

Searching for the bottom counterparts of $X(3872)$ and $Y(4260)$ via $\pi^+\pi^-\Upsilon$

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The $X(3872)$ and $Y(4260)$, among a host of charmoniumlike mesons, have rather unusual properties: the former has very small total width, the latter has large rate into $\pi^+\pi^-J/\psi$ channel. It would not be easy to settle between the many suggested explanations for their composition. We point out that discovering the bottom counterparts should shed much light on the issue. The narrow state can be searched for at the Tevatron via $p\bar{p} \rightarrow \pi^+\pi^-\Upsilon + X$, but the LHC should be much more promising. The state with large overlap with Υ can be searched for at B factories via radiative return $e^+e^- \rightarrow \gamma_{\text{ISR}} + \pi^+\pi^-\Upsilon$ on $Y(5S)$, or by $e^+e^- \rightarrow \pi^+\pi^-\Upsilon$ direct scan.

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I. INTRODUCTION

Owing to the unprecedented luminosities achieved at the B factories, heavy quarkonium spectroscopy is experiencing a renaissance. There is an X and a Y and a Z of 3940 MeV states produced via various mechanisms, but two states stand out especially: the $X(3872)$ and the $Y(4260)$, both observed in the $\pi^+\pi^-J/\psi$ channel.

The $X(3872)$ was discovered [1] by the Belle experiment in $\pi^+\pi^-J/\psi$ recoiling against K^+ from B^+ decay, and was quickly confirmed by the CDF [2] and D0 [3] experiments in $p\bar{p} \rightarrow \pi^+\pi^-J/\psi + \text{anything}$, as well as by [4] the BABAR experiment. The width is narrow, $\Gamma < 2.3$ MeV [1], and is consistent with experimental resolution. Subsequent studies strongly favor the 1^{++} quantum number [5]. With its mass just at the $D^0\bar{D}^{*0}$ threshold, theoretical interpretation has ranged from $D^0\bar{D}^{*0}$ molecule, 4 quark state, to charmonium hybrid.

The $Y(4260)$ was discovered [6] by the BABAR experiment in initial state radiation (ISR, or ‘‘radiative return’’) $e^+e^- \rightarrow \gamma_{\text{ISR}} + \pi^+\pi^-J/\psi$ events, hence is 1^{--} . The width is found to be around 90 MeV. What is peculiar is the large partial width for $Y(4260) \rightarrow \pi^+\pi^-J/\psi$. Furthermore, it falls at a local minimum of the $e^+e^- \rightarrow \text{hadrons}$ cross section. The state has been confirmed [7] by the CLEO-c experiment via $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ energy scan. Theoretical interpretations range from hybrid [8], 4 quark state [9], meson molecule [10] or baryonium [11], to conventional $\psi(4S)$ [12].

It is not our intention to comment on the various theoretical interpretations, which clearly needs more data and more debate to settle. Judging from the history of hadronic spectroscopy, it would not be easy for this to be conclusive. Rather, the intent of this short note is to point out where and how to find analogous states involving b quarks, dubbed the X_b and Y_b , respectively. Since m_b is much larger than m_c , observing such states would not only be spectacular, but should offer immense help to distinguish between models.

Clearly, the analogous search channel would be $\pi^+\pi^-\Upsilon$. We point out that the narrow state X_b can be searched for at the Tevatron (and better at the LHC). The 1^{--} state Y_b can be searched for at the B factories (and future Super B factory), either by ISR search on the $Y(5S)$, or by direct scan at $Y(5S)$ energies and beyond.

II. $\pi^+\pi^-\Upsilon$ SEARCH AT HADRON COLLIDERS

Let us first focus on the X_b . If this is a 1^{++} state, and unlike the $X(3872)$ case where there is now no analogue of the parent B meson, one can only think of searching at hadronic colliders.

