Homogeneous scalar field and the wet dark sides of the universe

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We study the possibility that a generalized real scalar field minimally coupled to gravity could explain *both* the galactic and the cosmological dark components of the universe. Within the framework of Einstein's Relativity we model static galactic halos by considering the most general action built from the scalar field and its first derivatives. Although the gravitational configuration is static, the scalar field may be either static, or homogeneous and linear in time. In the case of the static scalar field, the models we look at inevitably posses unphysical negative energies, and we are led to a sort of *no-go* result. In the case of the *homogeneous* scalar field, on the contrary, we find that compact objects with flat rotational curves and with the mass and the size of a typical galaxy can be successfully modeled and the Tully-Fisher relation recovered. We further show that the homogeneous scalar field deduced from the galactic halo spacetimes has an action compatible with the kinetic Unified Dark Matter models recently proposed by Scherrer. Therefore, such a homogeneous kinetic Unified Dark Matter not only may correctly mimic the galactic dynamics, but could also be used to model the present day accelerated expansion in the universe.

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I. INTRODUCTION

Scalar fields play a central role in modeling the universe. Being an essential ingredient in the inflationary scenario [1], the scalar fields might drive the initial accelerated expansion [2] necessary to solve some of the problems of the Cosmological standard model. To explain the recent observations of type Ia supernovae [3], supported by other independent observations (Cosmic Microwave Background (CMB) fluctuations, galaxy clustering), which suggest that we live in an accelerated universe dominated by some kind of unknown Dark Energy (DE) [4], scalar fields are again often evoked. One can basically classify the scalar fields studied in the literature into two main classes: the canonical ones [2,5], where the action is given by the sum of the standard kinetic and a potential term, and the more general actions which were recently introduced under the colloquial name of K-essence [6]. It is worthwhile mentioning that the noncanonical Lagrangians for the scalar field were studied earlier by Bekenstein and Milgrom [7] in connection with the Modified Newtonian Dynamics. They also appear naturally in the velocity potential formulation of the Relativistic Hydrodynamics [8].

One of the most challenging problems in theoretical cosmology and astrophysics is the so-called Dark Matter (DM) problem (see [9,10] for a review). Several observations, carried out at different scales, *suggest* the existence of an invisible component, whose presence can be only inferred through the gravitational effects. The main evidence for the existence of DM appears at galactic scales. The mass profile of a galaxy can be obtained by analyzing the rotation curves of the surrounding particles-stars. This was done in the early 1980s [11] by studying the velocity, obtained through the frequency shifts in the 21 cm emission line of neutral hydrogen clouds. If we consider that the circular orbits are stable due to the balance of the centrifugal and gravitational forces, the mass profile in a galaxy must be given by $M(r) = rv_c^2(r)/G$. It is observed that the velocity of rotation is approximately an increasing function of the distance till the edge of the visible galaxy, and remains nearly constant from there on. The existence of some unobserved matter with an energy density profile $\rho \propto r^{-2}$ is then inferred.

A different sign for the existence of DM appears at the scales of galactic clusters. In 1933, Zwicky [12] realized that considering the clusters as equilibrium systems, their age being much longer than their dynamical time-scale, the virial theorem $2\langle T\rangle + \langle U\rangle = 0$ applies and the mass of the cluster can be evaluated with the result: $M = \alpha\langle v^2\rangle R/G$. Here α is a constant of order unity which depends on the matter distribution, M is the mass of the cluster and R its radius. By determining the speed of motion of many galaxies within a cluster from the shifts of the spectral lines, Zwicky could infer the mass required to maintain the Coma cluster held together. Surprisingly, it was by 2 orders of magnitude greater than the observed one. Also the study of gravitational lenses in galaxies [13] and galaxy clusters [14] suggests that there should be more matter than the one obtained by only counting stars.

Yet more evidence for the existence of a DM component comes from cosmological scales. Recent data from the CMB [15] point out that we live in a flat ($k = 0$), or a nearly flat universe [16]. This agrees with the predictions of the inflationary scenario implying that one must have $\Omega \simeq 1$. The energy density of the universe, therefore, must be very close to the critical value $\rho_c \sim 10^{-29}$ g/cm^3, pointing toward the discrepancy with the observed density in the universe and the need of DM. Apart form this, DM seems to be an essential ingredient in producing the ob-

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served structures in the Universe [17], providing the gravitational potential depressions where the galaxies are nourished and formed.

Many candidates for the DM component have been proposed along the years [9]. They range from elementary particles such as WIMPs (neutrinos, axions, neutralino, gravitino, etc.) to the ones in which the DM is formed by compact objects such as primordial black holes or MACHOs (brown dwarfs or Jupiter like objects). If one thing is clear, however, is that the DM can not consist of baryonic matter alone. The nucleosynthesis and the abundances of light elements [18] restrict the amount of baryonic matter to $\rho_b \sim 0.01\rho_c$, therefore there must be something more out there to produce the observed spatially flat universe. A sensible model for DM should probably solve all the problems stated above at the same time, yet no model proposed so far does the job.

