Decay constants of *P*-wave heavy-light mesons from unquenched lattice QCD

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We review some decays that require knowledge of the decay constants of 0^+ heavy-light mesons. We compute the decay constants of *P*-wave heavy-light mesons from unquenched lattice QCD, with two degenerate flavours of sea quarks, at a single lattice spacing. The lightest sea quark mass used in the calculation is a third of the strange quark mass. For the charm-strange meson we obtain the decay constant: $f_{D_{s0^+}} = 340(110)$ MeV using our normalization conventions. We obtain the $f_{P_s}^{\mathrm{static}}$ (static-strange *P*-wave) decay constant as 302(39) MeV.

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I. INTRODUCTION

The decay constants of heavy-light *P*-wave mesons have a number of important uses in phenomenology [1,2]. In this paper we use unquenched lattice QCD to compute the decay constants of 0^+ heavy-light mesons, using heavy quarks with masses around that of the charm quark, and with heavy quarks in the static limit.

Because of heavy-quark symmetry, there are four *P*-wave ($L = 1$) heavy-light excited mesons, noted for short D^{**}, organised in two doublets, one doublet carrying a total angular momentum $j = 1/2$ and the other $j = 3/2$ for the light quarks. In the charm sector, we use the notation that the scalar $J^P = 0^+(j = 1/2)$ meson is D_{0^+} when the light quark is an up or a down quark and D_{s0^+} when it is a strange quark. In the static limit of HQET, the members of each of the two doublets are degenerate in mass.

The low mass of the $D_s(2317)$ meson, recently discovered by *BABAR* [3] and confirmed by other experiments, relative to the quark model predictions of Godfrey and Isgur [4,5] was originally a puzzle. The quantum numbers of $D_s(2317)$ are consistent with $J^P = 0^+$, but this needs confirmation [6]. There are some speculations that this state may be a molecule [7]. The theoretical studies to understand the $D_s(2317)$ have recently been reviewed by Colangelo *et al.* [8] and by Swanson [9]. The lattice QCD results for the mass of the D_{s0^+} state were tantalising close (within large errors) to the experimental mass of the $D_s(2317)$ state [10,11]. However, the lattice calculation of the D_{s0^+} meson in [12] did not agree with the mass of the $D_s(2317)$. As discussed by UKQCD [13] a hadronic state will couple to many different interpolating operators made out of quarks, antiquarks and glue, with the same quantum numbers as the hadron. In order to say whether a hadronic state is more like a molecule or $\bar{q}q$ state, the lattice calculation needs to also determine the amplitude for a hadronic state to be in a specific configuration of quarks and antiquarks. One possible way to determine the quark distribution of a hadronic state is to look at observables like form factors, the most basic being the decay constant. For example, in a simple picture of a molecular state, the decay constant should be suppressed relative to that of a bound state of quark and antiquark. An analysis of nonleptonic decays of the *B* meson using the factorisation hypothesis suggested that the decay constant of the $D_s(2317)$ could be significantly smaller than that of the pseudoscalar heavy-light meson [14,15]. This motivates a lattice calculation of the decay constant of the 0^+ heavy-light meson.

The $B \to D^{**}\pi$ decays are also relevant to clarify the so called '1/2 vs 3/2 puzzle' [16–18] Indeed, the heavy to heavy $B \to D_j^{**}$ transitions, with $j = 1/2$, $3/2$, are parametrized by the generalized Isgur-Wise functions $\tau_j(w)$, where $w = v_B \cdot v_{D_j^{**}}$. Several theoretical considerations using independent methods (sum rules derived from QCD [19], covariant quark model [20], lattice QCD [21,22], experimental data combined with naive factorization [17]), suggest that the $j = 3/2$ states dominate, with respect to the $j = 1/2$ states in $B \to D^{**}$ transitions, i.e. $\tau_{3/2}(1) > \tau_{1/2}(1)$. Nevertheless this prediction is in contradiction with some of the currently available experimental data. The solution for this puzzle still requires new inputs from both theory and experiment. From the theoretical side, the determination of the decay constant, $f_{D_{0^+}}$ of the D_{0^+} meson is needed to evaluate the decay width of two (out of the three) classes of $B \to D^{**}\pi$ decays, when naive factorization is assumed [17].