The crucial question is: What is the mass? From the fact that the $X(3872)$ is right at the $D^0\bar{D}^{*0}$ threshold, the analogy would be the $B\bar{B}^*$ threshold, which would be at 10604 MeV, regardless of B^0 or B^+ . This is of course just a guess [13]. By coupled s - and d -wave $B\bar{B}^*$ channels, some models predict [14,15] the X_b mass to be 10562 MeV, below the $B\bar{B}^*$ threshold, while the $X(3872)$ mass is brought about by couplings between the $D\bar{D}^*$, $\rho J/\psi$ and $\omega J/\psi$ channels [14]. Whether $M_{X_b} \sim 10604$ MeV or 10562 MeV, the available energy for the $\pi^+\pi^-$ system is over 1000 MeV, and one can check whether ρ is still dominant once X_b is observed.

Turning to X_b production, we first note that, at the Tevatron, J/ψ production from B decay is but a fraction of the total cross section for $p\bar{p} \rightarrow J/\psi + \text{anything}$, while $X(3872)$ production is consistent with $\psi(2S)$ in prompt production fraction. Therefore, in moving to the $X_b \rightarrow \pi^+\pi^-\Upsilon$ search, we assume that the X_b production mechanism is similar to prompt $X(3872)$ production. For $X(3872) \rightarrow \pi^+\pi^-J/\psi$ reconstruction at CDF [2], we will take the number to be ~ 3500 events per fb^{-1} .

Y production at Tevatron energies (for our purpose, we do not distinguish between 1.8 and 1.96 TeV) has been studied by both CDF and D0 [16,17]. Compared to J/ψ production [18], the cross section is smaller by almost 3 orders of magnitude. Assuming this fraction, together with the leptonic rate of Y being only 40% that of J/ψ , our very

rough estimate for the number of reconstructed $X_b \rightarrow \pi^+ \pi^- Y$ events is of order 20 for an integrated luminosity of order 8 fb^{-1} expected for the Tevatron Run II. Thus, the case appears to be marginal.

We caution that we could be off by an order of magnitude, so the direction should still be pursued at the Tevatron. We do not know, for example, the branching fraction of $X_b \rightarrow \pi^+ \pi^- Y$ compared with $X(3872) \rightarrow \pi^+ \pi^- J/\psi$, nor do we know the variation in production fraction with m_Q . The search program should start with reconstructing $Y(2S) \rightarrow \pi^+ \pi^- Y$. Making a similar estimate as above, taking into account the $Y(2S) \rightarrow \pi^+ \pi^- Y$ branching ratio compared with $\psi(2S) \rightarrow \pi^+ \pi^- J/\psi$, one expects ~ 100 reconstructed $Y(2S) \rightarrow \pi^+ \pi^- Y$ events for an integrated luminosity of order 8 fb^{-1} . If one cannot even establish $Y(2S)$, then it would be doubtful whether X_b can be found via the $\pi^+ \pi^- Y$ channel.

The situation should be much better at the LHC. It is not clear what is the actual ratio of inclusive Y vs J/ψ production, although it should be better than at Tevatron energies. The PYTHIA based simulation results of Ref. [19], extrapolating from fitted results to Tevatron measurements, suggest that the cross section for $\mathcal{B}(Y \rightarrow \mu^+ \mu^-) d\sigma(pp \rightarrow Y + X)/dp_T$ at LHC is roughly 1/10 that of $\mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) d\sigma(p\bar{p} \rightarrow J/\psi + X)/dp_T$ at the Tevatron. Thus, even with a few fb^{-1} at the LHC, the ATLAS and CMS experiments should be able to discover $X_b \rightarrow \pi^+ \pi^- Y$, if it exists and is as narrow as $X(3872)$. Once again, the benchmark test should be to reconstruct $Y(2S) \rightarrow \pi^+ \pi^- Y$, and to look for extra narrow states above it that do not fit the usual $Y(nS)$ spectrum.

The production of $b\bar{b}$ is enhanced in the forward direction at high energy hadronic colliders, and dedicated B experiments such as LHCb [20] have a forward detector design aimed at reconstructing both b hadrons. Thus, LHCb may be the best suited for the study of X_b . It is the only hadronic collider experiment that has particle identification and full calorimetry capabilities. Though not needed for $X_b \rightarrow \pi^+ \pi^- Y$ search, these should enable it to do a more complete study (such as $K^+ K^- Y$ or ωY) of bottomonium spectroscopy beyond the X_b , such as searching for d -wave mesons which branch into $\pi^+ \pi^- Y$. Once found, the J^{PC} quantum numbers can be established through, for example, partial wave analysis. Identifying more states would clearly help the interpretation.

