Solar system experiments do not yet veto modified gravity models

Valerio Faraoni[*]

Physics Department, Bishop's University Sherbrooke, Québec, Canada J1M 0C8

The dynamical equivalence between modified and scalar-tensor gravity theories is revisited and it is concluded that it breaks down in the limit to general relativity. A gauge-independent analysis of cosmological perturbations in both classes of theories lends independent support to this conclusion. As a consequence, the PPN formalism of scalar-tensor gravity and Solar System experiments do not veto modified gravity, as previously thought.

I. INTRODUCTION

There are many models in the literature aiming at explaining the observed acceleration of the cosmic expansion discovered with supernovae of type Ia [1] in conjunction with the recent cosmic microwave background experiments [2]. One class of models postulates that the universe is filled with unclustered dark energy comprising 70% of its energy content. This dark energy has exotic properties and, if its effective equation of state is truly such that $P < -\rho$ (where ρ and P are the dark energy density and pressure, respectively), as the observations seem to favor [3], it is even more exotic and it is called phantom energy or superquintessence. Phantom energy violates all of the energy conditions and is rather difficult to accept because of the possibility of instabilities, ghosts, and strange thermodynamic behavior [4]. As an alternative to such exotic physics, it has been suggested that perhaps gravity should be modified at large scales [5,6] by introducing in the gravitational sector terms nonlinear in the Ricci curvature R. This way, one can dispense entirely with exotic dark energy. Apart from this *ad hoc* justification, there are also motivations (and corrections) for nonlinear gravity from M-theory [7]. Scenarios based on this idea are called "modified gravity", "nonlinear theories", or "fourth-order gravity" [8]. In its simplest form, the action is

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{(m)} \quad (1)$$

where $\kappa \equiv 8\pi G$. The corresponding field equations are

$$f'R_{ab} - \frac{f}{2} g_{ab} = \nabla_a\nabla_b f' - g_{ab}\Box f' + \kappa T_{ab}^{(m)}, \quad (2)$$

where a prime denotes differentiation with respect to R. Whenever $f(R)$ is nonlinear in R, the Palatini variation treating the metric and the connection as independent variables produces field equations that are different from (2), which are obtained with the usual Einstein-Hilbert variation with respect to the metric only. The "Palatini approach" is widely used in cosmology, in addition to the

usual "metric formalism" [9]. Furthermore, if the matter part of the action $S^{(m)}$ also depends on the connection, one obtains a third possibility, metric-affine gravity theories [10,11]. In the following we consider the metric approach to modified gravity but the methods and the conclusions apply to the Palatini approach as well.

We briefly recall the dynamical equivalence between $f(R)$ gravity and scalar-tensor gravity [12–14] (for the dynamical equivalence between scalar-tensor theories see Ref. [15]). By introducing an auxiliary field ϕ the action (1) becomes

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[f(\phi) + \frac{df}{d\phi}(R - \phi) \right] + S^{(m)} \quad (3)$$

if $d^2f/d\phi^2 \neq 0$. This action integral can be written as

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [\psi(\phi)R - V(\phi)] + S^{(m)}, \quad (4)$$

where

$$\psi(\phi) \equiv \frac{df}{d\phi}, \qquad V(\phi) \equiv \phi \frac{df}{d\phi} - f(\phi). \quad (5)$$

This action describes a scalar-tensor theory of gravity ([16,17]—see [18] for a review) with Brans-Dicke parameter $\omega = 0$. The corresponding field equations are

$$G_{ab} = \frac{1}{\psi}\left(\nabla_a\nabla_b\psi - g_{ab}\Box\psi - \frac{V}{2}g_{ab}\right) + \frac{\kappa}{\psi}T_{ab}^{(m)}, \quad (6)$$

$$R\frac{d\psi}{d\phi} - \frac{dV}{d\phi} = 0. \quad (7)$$

Trivially, if $\phi = R$, the action (3) reduces to (1). Vice-versa, by varying the action (3) with respect to ϕ and assuming that $S^{(m)}$ is independent of ϕ, one obtains

$$(R - \phi)f''(\phi) = 0, \quad (8)$$

which yields $\phi = R$ provided that $f'' \neq 0$ (a prime now denotes differentiation with respect to ϕ). Similarly, one shows that $f(R)$ gravity in the Palatini formalism is equivalent to a $\omega = -3/2$ Brans-Dicke theory when the matter action is independent of the connection (e.g., [10]). This

─────────
[*]Electronic address: vfaraoni@cs-linux.ubishops.ca

dynamical equivalence between theories can be quite useful but it should not be abused. It has been used to constrain $f(R)$ gravity based on Solar System bounds on the post-Newtonian parameters of scalar-tensor gravity [14,19,20]. The underlying logic is that deviations from general relativity (GR) are not detected in our local spacetime neighborhood, therefore these deviations (if they exist) must be small. This "closeness of $f(R)$ gravity to GR" implies that $f(R)$ gravity reduces to GR in an appropriate limit, which we address here. While there is in principle no problem in taking this limit directly in modified gravity, the dynamical equivalence with scalar-tensor gravity breaks down in this limit. In fact, GR corresponds to $f(R) = R$ and $f'' \equiv 0$, while the dynamical equivalence *requires* $f'' \neq 0$. This fact has been overlooked and the dynamical equivalence has been used beyond its realm of validity in the limit to GR by advocating the parametrized post-Newtonian (PPN) formalism of scalar-tensor gravity. This procedure is invalid and it explains why opposite claims of compatibility/noncompatibility of $f(R)$ gravity with Solar System experiments occur in the literature—worse, even the Newtonian limit is the subject of dispute [19,20].

