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Systematics of heavy quarkonia from Regge trajectories on (n, M^2) and (M^2, J) planes

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We show that heavy quarckonium states, similar to light mesons, form Regge trajectories in (n, M^2) and (M^2, J) planes and the slope of these trajectories is independent on the quantum numbers of the mesons. This fact can be useful for the prediction of the masses of heavy quarkonia and the determination of the quantum numbers of the newly discovered states.

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I. INTRODUCTION

It is well known that for a wide range of potentials the only singularities of the nonrelativistic scattering amplitude are the poles in the orbital momentum l plane (Regge poles). The position of these poles depends on the energy, i.e. $l = \alpha(s)$, where $\alpha(s)$ is the Regge trajectory that corresponds to physical particles and resonances.

The collection of the values of the masses of light mesons $q\bar{q}$ is described fairly well by the linear Regge trajectories both on (n, M^2) and (M^2, J) planes (here M is the mass of the meson and n and J are its radial quantum number and total spin, respectively). The linearity of the dependence

$$n = \frac{1}{\mu_{q\bar{q}}^2} M^2 + n_0, \qquad J = \frac{1}{\mu_{q\bar{q}}^2} M^2 + J_0 \tag{1}$$

is typical for the "string" nature of the mesons. In the Refs. [1,2] the attempts were made to describe the relativistic string with fermions at the ends. In the limit of massless quarks, the relativistic string model leads to linear Regge trajectories $J = \alpha' M^2 + \alpha(0)$. When the quark masses are taken into account, the trajectory deviates from linear for lower states. Another reason for such deviation is the screening of the quark-antiquark potential because of the additional $q\bar{q}$-pair production. This effect is important for higher states of the mesons.

The spectrum of light mesons was analyzed in Refs. [3–5] and the Regge trajectories for these states were presented. Below we will perform the same analysis for heavy quarkonium states using the whole experimental information available at the moment. In particular, we are going to check whether the spectrum of heavy quarkonia can be described by some simple relations [similar to Eq. (1)] and determine the parameters of these relations. In our analysis we will try to keep these parameters universal and assume that they do not depend on the quantum numbers of the meson. Such an analysis could be useful both for the comparison of the experimental data with potential model predictions and for the determination of the parameters of regge trajectories for heavy quarkonium.

II. CHARMONIUM

The most extensively studied charmonium mesons are vector states ($J^{PC} = 1^{--}$). For these states the value of the total spin of the quark-antiquark pair is $S = 1$, while for the orbital momentum of this pair the values $L = 0$ and $L = 2$ are allowed, so the mixing of the states with different values of L is possible. This mixing can change the decay modes of the charmonium mesons, but does not change dramatically the values of their masses. Actually, we expect that this mixing gives the relative error of order $\Lambda_{\mathrm{QCD}}^2/m_c^2 \ll 1$, so we will neglect it in our paper.

In Fig. 1(a) and 1(b) we show the experimental values of the masses of vector charmonia with $L = 0, 2$ (labeled by filled stars) and the predictions presented in Ref. [6] (labeled by filled triangles). Since there are three predictions for the mass of each meson in this paper, we use different connecting line styles to distinguish them. It is seen clearly that the experimental values are fairly well described by linear Regge trajectories (shown by the straight line), and the mass of the meson M and its radial quantum number n satisfy the relation

$$M_n^2 = \mu_{c\bar{c}}^2 n + M_0^2. \tag{2}$$

Similar to the light-meson case, we expect that the slope coefficient $\mu_{c\bar{c}}^2$ is the same both for $L = 0$ and $L = 2$, while the parameter M_0 depends on L. Fitting the experimental data, we obtain the value

$$\mu_{c\bar{c}}^2 = (3.2 \pm 0.2)\ \mathrm{GeV}^2. \tag{3}$$

When determining the above uncertainty, we have used the value $\Lambda_{\mathrm{QCD}}^2/m_c$ for the errors in charmonium masses instead of experimental values. This choice looks reasonable, since we have neglected the mixing of mesons with $L = 0$ and $L = 2$. From Figs. 1(a) and 1(b), one can see that only one set of predictions, presented in [6], is in agreement with the linear parameterization.