In preparing for a search for X_b at LHCb, once again the benchmark test would be to reconstruct $Y(2S) \rightarrow \pi^+ \pi^- Y$. If LHCb can demonstrate this, given higher cross section for forward vs central $b\bar{b}$ production and a more specialized detector, it should be straightforward to find the X_b , if it exists, while LHCb may be able to discover other narrow states. It would be interesting to see LHCb shed light on heavy quarkonium spectroscopy, even though it was designed for flavor physics.

III. $\pi^+ \pi^- Y$ SEARCH AT $e^+ e^-$ COLLIDERS

The $Y(4260)$ (we shall denote it Y_c) was first observed [6] in radiative return $e^+ e^- \rightarrow \gamma_{\text{ISR}} + \pi^+ \pi^- J/\psi$, and confirmed [7] by direct $e^+ e^- \rightarrow \pi^+ \pi^- J/\psi$ scan. The observed width of 88 MeV is broad compared to $X(3872)$. Averaging over BABAR and CLEO, one has,

$$\Gamma(Y_c \rightarrow ee) \mathcal{B}(Y_c \rightarrow \pi^+ \pi^- J/\psi) \sim 6 \text{ eV}, \quad (1)$$

or $\mathcal{B}(Y_c \rightarrow ee) \mathcal{B}(Y_c \rightarrow \pi^+ \pi^- J/\psi) \sim 7 \times 10^{-8}$, which is larger than the case for $\psi(4040)$ and $\psi(4160)$. But since $Y_c(4260)$ falls at a dip in the $e^+ e^- \rightarrow \text{hadrons}$ cross section, barring subtle interference effects [12], presumably $\Gamma(Y_c \rightarrow ee) \ll \Gamma(\psi(4160) \rightarrow ee) \sim 770 \text{ eV}$. Hence, the partial width $\Gamma(Y_c \rightarrow \pi^+ \pi^- J/\psi)$ should be a few MeV or higher, much larger than typical charmonia.

For Y_b , one can contemplate production in radiative return $e^+ e^- \rightarrow \gamma_{\text{ISR}} \pi^+ \pi^- Y$, or by direct $e^+ e^- \rightarrow Y_b \rightarrow \pi^+ \pi^- Y$ scan. Search in the hadronic environment would be hampered by large background due to a sizable width. The question again is, what is the mass? Further, what is the width, and $\Gamma(Y_b \rightarrow ee) \mathcal{B}(Y_b \rightarrow \pi^+ \pi^- Y)$?

The results of the CLEO study [7] of 15 decay modes of the $Y_c(4260)$ are compatible with the hybrid charmonium picture [8], could be supportive of 4 quark states [9] if partners are seen, and disfavors all other proposals. Without advocating a hybrid interpretation, we take the $Q\bar{Q}g$ hybrid picture as a guide for discussing the mass of Y_b . Lattice studies have put the lowest $b\bar{b}g$ hybrid at around 10700–11000 MeV [21]. The 1^{--} quantum number is possible, with many other possible quantum numbers, including exotic ones such as 1^{-+} . The 1^{--} , however, can mix with standard s -wave mesons and may not be the lightest, but it is clearly the most accessible.

Lattice studies tend to give lightest $c\bar{c}g$ hybrid mass around 4400 MeV. If $Y_c(4260)$ is indeed dominantly a hybrid, by analogy the $b\bar{b}g$ hybrid lattice range could be scaled down to 10600–10900 MeV. This would make $Y(5S)$, at 10865 MeV, an excellent place to conduct $e^+ e^- \rightarrow \gamma_{\text{ISR}} Y_b \rightarrow \gamma_{\text{ISR}} + \pi^+ \pi^- Y$ search, aside from the main program of B_s studies. We shall take 10600, 10700 and 10800 MeV as nominal M_{Y_b} values for this purpose. We caution, however, that even with lattice studies of hybrids, there are uncertainties due to difference in numerical approach, scale uncertainty, as well as treatment of dynamic quarks. For example, some studies [22] find the lowest $b\bar{b}g$ hybrid mass to be ~ 10900 – 11000 MeV, while giving the right mass for $c\bar{c}g$ hybrid that is consistent with $Y_c(4260)$. If Y_b is heavier than 10900 MeV, then a direct scan would be more profitable.