Recently some suggestions to look at classical scalar fields as candidates for the cosmological DM, rather than concentrating on fundamental particles or some compact objects, were voiced [19]. Interesting models have been proposed in which a scalar field, apart from modeling the cosmological DM, can evolve to produce the accelerated expansion of the universe. These models in which both cosmological dark components (matter and energy) arise from the same field are commonly known as Unified Dark Matter models. Some of the proposals involve the so-called Chaplygin gas [20] and its generalizations [21]. In these models the universe expansion is driven by a perfect fluid with a phenomenological equation of state derived from a string theory motivated Born-Infeld Lagrangian [22]. The evolution of the cosmological background interpolates between a DM universe at early times and a DE dominated universe at the late epoch. Recent studies indicate, however, the existence of serious problems related with structure formation in these universes [23] (see however [24]). These problems appear mainly due to a high speed of sound for the cosmological perturbations during certain periods of the expansion, but may be alleviated in the so-called kinetic Unified Dark Matter (kUDM) models recently introduced by Scherrer [25], where a speed of sound verifying $c_s^2 \ll 1$ is guaranteed for all times. The actions in Scherrer's models depend only on the scalar field derivatives maintaining the shift symmetry of the scalar field, but otherwise are quite generic. Possible physical motivations for these actions are discussed in [6,26].

The main purpose of this paper is to analyze whether one could make a step further and model in a consistent way, not only the cosmological DM and DE, but also the DM one believes accounts for the observed flat rotation curves in spiral galaxies, with a *single* classical scalar field. We consider spherically symmetric static galactic halos and find that the generalized static scalar field configurations are not good candidates to model the galactic matter. The homogeneous scalar field, on the other hand, and one must admit somewhat surprisingly, leads to quite interesting results. Not only the fact that the scalar field is homogeneous, but rather the form of the action derived within the galactic halo, akin to that one proposed by Scherrer [25], indicates that such matter might be a perfect match for cosmology.

The paper is organized as follows. In Sec. II (and in the short Appendix) we briefly discuss the form of the line element appropriate for the problem of galactic halos. Section III deals with the matter field. Here we show that if the gradient of the scalar field is spacelike, the static spherically symmetric configurations of this field with the imposed flattened rotational curves are unphysical. This goes both for the canonical Quintessence and the noncanonical K-essence fields, indicating a *no-go* result. If, however, the gradient of the field is timelike, the field is homogeneous and linear in time, and the action for the scalar field contains only derivative terms but not the field itself, one can get physically reasonable configurations. The possibility of modeling the galactic halos with a homogeneous scalar field opens a way to consider this field as the one also responsible for the overall expansion of the universe. We find out that the model of scalar matter suggested by the dynamics of the galactic halos fits quite well with some simple unifying models suggested and studied in cosmology by various authors under different names: x-matter [27], *wet* dark fluid [28], or the matter with the so-called generalized linear equation of state [29] (see as well [30]). We study the fitting of the free parameters of the action with galactic observations and its implications for cosmology in the Section IV. Finally, in Sec. V, we discuss our results.

Throughout this paper we use the units in which $\hbar = c = 1$, $M_{\rm pl} = (8\pi G)^{-1/2}$ and the signature of the metric is taken to be $(-, +, +, +)$.

II. GALACTIC HALOS

We consider that a typical galaxy is formed by a thin disk of visible matter immersed in a large halo built of some unseen *exotic* matter, which can be conveniently described as static and spherically symmetric. This exotic matter would be the main contributor to the dynamics, so that the observed luminous matter can be treated as a test fluid from which information about the physics of the halo can be inferred. The observations indicate that the visible particles within the thin disk have rotation curves with velocities independent of their distance to the center of the galaxy [11,31].

One wants to know how much information about the halo can be figured out from the observed rotation curves. The Newtonian analysis mentioned in the previous section is only valid under certain conditions [32]: weak gravitational field, small velocities and small pressures. In this approximation, the gravitational field is given by the Newtonian potential, which can be completely determined from the observed rotation curves. The relativistic case is somewhat more complex.

The spherically symmetric and static halo in General Relativity is described by a line element conveniently parametrized in *curvature coordinates* as [33]:

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{1}{1-\frac{2m(r)}{r}}dr^2 + r^2 d\Omega^2. \tag{1}$$

In the equation above $d\Omega^2$ is the metric of a unit sphere, and there are two metric functions: $\phi(r)$, known as the *gravitational potential*, and $m(r)$, known as the *effective gravitational mass*. In the Newtonian limit these two functions coincide with their usual interpretations. We distinguish the dynamical mass [34] (or *pseudomass* [32]) $M(r)$ described in the Introduction and obtained from the observed rotation curves, and the effective gravitational mass $m(r)$, defined with the help of the rr component of the metric tensor and given in the expression (1). It is worthwhile to keep in mind that these are two *different* concepts and in general take different values.

The most general static spherically symmetric metric with flat rotation curves is given by (see the Appendix A and Refs. [34–37]):

$$ds^2 = -\left(\frac{r}{r_\star}\right)^l dt^2 + A(r)dr^2 + r^2 d\Omega^2. \tag{2}$$

Here $r_\star$ is a constant parameter with dimensions of length and $l = 2(v_c/c)^2$. Therefore, the domain of the parameter l is restricted to $0 < l < 2$. To determine the metric function $A(r)$, however, one needs to know more about the matter content. It is interesting to point out that the form of the line element (2), as it stands, is generic in the sense that it does not depend on the metric theory of gravity used, nor it depends on the matter content. To obtain it one only assumes that the rotation curves are flat. In short, the profile of the rotation curves gives us the chance of reproducing completely the 00 component of the metric tensor—the function $\phi(r)$, but tells nothing about the other independent component—the function $m(r)$ [32,34–37]. If we want to obtain some information about the function $m(r)$, one must assume either the nature of the matter dominating the configuration, or deduce it from different observations, for example, gravitational lensing [32] etc.