In Table I we show the values of this parameter, the experimental values of vector meson masses, and our

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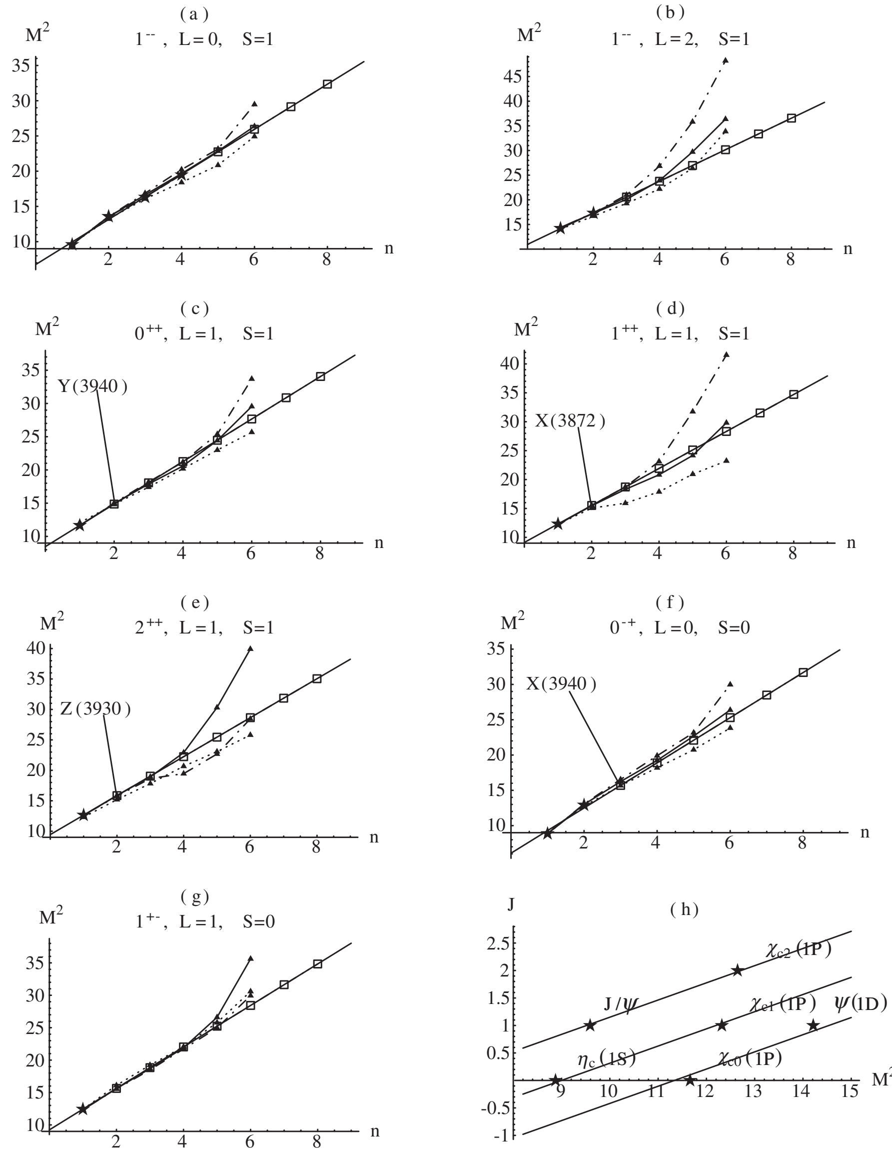

FIG. 1. Regge trajectories for charmonium mesons. Filled stars—experimental values; filled triangles—the results of Ref. [6]; open squares—our predictions.

predictions for the masses of excited mesons obtained with the help of Eq. (2).

The experimental information about the mesons with other values of L and/or S is not as good, since the observation of these states is more complicated. For each of the sets of the values of these quantum numbers, the mass of at least one meson is, however, known, so we can determine the parameter M_0 in the relation (2) and, using a known value of the slope coefficient μ^2, construct the corresponding Regge trajectory. In Table I we give the experimental quarkonia masses, the values of the parameter M_0, and our predictions for the masses of the excited states. The parameter $\mu^2_{c\bar{c}}$ was assumed to be universal for all charmonium trajectories. The reason for such an assumption is that one can observe a similar universality in a light-meson case. One of the aims of our paper was to check whether the slope coefficient is universal in the case of heavy quarkonia. The trajectories are shown in Figs. 1(c)–1(g).