It is reasonable to assume that $\Gamma(Y_b \rightarrow \pi^+ \pi^- Y)$ is comparable to $\Gamma(Y_c(4260) \rightarrow \pi^+ \pi^- J/\psi)$. For the total width, taking $\Gamma_{Y_b} \sim \Gamma_{Y_c} \sim 100 \text{ MeV}$ is also reasonable. But Γ_{Y_b} could be narrower. For example $Y_c \rightarrow D\bar{D}^* \pi$ is not forbidden, but $Y_b \rightarrow B\bar{B}^* \pi$ could be hampered by phase space if Y_b is lighter than $Y(5S)$. A narrower width

could compensate for the suppression of $\Gamma(Y_b \rightarrow ee)$ due to b quark charge. We therefore take Eq. (1) and estimate that $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y) \lesssim 6$ MeV.

The Belle experiment has performed an engineering run on $Y(5S)$ in 2005 with 1.86 fb^{-1} data [23], and has accumulated over 20 fb^{-1} just before 2006 summer shutdown. The ISR cross section for $e^+e^- \rightarrow \gamma_{\text{ISR}}Y_b \rightarrow \gamma_{\text{ISR}} + \pi^+\pi^-Y$ on $Y(5S)$ resonance, in the narrow Y_b width approximation and leading order in α , is [24]

$$\sigma_{\text{ISR}} \simeq 3.6 \times 10^7 \frac{\Gamma_{ee}\mathcal{B}_{\pi^+\pi^-Y}}{M_{Y_b}} \frac{1}{x} \left(1 - x + \frac{x^2}{2}\right) \text{ pb}, \quad (2)$$

where $x = 1 - M_{Y_b}^2/s$ is the energy fraction carried away by the ISR photon (usually not observed) in the CM frame. The cross sections for our representative values of $M_{Y_b} = 10600, 10700$ and 10800 MeV are given in Table I.

Radiative return cross section is $\mathcal{O}(\alpha)$ suppressed, but one might enjoy a longer run on the $Y(5S)$ for reasons of B_s physics. One could also gain in $1/E_\gamma$ enhancement when Y_b is closer to $Y(5S)$, though the narrow width approximation may start to be questionable. However, we do not know the width for Y_b , so we just use Table I as a rough guide. With 30 fb^{-1} on $Y(5S)$, assuming $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$ is similar Eq. (1), even for $M_{Y_b} \sim 10600$ MeV one expects close to $600 \pi^+\pi^-\ell^+\ell^-$ events, where $\ell = e, \mu$ and $m_{\ell\ell}$ reconstructing to M_Y . Thus, even for $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$ as low as 1 eV , one can get similar significance for Y_b as the *BABAR* discovery of $Y(4260)$, where 125 events were obtained from 211 fb^{-1} data on the $Y(4S)$. It seems that ISR return on $Y(5S)$ would definitely find the corresponding Y_b if it is lighter in mass.

One could also directly scan for $e^+e^- \rightarrow Y_b \rightarrow \pi^+\pi^-Y$, which would likely be the only option for Y_b heavier than $Y(5S)$. The cross section is

$$\sigma_0(s) \simeq \frac{12\pi\mathcal{B}_{ee}\mathcal{B}_{\pi^+\pi^-Y}}{s} \sim \frac{1027}{M_{Y_b}^2(\text{GeV})} \text{ pb} \sim 9 \text{ pb}, \quad (3)$$

where $s = M_{Y_b}^2$, and we have taken $\mathcal{B}_{ee}\mathcal{B}_{\pi^+\pi^-Y}$ to be the same as for $Y_c(4260)$. With just 13.2 pb^{-1} on the $Y(4260)$, CLEO was able to observe [7] a clean signal of $37 \pi^+\pi^-J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-$ events with little background, measuring $\sigma_0(e^+e^- \rightarrow \pi^+\pi^-J/\psi) \simeq 58 \text{ pb}$, which is

