

Structure formation on the brane: A mimicry

Supratik Pal

Department of Physics and Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur 721 302, India
and Relativity and Cosmology Research Centre, Department of Physics, Jadavpur University, Kolkata 700 032, India
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We show how braneworld cosmology with bulk matter can explain structure formation. In this scenario, the nonlocal corrections to the Friedmann equations supply a Weyl fluid that can dominate over matter at late times due to the energy exchange between the brane and the bulk. We demonstrate that the presence of the Weyl fluid radically changes the perturbation equations, which can take care of the fluctuations required to account for the large amount of inhomogeneities observed in the local universe. Further, we show how this Weyl fluid can mimic dark matter. We also investigate the bulk geometry responsible for the scenario.

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I. INTRODUCTION

Observations suggest that the expanding universe is homogeneous and isotropic at scales larger than $150 h^{-1} \text{Mpc}$. But the large amount of inhomogeneities observed in the local universe needs sufficient and convincing explanation. The usual cosmological models based on the standard Friedmann equations require that the baryonic matter fluctuation $\delta > 1$ today, implying $\delta > 10^{-3}$ at the time of recombination. This is in direct contradiction with cosmic microwave background (CMB) observations by over an order of magnitude. This limitation of standard cosmology leads to an inevitable prediction of dark matter [1]. CMB observations [2] suggest that $\Omega_{\text{baryon}} h^2 \approx 0.02$ i.e., only about 4% of cosmic density is baryonic. Hence dark matter, if it exists, has to be the dominant nonbaryonic component, contributing to as much as 1/3 of cosmic density [3]. It is proposed that its nonbaryonic nature helps it decouple from radiation, resulting in a growth of structure that starts much before the hydrogen recombination. But there are several problems associated with dark matter, the most pronounced of which being its ill-response to detection by series of experiments. Though the experiments of DAMA group [4] have reported in favor of its existence in the form of weakly interacting massive particles [5], similar searches by a number of other groups, such as CDMS [6], CRESST [7], EDELWEISS [8] have yielded negative results (for recent results, see [9]). So, there arise questions such as whether dark matter exists at all or it is the gravity sector, rather than the matter sector, that needs modifications. This leads people to consider modified gravity theories, e.g., modified Newtonian dynamics (MOND) [10], bifurcating gravity [11], phantom cosmology [12], etc.

One such modified gravity theory is the braneworld gravity [13] which has opened up new avenues of explaining the observations with the help of a modified version of the standard Einstein equation [14]. In this scenario, when

the bulk consists of matter, the bulk metric for which the FRW geometry on the brane is recovered, is given by a higher dimensional generalization of the *radiative* Vaidya black hole [15] that exchanges energy with the brane [16,17]. In presence of bulk matter the *modified* Einstein equation on the brane reads [16]

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + 6 \frac{\kappa^2}{\lambda} \mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu} + \mathcal{F}_{\mu\nu}, \quad (1.1)$$

where $\mathcal{S}_{\mu\nu}$, $\mathcal{E}_{\mu\nu}$, and $\mathcal{F}_{\mu\nu}$ are the quadratic contribution from brane energy-momentum tensor, the projected bulk Weyl tensor and the projected bulk energy-momentum tensor on the brane, respectively.

Incorporating all the braneworld corrections, one can conveniently express the Friedmann equations on the brane as

$$H^2 = \frac{\kappa^2}{3} \rho^{\text{eff}} + \frac{\Lambda}{3} - \frac{k}{a^2} \quad (1.2)$$

$$\dot{H} = -\frac{\kappa^2}{2} (\rho^{\text{eff}} + p^{\text{eff}}) + \frac{k}{a^2}, \quad (1.3)$$

where the *effective* density and pressure are given by [18]

$$\rho^{\text{eff}} = \rho + \frac{\rho^2}{2\lambda} + \frac{C(t)}{a^4} \quad (1.4)$$

$$p^{\text{eff}} = p + \frac{\rho}{2\lambda} (\rho + 2p) + \frac{C(t)}{3a^4}. \quad (1.5)$$

The term $\rho^* = C(t)/a^4$ is the combined effect of $\mathcal{E}_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$, and is called the ‘‘Weyl fluid,’’ that supplies an additional perfect fluidlike effect to the usual brane fluid. Further, the Weyl parameter $C(t)$ is related to the black hole mass, that results in energy-exchange between the bulk and the brane so that the individual matter-conservation is no longer valid but the total mass-energy of the bulk-brane system is now conserved. Consequently, the matter conservation equation on the brane is no longer

*Electronic address: supratik@cts.iitkgp.ernet.in

a sacrosanct equation. Rather, it is modified to the *non-conservation equation* [19]

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -2\psi, \quad (1.6)$$

where $\psi = dm/dv$, with $m(v)$ the mass of the Vaidya black hole. It shows how the brane either loses ($\psi > 0$) or gains ($\psi < 0$) energy in exchange with the bulk black hole. To a braneworld observer, the term ψ is the quantitative estimate of the brane-projection of the bulk energy density.