In Fig. 1(h) we show the Regge trajectories on the (J, M^2) plane for the ground states of charmonia

$$J = \alpha(M^2) = \alpha' M^2 + \alpha(0). \tag{4}$$

Here we used the value of the slope coefficient from the proceeding analysis:

$$\alpha' = \frac{1}{\mu^2_{c\bar{c}}} = (0.31 \pm 0.02)\ \mathrm{GeV}^{-2}.$$

It should be noted that this coefficient is different from the one in the light-meson case. The interceptions $\alpha(0)$ for

TABLE I. The parameters of the Regge trajectories and masses of the mesons for the $(c\bar{c})$ sector. Our predictions for excited charmonium masses are shown in bold.

	n	Data	M_0, GeV	M_n, GeV		n	Data	M_0, GeV	M_n, GeV
	1	3.097		3.15 ± 0.07		1	3.51		3.51 ± 0.08
	2	3.68609		3.63 ± 0.09		2			$\mathbf{3.94 \pm 0.09}$
	3	4.04		4.04 ± 0.09		3			$\mathbf{4.33 \pm 0.1}$
1^{--}	4	4.415	2.6	4.42 ± 0.1	1^{++}	4		3.02	$\mathbf{4.68 \pm 0.1}$
$L = 0$	5			$\mathbf{4.77 \pm 0.1}$	$L = 1$	5			$\mathbf{5.01 \pm 0.1}$
$S = 1$	6			$\mathbf{5.09 \pm 0.1}$	$S = 1$	6			$\mathbf{5.32 \pm 0.1}$
	7			$\mathbf{5.4 \pm 0.1}$		7			$\mathbf{5.61 \pm 0.1}$
	8			$\mathbf{5.69 \pm 0.1}$		8			$\mathbf{5.89 \pm 0.1}$
	1	3.77		3.76 ± 0.09		1	3.556		3.56 ± 0.08
	2	4.16		4.17 ± 0.1		2			$\mathbf{3.98 \pm 0.09}$
	3			$\mathbf{4.53 \pm 0.1}$		3			$\mathbf{4.36 \pm 0.1}$
1^{--}	4		3.31	$\mathbf{4.87 \pm 0.1}$	2^{++}	4		3.07	$\mathbf{4.72 \pm 0.1}$
$L = 2$	5			$\mathbf{5.19 \pm 0.1}$	$L = 1$	5			$\mathbf{5.04 \pm 0.1}$
$S = 1$	6			$\mathbf{5.49 \pm 0.1}$	$S = 1$	6			$\mathbf{5.35 \pm 0.1}$
	7			$\mathbf{5.78 \pm 0.1}$		7			$\mathbf{5.64 \pm 0.1}$
	8			$\mathbf{6.05 \pm 0.1}$		8			$\mathbf{5.92 \pm 0.1}$
	1	3.415		3.42 ± 0.08		1	2.979		3.05 ± 0.07
	2			$\mathbf{3.86 \pm 0.09}$		2	3.594		3.53 ± 0.08
	3			$\mathbf{4.25 \pm 0.1}$		3			$\mathbf{3.96 \pm 0.09}$
0^{++}	4		2.91	$\mathbf{4.61 \pm 0.1}$	0^{-+}	4		2.47	$\mathbf{4.35 \pm 0.1}$
$L = 1$	5			$\mathbf{4.95 \pm 0.1}$	$L = 0$	5			$\mathbf{4.7 \pm 0.1}$
$S = 1$	6			$\mathbf{5.26 \pm 0.1}$	$S = 0$	6			$\mathbf{5.03 \pm 0.1}$
	7			$\mathbf{5.56 \pm 0.1}$		7			$\mathbf{5.34 \pm 0.1}$
	8			$\mathbf{5.84 \pm 0.1}$		8			$\mathbf{5.63 \pm 0.1}$

different trajectories were found to be

$$\underline{J/\psi(1S), \chi_{c2}(1P)}: \alpha(0) = -2,$$
$$\underline{\eta_c(1S), \chi_{c1}(1S), h_c(1P)}: \alpha(0) = -2.8,$$
$$\underline{\chi_{c0}(1P), \psi(1D)}: \alpha(0) = -3.5.$$

These values are almost in the range $|\alpha_c(0)| \geq 3 \div 4$ that was obtained in Ref. [7] with the help of QCD sum rules. The intercept of the parent trajectory was also presented in [8], where $\alpha(0)$ was expressed through the value of the ψ-meson at the origin. In that work the value $\alpha_c(0) = -3.5 \pm 0.6$ was found and it agrees well with the results of the present paper.