The limit to GR of the equivalent scalar-tensor theory is more general than the weak field limit: it includes the strong field regime and it turns out to be a singular limit, as shown below. Moreover, the limit of Brans-Dicke theory to GR in vacuum, or in the presence of "conformal" matter (i.e., with vanishing trace of the stress-energy tensor), is riddled with problems. Therefore, one must be particularly careful in basing all of one's conclusions on the compatibility with Solar System experiments (which test the gravitational field *in vacuo* or at very low densities) solely on the equivalence with scalar-tensor gravity. In the following section we discuss the direct limit of $f(R)$ gravity to GR without using the equivalence with scalar-tensor gravity, and then the corresponding limit to GR of the equivalent scalar-tensor gravity, showing the problems arising in this last situation. We do not want to commit ourselves to specific choices of the function $f(R)$ (e.g., the CDTT model [5,6], etc.) but we consider a general form of the function $f(R)$.

II. THE LIMIT OF $f(R)$ GRAVITY TO GENERAL RELATIVITY

Perhaps the easiest way to consider the limit of $f(R)$ gravity to Einstein's theory consists of introducing a small parameter ϵ such that $f(R)$ can be expressed as [21]

$$f(R) = R + \epsilon\varphi(R). \tag{9}$$

The action of GR $S_{\mathrm{GR}} = (2\kappa)^{-1} \int d^4x \sqrt{-g}R + S^{(m)}$ is obtained in the limit $\epsilon \to 0$ [22]. The field equations become

$$(1 + \epsilon\varphi')R_{ab} - \frac{1}{2}(R + \epsilon\varphi)g_{ab}$$
$$= \epsilon\nabla_a\nabla_b\varphi' - \epsilon g_{ab}\Box\varphi' + \kappa T_{ab}^{(m)} \tag{10}$$

which, in the limit $\epsilon \to 0$, formally reduce to the Einstein equations $G_{ab} = \kappa T_{ab}^{(m)}$. So, there is no problem in taking the limit of $f(R)$ gravity to GR directly. Let us consider now the "equivalent" scalar-tensor theory (4). Although the conventional way to obtain this limit is letting the Brans-Dicke parameter ω go to infinity, here ω is fixed to be zero. The limit to GR can again be obtained by letting ϵ go to zero in the equations of scalar-tensor theory, which is equivalent to taking the limit $\phi \to$ constant. Assuming that $f(R)$ is given by Eq. (9), the nonlinear theory (1) is equivalent to (3) with

$$\psi(\phi) = 1 + \epsilon\varphi'(\phi), \qquad V(\phi) = \epsilon(\varphi'\phi - \varphi) \tag{11}$$

provided that $f'' \neq 0$. Now, in the limit $\epsilon \to 0$, $f'' = \epsilon\varphi'' \to 0$ and the equivalence is broken. Formally, the field Eq. (6) reduces to the Einstein equation while (7) is identically satisfied. There are however, problems with this procedure. One should also find the asymptotic behavior of the field ϕ as $\epsilon \to 0$. The situation is analogous to the standard limit to GR of Brans-Dicke theory, the prototype of scalar-tensor theories, in which GR is usually obtained by taking the limit $\omega \to \infty$. The standard textbook presentation provides the asymptotic behavior of the Brans-Dicke field ϕ_{BD}:

$$\phi_{\mathrm{BD}} = \phi_0 + \mathrm{O}\left(\frac{1}{\omega}\right), \tag{12}$$

where ϕ_0 is a constant [26]. But when the trace of the energy-momentum tensor of matter $T^{(m)}$ vanishes, the $\omega \to \infty$ limit fails to give back GR—this phenomenon is reported for a number of exact Brans-Dicke solutions [27] and is identified as a general feature of Brans-Dicke theory explained by a restricted conformal invariance enjoyed by the theory when $T^{(m)} = 0$ [28]. This anomalous behavior is intimately linked with the asymptotics displayed by the Brans-Dicke field in these situations [28,29,33]

$$\phi_{\mathrm{BD}} = \phi_0 + \mathrm{O}\left(\frac{1}{\sqrt{\omega}}\right) \qquad (T^{(m)} = 0). \tag{13}$$

