quarks. For sufficiently heavy quarks, it is known that a weakly first order finite-temperature phase transition persists, even though fundamental representation quarks break the $Z(3)$ center symmetry and no local order parameter for the transition is available. Therefore, dynamical quarks may be viewed as adding soft $Z(3)$ symmetry-breaking terms to the effective theory. Such terms will lift the degeneracy of the minima of the potential, leaving the minimum with a real expectation value for the trace of the Wilson line as the unique global minimum. Matching, at high temperature, to EQCD with a nonzero number of quark flavors should be straightforward. The resulting effective theory should capture some of the $Z(3)$ physics postulated to be important in the full theory even at nonzero N_f [8]. Pursuing this generalization will be left for future work.

APPENDIX A: MINIMIZING THE POTENTIAL

The variation of the hard potential $V_0(\mathcal{Z})$, defined in Eq. (3.4), is

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[18]Such simulations are currently underway [24].

$$\delta V_0 = \mathrm{tr}[\delta Z \{c_1 Z^\dagger + c_2 Z^{-1} \det Z + 2c_3 Z^\dagger Z Z^\dagger\}] + \mathrm{H.c.} \tag{A1}$$

and this vanishes when Z satisfies

$$c_1 Z^\dagger Z + c_2 \det Z + 2c_3 (Z^\dagger Z)^2 = 0. \tag{A2}$$

Subtracting this equation from its Hermitian conjugate shows that $\det Z$ must be real at extrema. For $c_2 < 0$, which we have assumed, one may see directly from the potential (3.8) that the minimum of $V_0(Z)$ will occur for matrices with positive determinant.

Let λ^2 denote a (positive, real) eigenvalue of the positive-definite matrix $Z^\dagger Z$. To satisfy relation (A2), these eigenvalues must obey

$$2c_3 \lambda^4 + c_1 \lambda^2 + c_2 \det Z = 0. \tag{A3}$$

For $c_2 < 0 < c_3$, and a positive determinant of Z, this has a unique positive root,

$$\lambda^2 = \frac{-c_1 + \sqrt{c_1^2 - 8c_2 c_3 \det Z}}{4c_3}. \tag{A4}$$

Therefore, all eigenvalues of $Z^\dagger Z$ are equal, implying that this matrix is proportional to the unit matrix. Consequently, Z must equal a special unitary matrix times λ. So $\det Z = \lambda^3$ and condition (A4) becomes $\lambda^2 = [-c_1 + \sqrt{c_1^2 - 8c_2 c_3 \lambda^3}]/(4c_3)$. The solutions of this equation are $\lambda = 0$ and $\lambda = \lambda_\pm$ where

$$\lambda_\pm \equiv \frac{-c_2 \pm \sqrt{c_2^2 - 8c_1 c_3}}{4c_3}. \tag{A5}$$

These nonzero roots are real if $c_2^2 > 8c_1 c_3$. Evaluating V_0 at its three possible extrema, one finds that the root λ_+ corresponds to a global minimum provided $c_2 < 0$ and $c_2^2 > 9c_1 c_3$. Assuming c_2 satisfies these conditions, $V_0(Z)$ is minimized when $Z = \lambda_+ \Omega$ for arbitrary $\Omega \in SU(3)$. When $c_1 > 0$ there is a local minimum at $Z = 0$ and the condition $c_2^2 > 9c_1 c_3$ implies that c_2 must be less than $-3\sqrt{c_1 c_3}$. But if $c_1 < 0$ then $Z = 0$ is a local maximum and c_2 must merely be negative. If $c_1 > 0$ and $8c_1 c_3 < c_2^2 < 9c_1 c_3$, then $Z = 0$ is the global minimum of the potential and $Z = \lambda_+ \Omega$ is only a local minimum.

To analyze the soft potential $V_1(Z)$, given in Eq. (3.9), it is convenient to insert the singular value decomposition $M = LmR^\dagger$. Here L and R are unitary matrices, while m is diagonal, real, and positive. Only the cubic terms in $V_1(Z)$ depend on the unitary matrices L and R, and it is easy to see that $\tilde{c}_2(\mathrm{tr}[M^3] + \mathrm{tr}[(M^\dagger)^3])$ is minimized for $L = R$ if $\tilde{c}_2 < 0$, or $L = -R$ if $\tilde{c}_2 > 0$. Therefore, it is sufficient to minimize $V_1(Z)$ for M a diagonal real matrix. This gives

$$V_1(Z) = \sum_i (\tilde{c}_1 \mu_i^2 + 2\tilde{c}_2 \mu_i^3 + \tilde{c}_3 \mu_i^4), \tag{A6}$$

where $\{\mu_i\}$ are the real eigenvalues of M. Solving for zeros of this expression, one finds that $V_1(Z)$ vanishes only at $M = 0$, and is strictly positive for all nonzero M, provided $\tilde{c}_1$ and $\tilde{c}_3$ are positive and $\tilde{c}_1 \tilde{c}_3 > \tilde{c}_2^2$, which we assume.