It is clear that the positions of the physical states are described by the linear Regge trajectories with the universal slope with pretty good accuracy. It should be mentioned that for the leading Regge trajectory, the exchange degeneration holds, i.e. J/ψ and χ_{c2} mesons lie on one trajectory. This fact is not the result of the parameter fit but an automatic consequence of the correct description of the charmonia spectrum in an (n, M^2) plane. It should be mentioned that the exchange degeneracy was used earlier for the determination of the intercept $\alpha(0)$; this is important for the determination of the c-quark wave function in J/ψ, η_c, and D mesons [9–12].

III. NEW STATES

Recently some new particles were observed. The production and decay channels of these particles indicate that they are excited charmonium states, so it would be interesting to consider the question of the disposition of these particles on the Regge trajectories presented in the previous section.

A. *X*(3872)

The first of such particles was the $X(3872)$ meson. The properties of this state were widely discussed in the literature [13–16].

$X(3872)$ was observed by the Belle [17], CDFII [18], D0 [19] and *BABAR* [20] collaborations in the decay

$$B^{\pm} \to K^{\pm} X(3872) \to K^{\pm} \pi^{+} \pi^{-} J/\psi,$$

and the world average of its mass is

$$M_X = 3871.9 \pm 0.5 \text{ MeV}.$$

The further experimental study showed that besides the $X \to \pi^+\pi^- J/\psi$ decay, the decays $X(3872) \to \gamma J/\psi$ [21], $X(3872) \to \pi^+\pi^-\pi^0 J/\psi$ [21], and $X(3872) \to D^0 \bar{D}^0 \pi^0$ [15] also exist and the branching fractions of these channels are linked by the relations

$$\begin{aligned} \mathrm{Br}(B^{\pm} \to K^{\pm}X)\mathrm{Br}(X \to \gamma\psi) &= (1.8 \pm 0.6 \pm 0.1) \times 10^{-6}, \\ \mathrm{Br}(B^{\pm} \to K^{\pm}X)\mathrm{Br}(X \to \pi^{+}\pi^{-}\psi) &= (1.3 \pm 0.3) \times 10^{-5}, \\ \mathrm{Br}(B^{\pm} \to K^{\pm}X)\mathrm{Br}(X \to D^{0}\bar{D}^{0}\pi^{0}) &= (2.2 \pm 0.7 \pm 0.4) \times 10^{-4}, \\ \frac{\mathrm{Br}(X \to \pi^{+}\pi^{-}\pi^{0}\psi)}{\mathrm{Br}(X \to \pi^{+}\pi^{-}\psi)} &= 1.0 \pm 0.4 \pm 0.3. \end{aligned} \tag{5}$$

For the total width of this state only the upper boundary is known: $\Gamma_X < 2.3$ MeV. If we assume that there are no other significant decay modes, then we can determine the branching fractions of the reactions listed above and set the upper bounds on their widths:

$$\begin{aligned} \underline{X \to \gamma\psi}&:\mathrm{Br} \approx (7.4 \pm 0.4) \times 10^{-3}, & \Gamma &< (17 \pm 9)\ \mathrm{keV}, \\ \underline{X \to \pi^{+}\pi^{-}\psi}&:\mathrm{Br} \approx (5.3 \pm 0.8)\%, & \Gamma &< (0.12 \pm 0.02)\ \mathrm{MeV}, \\ \underline{X \to \pi^{+}\pi^{-}\pi^{0}\psi}&:\mathrm{Br} \approx (5.3 \pm 4.6)\%, & \Gamma &< (0.12 \pm 0.11)\ \mathrm{MeV}, \\ \underline{X \to D^{0}\bar{D}^{0}\pi^{0}}&:\mathrm{Br} \approx (90 \pm 2.5)\%, & \Gamma &< (2.1 \pm 0.6)\ \mathrm{MeV}. \end{aligned} \tag{6}$$

Since the decay $X(3872) \to \gamma\psi$ is allowed, the charge parity of $X(3872)$ should be positive. The angular distribution in the $X(3872) \to \pi^{+}\pi^{-}\psi$ channel rules out the possibility of the scalar meson [22].

Among the particles listed in the Table I $\chi_{c0}(2P)$, $\chi_{c1}(2P)$, and $\eta_c(3S)$ mesons have masses that are closest to the mass of $X(3872)$. Since the case of the negative charge conjugation parity is forbidden by $X(3872) \to \gamma\psi$ decay and the case of the scalar meson contradicts the angular distributions, the only variant left is $X(3872) = \chi_{c1}(2P)$.