Similarly, the examination of the asymptotics of the scalar field ϕ as the parameter ϵ tends to zero should provide a useful check of the limiting procedure. In the scalar-tensor equivalent of $f(R)$ gravity, the Brans-Dicke parameter is fixed to be $\omega = 0$ in the metric formalism ($\omega = -3/2$ in the Palatini formalism) and we must necessarily come up with a different way of taking the limit to GR, hence the possibility considered of letting ϵ going to zero while ϕ becomes constant. In this case, we should obtain a reasonable asymptotic behavior for the fields g_{ab} and ϕ, say

$$g_{ab} = g_{ab}^{(\mathrm{GR})} + \epsilon h_{ab}, \qquad \phi = \phi_0 + r(\epsilon), \tag{14}$$

where $g_{ab}^{(GR)}$ is the general-relativistic metric, ϕ_0 is a constant, and the remainder $r(\epsilon)$ tends to zero as $\epsilon \to 0$. However, this is not the case. By inserting Eq. (9) into Eq. (6) one obtains

$$G_{ab} = \frac{\epsilon}{1 + \epsilon\varphi'}\left[\nabla_a\nabla_b\varphi' - g_{ab}\Box\varphi' + \frac{1}{2}(\varphi - \phi_0\varphi')g_{ab}\right]$$
$$+ \frac{\kappa T_{ab}^{(m)}}{1 + \epsilon\varphi'}, \tag{15}$$

while

$$\Box\phi = -\frac{\epsilon\varphi'''}{1 + \epsilon\varphi'}\nabla^c\phi\nabla_c\phi + \frac{\epsilon(\phi\varphi' - 2\varphi) - \phi}{3\epsilon\varphi''}$$
$$+ \frac{1 + \epsilon\varphi'}{\epsilon\varphi''}\kappa T^{(m)}, \tag{16}$$

where the indices are raised and lowered with $g_{ab}^{(GR)}$. Further substitution of Eq. (14) yields, in the limit $\epsilon \to 0$,

$$r(\epsilon) = O\left(\frac{1}{\epsilon}\right). \tag{17}$$

The remainder $r(\epsilon)$ diverges instead of vanishing: $\epsilon \to 0$ is a singular limit of the equivalent scalar-tensor version of $f(R)$ gravity while the direct limit $\epsilon \to 0$ of $f(R)$ gravity does not lead to this problem. Again, this reflects the breakdown of the dynamical equivalence in the limit to GR in which $f'' \to 0$. Note that the procedure employed here parallels the procedure used to obtain the estimate (12) for Brans-Dicke theory [26,31,33].

III. STABILITY OF DE SITTER SPACE IN MODIFIED AND SCALAR-TENSOR GRAVITY

In this section we consider the cosmological dynamics of modified gravity. By assuming the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{18}$$

in comoving coordinates (t, x, y, z), the field Eqs. (2) of modified gravity reduce to

$$H^2 = \frac{1}{3f'(R)}\left\{\frac{f(R) - Rf'(R)}{2} - 3H\dot{R}f''(R) + \kappa\rho^{(m)}\right\}, \tag{19}$$

$$2\dot{H} + 3H^2 = -\frac{1}{f'(R)}\left\{(\dot{R})^2f'''(R) + 2H\dot{R}f''(R)\right.$$
$$\left. + \ddot{R}f''(R) - \frac{1}{2}[f(R) - Rf'(R)] + \kappa P^{(m)}\right\}. \tag{20}$$

Consider, for simplicity, the situation in which the curvature terms dominate the dynamics and $\rho^{(m)}$ and $P^{(m)}$ are negligible. Then, the scale factor $a(t)$ enters the field equations only through the Hubble parameter $H \equiv \dot{a}/a$

and it is natural to use H as the dynamical variable. The field equations are of fourth order in a (hence the name "fourth order gravity" given to $f(R)$ theories) and of third order in H. The main result of this section is that the $\omega = 0$ scalar-tensor theory does not provide the same stability condition derived in $f(R)$ gravity, but two different ones according to the type of perturbations considered. Therefore, these two theories are inequivalent with regard to stability.

The equilibrium points in the phase space $(H, \dot{H}, \ddot{H})$ are de Sitter spaces characterized by constant Hubble parameter H_0 given by

$$H_0^2 = \frac{f_0}{6f_0'} \tag{21}$$

and $R_0 = 12H_0^2$. The stability of these de Sitter spaces with respect to both homogeneous and inhomogeneous perturbations was studied in Ref. [34], with the result that the stability conditions with respect to both types of perturbations coincide and, in our notations, are expressed by [35]

$$\frac{(f_0')^2 - 2f_0f_0''}{f_0'f_0''} \geq 0. \tag{22}$$

The study of stability with respect to inhomogeneous perturbations, which are inherently gauge-dependent, requires the use of a gauge-invariant formalism. It is counter-intuitive that the stability condition with respect to inhomogeneous perturbations is not more restrictive than the corresponding stability condition with respect to homogeneous perturbations. This is not the case, for example, in scalar-tensor gravity. Stability in scalar-tensor gravity was also studied in Refs. [34,36], but the cases $\omega = 0$ and $\omega = -3/2$ were excluded by the analysis. In the following subsections we study the stability of the de Sitter equilibrium points in the $\omega = 0$ equivalent of metric $f(R)$ gravity. As expected, the stability condition with respect to inhomogeneous perturbations turns out to be different from the stability condition with respect to homogeneous ones.