In the observed galaxies the orbiting particles are nonrelativistic, therefore l is a small parameter close to zero ($l < 10^{-5}$). The gravitational field far away from the center, where supermassive black holes are expected to "hide", is also small ($2m(r)/r \ll 1$ and $2\phi(r) \ll 1$) [38]. The pressure inside the halo, in the standard models of galaxies, is also usually assumed to be close to zero ($p \ll \rho$). If these three conditions are met, the Newtonian approximation applies and the parameters $m(r)$ and $M(r)$ do coincide. Nevertheless, depending on the choice of matter the last of the three above assumptions does not always correspond to physical reality. Matter based on "string fluids" [39] or on scalar fields [36,37,40,41] could exert a significant amount of pressure and the Newtonian approximation would no longer apply. An expression for the dynamical mass in a first post-Newtonian approximation, which shows explicitly the discrepancies between $M(r)$ and $m(r)$, is given in [32].

III. THE DARK MATTER COMPONENT

To proceed any further one should specify the matter which makes up the galactic halo. To model the DM we use the most general action for a minimally coupled scalar field constructed from the scalar field and its first derivatives:

$$S = \int d^4x \sqrt{-g}\,\mathcal{L}(\varphi, X). \tag{3}$$

Here $\mathcal{L}(\varphi, X)$ is the Lagrangian density and X the kinetic scalar defined by $X \equiv -1/2 g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. These kind of actions are usually referred to as K-field, generalized scalar field or, in a cosmological setting, K-essence [6]. Special cases discussed in the literature are the factorizable K-field $\mathcal{L} = K(\varphi)F(X)$ [42] and the purely kinetic K-field $\mathcal{L} = F(X)$ [25]. The canonical scalar field $\mathcal{L} = X - V(\phi)$ can be always rewritten as a factorizable K-field by a field redefinition.

The energy-momentum tensor for the action (3) is:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\partial\mathcal{L}}{\partial X}\partial_\mu\varphi\partial_\nu\varphi + \mathcal{L}g_{\mu\nu}.$$

Here one runs into two *qualitatively* different situations [43]. If the derivative term $\partial_\mu\varphi$ is timelike ($X > 0$), as it is usual in cosmology, we can identify the energy-momentum tensor with a perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$ by the formal relations:

$$u_\mu = \frac{\partial_\mu\varphi}{\sqrt{2X}}, \qquad p = \mathcal{L}, \qquad \rho = 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L}. \tag{4}$$

Here u_μ is the 4-velocity, ρ the energy density and p the pressure. On the other hand, if the derivative term is spacelike ($X < 0$), this identification is no longer attainable. However, we can still identify the energy-momentum tensor with an anisotropic fluid $T_{\mu\nu} = (\rho + p_\perp)u_\mu u_\nu + p_\perp g_{\mu\nu} + (p_\parallel - p_\perp)n_\mu n_\nu$. Now, the formal relations are:

$$n_\mu = \frac{\partial_\mu\varphi}{\sqrt{-2X}}, \qquad p_\perp = -\rho = \mathcal{L}, \qquad p_\parallel = \mathcal{L} - 2X\frac{\partial\mathcal{L}}{\partial X}. \tag{5}$$

There are two different pressures in this case, one in the direction parallel to n_μ ($p_\parallel$) and the other one in an orthogonal direction ($p_\perp$). It is important to stress, however, that the equality $p_\perp = -\rho$ for this sort of matter always holds, implying that the Newtonian approximation is no longer valid.

The metric and the energy-momentum tensors are related through the Einstein Equations $G_{\mu\nu} = M_{\rm pl}^{-2} T_{\mu\nu}$, and since we are only interested here in static, spherically symmetric configurations (1), restrictions on the function $\varphi(t, r)$ and on the form of the action $\mathcal{L}(\varphi, X)$ appear. The symmetries imply a diagonal Einstein and therefore a diagonal energy-momentum tensor, forcing the vanishing of the $T_{\rm tr}$ component. Therefore, either the scalar field is strictly static $\varphi = \varphi(r)$, or it is strictly homogeneous $\varphi = \varphi(t)$. The inhomogeneous time dependent scalar fields are not allowed by the symmetries imposed on the halo. Furthermore, the staticity of the spacetime ensures that the energy density and the pressure must be independent of time. This implies that for the case $\varphi = \varphi(t)$ the action can only depend on the field derivatives $\mathcal{L}(\varphi, X) = F(X)$, but not on the field itself. Moreover the scalar field φ may only be linear in time, $\varphi = at$. It is interesting to observe that in the last case the spacetime does not inherit the symmetries of the fundamental field producing the geometry, and while the metric remains static, the fundamental scalar field may be time dependent. Similar, but rather an inverse situation was discussed in the case of a tilted homogeneous string cosmology [44], where the homogeneous geometry was produced by an inhomogeneous scalar dilaton.