There are, however, some arguments against this assignment. First of all, the upper bound for the width of the radiative decay $\Gamma(X(3872) \to \gamma\psi) < 17$ keV is less than theoretical predictions (for example, in [6] one can find the values $\Gamma[\chi_{c1}(2P) \to \gamma\psi] = 30 \div 60$ keV). A second argument is that the decay $X(3872) \to \rho\psi \to \pi^{+}\pi^{-}\psi$ implies that $X(3872)$ is an isovector, so it cannot be a charmonium. In Ref. [14] an alternative model is considered. According to this work $X(3872)$ is a loosely bound $D^0\bar{D}^{0*}$ molecule (deuson). The mass of $X(3872)$ is surprisingly close to the $D^0\bar{D}^{0*}$ threshold

$$M_{D^0} + M_{D^{0*}} = 3871.2 \pm 0.6\ \mathrm{MeV},$$

and in the case of zero orbital momentum of the mesons in this molecule its quantum numbers should be equal to $J^{PC} = 1^{++}$. This assumption explains well both the isospin violation and the smallness of the radiative decay width.

It should be mentioned that such an explanation has some serious drawbacks. First, the $X(3872)$ mass is above the $D^0\bar{D}^{0*}$ threshold. Second, the production probability of such a molecule should be smaller than the experimental value

$$\mathrm{Br}\,(B^{+} \to K^{+}X) = (2.5 \pm 1.0) \times 10^{-4}.$$

This value was obtained from the experimental results (5) and (6) and has the same order of magnitude as the production probabilities of other charmonium states in similar decays [15]. The smallness of the production rate is caused by the fact that the size of the loosely bound molecule should be lager than that of the $(c\bar{c})$ state, and the value of its wave function at the origin should therefore be smaller [23]. This problem could be avoided if we assume that $X(3872)$ is a mixture of the charmonium and the $D\bar{D}^*$ molecule [in the weak decay of a B meson, a $(c\bar{c})$ component of this mixture is produced, and the isospin violation in the decay $X(3872) \to \rho\psi$ is explained by the presence of the DD^* component], but the other partner of $X(3872)$ was never observed.

We think that the other explanation looks more plausible. In Refs. [13,24] it was shown that closeness of $D\bar{D}^*$ thresholds (the mass of X is a little bit larger than the $D^0\bar{D}^{*0}$ threshold and a little bit smaller than the D^+D^{*-} one) increases the probability of the mixing of $\chi_{c1}(2P)$ with other charmonium states. Such a mixing leads to the decrease of the radiative decay width and can be a reason for the isospin violation.

B. $Z(3930)$, $X(3940)$, $Y(3940)$

The state $Z(3930)$ was observed by the Belle Collaboration in the reaction $\gamma\gamma \to D\bar{D}$ [25] and its mass equals

$$M_{Z(3930)} = 3931 \pm 4 \pm 2\ \mathrm{MeV}.$$

The best candidates with the required charge parity from the particles listed in Table I are $\chi_{c1}(2P)$, $\eta_c(3S)$, and $\chi_{c2}(2P)$ mesons. The production and decay channels rule out the first two variants, so we assign the quantum numbers $J^{PC} = 2^{++}$ to $Z(3900)$.

The $X(3940)$ particle was observed by the Belle Collaboration recoiling against ψ in the e^+e^- collision [26]. The mass of this particle is

$$M_{X(3940)} = 3943 \pm 6 \pm 6\ \mathrm{MeV},$$

and the dominant decay channel is $X(3940) \to D^*\bar{D}$. The best variants for this state are $\chi_{c1}(2P)$, $h_c(2P)$, and $\eta_c(3S)$

mesons. Production and decay channels exclude the first two variants, so we choose the last assignment for $X(3940)$. It is interesting to mention that our prediction for the $\eta_c(3S)$ mass is only 19 MeV above the mass of $X(3940)$, while the predictions of other models are 4040–4060, i.e. approximately 100 MeV too high [14].