A. Stability with respect to homogeneous perturbations in $\omega = 0$ scalar-tensor gravity

In the $\omega = 0$ scalar-tensor theory described by the action (3) the field equations become, in the metric (18) and in the absence of matter,

$$3f'H^2 = \frac{1}{2}(\phi f' - f) - 3Hf''\dot{\phi}, \tag{23}$$

$$-2f'\dot{H} = f'''(\dot{\phi})^2 + f''\ddot{\phi} - Hf''\dot{\phi}, \tag{24}$$

$$f''R - V' = 0. \tag{25}$$

Manipulation of Eqs. (23) and (24) leads to the Klein-Gordon-like equation for ϕ

$$\ddot{\phi} + 3H\dot{\phi} = \frac{1}{3f''}[-f'R - 3f'''(\dot{\phi})^2 + 2(\phi f' - f)]. \tag{26}$$

The equilibrium points in this picture correspond to de Sitter spaces with constant scalar field given by

$$H_0^2 = \frac{f_0}{6f_0'}, \qquad \phi_0 = \frac{2f_0}{f_0'} = R_0. \qquad (27)$$

Homogeneous perturbations of the de Sitter fixed points are described by

$$H(t) = H_0 + \delta H(t), \qquad \phi(t) = \phi_0 + \delta\phi(t), \qquad (28)$$

and obey the evolution equations

$$12H_0 f_0' \delta H + (6H_0^2 - \phi_0)f_0'' \delta\phi + 6H_0 f_0'' \delta\dot\phi = 0, \quad (29)$$

$$- 2f_0' \delta\dot H = f_0'' \delta\ddot\phi - H_0 f_0'' \delta\dot\phi \qquad (30)$$

$$6f_0'' \delta\dot H + 24H_0 f_0'' \delta H + (12H_0^2 f_0''' - \phi_0 f_0''' - f_0'')\delta\phi = 0, \qquad (31)$$

where the constraint $\phi = R$ has been used. By eliminating $\delta\dot H$ and using the zero-order equations for de Sitter space, one obtains

$$\delta\ddot\phi + 3H_0 \delta\dot\phi + \frac{1}{3f_0'(f_0'')^2}[(f_0')^2 - 2f_0]\delta\phi = 0. \quad (32)$$

Stability is achieved, and the perturbations $\delta\phi$ do not run away, when the effective mass squared in this harmonic oscillator equation is non-negative, i.e.,

$$\frac{(f_0')^2 - 2f_0}{f_0'} \geq 0. \qquad (33)$$

This is the desired stability condition with respect to homogeneous perturbations in the $\omega = 0$ scalar-tensor theory that is supposed to be equivalent to $f(R)$ gravity. This condition is different from (22) showing that, at best, the equivalence should be treated with care. The stability condition (33) should be compared with the stability condition with respect to inhomogeneous perturbations, which we derive in the next subsection.

B. Inhomogeneous perturbations in $\omega = 0$ scalar-tensor gravity

The analysis of inhomogeneous perturbations necessarily requires a gauge-independent formalism. We adopt the covariant and gauge-invariant formalism of Bardeen-Ellis-Bruni-Hwang [37] in the version of Hwang valid for generalized gravity [38]. The metric perturbations are given by

$$g_{00} = -a^2(1 + 2AY), \qquad (34)$$

$$g_{0i} = -a^2 BY_i, \qquad (35)$$

$$g_{ij} = a^2[h_{ij}(1 + 2H_L) + 2H_T Y_{ij}], \qquad (36)$$

where Y, Y_i, and Y_{ij} are the scalar, vector, and tensor harmonics, respectively, satisfying

$$\bar\nabla_i \bar\nabla^i Y = -k^2 Y, \qquad Y_i = -\frac{1}{k}\bar\nabla_i Y, \qquad (37)$$

$$Y_{ij} = \frac{1}{k^2}\bar\nabla_i\bar\nabla_j Y + \frac{1}{3}Y h_{ij}. \qquad (38)$$

Here h_{ij} is the three-dimensional metric of the FLRW background and the operator $\bar\nabla_i$ is the covariant derivative associated with h_{ij}, while k is an eigenvalue. The gauge-invariant variables used are Bardeen's potentials and the Ellis-Bruni variable

$$\Phi_H = H_L + \frac{H_T}{3} + \frac{\dot a}{k}\left(B - \frac{a}{k}\dot H_T\right), \qquad (39)$$

$$\Phi_A = A + \frac{\dot a}{k}\left(B - \frac{a}{k}\dot H_T\right) + \frac{a}{k}\left[\dot B - \frac{1}{k}(a\dot H_T)^\cdot\right], \qquad (40)$$

$$\Delta\phi = \delta\phi + \frac{a}{k}\dot\phi\left(B - \frac{a}{k}\dot H_T\right), \qquad (41)$$

with Δf and ΔR defined similarly to the last equation. The first order equations of motion for the gauge-invariant perturbations can be found in Ref. [38]; they simplify considerably in the de Sitter background, reducing to

$$\Delta\ddot\psi + 3H_0\Delta\dot\psi + \left(\frac{k^2}{a^2} - 4H_0^2\right)\Delta\psi + \frac{\psi_0}{3}\Delta R = 0, \quad (42)$$

$$- \dot\Phi_H + H_0\Phi_A = \frac{1}{2}\left(\frac{\Delta\dot\psi}{\psi_0} - H_0\frac{\Delta\psi}{\psi_0}\right), \qquad (43)$$

$$\left(\frac{k}{a}\right)^2 \Phi_H = -\frac{k^2}{2a^2}\frac{\Delta\psi}{\psi_0}, \qquad (44)$$