The goal of the present article is to show that the bulk-brane energy exchange results in the growth of the Weyl fluid at late times. Following Newtonian analysis of perturbations from gravitational instability, which is the simplest yet logical analysis of gravitational perturbations, we demonstrate that the Weyl fluid can mimic dark matter in explaining structure formation. That the braneworld gravity can be a very good alternative to dark matter in astrophysical contexts of clusters and galaxies was proposed in [20,21]. In this article we address the cosmological sector of dark matter.

The plan of the paper is as follows. In Sec. II, we obtain the perturbation equations for the effective perfect fluid on the brane. Sec. III is devoted to the solution of the effective perturbation equations with the help of Weyl fluid that mimics dark matter, followed by a comparative analysis with the other dark matter models from standard cosmology as well as modified gravity theories. We construct the bulk geometry for this setup in Sec. IV. Finally, we summarize our results and discuss some open issues.

II. EFFECTIVE PERTURBATION EQUATIONS

Since our focus is on the late time behavior in the *matter-dominated* era, we restrict ourselves to the analysis of the zero brane cosmological constant scenario. Hence, the equations of hydrodynamics that involve the quadratic brane correction and the Weyl fluid correction to the brane perfect fluid, are

$$\frac{\partial \rho^{\text{eff}}}{\partial t} + \vec{\nabla} \cdot (\rho^{\text{eff}} \vec{v}^{\text{eff}}) = 0, \quad (2.1)$$

$$\frac{\partial \vec{v}^{\text{eff}}}{\partial t} + (\vec{v}^{\text{eff}} \cdot \vec{\nabla}) \vec{v}^{\text{eff}} = -\frac{\vec{\nabla} p^{\text{eff}}}{\rho^{\text{eff}}} - \vec{\nabla} \Phi^{\text{eff}}, \quad (2.2)$$

$$\nabla^2 \Phi^{\text{eff}} = 4\pi G \rho^{\text{eff}}, \quad (2.3)$$

where \vec{v}^{eff} is the velocity field in the *effective* perfect fluid. It should be noted that the term Φ^{eff} is not the usual Newtonian potential but the effective gravitational potential which is the resultant effect of the Newtonian as well as the relativistic potential. The latter plays a crucial role in the braneworld context and has been discussed in detail in [20]. We next consider small perturbations

$$\rho^{\text{eff}}(\vec{x}, t) = \bar{\rho}^{\text{eff}}(t)(1 + \delta^{\text{eff}}(\vec{x}, t)), \quad (2.4)$$

$$\Phi^{\text{eff}}(\vec{x}, t) = \Phi_0^{\text{eff}} + \phi^{\text{eff}}, \quad (2.5)$$

where $\bar{\rho}^{\text{eff}}(t)$ and Φ_0^{eff} are, respectively, the unperturbed effective density and effective potential and δ^{eff} and ϕ^{eff} are their corresponding fluctuations. It is worthwhile to mention the significant difference of the density fluctuation of the braneworld cosmology from that of the standard cosmology. In the standard cosmology, δ is the fluctuation of baryonic matter only. On the contrary, in the braneworld cosmology, δ^{eff} is the sum total of the fluctuations of baryonic matter and of the contribution from braneworld corrections.