A. The static field $\varphi = \varphi(r)$

In this case the derivative of the field is spacelike ($X < 0$) and the scalar field, therefore, behaves as an anisotropic fluid (5). The Einstein equations for the tt, rr and $\theta\theta$ components are:

$$\frac{1}{Ar^2}\left[\frac{A'}{A} r + A - 1\right] = -M_{\rm pl}^{-2}\mathcal{L}, \tag{6}$$

$$\frac{1}{Ar^2}[(l+1) - A] = -M_{\rm pl}^{-2}\left[2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L}\right], \tag{7}$$

$$\frac{1}{4Ar^2}\left[(l+2)\frac{A'}{A} r - l^2\right] = -M_{\rm pl}^{-2}\mathcal{L}. \tag{8}$$

Here the prime denotes differentiation with respect to the variable r. The above three equations imply automatically the matter conservation equation. Combining (6) and (8) we obtain a differential equation for the metric component $A(r)$:

$$(l-2)\frac{A'}{A} r - 4A - (l^2 - 4) = 0,$$

whose general solution is given by:

$$A(r) = \frac{a_i}{1 \pm (\frac{R_i}{r})^{b_i}}. \tag{9}$$

Here R_i is a positive arbitrary integration constant with dimensions of length. We have introduced the subindex i to distinguish between the cases $\varphi = \varphi(r)$ ($i = r$) and $\varphi = \varphi(t)$ ($i = t$) of the following subsection. The strictly positive parameters a_r and b_r are given by:

$$a_r = \frac{4 - l^2}{4}, \qquad b_r = 2 + l.$$

Simple analytic solutions can be easily obtained for the special case $R_r = 0$. Such are the solutions for the canonical scalar field with an exponential potential [37], the factorizable K-field with a potential of the form $K(\varphi) = 1/\varphi^2$ and arbitrary $F(X)$ [45] or the purely kinetic case with $F(X) \propto X^{-2/l}$ [45]. We do not give the explicit solutions here due to their scarce physical relevance. For all these solutions the metric function $A(r)$ is a constant and the energy density and the pressure are obtained directly from Eqs. (6) and (7) using the definitions given in (5):

$$p_\perp = -\rho = \frac{l^2}{4 - l^2}\left(\frac{M_{\rm pl}}{r}\right)^2, \qquad p_\parallel = \frac{l(l+4)}{4 - l^2}\left(\frac{M_{\rm pl}}{r}\right)^2.$$

It is easy to see that the energy density is always negative, therefore these solutions must be considered unphysical. This affects as well the solution presented in [37], where the energy density sign (their expression (19)) is erroneous. Note that the energy density and the pressure are directly read from the line element and as long as the galactic rotational velocities are nontachyonic one can not escape the negative energy densities.

For the general case ($R_r \neq 0$) the situation is more complex and exact analytic expressions for the action can not be easily given. However, certain observations about the general behavior of the solution can still be made. In the asymptotic limit $r \gg R_r$ the general solution converges to the special case $R_r = 0$ mentioned above, so the negative energy densities persist. These solutions, nevertheless, could still be useful if their behavior were physical for some range $r < r_{\rm crit}$ say, and then matched to a different solution. However, unfortunately, this is not the case. For the $-$ branch of the Eq. (9) the energy density remains always negative. For the $+$ branch, positive energy densities appear for some range $r < r_{\rm crit}$, but the effective gravitational mass $m(r)$ defined by (1) remains always negative, so the solution is again unphysical. The occurrence of the negative mass with a positive energy density signals the presence of a singularity with an infinite negative mass in the center, analogous to the one discussed by Bonnor in [46].

Thus, one may consider this subsection as one leading to a kind of *no-go* result for the static scalar fields $\varphi = \varphi(r)$. Once we have assumed the form of the line element suitable for the description of flat rotation curves (Section II), the minimally coupled generalized *static* scalar field is found unfit to play the role of the dark matter in the galactic halos.

B. The homogeneous field $\varphi = at$

The configuration $\varphi = at$, as mentioned above, is only possible if the action (3) is of the purely kinetic form $\mathcal{L}(\varphi, X) = F(X)$—no potential term. In this case, one may interpret the scalar field as an irrotational isentropic perfect fluid in a disguise [47], although the equation of state $p = p(\rho)$ does not have to be of a simple form. The action itself can be thought of as the hydrodynamical action written in terms of the velocity potential [8,47].

The tt, rr and $\theta\theta$ components of the Einstein equations are now:

$$\frac{1}{Ar^2}\left[\frac{A'}{A}r + A - 1\right] = M_{\rm pl}^{-2}\left[2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L}\right], \tag{10}$$

$$\frac{1}{Ar^2}[(l+1) - A] = M_{\rm pl}^{-2}\mathcal{L}, \tag{11}$$

$$\frac{1}{4Ar^2}\left[-(l+2)\frac{A'}{A}r + l^2\right] = M_{\rm pl}^{-2}\mathcal{L}. \tag{12}$$

Combining the expressions (11) and (12), we obtain a differential equation for the function $A(r)$:

$$(l+2)\frac{A'}{A}r - 4A - [l^2 - 4(l+1)] = 0,$$

whose general solution is again given by (9), where the parameters a_t and b_t are now:

$$a_t = -\frac{l^2 - 4(l+1)}{4}, \qquad b_t = \frac{l^2 - 4(l+1)}{l+2}.$$

Unlike in the static case, however, while the parameter a_t still remains positive, the parameter b_t becomes negative.

For the special case when the constant $R_t = \infty$ an analytic solution can be easily found. The action is given by $F(X) \propto X^{2/l}$, and the energy density and the pressure are:

$$\rho = \frac{l(l-4)}{l^2 - 4(l+1)}\left(\frac{M_{\rm pl}}{r}\right)^2, \qquad p = \frac{-l^2}{l^2 - 4(l+1)}\left(\frac{M_{\rm pl}}{r}\right)^2,$$

which are both positive. Interpreted as a fluid with a constant barotropic index $p = w\rho$ one gets $w = l/(4 - l)$ $(0 < w < 1)$, so that for the observed galaxies $(l \ll 1)$ the fluid is nearly "dust". This solution is analogous to the infinite *isothermal sphere*.