A recent summary [17] of results for the decay constant, showed that estimates for $f_{D_{0^+}}$ lay between 122 and 417 MeV. This summary also reported that the decay constant of 0^+ heavy-light mesons in the static limit was higher than at the charm mass. This required confirmation, because the heavy-quark mass dependence was obscured by model dependence. Some factorisation schemes [23] for

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nonleptonic decays of B mesons use the static limit as the leading term, hence computing the decay constant of the P-wave static-light meson is also important for this reason. The evaluation of the $1/m_Q$ corrections to the heavy-quark limit of the various quantities describing $B \rightarrow D_j^{**}$ transitions is crucial since simple estimates tend to suggest the presence of large corrections [17].

There is a huge effort in lattice gauge theory that aims to compute the decay constant of the heavy-light pseudoscalar mesons, because they are crucial for determining CKM matrix elements [24,25]. The decay constant of the heavy-light vector meson has been computed in [26] in order to test the HQET scaling relations.

There has been no published work (apart from a private communication reported in [17]) on using lattice QCD to compute the decay constants of P-wave heavy-light mesons. The study of the wave-function of P-wave static-light mesons [27], using quenched lattice QCD, did not extract a decay constant. UKQCD has studied the charge and matter radial distributions of P-wave static-light mesons [28,29].

In our calculations, we used the nonperturbatively improved clover action with clover coefficient of 2.0171 for the light quarks. The Wilson gauge action was used. The lattice volume was 16^3 32 and $\beta = 5.2$. There were two flavours of sea quarks. The κ values are tabulated with the results. The sea quark masses span from the strange quark to one third of the strange quark mass. The details of the light hadron spectroscopy can be found in [30,31].

The plan of the paper is that in Sec. II, we first discuss our results using clover fermions with quark masses around the charm value. Next in Sec. III, we present data using static heavy quarks. In the final Sec. IV we compare our results to other determinations and make some remarks about the heavy-quark mass dependence of the decay constants.

II. DECAY CONSTANTS AROUND THE CHARM MASS

We have used the unquenched data from [10,32] to study the decay constant of the 0^+ meson around the charm mass with clover quarks.

The data set used the sea $\kappa = 0.1350$ and three valence light quark masses using κ-values of 0.1340, 0.1345 and 0.1350. The κ values for the heavy quarks were: 0.113, 0.119 and 0.125. Using $r_0 = 0.5$ fm, the inverse lattice spacing is 1.88 GeV [30]. The sea quark mass was kept fixed at the mass of the strange quark, hence this is a partially quenched calculation. The data sample included 394 gauge configurations separated by 20 trajectories. Adjacent data were binned together. A smearing matrix of order two, using local and fuzzed basis states, was fitted to a factorizing fit model [30].

The decay constant of the D_{0^+} meson can be defined by equation,

$$\langle 0 \mid V_\mu^{cq} | D_{0^+} \rangle = i p_\mu g_{0+} \tag{1}$$

where V_μ^{cq} is the vector current with two nondegenerate quark flavours. This uses the $V - A$ structure of the charged weak quark current and parity conservation. We used a unit gamma matrix between the heavy and light quark fields as the interpolating operator for the D_{0^+} state. Unfortunately, we did not measure the lattice correlators with γ_0 at the sink and 1 at the source that would be required to use Eq. (1) directly. In principle due to the number of gamma matrices, there are 256 different local meson correlators. Many lattice QCD codes do not compute every possible correlator, because some correlators are zero by symmetries. The correlator with a γ_0 at the sink and a unit matrix at the source is zero for degenerate quarks because of charge conjugation.