TABLE I. Cross section for $e^+e^- \rightarrow \gamma_{\text{ISR}}Y_b \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-Y$ on the $Y(5S)$, and for direct $e^+e^- \rightarrow Y_b \rightarrow \pi^+\pi^-Y$, for $M_{Y_b} = 10600, 10700$ and 10800 MeV, in the narrow width approximation. We take $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$ to be 6 eV , comparable to Eq. (1). For higher values of M_{Y_b} , ISR from $Y(5S)$ ceases to be feasible, but direct scan can still be done, with only a slight drop in cross section with s .

process	10600	10700	10800
$e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-Y$	0.4 pb	0.6 pb	1.6 pb
$e^+e^- \rightarrow \pi^+\pi^-Y$	9.1 pb	9.0 pb	8.8 pb

consistent with Eq. (1). If Eq. (1) holds approximately for $Y_b \rightarrow \pi^+\pi^-Y$, even though $\mathcal{B}(Y \rightarrow \ell\ell) \simeq 0.4\mathcal{B}(J/\psi \rightarrow \ell\ell)$, the 30 pb^{-1} per energy scan performed by Belle for $\sqrt{s} = 10825, 10845, 10865, 10885$ and 10905 MeV for the $Y(5S)$ engineering run [23] can already be very useful. This is especially so if the scan can be repeated to cover fully the 10700 – 11000 MeV range, maybe with 30 – 50 MeV steps, assuming that Γ_{Y_b} is not drastically different from Γ_{Y_c} . But since we do not really know $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$ nor Γ_{Y_b} , discovery may still come first from radiative return studies.

IV. DISCUSSION AND CONCLUSION

For the narrow X_b state, one needs high center of mass energy to produce the heavy quarkonia of interest, especially for the case of bottomonia. One also needs to associate the quarkonia with a $\pi^+\pi^-$ pair to form the exotic meson. As was the case for $X(3872)$, only the Tevatron was able to confirm the discovery by the B factories. Note that broad states would be too hard to establish in a high background environment, even for the Tevatron and the LHC. Combinatoric background would be much higher for heavy ion collisions.

We have discussed the mass range of 10560 – 10600 MeV motivated by $B\bar{B}^*$ threshold. But for the actual hadronic collider search, one should certainly aim for a broader range. Since the case is marginal at the Tevatron, discovery may have to wait for the LHC. Whether Tevatron or LHC, $Y(2S) \rightarrow \pi^+\pi^-Y$ reconstruction should be studied. We remark that many new states, such as d -wave mesons, are not quite accessible at e^+e^- machines because of suppressed e^+e^- widths. Detectors at the LHC, especially the forward design of LHCb, have good potential for discovering other narrow bottomonia beyond the X_b via $\pi^+\pi^-Y$ (and charmonia via $\pi^+\pi^-J/\psi$).

More immediately accessible at the B factories is the broad Y_b state that decays prominently into $\pi^+\pi^-Y$. We have used the 10600 – 10800 MeV range motivated by hybrid $Q\bar{Q}g$ picture to illustrate the efficacy of radiative return or direct scan search, in part because of the available $Y(5S)$ data at Belle. But again, the target range should be broader, as M_{Y_b} could be in 10900 – 11000 MeV range. Furthermore, model pictures such as 4-quark states should also be kept in mind, and Y_b mass above 11000 MeV is not impossible. But it would be difficult to persuade B factories to run at such energies.

The best case would be if Y_b is below the $Y(5S)$. Unless $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$ is much less than 6 eV (Eq. (1)), the state is likely to be discovered in a 30 fb^{-1} or so data run. Knowing the mass and width, one can then do the direct $e^+e^- \rightarrow \pi^+\pi^-Y$ scan and search for other channels such as K^+K^-Y or $\pi^0\pi^0Y$. If Y_b does not show up in radiative return on $Y(5S)$, besides the possibility of suppressed $\Gamma(Y_b \rightarrow ee)\mathcal{B}(Y_b \rightarrow \pi^+\pi^-Y)$, it is possible that Y_b is heavier. A quick scan with 30 pb^{-1} each for

the energies 10940, 10980, 11020, 11060, 11100 MeV could complement the existing scan around $Y(5S)$, i.e. 10825, 10845, 10865, 10885 and 10905 MeV, and could extend the discovery potential. There is a good chance that the Y_b could be discovered soon.

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