In the general case $(R_t \neq \infty)$ the solution is rather more interesting. We will be interested in the $-$ branch of the Eq. (9). In this case, the complete solution can be derived from a scalar field obeying the following two-parameter family of K-actions:

$$F(X) = \left(\frac{M_{\rm pl}}{R_t}\right)^2[p_1^N p_2^2]^{1/(N+2)}[X^{2/l} - X^{-N/l}]. \tag{13}$$

For these fluids, the energy density and the pressure are:

$$\begin{aligned}\rho &= \rho_1\left(\frac{M_{\rm pl}}{r}\right)^2 + \rho_2\left(\frac{M_{\rm pl}}{R_t}\right)^2\left(\frac{r}{R_t}\right)^N,\\ p &= p_1\left(\frac{M_{\rm pl}}{r}\right)^2 - p_2\left(\frac{M_{\rm pl}}{R_t}\right)^2\left(\frac{r}{R_t}\right)^N,\end{aligned} \tag{14}$$

where ρ_1, ρ_2, p_1, p_2 and N are positive constants given by:

$$\rho_1 = \frac{-l(4-l)}{l^2 - 4(l+1)}, \qquad \rho_2 = \frac{-4(6 + 5l - l^2)}{(l+2)[l^2 - 4(l+1)]},$$
$$p_1 = \frac{-l^2}{l^2 - 4(l+1)}, \qquad p_2 = \frac{-4(l+1)}{l^2 - 4(l+1)},$$
$$N = \frac{l(2-l)}{l+2}.$$

The solutions are only valid for $r < R_t$ due to the behavior of the metric component given by the Eq. (9), and are in fact the Tolman type V solutions [48]. As pointed out by Tolman, and as is usual for solutions of this kind, an explicit equation of state $p = p(\rho)$ can not be found. It is interesting, however, to point out that it has been possible to obtain a simple action which describes the fluid (13) without referring to an explicit equation of state. In fact, a simple action can be often given in many instances for which the explicit equation of state $p(\rho)$ is not available.

Further, these solutions may be used to describe compact objects of finite size. The expression (14) indicates that the pressure is positive until some value r_0 is reached, then it vanishes and changes sign. It is thus possible to match the solution with an exterior Schwarzschild vacuum for $r > r_0$, defining $r = r_0$ as the halo external surface. We will use the subindex 0 to refer to the values evaluated on this surface. The expressions for the radius r_0 and for the effective gravitational mass $m_0 = 4\pi\int_0^{r_0}\rho(r)r^2dr$ of the compact objects are then given (in physical units) by:

$$r_0 = \left(\frac{p_1}{p_2}\right)^{1/(2+N)}R_t, \qquad m_0 = \frac{l}{2(l+1)}\frac{c^2 r_0}{G}.$$

The constant $r_\star$ of Eq. (2) is now fixed by the matching to the exterior vacuum solution and is given by $r_\star^l = (l+1)r_0^l$. Since $r_0 < R_t$, it is always possible to construct these solutions. We are only interested in the behavior of the fluid within the halo, which in turn restricts the domain of the function $F(X)$ to $X \geq 1$.

The expressions above are *exact*. However, we are more interested in a *rule of thumb* to work with, and since the observed rotation velocities in the galaxies are small compared to the speed of light, we have found it convenient to proceed working to first order in l. To this order we propose the simplified action

$$F(X) = \left(\frac{M_{\rm pl}}{R_t}\right)^2[X^{2/l} - 1], \tag{15}$$

which fits amazingly well with (13) within range $X \geq 1$, where Eq. (13) applies. The result above is central to this paper. It gives a simple equation of state, or rather an action for the homogeneous scalar field, which can be used to model the DM in the galaxies. Equation (15) should not be seen as a formal limit of Eq. (13), but rather as a suggested, or guessed action describing the matter. This matter approximates Eq. (13) within the halo, and therefore reproduces the desired geometry. Moreover, similar actions are used in cosmology under the different names of x-matter [27], wet dark fluid [28] and matter with the generalized linear equation of state [29]. Thus, the action (15) may potentially serve as a unified matter description both for cosmology, on one hand, and within the galactic halo, on the other. The two arbitrary constants in the model: R_t and l, must be determined from the observations. A brief estimate of the order of magnitude of these parameters will be the task of the following section.

IV. FITTING THE MODEL

A. Galaxies

We first adjust the two free parameters of the equation of state to fit a typical galaxy. As we have mentioned in the Sec. II, the parameter l is directly related to the rotation velocities. We take $l \sim 10^{-5}$. The size and the mass of a typical galaxy are then given by the first order expressions:

$$r_0 \simeq \frac{l}{2} R_t, \qquad m_0 \simeq \frac{l}{2} \frac{c^2 r_0}{G}.$$

If we take R_t of the order $R_t \sim 3000$ Mpc one obtains a compact object with $r_0 \sim 15$ Kpc and $m_0 \sim 10^{12} M_\odot$, compatible with the size and the mass for a typical galactic halo. Curiously enough we had to assume the constant R_t of the order of the size of the observable universe to fit the observations. The last two equations can be combined to obtain a relation between the mass of a galaxy and the velocity of the orbiting particles:

$$m_0 \simeq \frac{R_t}{c^2 G} v_c^4.$$

The equation above is nothing else but the Tully-Fisher relation ($m_0 = \kappa v_c^4$), where the constant of proportionality κ is determined by the parameter R_t of the model.