$$\Phi_A + \Phi_H = -\frac{\Delta\psi}{\psi_0}, \qquad (45)$$

$$\ddot H_T + 3H_0\dot H_T + \frac{k^2}{a^2}H_T = 0, \qquad (46)$$

$$\ddot\Phi_H + 3H_0\dot\Phi_H - H_0\dot\Phi_A - \frac{V_0}{2\psi_0}\Phi_A$$
$$= -\frac{1}{2}\left[\frac{\Delta\ddot\psi}{\psi_0} + 2H_0\frac{\Delta\dot\psi}{\psi_0} - \frac{V_0}{2\psi_0^2}\Delta\psi\right], \qquad (47)$$

and

$$\Delta R = 6\left[\ddot\Phi_H + 4H_0\dot\Phi_H + \frac{2}{3}\frac{k^2}{a^2}\Phi_H - H_0\dot\Phi_A\right.$$
$$\left. + \left(\frac{k^2}{3a^2} - 4H_0^2\right)\Phi_A\right], \qquad (48)$$

where $a = a_* e^{H_0 t}$ is the unperturbed scale factor. Equation (46) exhibits a positive effective mass squared

k^2/a^2 for the tensor modes H_T, therefore de Sitter spaces are always stable with respect to these modes, to linear order. By using Eq. (45) to eliminate Φ_A and Taylor-expanding $\Delta\psi = \psi_0'\Delta\phi + \ldots$, one easily obtains

$$\Phi_H = \Phi_A = -\frac{\psi_0'}{2\psi_0}\Delta\phi \qquad (49)$$

and

$$\Delta R = -\frac{3\psi_0'}{\psi_0}\left[\Delta\ddot{\phi} + 3H_0\Delta\dot{\phi} + \left(\frac{k^2}{a^2} - 4H_0^2\right)\Delta\phi\right]. \quad (50)$$

By using the fact that $\Delta R = \Delta\phi$, the equation for the scalar perturbations $\Delta\phi$ is obtained:

$$\Delta\ddot{\phi} + 3H_0\Delta\dot{\phi} + \left(\frac{k^2}{a^2} - 4H_0^2 + \frac{\psi_0}{3\psi_0'}\right)\Delta\phi = 0. \quad (51)$$

The term $(k/a)^2$ can be neglected at late times when a diverges exponentially fast and the stability condition for the perturbations $\Delta\phi$ (and $\Phi_H = \Phi_A \propto \Delta\phi$) is therefore

$$\frac{(f_0')^2 - 2f_0 f_0''}{f_0' f_0''} \geq 0. \qquad (52)$$

This is the desired stability condition with respect to inhomogeneous perturbations. It is different from the condition for homogeneous perturbations (33) and it coincides with (22). Not only one of the stability conditions with respect to homogeneous and inhomogeneous perturbations (33) and (52) of the $\omega = 0$ scalar-tensor equivalent of $f(R)$ gravity fails to coincide with the condition (22), but they also differ from each other. The purported equivalence is not a true equivalence. The reason should be looked for in the fact that, while in a true scalar-tensor theory the Brans-Dicke-like field ϕ is a true dynamical field whose evolution is only ruled by the dynamical field equations, in the theory considered here ϕ is forced to obey the additional constraint $\phi = R$, thus limiting its natural evolution. In other words, we have gone from fourth order equations to second order equations by adding a scalar degree of freedom to the theory, which has now spin 0 content in addition to spin 2, but the scalar degree of freedom is somehow constrained by the condition $\Delta\phi = \Delta R$.

IV. DISCUSSION AND CONCLUSIONS

While the limit to GR $\epsilon \to 0$ in the action (1) of modified gravity presents no problems of principle, the limit to GR of the equivalent scalar-tensor theory is ill-defined. In view of the problems presented by this limit in Brans-Dicke theory, special care is advised when using the dynamically equivalent scalar-tensor theory to analyze the weak field limit of modified gravity. As shown above, the limit to GR of the equivalent $\omega = 0$ Brans-Dicke theory is a singular one, illustrating the fact that the equivalence

breaks down in this limit. *A posteriori* it is easy to see that this is implied by the fact that $f''(R) \to 0$ in this limit.

Another issue is the following: if the dynamical equivalence were to hold in the limit to GR, the experimental bound $\omega > 40000$ [39] would be in violent conflict with the values of the Brans-Dicke parameter $\omega = 0$ or $-3/2$ obtained, unless the field ϕ is short-ranged. The effective mass of ϕ is given by

$$m_{\text{eff}}^2 = V'' = \epsilon(\varphi'' + \phi\varphi''') \qquad (53)$$

and vanishes as $\epsilon \to 0$, making it impossible to suppress the violation of the bounds on ω, for *any* form of the function $f(R)$. This contradicts the results of Refs. [20,40] which support the viability of the weak field limit of the theory for specific forms of $f(R)$. As it turns out, the PPN limit of the associated scalar-tensor theory bears no relation to the weak field limit of the original modified gravity theory.

As a consequence, the conclusion [14,19] that modified gravity always violates the stringent Solar System bounds on scalar-tensor gravity [39] is invalid. The issue of the correct Newtonian and post-Newtonian limit of such theories is still open and should be approached directly without using the dynamical equivalence discovered in Refs. [12,13], which is still useful for other purposes. The regime that is more interesting, however, is the one in which the deviations from GR in the Solar System are small. A complete study of the PPN limit of general modified gravity scenarios without resorting to the equivalent scalar-tensor theory (initiated in Refs. [20,23]) will be presented elsewhere.