We proceed by expressing in terms of comoving coordinates and neglecting terms of second or higher order, which results in the following set of simplified perturbation equations

$$\frac{\partial \delta^{\text{eff}}}{\partial t} + \frac{1}{a} \vec{\nabla}_r \cdot \vec{u}^{\text{eff}} = 0, \quad (2.6)$$

$$\frac{\partial \vec{u}^{\text{eff}}}{\partial t} + \frac{\dot{a}}{a} \vec{u}^{\text{eff}} = -\frac{1}{a} \frac{\vec{\nabla}_r p^{\text{eff}}}{\bar{\rho}^{\text{eff}}} - \frac{1}{a} \vec{\nabla}_r \phi^{\text{eff}}, \quad (2.7)$$

$$\nabla_r^2 \phi^{\text{eff}} = 4\pi G a^2 \bar{\rho}^{\text{eff}} \delta^{\text{eff}}, \quad (2.8)$$

and express the solution in terms of Fourier transform

$$\delta^{\text{eff}}(\vec{x}, t) = \sum_k \delta_k^{\text{eff}}(t) e^{i\vec{k} \cdot \vec{x}} \quad (2.9)$$

$$\delta_k^{\text{eff}}(t) = \frac{1}{V} \int \delta^{\text{eff}}(\vec{x}, t) e^{-i\vec{k} \cdot \vec{x}} d^3 \vec{x}. \quad (2.10)$$

Further, assuming the effective pressure to be a function of the effective density alone, the equations of hydrodynamics now transform into a linear perturbation equation for δ_k^{eff} :

$$\frac{d^2 \delta_k^{\text{eff}}}{dt^2} + 2\frac{\dot{a}}{a} \frac{d\delta_k^{\text{eff}}}{dt} - (4\pi G \bar{\rho}^{\text{eff}} - (c_{\text{eff}}^2 k/a)^2) \delta_k^{\text{eff}} = 0, \quad (2.11)$$

where $c_{\text{eff}}^2 = \frac{\dot{p}^{\text{eff}}}{\dot{\rho}^{\text{eff}}} = [c_s^2 + \frac{\rho+p}{\rho+\lambda} + \frac{4\rho^*}{9(\rho+p)(1+\rho/\lambda)}] \times [1 + \frac{4\rho^*}{3(\rho+p)(1+\rho/\lambda)}]^{-1}$ is the effective sound speed squared [19].

III. SOLUTIONS WITH THE WEYL FLUID

The above perturbative analysis can account for the required amount of gravitational instability if the Weyl density redshifts more slowly than baryonic matter density, so that even if it starts from a small initial value, it can eventually dominate over matter. Now, the nature and evolution of the Weyl fluid is governed by Eq. (1.6) via the 4D Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$. This gives

$$\dot{\rho}^* + 4\frac{\dot{a}}{a}\rho^* = \mathcal{Q}, \quad (3.1)$$

where \mathcal{Q} is a coupling term which can be calculated, on principle, if the projected bulk energy density ψ is known. But in practice, since no one can fix the exact bulk geometry *a priori*, one has to take an ansatz for \mathcal{Q} so far as it is physically reasonable and consistent with the brane equations. One such ansatz has been considered in [22] for a dilaton field in the bulk. Let us here take an ansatz $\mathcal{Q} = \alpha H \rho^*$ ($\alpha > 0$), for which the Weyl fluid behaves like

$$\rho^* \propto \frac{1}{a^{(4-\alpha)}}. \quad (3.2)$$

Thus the Weyl parameter is given by $C(t) = C_0 a^\alpha(t)$, where C_0 is its initial value at the matter-dominated epoch. Obviously, the Weyl fluid is strictly radiationlike only if $\alpha = 0$, i.e. for matter-free bulk scenario. But for the bulk with matter, the nature of the Weyl fluid depends on the coupling strength α . The more the coupling strength α (within the range $1 < \alpha < 4$), the more the dominance of the Weyl fluid over matter. Hence the Weyl fluid can mimic dark matter for $\alpha \approx 1$. However, calculations of CMB anisotropies from the present model may lead to a better quantitative estimation for α . From Eq. (1.6), $\alpha > 0 \Rightarrow \psi > 0$ reveals that the brane loses energy to the bulk black hole. The increase of black hole mass is felt by a brane-world observer through the projected bulk energy density. Consequently, it results in the growth of the Weyl density at the expense of brane energy.