Now, the action we propose to describe the DM in the halos of the spiral galaxies (15) with the parameters $R_t \sim 10^3$ Mpc and $l \sim 10^{-5}$ may work. But then one may possibly suggest to identify the homogeneous scalar field with the velocity potential of some perfect fluid in the galaxy and just stop here, instead of further promoting it to cosmic scales. If the equation of state we propose were related exclusively to the galactic matter, it would seem unlikely, that the same action parametrized by the same parameters could fit the global universal expansion. The presence of the constant R_t of the order of the size of the observable universe in the parametrization of the galactic DM action signals as well a tentative relation to cosmology. More interestingly, the action (15) does fit favorably with cosmology, and this is a nontrivial result.

B. Cosmology

Consider now the action (15) in the setting of a homogeneous and isotropic universe. Working still to first order in l, the pressure and the energy density are easily obtained using the relations given in (4):

$$p = \left(\frac{M_{\rm pl}}{R_t}\right)^2 [X^{2/l} - 1], \qquad \rho \simeq \left(\frac{M_{\rm pl}}{R_t}\right)^2 \left[\frac{4}{l} X^{2/l} + 1\right].$$

For convenience it is useful to define the parameters w and c_s^2, the barotropic index and the velocity of sound, respectively, given by the following expressions :

$$w = \frac{F(X)}{2XF'(X) - F(X)} \simeq \frac{X^{2/l} - 1}{\frac{4}{l} X^{2/l} + 1}, \qquad c_s^2 = \frac{F'(X)}{2XF''(X) + F'(X)} \simeq \frac{l}{4}. \tag{16}$$

The barotropic index w defines the evolution of the scale factor of such a universe, while the sound speed c_s^2 gives the evolution of the first order small perturbations [49]. There is no need to write down the solutions of the Einstein Equations to see how the model behaves. The scalar field evolution equation is given by:

$$\dot{X} + 6Hc_s^2(X)X = 0.$$

Here $H = \dot{a}/a$ is the Hubble constant, a is the scale factor and the dot has the usual meaning. While the universe expands ($H > 0$) and given that the square of the speed of sound (16) remains always positive, the kinetic scalar X is a decreasing function of time. The barotropic index evolves from dust ($w \simeq 0$) at early times ($X \gg 1$), to a "cosmological constant" ($w \simeq -1$) at late times ($X \ll 1$). The speed of sound, however, always remains small, $c_s^2 \ll 1$. Even during the late epoch, when the fluid approaches the "cosmological constant" like equation of state, the sound speed remains well below the speed of light. The fact that the DE density perturbations propagate with low velocities makes this model distinguishable from the standard ΛCDM, where a purely nonclustering Cosmological Constant would produce a different pattern at large angular scales in the CMB fluctuations [50,51].

The energy density for the effective value of the "cosmological constant" is determined by the parameter R_t fixed in the previous subsection against the galactic data and is given by

$$\rho_\Lambda = \frac{c^2}{8\pi G R_t^2} = 6.5 \cdot 10^{-30}\ \mathrm{g/cm^3}.$$

The value of the "cosmological constant" then becomes $\Lambda_{\rm eff} = R_t^{-2}$ which gives $\Lambda_{\rm eff} \sim 10^{-52}\ \mathrm{cm^{-2}}$ (cf. [4]).

We also obtain that there exists a relation between the value of the cosmological constant and the Tully-Fisher proportionality factor κ:

$$\kappa^2 \Lambda_{\rm eff} = \frac{1}{G^2 c^4},$$

determined only by the fundamental constants G and c.

V. DISCUSSION

The flattening of the rotation curves in spiral galaxies, the missing mass in the galactic clusters, the spatial flatness of the universe, the structure formation and the present day acceleration all point out to the existence of a Dark Side in the Universe. It is possible that each of the above mentioned problems has a separate solution; it would be physically more appealing, though, if the solution were unique.

Our Universe is homogeneous and isotropic on large scales. On galactic scales, however, the observed matter distribution is nonuniform. This itself, does not exclude the possibility of the presence of a homogeneous scalar field on scales of galactic halos in addition to a nonuniform ordinary matter distribution.

We have started the technical part of the paper by looking at possible static configurations of scalar field compatible with flat rotational curves, and have concluded that if the scalar field is static no physically interesting solutions result.

If the scalar field is homogeneous, however, one may construct models of galactic halos based on exact solutions of the Einstein Equations which have flat rotational curves. The matter in these galactic halos is described by an action which only depends on derivative terms of the scalar field. We interpret this intragalactic scalar field as an integral part of a global homogeneous field driving the expansion of the universe. We do so, encouraged by the fact that the homogeneous scalar field we deduce from the galactic rotational curves makes a good match to cosmology.

The model for the homogeneous field we have read from the galactic halo metric is of the form which belongs to a class of models which could solve the Dark Matter and the Dark Energy problem in cosmology—the class of models suggested by Scherrer [25]. Of course, this could be a pure coincidence: the same action for the homogeneous scalar field resolves it all. Nonetheless, we believe it is worth exploring this possibility further.