Nontrivial minima of the hard potential $V_0(Z)$ [i.e., $Z/\lambda_+ \in SU(3)$] coincide with minima of the soft potential $V_1(Z)$ [i.e., configurations satisfying $Z \propto \mathbb{1}$ so that $M = 0$] when Z equals λ_+ times an element of the center of $SU(3)$. So the global minima of the complete potential $V_0(Z) + g_3^2 V_1(Z)$ (when $c_2^2 > 9c_1 c_3$) lie at

$$Z = e^{2\pi i n/3} \lambda_+ \mathbb{1}, \qquad n \in \{0, 1, 2\}. \tag{A7}$$

Using the decomposition (3.12) of Z and the resulting relations (3.15) between the coefficients in $V_0(Z)$ and the tree-level heavy masses, one may translate the above conditions on the coefficients into equivalent conditions on m_ϕ and m_χ (which are the tree-level masses for fluctuations in the magnitude and phase of $\mathrm{tr} Z$, respectively). The results may be summarized as follows.

(1) If $m_\chi^2 > 0$ and $m_\chi^2 + 3m_\phi^2 > 0$ then $c_2 < 0 < c_3$. These conditions are required so that $V_0(Z)$ is bounded below, and favors matrices with positive determinant.

(2) If $m_\phi^2 > m_\chi^2/3$ then c_1 is negative and there is a local maximum at $Z = 0$ and three degenerate global minima at $Z = \lambda_+ e^{2\pi i n/3} \mathbb{1}$.

(3) If $m_\chi^2/9 < m_\phi^2 < m_\chi^2/3$ then $c_1 > 0$ and $c_2 < -3\sqrt{c_1 c_3}$, implying that there is a local minimum at $Z = 0$ and three degenerate global minima at $Z = \lambda_+ e^{2\pi i n/3} \mathbb{1}$.

(4) If $0 < m_\phi^2 < m_\chi^2/9$ then $c_1 > 0$ and $-3\sqrt{c_1 c_3} < c_2 < -\sqrt{8c_1 c_3}$, in which case there is a global minimum at $Z = 0$ and three degenerate local minima at $Z = \lambda_+ e^{2\pi i n/3} \mathbb{1}$.

APPENDIX B: EXPANSION IN SHIFTED FIELDS

Expressing the potential (3.7) in terms of the shifted fields (3.12) which describe fluctuations away from the minimum gives

$$g_3^{-2} V(Z) = V_{\min} + \tfrac{1}{2} m_\phi^2 \phi^2 + \tfrac{1}{2} m_\chi^2 \chi^2 + m_h^2 \, \mathrm{tr}(h^2) + V_{\mathrm{int}}(\phi, \chi, h, a), \tag{B1}$$

with

$$V_{\min} = \tfrac{1}{54}(9c_1 + v c_2) v^2 / g_3^2, \tag{B2}$$

and

$$
\begin{aligned}
V_{\text{int}}(\phi, \chi, h, a) = & -\frac{g_3}{\sqrt{6}v}\Bigg\{\Big(3c_1 + \frac{2}{3}vc_2\Big)\phi^3 + (3c_1 + 2vc_2)\phi\chi^2 + (6c_1 + vc_2)\phi\,\text{tr}(a^2) + (18c_1 + 7vc_2)\phi\,\text{tr}(h^2) \\
& + (12c_1 + 2vc_2)\chi\,\text{tr}(ah) + \sqrt{6}(6c_1 + 4vc_2)\,\text{tr}(a^2 h) - \sqrt{6}\Big(6c_1 + \frac{4}{3}vc_2\Big)\text{tr}(h^3)\Bigg\} \\
& - \frac{g_3^2}{v^2}(3c_1 + vc_2)\Bigg\{\frac{1}{8}(\phi^2 + \chi^2)^2 + \frac{1}{2}\phi^2\,\text{tr}(a^2 + 3h^2) + 2\phi\chi\,\text{tr}(ah) + \frac{1}{2}\chi^2\,\text{tr}(3a^2 + h^2) \\
& + 2\phi\,\text{tr}(a^2 h + h^3) + 2\chi\,\text{tr}(a^3 + ah^2) + \frac{3}{2}\,\text{tr}(a^4 + 4a^2h^2 - 2(ah)^2 + h^4)\Bigg\} + g_3^2\{\tilde{c}_1\,\text{tr}(a^2 + h^2)\} \\
& - g_3^3\{2\tilde{c}_2\,\text{tr}(3a^2h - h^3)\} + g_3^4\{\tilde{c}_3\,\text{tr}(a^4 + 4a^2h^2 - 2(ah)^2 + h^4)\}, \quad\quad\quad (B3)
\end{aligned}
$$

Here c_3 has been expressed in terms of v using Eq. (3.11). The tree-level masses are

$$
m_\phi^2 = -2c_1 - \tfrac{1}{3}vc_2, \qquad m_\chi^2 = -vc_2,
$$
$$
m_h^2 = -2c_1 - \tfrac{4}{3}vc_2. \qquad\qquad (B4)
$$

Expressed in terms of these masses, $V_{\text{min}} = -\frac{v^2}{108}(9m_\phi^2 - m_\chi^2)/g_3^2$. This shows directly that $m_\phi^2 > \frac{1}{9}m_\chi^2$ is necessary for the global minima to lie at the nontrivial Z(3) extrema.

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