We are now in a position of dealing with the perturbation equation (2.11). This involves the fluctuation of ρ^{eff} which is a sum-total of three quantities given by Eq. (1.4). Of them, the quadratic correction term $\rho^2/2\lambda$ comes into play at the physics of early universe such as during inflation (where $\rho \gg \lambda$) [23] but it contributes very little at the present era since $\rho \ll \lambda > (100 \text{ GeV})^4$. Hence, for all practical purpose, the effective density at late times can be approximated as

$$\rho^{\text{eff}} \approx \rho + \rho^*. \quad (3.3)$$

What turns out from the above equation is that along with the usual matter density, here we have an additional (Weyl) density contributing to the total density that governs the perturbation equation (2.11). Separating the baryonic (matter) part from the nonbaryonic (Weyl) part of Eq. (2.11) now yields two wave equations

$$\frac{d^2 \delta_B}{dt^2} + 2\frac{\dot{a}}{a} \frac{d\delta_B}{dt} = 4\pi G \bar{\rho}_B \delta_B + 4\pi G \bar{\rho}^* \delta^* \quad (3.4)$$

$$\frac{d^2 \delta^*}{dt^2} + 2\frac{\dot{a}}{a} \frac{d\delta^*}{dt} = 4\pi G \bar{\rho}^* \delta^* + 4\pi G \bar{\rho}_B \delta_B, \quad (3.5)$$

where δ_B and δ^* are the fluctuations of baryonic matter and Weyl fluid, respectively, and we have neglected the term involving sound speed because of the growing fluc-

tuations. Consequently, with $\Omega_b \ll \Omega^*$, the relevant growing mode solution for Eq. (3.5), as a function of the redshift, turns out to be

$$\delta^*(z) = \delta^*(0)(1+z)^{-1}. \quad (3.6)$$

Substituting the above expression in the fluctuation Eq. (3.4) of baryonic density gives

$$\frac{d^2 \delta_B}{dt^2} + 2\frac{\dot{a}}{a} \frac{d\delta_B}{dt} = 4\pi G \bar{\rho}^* \delta^*(0)(1+z)^{-1}. \quad (3.7)$$

Further, we know that though the universe evolves differently at early times, the standard cosmological solution for the scale factor is recovered in RS-II type braneworld gravity at late times [16,24]. So, the late time behavior for a spatially flat brane is given by

$$a(t) = \left(\frac{3}{2} H_0 t\right)^{2/3(w+1)}. \quad (3.8)$$

With this scale factor and considering $\Omega^* \approx 1$ at present time, Eq. (3.7) now takes the form

$$a^{3/2} \frac{d}{da} \left(a^{-1/2} \frac{d\delta_B}{da} \right) + 2 \frac{d\delta_B}{da} = \frac{3}{2} \delta^*(0). \quad (3.9)$$

A typical solution for the above equation is given by

$$\delta_B(z) = \delta^*(z) \left(1 - \frac{1+z}{1+z_N} \right), \quad (3.10)$$

where we have used the standard relation of the scale factor with the redshift function $a \propto (1+z)^{-1}$. Note that though the above relation is a look-alike of the standard cosmological relation, physically it is completely different. Unlike the usual dark matter fluctuation, here δ^* is the fluctuation of Weyl density that arises naturally in brane-world context.

Let us now analyze some of the basic features of the present model. Equation (3.10) reveals that at $z \rightarrow z_N$, the baryonic fluctuation $\delta_B \rightarrow 0$ while δ^* remains finite. This implies that even if the baryonic fluctuation is very small at a redshift of $z_N \approx 1000$, as confirmed by CMB data [2], the fluctuations of the Weyl fluid had a finite amplitude during that time. At $z \ll z_N$ the baryonic matter fluctuations are of equal amplitude as the Weyl fluid fluctuations. This explains the structures we see today. Further, in this perturbative analysis, no extra matter (e.g. dark matter) has to be put by hand in order to explain the structures we see today.

The *effective* equation of state parameter is given by

$$w^{\text{eff}} = \frac{p^{\text{eff}}}{\rho^{\text{eff}}} = \frac{p + \rho(\rho + 2p)/2\lambda + C(t)/3a^4}{\rho + \rho^2/2\lambda + C(t)/a^4} \quad (3.11)$$

which, in the matter-dominated era, can be approximated as

$$w^{\text{eff}} \approx \frac{1}{3(1 + Ca^{1-\alpha})}. \quad (3.12)$$

Clearly, it bears significant difference from the equation of

state of cold dark matter ($w = 0$). In this sense the term ‘‘mimicry’’ may sound a bit misleading, though here we mean that it is the perturbative nature of evolution that is being mimicked by the Weyl fluid. Further, w^{eff} does not cross the phantom divider line ($w < -1$) [25] at least during the matter-dominated era in which we are interested. Hence this model is favored by SNLS [26]. Moreover, the braneworld scenario provides us with a new window for the cosmic coincidence problem. Here the evolution of the universe may not be a cosmological constant (or dark energy) effect at all, rather an outcome of the leakage of gravitational signal into the extra dimension. Hence the focus now shifts from the coincidence problem to the question: how can the late-acceleration of the universe be explained by the modified Friedmann equations?