Another apparent problem with the limit to GR lies in the fact that ϕ must become constant: because $\phi = R$, were this limit correct, it could only reproduce solutions with constant Ricci curvature (which includes vacuum solutions and solutions sourced by conformal matter). Although this could work for vacuum solutions used to describe Solar System experiments, it is by no means acceptable to have a limit to GR valid only for special solutions: the limit must apply to the general theory. However, we believe that this second problem is not very relevant because, when f'' vanishes in Eq. (8) in this limit, the equality $\phi = R$ is no longer enforced.

Finally, the viability of modified gravity scenarios does not rely only on its correct weak field limit: other issues are the presence of ghosts and instabilities and, of course, a correct cosmological dynamics. These have been considered separately in the literature [41].

[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); **118**, 2668 (1999); Astrophys. J. **560**, 49 (2001); **607**, 665 (2004); S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998); Astrophys. J. **517**, 565 (1999); J. L. Tonry *et al.*, Astrophys. J. **594**, 1 (2003); R. Knop *et al.*, Astrophys. J. **598**, 102 (2003); B. Barris *et al.*, Astrophys. J. **602**, 571 (2004).

[2] A. D. Miller *et al.*, Astrophys. J. Lett. **524**, L1 (1999); P. de Bernardis *et al.*, Nature (London) **404**, 955 (2000); A. E. Lange *et al.*, Phys. Rev. D **63**, 042001 (2001); A. Melchiorri, L. Mersini, C. J. Odman, and M. Trodden, Astrophys. J. Lett. **536**, L63 (2000); S. Hanany *et al.*, Astrophys. J. Lett. **545**, L5 (2000); D. N. Spergel *et al.*, Astrophys. J. (Suppl.) **148**, 175 (2003); C. L. Bennett *et al.*, Astrophys. J. (Suppl.) **148**, 1 (2003); T. J. Pearson *et al.*, Astrophys. J. **591**, 556 (2003); A. Benoit *et al.*, Astron. Astrophys. **399**, L25 (2003).

[3] J. S. Alcaniz, Phys. Rev. D **69**, 083521 (2004); A. Melchiorri, L. Mersini, C. J. Odman, and M. Trodden, Phys. Rev. D **68**, 043509 (2003); S. Hannestad and E. Mortsell, Phys. Rev. D **66**, 023510 (2002); U. Alam *et al.*, Mon. Not. R. Astron. Soc. **354**, 275 (2004); S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **70**, 043531 (2004); T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. **429**, 807 (2005).

[4] S. Capozziello, S. Nojiri, and S. D. Odintsov, Phys. Lett. B **632**, 597 (2006); S. Nojiri and S. D. Odintsov, hep-th/0506212; Phys. Rev. D **72**, 023003 (2005); V. Faraoni, Class. Quant. Grav. **22**, 3235 (2005); W. Fang *et al.*, Int. J. Mod. Phys. D **15**, 199 (2006); P. F. Gonzalez-Diaz and J. A. Jimenez-Madrid, Phys. Lett. B **596**, 16 (2004); M. G. Brown, K. Freese, and W. H. Kinney, astro-ph/0405353; E. Elizalde, S. Nojiri, and S. D. Odintsov Phys. Rev. D **70**, 043539 (2004); Phys. Lett. B **574**, 1 (2003); Phys. Rev. D **70**, 043539 (2004); J.-G. Hao and X.-Z. Li, Phys. Lett. B **606**, 7 (2005); Phys. Rev. D **68**, 043501 (2003); **69**, 107303 (2004); J. M. Aguirregabiria, L. P. Chimento, and R. Lazkoz, Phys. Rev. D **70**, 023509 (2004); Y.-S. Piao and Y.-Z. Zhang, Phys. Rev. D **70**, 063513 (2004); P. F. Gonzalez-Diaz, Phys. Rev. D **68**, 021303(R) (2003); Phys. Rev. D **69**, 063522 (2004); Phys. Lett. B **586**, 1 (2004); H. Q. Lu, Int. J. Mod. Phys. D **14**, 355 (2005); V. B. Johri, Phys. Rev. D **70**, 041303 (2004); H. Stefancić, Phys. Lett. B **586**, 5 (2004); D. J. Liu and X. Z. Li, Phys. Rev. D **68**, 067301 (2003); J. G. Hao and X. Z. Li, Phys. Rev. D **69**, 107303 (2004); M. P. Dabrowski, T. Stachowiak, and M. Szydlowski, Phys. Rev. D **68**, 103519 (2003); E. Babichev, V. Dokuchaev, and Yu. Eroshenko, Phys. Rev. Lett. **93**, 021102 (2004); Z. K. Guo, Y. S. Piao, and Y. Z. Zhang, Phys. Lett. B **594**, 247 (2004); J. M. Cline, S. Jeon, and G. D. Moore, Phys. Rev. D **70**, 043543 (2004); S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003); V. Faraoni, Ann. Phys. (NY) **317**, 366 (2005); V. Faraoni, M. N. Jensen, and S. A. Theuerkauf, Class. Quant. Grav. **23**, 4215 (2006); L. Mersini, M. Bastero-Gil, and P. Kanti, Phys. Rev. D **64**, 043508 (2001); M. Bastero-Gil, P. H. Frampton, and L. Mersini, Phys. Rev. D **65**, 106002 (2002); P. H. Frampton, Phys. Lett. B **555**, 139 (2003).