Colangelo *et al.* [1] define the decay constant (g_{0+}) using Eq. (2).

$$\langle 0 \mid \bar{c} q | D_{0^+} \rangle = \frac{M_{D_{0^+}}^2}{m_c} g_{0+} \tag{2}$$

where m_c is the mass of the charm quark. Equation (2) can be related to Eq. (1) since the divergence of the vector current for nondegenerate quarks is proportional to the scalar density. The derivation of Eq. (2) from Eq. (1) assumes that the mass of the light quark can be neglected relative to that of the charm quark.

From a lattice QCD perspective, it is more natural to define the decay constant using Eq. (3).

$$\langle 0 \mid \bar{c} q | D_{0^+} \rangle = M_{D_{0^+}} f_{0+} \tag{3}$$

The determination of the mass of the charm quark can have quite large systematic errors at a fixed lattice spacing [32], so we prefer the definition of the decay constant with no explicit factor of the charm mass. To convert from our normalization of the decay constant f_{0+} to that of g_{0+}, we note that for this data set UKQCD obtained $m_c(m_c)^{\overline{MS}} = 1.247(3)^{+20}_{-4}$ GeV [32], using a quark mass defined using the Fermilab heavy-quark formalism [33].

The matrix element is related to the coupling amplitude Z_i in the fit to the 0^+ to 0^+ meson correlator.

$$Z_i = \frac{\langle 0 \mid \bar{c} q | D_{0^+} \rangle}{\sqrt{2 M_{D_{0^+}}}} \tag{4}$$

A simple linear fit model was used to extrapolate and interpolate in the quark masses. We first interpolated to the strange quark mass [34], or extrapolated to the light quark mass (at κ_{crit}). The decay constants were then interpolated to the value of the mass of the charm quark [32].

In Fig. 1 we report an effective mass plot for the local-local and fuzzed-local correlators used in the analysis. In Fig. 2 we plot the bare decay constant as a function of the bare light quark mass in physical units with mass of the heavy-quark fixed. Similarly, Fig. 3 shows the bare lattice decay constant as a function of the heavy-quark mass, with

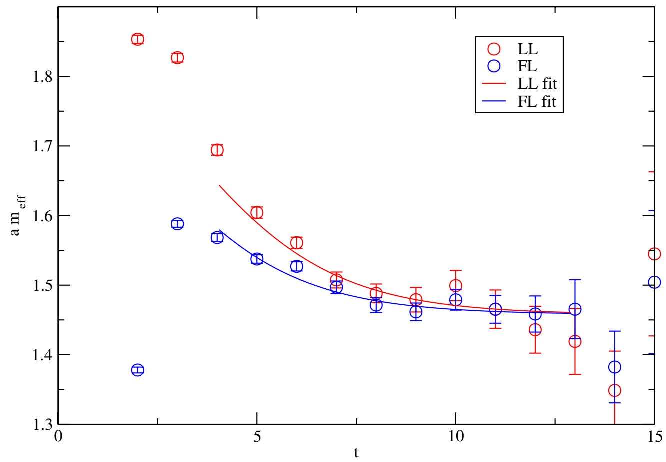

FIG. 1 (color online). Effective mass plot and fitted model for the correlators with heavy $\kappa = 0.113$ and light $\kappa = 0.1350$.

the light quark mass fixed at the mass of the strange quark. The dependence of the decay constant on the mass of the quarks is very mild. We discuss some of the issues in the chiral extrapolations in Sec. III.