The most interesting feature of the model is, as we see it, that the two free parameters in the K-action, when fixed against the galactic data, imply a reasonable value for the cosmological constant which is unrelated to the fit. There also exist the possibility to differentiate the model from the standard ΛCDM due to a different clustering of the Dark Energy in our model which would leave observable imprints on the Cosmic Microwave Background.

Some words of caution should be spelled out. One should of course take the global solution to the DM problem we suggest as a toy model. The action for the cosmological fluid is given by the Eq. (15) with the parameters fixed by the galactic curves. Now, while the constant R_t is universal in the model, different galaxies would have different rotation velocities and therefore different values for l. But in cosmology one must fix the value of l presumably globally. One may speculate that the global value of the constant l in the cosmological equation of state is determined as some sort of average, or is given by the first principles of an underlying theory. Be what may, however, it is encouraging to find out that there exists the possibility that a *single* matter ingredient can be used to model the Dark Side of the universe on all scales.

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APPENDIX: DETERMINATION OF THE g_{00} COMPONENT OF THE METRIC TENSOR FROM THE ROTATION CURVES

In this Appendix we briefly summarize how one can reconstruct some metric functions from the observed rotation curves [34–37]. We consider the equations of motion for an orbiting particle of mass m immersed in a static spherically symmetric gravitational field (1). For this purpose we choose, as usual, the coordinate system so that the orbit is contained in the equatorial plane $\theta = \pi/2$. The two conserved quantities, associated with the two Killing vectors of the metric $\xi_t = \partial/\partial t$ and $\xi_\phi = \partial/\partial\phi$, are, respectively, the energy $E \equiv -p \cdot \xi_t = -p_0$ and the angular momentum $L \equiv p \cdot \xi_\phi = p_\phi$, where p is the 4-momentum of the particle. The equation of motion for the radial coordinate is obtained from the constraint $p \cdot p + m^2 = 0$ by introducing the values for E and L given above, and can be written as:

$$f(r)\dot{r}^2 = \tilde{E}^2 - \tilde{V}^2_{\rm eff}(r),$$

with $f(r)$ and $\tilde{V}^2_{\rm eff}(r)$ given by:

$$f(r) = \frac{e^{2\phi(r)}}{1 - \frac{2m(r)}{r}}, \qquad \tilde{V}^2_{\rm eff}(r) = e^{2\phi(r)}\left(1 + \frac{\tilde{L}^2}{r^2}\right).$$

The symbol $\sim$ indicates quantities evaluated per unit mass.

The circular orbits are obtained by imposing $\dot{r} = 0$ ($\tilde{V}_{\rm eff}(r) = \tilde{E}$) and $\ddot{r} = 0$ ($\tilde{V}'_{\rm eff}(r) = 0$). This gives the following expressions for $\tilde{E}$ and $\tilde{L}$:

$$\tilde{E}^2 = \frac{e^{2\phi}}{1 - r\phi'}, \qquad \tilde{L}^2 = \frac{r^3\phi'}{1 - r\phi'}.$$

Using the definition for the tangential component of the velocity (equation (25.20) in Ref. [33]) and the expressions for $\tilde{E}$ and $\tilde{L}$ above, we obtain:

$$v_c^2 = \frac{p_\phi^2}{r^2 E_{\rm local}^2} = r\phi'.$$

Here v_c is measured in units of c ($v_c \to v_c/c$) and $E_{\rm local}$ is the energy of the particle measured by a local observer at rest. From the behavior of the rotation curves $v_c(r)$ the function $\phi(r)$ can be directly integrated and the 00 component of the metric tensor obtained:

$$g_{00}(r) = -\exp\left(2\int \frac{v_c^2(r)}{r} dr\right). \tag{A1}$$

For a galaxy with flat rotation curves ($v_c(r) = \text{const.}$), one obtains:

$$g_{00}(r) = -\left(\frac{r}{r_\star}\right)^l, \tag{A2}$$

where $r_\star$ is an integration constant with dimensions of length and l is given by $l = 2v_c^2$.

Two comments are in order. First, that the form of the rotation curve completely determines the 00 component of the metric tensor through the expression (A1). For the special case of flat rotation curves, we obtain (A2). Second, that the rotation curves tell nothing about the other independent component of the metric tensor (the rr component). To obtain the rr component of the line element, one needs to assume either the nature of the matter dominating the configuration, or deduce it from other type of observations, for example, gravitational lensing [32] etc.

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981); A. Linde, Phys. Lett. B **108**, 389 (1982).

[2] A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990); A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, England, 2000).

[3] A. G. Riess *et al.*, Astrom. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[4] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); T. Padmanabhan, Phys. Rep. **380**, 235 (2003); E. J. Copeland, M. Sami, and S. Tsujikawa, hep-th/0603057.

[5] E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998); A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999); J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro, and M. Abney, Rev. Mod. Phys. **69**, 373 (1997).

[6] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).

[7] J. Bekenstein and M. Milgrom, Astrophys. J. **286**, 7 (1984).

[8] B. F. Schutz, Phys. Rev. D **2**, 2762 (1970); B. F. Schutz and R. Sorkin, Ann. Phys. (N.Y.) **107**, 1 (1977); J. D. Brown, Class. Quant. Grav. **10**, 1579 (1993); R. Jackiw, physics/0010042.

[9] G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).

[10] V. Sahni, atro-ph/0403324.

[11] V. Rubin, W. Ford, and N. Thonnard, Astrophys. J. **238**, 471 (1980).