[5] S. Capozziello, S. Carloni, and A. Troisi, astro-ph/0303041.

[6] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).

[7] S. Nojiri and S. D. Odintsov, Phys. Lett. B **576**, 5 (2003); S. Nojiri, S. D. Odintsov, and M. Sami, hep-th/0605039.

[8] S. Capozziello, V. I. Cardone, S. Carloni, and A. Troisi, Int. J. Mod. Phys. D **12**, 1969 (2003); S. Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003); D. N. Vollick, Phys. Rev. D **68**, 063510 (2003); S. Carloni, P. K. S. Dunsby, S. Capozziello, and A. Troisi, Class. Quant. Grav. **22**, 4839 (2005); D. A. Easson, Int. J. Mod. Phys. A **19**, 5343 (2004); D. A. Easson, F. P. Schuller, M. Trodden, and M. N. R. Wohlfarth, Phys. Rev. D **72**, 043504 (2005); G. J. Olmo and W. Komp, gr-qc/0403092; È. È. Flanagan, Phys. Rev. Lett. **92**, 071101 (2004); Class. Quant. Grav. **21**, 417 (2004); **21**, 3817 (2004); M. Ishak, A. Upadhye, and D. N. Spergel, astro-ph/0507184; S. Nojiri and S. D. Odintsov, Phys. Lett. B **576**, 5 (2003); **599**, 137 (2004); G. Allemandi, A. Borowiec, and M. Francaviglia, Phys. Rev. D **70**, 103503 (2004); A. Lue, R. Scoccimarro, and G. Starkman, Phys. Rev. D **69**, 044005 (2004); T. Koivisto, gr-qc/0505128; M. Sami, A. Toporensky, P. V. Tretjakov, and S. Tsujikawa, Phys. Lett. B **619**, 193 (2005); K. A. Bronnikov and M. S. Chernakova, gr-qc/0503025; M. C. B. Abdalla, S. Nojiri, and S. D. Odintsov, Class. Quant. Grav. **22**, L35 (2005); L. Amendola, D. Polarski, and S. Tsujikawa, astro-ph/0603703.

[9] D. N. Vollick, Phys. Rev. D **68**, 063510 (2003); Class. Quant. Grav. **21**, 3813 (2004); X. H. Meng and P. Wang, Class. Quant. Grav. **20**, 4949 (2003); **21**, 951 (2004); Phys. Lett. B **584**, 1 (2004); E. E. Flanagan, Phys. Rev. Lett. **92**, 071101 (2004); T. Koivisto, Phys. Rev. D **73**, 083517 (2006); Class. Quant. Grav. **23**, 4289 (2006); T. Koivisto and H. Kurki-Suonio, Class. Quant. Grav. **23**, 2355 (2006); P. Wang, G. M. Kremer, D. S. M. Alves, and X. H. Meng, Gen. Relativ. Gravit. **38**, 517 (2006); G. Allemandi, M. Capone, S. Capozziello, and M. Francaviglia, Gen. Relativ. Gravit. **38**, 33 (2006).

[10] T. P. Sotiriou, gr-qc/0604028; gr-qc/0509029; gr-qc/0512017.

[11] T. P. Sotiriou and S. Liberati, gr-qc/0604006; N. J. Poplawski, Class. Quant. Grav. **23**, 2011 (2006); gr-qc/051107.

[12] P. Teyssandier and P. Tourrenc, J. Math. Phys. (N.Y.) **24**, 2793 (1983).

[13] D. Wands, Class. Quant. Grav. **11**, 269 (1994).

[14] T. Chiba, Phys. Lett. B **575**, 1 (2003).

[15] D. F. Torres and H. Vucetich, Phys. Rev. D **54**, 7373 (1996).

[16] C. H. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[17] P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970); K. Nordvedt, Astrophys. J. **161**, 1059 (1970).

[18] V. Faraoni, *Cosmology in Scalar-Tensor Gravity* (Kluwer Academic, Dordrecht, 2004).

[19] M. E. Soussa and R. P. Woodard, Gen. Relativ. Gravit. **36**, 855 (2004); R. Dick, Gen. Relativ. Gravit. **36**, 217 (2004); A. E. Dominguez and D. E. Barraco, Phys. Rev. D **70**, 043505 (2004); D. A. Easson, Int. J. Mod. Phys. A **19**, 5343 (2004); G. J. Olmo, Phys. Rev. Lett. **95**, 261102 (2005); Phys. Rev. D **72**, 083505 (2005); Phys. Rev. D

72, 083505 (2005); Phys. Rev. D **72**, 083505 (2005); I. Navarro and K. Van Acoleyen, Phys. Lett. B **622**, 1 (2005); G. Allemandi, M. Francaviglia, M. L. Ruggiero, and A. Tartaglia, Gen. Relativ. Gravit. **37**, 1891 (2005); J. A. R. Cembranos, Phys. Rev. D **73**, 064029 (2006); S. Capozziello and A. Troisi, Phys. Rev. D **72**, 044022 (2005); T. Clifton and J. D. Barrow, Phys. Rev. D **72**, 103005 (2005).

[20] T. P. Sotiriou, gr-qc/0507027.