A comparative study of our model with the other dark matter models and modified gravity theories is instructive. Equation (3.12) reveals that our model is distinct from the widely popular Λ CDM model [5] so far as the equation of state is concerned. However, quite surprisingly, the scale factor bears strong similarity with that of the matter-dominated phase of Λ CDM. Hence, though they have some common features, whether or not the present model will eventually evolve into Λ CDM cosmology at very late times needs further investigation. As of now, this is an open question as obvious from the preceding discussions. The alternative gravity theory MOND [10], is based on modifying Newton’s law. Though Bekenstein has recently proposed a relativistic MOND [27], a more convincing and satisfactory relativistic version bearing important features such as lensing is yet to come. In comparison, the braneworld model is based on purely relativistic idea and hence is potentially more advanced. Another alternative cosmology is the bifurcating theory [11] where the effective Lagrangian bifurcates into several branches, thereby giving a possibility of unifying dark matter and dark energy. However, here a scalar field is still required to model dark matter. The model discussed in the present article does not need any such scalar field. Here the bulk-brane geometry plays the trick. Phantom cosmology [12] can also provide an unified description but it has a wrong sign in the kinetic term that needs convincing physical explanation.

IV. BULK GEOMETRY

When the bulk consists of matter, the most general bulk metric for which a cosmological (FRW) metric on the brane is recovered, is given by a radiative black hole which is a 5-dimensional generalization [16,17,19] of Vaidya black hole [15]. In terms of transformed (null) coordinate $v = t + \int dr/f$, the bulk metric can be written as

$$dS_5^2 = -f(r, v)dv^2 + 2drdv + r^2d\Sigma_3^2, \quad (4.1)$$

where Σ_3 is the 3-space. For a spatially flat brane, the function $f(r, v)$ is given by

$$f(r, v) = \frac{r^2}{l^2} - \frac{m(v)}{r^2} \quad (4.2)$$

with the length scale l related to the bulk (negative) cosmological constant by $\Lambda_5 = -6/l^2$ and $m(v)$ is the variable mass of the Vaidya black hole. This bulk black hole metric is a solution of the 5-dimensional Einstein equation with the bulk energy-momentum tensor

$$T_{MN}^{\text{bulk}} = \psi q_M q_N, \quad (4.3)$$

where q_M are the ingoing null vectors and $\psi = dm/dv$ is the rate of incoming radial energy flow to the black hole.

Now, the black hole mass is the rescaled Weyl parameter which is further related to the scale factor by Eq. (3.2)

$$m(t) = \frac{\kappa^2}{3} C(t) \propto a^\alpha(t). \quad (4.4)$$

Hence, with the scale factor of Eq. (3.8), the on-brane mass of the bulk black hole turns out to be

$$m(t) = m_0 t^{2\alpha/3(w+1)}, \quad (4.5)$$

where m_0 is the black hole mass at the onset of matter-dominated era, which is given by

$$m_0 = C_0 \frac{\kappa^2}{3} \left(\frac{3}{2} H_0 \right)^{2\alpha/3(w+1)}. \quad (4.6)$$

Since t is the proper time on the brane, Eq. (4.5) gives the on-brane mass $m(t)$. In order to obtain the black hole geometry from the point of interest of a braneworld observer, we have to find out the off-brane mass $m(v)$. However, an exact expression for the bulk geometry can never be obtained purely from the brane data but an approximate expression for the same at the vicinity of the brane can be obtained by following a perturbative brane-based approach.

At low energy, the Friedmann brane, located outside the ‘‘event horizon,’’ moves radially in the bulk. Its radial trajectory is given by the geodesic $r(t)$, which reduces to the scale factor $a(t)$ at the brane-location. To a braneworld observer, a brane moving radially outwards in the bulk is identical to an expanding brane [28]. Hence, the function $f(r, v)$ at the brane-location reduces to

$$f(r, v)|_{\text{brane}} = \frac{r^2}{l^2} - C_0 \frac{r^\alpha(t)|_{\text{brane}}}{r^2} = \frac{r^2}{l^2} - C_0 \frac{a^\alpha(t)}{r^2}, \quad (4.7)$$

where $r(t)|_{\text{brane}}$ is the radial trajectory at the brane-location. Thus, the null coordinate v turns out to be

$$v = t + \frac{l}{2\sqrt{m_0}t^{\alpha/3(w+1)}} \left[\frac{1}{2} \ln \left(\frac{r/l - \sqrt{m_0}t^{\alpha/3(w+1)}}{r/l + \sqrt{m_0}t^{\alpha/3(w+1)}} \right) + \tan^{-1} \frac{r/l}{\sqrt{m_0}t^{\alpha/3(w+1)}} \right]. \quad (4.8)$$