To convert the lattice number into the $\overline{MS}$ scheme we use tadpole improved perturbation theory to one loop order (the required expressions are listed in [35,36]).

$$Z_S = u_0\Big(1 + \alpha_s\Big(\frac{1}{\pi}\log(\mu a)^2 - 1.002\Big)\Big) \qquad (5)$$

where u_0 is the fourth root of the plaquette. The mass dependent improvement factor $(1 + b_S \frac{m_i + m_j}{2})$ was multiplied into the scalar current. We used the one loop expression for b_S [37],

$$b_S = (1 + \alpha_s 1.3722) \qquad (6)$$

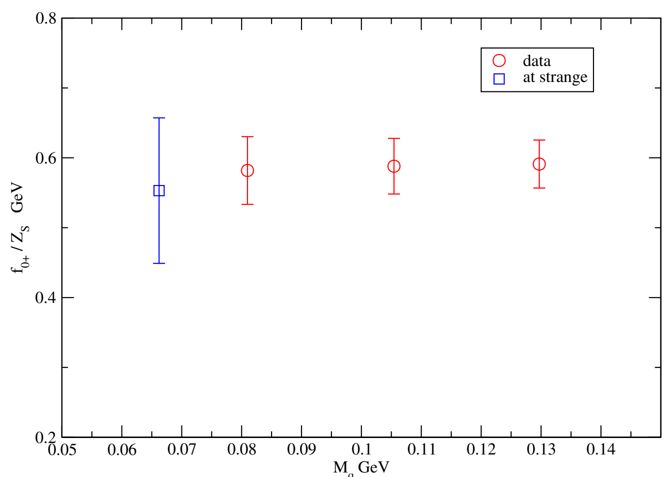

FIG. 2 (color online). Mass dependence of the *P*-wave decay constant defined in Eq. (3) at the fixed heavy-quark mass with $\kappa = 0.113$, as a function of the bare light quark mass in physical units. The estimate of the decay constant at the mass of the strange quark is also shown.

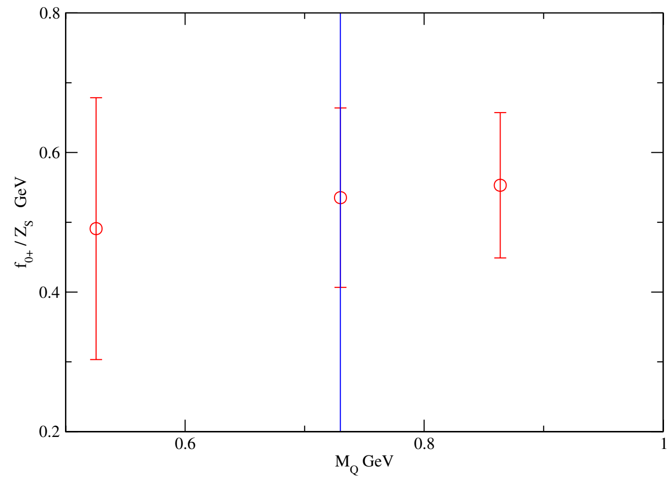

FIG. 3 (color online). Mass dependence of the *P*-wave decay constant defined in Eq. (3) at the strange quark mass, as a function of the bare heavy-quark mass in physical units. The vertical line is the estimate for the charm quark mass.

and the same prescription for the coupling as used in UKQCD's estimate of the mass of the charm quark [32]. We use $\mu = a^{-1}$ to determine Z_S.

In lattice units we obtain $f_{D_{0^+}}/Z_S = 0.280(70)$ and $f_{D_{s0^+}}/Z_S = 0.270(90)$. Using $r_0 = 0.5$ fm, and Z_S from Eq. (5), we obtain $f_{D_{0^+}} = 360(90)$ MeV and $f_{D_{s0^+}} = 340(110)$ MeV. In principle, we should estimate the effect of the lattice spacing dependence and higher order terms in the perturbative expansion, as we did when we estimated the mass of the bottom quark [38]. However, because the statistical errors are of the order of 30%, the statistical errors will clearly dominate. For example, a typical estimate of the systematic error due to the different ways of determining the lattice spacing in a lattice calculation with these parameters would be to vary the value of r_0 between 0.5 fm and 0.55 fm. This systematic error is a 10% effect.