Belle also has observed one more particle in the region of 3940 MeV [27]—$Y(3940)$. This meson was observed in the decay

$$B \to KY(3940) \to K\omega\psi,$$

and its mass is equal to

$$M_{Y(3940)} = 3943 \pm 11 \pm 13 \text{ MeV}.$$

The decays $Y(3940) \to D^{(*)}\bar{D}$ have not been seen, so it is possible that $X(3940)$ and $Y(3940)$ are distinct states. Because of the existence of $Y(3940) \to \omega\psi$, the charge parity of $Y(3940)$ should be positive. The best variants for this particle from Table I are $\chi_{cJ}(2P)$ mesons. Since the assignments for $\chi_{c1,2}(2P)$ are already chosen, we set $Y(3940) = \chi_{c0}(2P)$.

IV. BOTTOMONIUM

In the previous sections we have shown that the charmonium states are described fairly well by the linear Regge trajectories on (n, M^2) and (M^2, J) planes with the slope coefficient

$$\mu^2_{c\bar{c}} = 3.2 \text{ GeV}^2, \qquad \alpha' = \frac{1}{\mu^2_{c\bar{c}}}.$$

The similar analysis for bottomonium states [i.e. the $(b\bar{b})$ mesons] shows that in this case the linear Regge trajectories describe the mass spectrum unsatisfactorily. From Fig. 2(a), where all known vector bottomonia with the value of the orbital momentum $L = 0$ are shown, one can see that the slope of the Regge trajectory is not constant, but decreases with the increase of the radial quantum number n, so the linear Regge trajectory contradicts the experimental data. This fact should not be a big surprise. The linear Regge trajectories are typical for the string model of the quark-antiquark interaction under the assumption that the mass of the quarks tends to zero. Such an assumption is justified well for light mesons and agrees well with the experimental results. In the previous sections it was shown that the Regge trajectories of charmonium states are also linear, although the slope of these trajectories is different from that in the light-meson sector. In the case of $(b\bar{b})$ mesons, however, we see the significant deviations form the linearity. The possible reason of such deviation is that the massless quark approximations fails in this case. In the series of works (for example [1,2,28,29]), the results obtained in the framework of the string model with massive quarks are presented and these results are in qualitative agreement with the experimental picture.

The same result can be obtained in the framework of the potential models. Let us consider the widely used Cornell

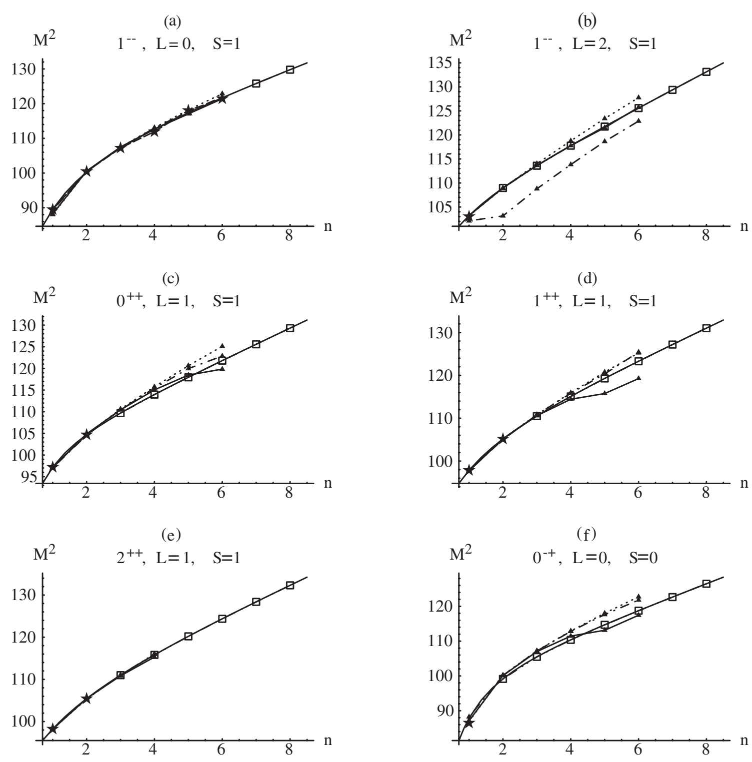

FIG. 2. Regge trajectories for bottomonium states. Filled stars—experimental values; filled triangles—prediction presented in Ref. [6]; open squares—our predictions.

potential of the quark-antiquark interaction [30]:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r + c. \tag{7}$$