The off-brane mass $m(v)$ at the vicinity of the brane can be found out by expanding $m(v)$ in Taylor series around its on-brane value $m(t)$ as

$$m(v) = m(t) + \left[\frac{\partial m}{\partial t_1} \right]_{t_1=t} \int \frac{dr}{f} + \frac{1}{2} \left[\frac{\partial^2 m}{\partial t_1^2} \right]_{t_1=t} \left(\int \frac{dr}{f} \right)^2 + \dots \quad (4.9)$$

Hence in the present scenario, $m(v)$ can be well approximated as

$$m(v) \approx m_0 t^{2\alpha/3(w+1)} + \frac{l\alpha\sqrt{m_0}}{3(w+1)} t^{-1+\alpha/3(w+1)} \times \left[\frac{1}{2} \ln \left(\frac{r/l - \sqrt{m_0} t^{\alpha/3(w+1)}}{r/l + \sqrt{m_0} t^{\alpha/3(w+1)}} \right) + \tan^{-1} \frac{r/l}{\sqrt{m_0} t^{\alpha/3(w+1)}} \right]. \quad (4.10)$$

One can now substitute the above expression for $m(v)$ into Eq. (4.2) to get the function $f(r, v)$ near the brane. It is straightforward to construct the relevant bulk metric at the vicinity of the brane by using together the function $f(r, v)$ and the coordinate v given in Eq. (4.8). To avoid the lengthy terms involved, we skip the final expression for the metric. From the physical point of view, it is not required either. The null coordinate v and $m(v)$ given in Eqs. (4.8) and (4.10) respectively, will carry all the informations about the bulk geometry to the brane.

V. SUMMARY AND DISCUSSIONS

In this article, we have shown that the braneworld cosmology with bulk matter can explain structure formation. When the bulk is constituted of matter, then the effective Einstein equation on the brane gives rise to a quantity that can act as an additional perfect fluid. This so-called ‘‘Weyl fluid’’ is the combined effect of the bulk Weyl tensor and bulk energy-momentum tensor, projected onto the brane. We have shown that the nature of the Weyl fluid depends on the energy-exchange between the brane and the bulk so that for strong bulk-brane coupling, it can dominate over ordinary matter. Furthermore, following Newtonian analysis of gravitational instability, we have shown that this Weyl fluid can account for the required amount of fluctuations in order to explain structure formation. Thus we conclude that the Weyl fluid can mimic dark matter in

structure formation with the advantage that nowhere we need to introduce any *ad hoc* extra matter such as dark matter. We have also investigated the bulk geometry given by a radiative Vaidya black hole and obtained the geometric quantities relevant for a braneworld observer.

Throughout this article, we stick to the perturbations from the Newtonian gravitational instability. Here in no way we tried to study in details the braneworld perturbations as done for matter-free bulk scenario in numerous papers [29–32]. Rather, we tried to see if the Weyl fluid can mimic dark matter in cosmological context with the simplest yet logical analysis of gravitational perturbations. The detailed study of scalar, metric, curvature, vector and tensor perturbations as well as theoretical studies of CMB anisotropies with the Weyl fluid behaving as dark matter are left for future works.

An interesting issue is to study gravitational lensing that serves as a probe of structures. Significant difference in the bending angle due to the difference in the potentials has been reported in [20,21]. It is to be seen how the effective potential affects cosmological lensing. A comparative study of the lensing effects with those of the other dark matter models, e.g. scalar field haloes [33] and verification from [34] will reveal which properties of dark matter can be reflected by Weyl fluid.

Finally, we have analyzed structure formation for a flat universe without considering any accelerated scale factor. Having realized that braneworld gravity can account for $\sim 30\%$ of cosmic density usually attributed to dark matter, one can pay attention to another $\sim 70\%$ which is considered to be dark energy with negative pressure. There currently exists some braneworld models of dark energy [35] consistent with supernova data. We expect that this perturbative analysis can be applied to those models, thereby providing an unified description of dark matter and dark energy from braneworld gravity, with excitingly new features.

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