[12] F. Zwicky, Helv. Phys. Acta **6**, 110 (1933).

[13] L. V. Koopmans and T. Treu, Astrophys. J. **583**, 606 (2003); J. Holopainen, C. Flynn, A. Knebe, S. P. Gill, and B. K. Gibson, Mon. Not. R. Astron. Soc. **368**, 1209 (2006).

[14] J. A. Tyson, G. P. Kochanski, and I. P. Dell'Antonio, Astrophys. J. Lett. **498**, 107 (1998).

[15] W. Hu and S. Dodelson, Annu. Rev. Astron. Astrophys. **40**, 171 (2002).

[16] P. de Bernadis *et al.*, Nature (London) **405**, 955 (2000); L. Page, astro-ph/0402547.

[17] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992); T. Padmanabhan, *Structure Formation in the Universe* (Cambridge University Press, Cambridge, England, 1993); V. F. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, England, 2005); J. Silk, astro-ph/0603209.

[18] G. Steigman, astro-ph/0501591.

[19] V. Sahni and L. Wang, Phys. Rev. D **62**, 103517 (2000); T. Matos, F. S. Guzman, and L. A. Ureña-Lopez, Class. Quant. Grav. **17**, 1707 (2000); P. J. E. Peebles, astro-ph/0002495; J. Goodman, astro-ph/0003018; J. McDonald, Phys. Rev. D **50**, 3637 (1993); O. Bertolami and F. M. Nunes, Phys. Lett. B **452**, 108 (1999).

[20] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001); N. Bilic, G. B. Tupper, and R. D. Viollier, Phys. Lett. B **535**, 17 (2002).

[21] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).

[22] A. Sen, J. High Energy Phys. 04 (2002) 048; 07 (2002) 065; G. W. Gibbons, Nucl. Phys. **B514**, 603 (1998).

[23] H. B. Sandvik, M. Tegmark, M. Zaldarriga, and I. Waga, Phys. Rev. D **69**, 123524 (2004); R. Bean and O. Dore, Phys. Rev. D **68**, 023515 (2003).

[24] L. Amendola, F. Finelli, C. Burigana, and D. Carturan, J. Cosmol. Astropart. Phys. 07 (2003) 005; M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **70**, 083519 (2004); N. Bilic, G. B. Tupper, and R. D. Viollier, hep-th/0504082.

[25] R. J. Scherrer, Phys. Rev. Lett. **93**, 011301 (2004).

[26] N. Arkani-Hamed, H. C. Cheng, M. A. Luty, and S. Mukohyama, J. High Energy Phys. 05 (2004) 074.

[27] T. Chiba, N. Sugiyama, and T. Nakamura, Mon. Not. R. Astron. Soc. **289**, L5 (1997).
[28] R. Holman and S. Naidu, astro-ph/0408102.
[29] E. Babichev, V. Dokuchaev, and Yu Eroshenko, Class. Quant. Grav. **22**, 143 (2005).
[30] S. Nojir and S. D. Odintsov, Phys. Rev. D **70**, 103522 (2004); S. Capozziello, S. Nojir, and S. D. Odintsov, Phys. Lett. B **634**, 93 (2006).
[31] M. Persic, P. Salucci, and F. Stel, Mon. Not. R. Astron. Soc. **281**, 27 (1996).
[32] T. Faber and M. Visser, astro-ph/0512213.
[33] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, CA, 1973).
[34] K. Lake, Phys. Rev. Lett. **92**, 051101 (2004).
[35] S. Bharadwaj and S. Kar, Phys. Rev. D **68**, 023516 (2003).
[36] U. Nucamendi, M. Salgado, and D. Sudarky, Phys. Rev. D **63**, 125016 (2001).
[37] T. Matos, F. S. Guzman, and D. Núñez, Phys. Rev. D **62**, 061301(R) (2000).
[38] Schödel *et al.*, Nature (London) **418**, 694 (2002).
[39] H. H. Soleng, Gen. Relativ. Gravit. **27**, 367 (1995).
[40] J. A. Gonzalez and D. Sudarsky, gr-qc/0405097.
[41] C. Armendariz-Picon and E. A. Lim, J. Cosmol. Astropart. Phys. 08 (2005) 007.
[42] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000); T. Chiba, T. Okabe, and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000).
[43] R. Tabensky and A. H. Taub, Commun. Math. Phys. **29**, 61 (1973).
[44] D. Clancy, A. Feinstein, J. E. Lidsey, and R. Tavakol, Phys. Lett. B **451**, 303 (1999).
[45] A. Diez-Tejedor and A. Feinstein (unpublished).
[46] W. B. Bonnor, J. Phys. A: Math. Gen. **10**, 1673 (1977).
[47] A. Diez-Tejedor and A. Feinstein, Int. J. Mod. Phys. D **14**, 1561 (2005).
[48] R. Tolman, Phys. Rev. **55**, 364 (1939).
[49] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999); E. P. Liang, Astrophys. J. **204**, 235 (1976).
[50] D. Giannakidis and W. Hu, Phys. Rev. D **72**, 063502 (2005).
[51] J. K. Erickson, R. R. Caldwell, P. J. Steinhardt, C. Armendariz-Picon, and V. Mukhanov, Phys. Rev. Lett. **88**, 121301 (2002); S. DeDeo, R. R. Caldwell, and P. J. Steinhardt, Phys. Rev. D **67**, 103509 (2003); R. Bean and O. Dore, Phys. Rev. D **69**, 083503 (2004).