This potential combines the main characteristics of the quark interaction known from QCD. At small values of the distance between quark and antiquark $r \ll r_0 \sim \sqrt{\alpha_s/\kappa} \sim 0.4$ fm, the leading term of this expression is the first one and we observe the asymptotical freedom. For large distances, on the contrary, the second term gives the main contribution and we observe the confinement. For the light mesons and charmonia the second case holds, so we can neglect the Coulombic term in expression (9) and obtain the linear Regge trajectories on (n, M^2) and (J, M^2) planes

$$M^2 = 8\kappa\left(2n + J + \frac{3}{2}\right) + \text{const} \tag{8}$$

that are in good agreement with the experimental data. For the lower bottomonium states, on the contrary, the first case takes place. After neglecting the second term in expression (7), we get

$$M^2 = -\frac{64\alpha_s^2 m^2}{9}\frac{1}{(n + J + 1)^2} + \text{const.} \tag{9}$$

Linking expressions (8) and (9), one can obtain the interpolation formula [31]

$$M^2 = 8\kappa\left(2n + J + \frac{3}{2}\right) - \frac{b^2}{(n + J + 1)^2} + M_0^2, \tag{10}$$

that joins both two limits. For large values of the quantum numbers the trajectory turns into linear with the slope

$$\mu_{b\bar{b}}^2 = 16\kappa,$$

and for small values of the quantum numbers the spectrum of the nonrelativistic quarkonium is restored.

In Fig. 2 we show the Regge trajectories obtained with the help of the formula (10) for all bottomonium states. It should be mentioned that the slope coefficient

$$\mu_{b\bar{b}}^2 = (4.1 \pm 0.3)\ \text{GeV}^2$$

is universal for all of these trajectories (though different from $\mu_{c\bar{c}}^2$ and $\mu_{q\bar{q}}^2$), while the parameters b and M_0 depend on the spin and parities of the particle. It is clear that the experimental states lie well on these trajectories and deviate from linear ones for small values of the radial quantum number n. The values of these parameters, as well as known experimental masses, and the predictions obtained with the help of Eq. (10) are presented in Table II.

TABLE II. Regge trajectories parameters and masses for $(b\bar{b})$ mesons. Our predictions for excited bottomonium masses are shown in bold.

	n	Data	M_0, GeV	M_n, GeV		n	Data	M_0, GeV	M_n, GeV
	1	9.46		9.46 ± 0.1		1	9.892		9.89 ± 0.1
	2	10.023		10. ± 0.1		2	10.255		10.3 ± 0.1
	3	10.355		10.4 ± 0.1		3			**10.5 ± 0.1**
1^{--}	4	10.58	$b = 12.1$	10.6 ± 0.1	1^{++}	4		$b = 8.68$	**10.7 ± 0.1**
$L = 0$	5	10.865	$M_0 = 9.87$	10.8 ± 0.1	$L = 1$	5		$M_0 = 9.9$	**10.9 ± 0.1**
$S = 1$	6	11.02		11. ± 0.1	$S = 1$	6			**11.1 ± 0.1**
	7			**11.2 ± 0.1**		7			**11.3 ± 0.1**
	8			**11.4 ± 0.2**		8			**11.4 ± 0.2**
	1	10.15		10.2 ± 0.1		1	9.912		9.91 ± 0.1
	2			**10.4 ± 0.1**		2	10.268		10.3 ± 0.1
	3			**10.7 ± 0.1**		3			**10.5 ± 0.1**
1^{--}	4		$b = 6.58$	**10.9 ± 0.1**	2^{++}	4		$b = 12.5$	**10.8 ± 0.1**
$L = 2$	5		$M_0 = 9.99$	**11. ± 0.1**	$L = 1$	5		$M_0 = 9.9$	**11. ± 0.1**
$S = 1$	6			**11.2 ± 0.1**	$S = 1$	6			**11.2 ± 0.1**
	7			**11.4 ± 0.2**		7			**11.3 ± 0.1**
	8			**11.5 ± 0.2**		8			**11.5 ± 0.2**
	1	9.859		9.86 ± 0.1		1	9.3		9.37 ± 0.1
	2	10.232		10.2 ± 0.1		2			**9.96 ± 0.1**
	3			**10.5 ± 0.1**		3			**10.3 ± 0.1**
0^{++}	4		$b = 5.26$	**10.7 ± 0.1**	0^{-+}	4		$b = 7.43$	**10.5 ± 0.1**
$L = 1$	5		$M_0 = 9.89$	**10.9 ± 0.1**	$L = 0$	5		$M_0 = 9.76$	**10.7 ± 0.1**
$S = 1$	6			**11. ± 0.1**	$S = 0$	6			**10.9 ± 0.1**
	7			**11.2 ± 0.1**		7			**11.1 ± 0.1**
	8			**11.4 ± 0.2**		8			**11.2 ± 0.1**