[21] A cosmological constant Λ can be added to the linear part of $f(R)$ but we omit it because modified gravity was introduced as an alternative to dark energy and the cosmological constant.

[22] This procedure has been considered for the specific scenario $f(R) = R - \mu^4/R$ [23], where the limit $\mu \to 0$ is effectively replaced by an expansion in the small parameter $\mu \approx H_0 \approx 10^{-33}$ eV analogous to ϵ. However, this scenario is not viable due to instabilities in the matter [24] and the gravity [25] sectors unless a positive power of R is added to the Lagrangian.

[23] C.-G. Shao, R.-G. Cai, B. Wang, and R.-K. Su, Phys. Lett. B **633**, 164 (2006).

[24] A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).

[25] V. Faraoni and S. Nadeau, Phys. Rev. D **72**, 124005 (2005).

[26] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[27] T. Matsuda, Prog. Theor. Phys. **47**, 738 (1972); C. Romero and A. Barros, Astrophys. Space Sci. **192**, 263 (1992); DF-CCEN-UFPb Report No. 9, 1992; Phys. Lett. A **173**, 243 (1993); Gen. Relativ. Gravit. **25**, 491 (1993); F. M. Paiva, M. Reboucas, and M. MacCallum, Class. Quant. Grav. **10**, 1165 (1993); F. M. Paiva and C. Romero, Gen. Relativ. Gravit. **25**, 1305 (1993); M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. D **51**, 4236 (1995); N. Banerjee and S. Sen, Phys. Rev. D **56**, 1334 (1997); L. A. Anchordoqui, D. F. Torres, M. L. Trobo, and G. S. Birman, Phys. Rev. D **57**, 829 (1998).

[28] V. Faraoni, Phys. Lett. A **245**, 26 (1998); Phys. Rev. D **59**, 084021 (1999).

[29] Recent developments on this issue include the discovery of certain solutions corresponding to energy-momentum tensor with $T^{(m)} \neq 0$ which also fail to converge to the corresponding solutions of Einstein's theory when $\omega \to \infty$ [30,31], and even the conjecture that the failure to achieve the limit to GR when ω becomes large may be a generic feature of Brans-Dicke gravity [31,32].

[30] L. A. Anchordoqui, S. P. Bergliaffa, M. L. Trobo, and G. S. Birman, Mod. Phys. Lett. A **14**, 1105 (1999).

[31] B. Chauvineau, Class. Quant. Grav. **20**, 2617 (2003).

[32] A. Bhadra and K. K. Nandi, Phys. Rev. D **64**, 087501 (2001).

[33] N. Banerjee and S. Sen, Phys. Rev. D **56**, 1334 (1997).

[34] V. Faraoni, Phys. Rev. D **72**, 061501(R) (2005).

[35] The study of physically different instabilities yields surprisingly similar stability conditions. See Refs. [42–47] for alternative studies of the stability of de Sitter space.

[36] V. Faraoni and M. N. Jensen, Class. Quant. Grav. **23**, 3005 (2006).

[37] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980); G. F. R. Ellis and M. Bruni, Phys. Rev. D **40**, 1804 (1989); G. F. R. Ellis, J. C. Hwang, and M. Bruni, Phys. Rev. D **40**, 1819 (1989); G. F. R. Ellis, M. Bruni, and J. C. Hwang, Phys. Rev. D **42**, 1035 (1990).

[38] J. C. Hwang, Class. Quant. Grav. **7**, 1613 (1990); **14**, 1981 (1997); 3327;**15**, 1401 (1998); 1387; Phys. Rev. D **42**, 2601 (1990); **53**, 762 (1996); J. C. Hwang and H. Noh, Phys. Rev. D **54**, 1460 (1996).

[39] B. Bertotti, L. Iess, and P. Tortora, Nature (London) **425**, 374 (2003).

[40] S. Capozziello, A. Stabile, and A. Troisi, gr-qc/0603071.

[41] A. Núñez and S. Solganik, Phys. Lett. B **608**, 189 (2005); hep-th/0403159; T. Chiba, J. Cosmol. Astropart. Phys. 05 (2005) 008; P. Wang, Phys. Rev. D **72**, 024030 (2005); A. De Felice, M. Hindmarsh, and M. Trodden, astro-ph/0604154; G. Calcagni, B. de Carlos, and A. De Felice, hep-th/0604201; R. Gannouji, D. Polarski, A. Ranquet, and A. A. Starobinsky, astro-ph/0606287.

[42] I. Navarro and K. van Acoleyen, J. Cosmol. Astropart. Phys. 03 (2006) 008.

[43] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, J. Cosmol. Astropart. Phys. 02 (2005) 010; J. Phys. A **39**, 6245 (2006).

[44] A. Dolgov and D. N. Pelliccia, Nucl. Phys. **B734**, 208 (2006).

[45] V. Müller, H.-J. Schmidt, and A. A. Starobinsky, Phys. Lett. B **202**, 198 (1988); H.-J. Schmidt, Class. Quant. Grav. **5**, 233 (1988); A. Battaglia Mayer and H.-J. Schmidt, Class. Quant. Grav. **10**, 2441 (1993).

[46] J. D. Barrow and S. Hervik, Phys. Rev. D **73**, 023007 (2006).

[47] O. Bertolami, Phys. Lett. B **186**, 161 (1987).