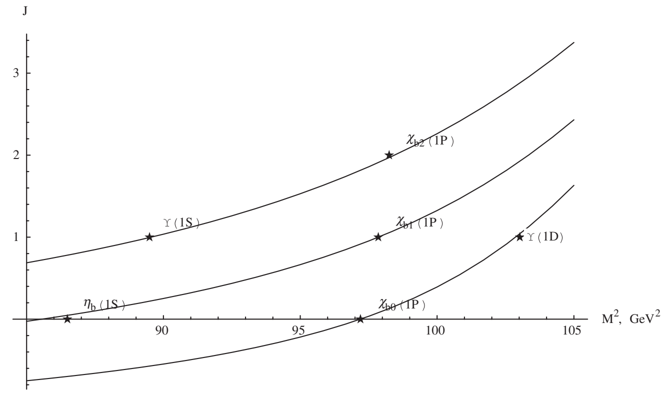

FIG. 3. Regge trajectories for bottomonium states.

Up to now we have used Eq. (10) for the construction of the Regge trajectories on an (n, M^2) plane and the spin of the particle for each trajectory was fixed. This formula can be used also for the construction of the trajectories on an (M^2, J) plane (Chew-Frautchi plot). To do this one needs to fix the radial quantum number n and solve Eq. (10) for the spin of the particle. There are three solutions to this equation, but only one of them is physically sensible (the others give negative values for the real part of J and we will not consider them here). As it was shown earlier, in the case of charmonium states the parameters of the Regge trajectories on (n, M^2) and (M^2, J) planes coincide, so we have checked this property for bottomonia. The trajectories for lightest bottomonium mesons (i.e. $n = 1$) are shown in Fig. 3 and one can see that they agree well with the experimental data. One can observe that the exchange degenerations holds for the 1^{--} and 2^{++} states, as it was in the $(c\bar{c})$ case. We did not perform any fits to obtain the values for the parameters of these trajectories. Instead of this we used the parameters for an S wave vector bottomonia [Fig. 2(a)] for the upper curve, and 1^{++} and 0^{++} bottomonia [Figs. 2(c) and 2(d)] for the middle and lower curves, respectively. The interceptions of these trajectories are equal to

$$\underline{\Upsilon(1S), \chi_{b2}(1P)}: \alpha(0) = -0.88,$$

$$\underline{\eta_b(1S), \chi_{b1}(1P)}: \alpha(0) = -1.2,$$

$$\underline{\chi_{b0}(1P), \Upsilon(1D)}: \alpha(0) = -1.5.$$

These estimates, unfortunately, are not reliable. According to the QCD sum rules restrictions [7] for the parent Regge trajectory we have

$$\alpha_b(0) < -7 \div -8.$$

The value of the same order of magnitude was presented in Ref. [8], where the intercept $\alpha_b(0)$ was expressed though the leptonic width of the $\Upsilon(1S)$ meson:

$$\alpha_b(0) = -11.5 \pm 0.5.$$

There is also another argument against our values of the interceptions. It seems logical that the absolute value of the intercept of the corresponding Regge trajectory should be larger for larger quark masses.

V. CONCLUSION

The new states discovered recently in the $(c\bar{c})$ sector open the question of their classification and the reliable prediction of heavy quarkonia masses. Such predictions were obtained in the framework of different potential or lattice models (see for example [6,32–36]), but the results of these calculations depend strongly on the choice of the model parameters. For the mass of the $\chi_{c0}(2P)$ meson, for example, one can find in the literature the values from 3.822 GeV [6] to 4.080 GeV [33]. Since there is no reason to prefer some prediction, it seems important to have some independent criterion that can help to choose the right value.

We think that this criterion could be the positioning of the mesons to the respective Regge trajectories on (n, M^2) and (J, M^2) planes (here n is the radial quantum number of the meson and J and M are its spin and mass). It is well known that the masses of the light mesons can be described with pretty good accuracy by the linear trajectories. In this paper we show that this is, with minor changes, valid also for heavy quarkonia. Namely, the charmonia Regge trajectories are the straight line with the slope as common for all charmonium mesons. In this paper we have used these trajectories to position to them recently discovered particles $X(3872)$, $Z(3930)$, $X(3940)$, and $Y(3940)$.

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