We now discuss the central value of r_0 used. When r_0, a number derived from the heavy-quark potential, was introduced by Sommer [39], he quoted a value of r_0 about 0.49 fm. The value of r_0 can also be determined from lattice calculations. As reviewed in [10], unquenched calculations with similar parameters to those used here compute a value of r_0 between 0.5 and 0.55 fm. The issue is not so clear because the HPQCD collaboration, using gauge configurations from the MILC collaboration, obtain a value of $r_0 = 0.469(7)$ fm. As UKQCD has argued [38], for calculations with these parameters it better to use a value of r_0 closer to 0.5 fm because this is the value determined from similar calculations. The MILC/HPQCD value for r_0 is with in 10% of 0.5 fm.

III. DECAY CONSTANTS IN THE STATIC LIMIT

This part of the work is a continuation of UKQCD's study of static-light mesons. The static-light meson spectrum was reported in [13], and then used to extract the mass of the bottom quark [38]. Here we use the extended data set that was used to look for chiral logs in the f_B decay constant [40].

In addition to using the standard static action of Eichten and Hill, we also report data using one of the static actions developed by the ALPHA collaboration [41] with improved signal to noise ratio. The exact static actions are described in [40].

Here we focus on the meson doublet with total light quark momentum of $j = 1/2$, we call this the P_- static-light meson [13]. As corrections to the static limit are included the P_- state will split into a 0^+ state and a 1^+ state. In this paper, the decay constant of the P_- static-light meson is called $f_P^{\rm static}$. The $f_P^{\rm static}$ decay constant is defined by the matrix element in equation

$$\langle 0 \mid V_\mu \mid P_-(p)\rangle = i p_\mu f_P^{\rm static} \tag{7}$$

where V_μ is the vector current [1,2]. The simple spin structure of the static quark means the same decay constant is obtained if in Eq. (7) the vector current (with $\mu = 0$) is replaced with a scalar current. The $f_P^{\rm static}$ matrix element is extracted from the amplitudes in the two point correlator

$$C(t) = \sum_x \langle 0 \mid V_0(x,t)\Psi_B^\dagger(x,0) \mid 0\rangle \tag{8}$$

$$\to Z_L^{\rm static} Z_{\Psi_B} \exp(-a\mathcal{E}t), \tag{9}$$

where Ψ_B is the interpolating operator for the P_- static-light meson and ground state dominance is shown in Eq. (9). In practise we use a basis of smearing functions to do variational smearing [13]. The $Z_L^{\rm static}$ amplitude is related to the $f_P^{\rm static}$ decay constant

$$f_P^{\rm static} = Z_L^{\rm static}\sqrt{\frac{2}{M_{B_{0^+}}}} Z_S^{\rm static} \tag{10}$$

where $Z_S^{\rm static}$ is a perturbative matching factor that we discuss below. This is an equivalent definition to the one used for the pseudoscalar heavy-light decay constant ($f_B^{\rm static}$).

It is traditional to match the results of a lattice static-light calculation to continuum QCD via two steps [42]. QCD is matched to the continuum static theory. The lattice static theory is then matched to the static continuum theory.

The matching of the static continuum theory to the static-lattice theory has been done by Eichten and Hill [43] and by Borrelli and Pittori [44] for the clover action. This matching was done for a heavy-light current with arbitrary gamma matrix (Γ). For the definition of the decay constant in Eq. (7), we do the perturbative matching assuming $\Gamma = \gamma_0$. The matching between the continuum static theory and the static-lattice theory is via $Z(\Gamma)$.

$$Z(\Gamma) = 1 + \frac{g^2}{12\pi^2}\left(\frac{3}{2}\log(\mu^2 a^2) + 5/4 - A_\Gamma\right) \tag{11}$$

where

$$A_\Gamma = d_1 + (d_2 - d^1)G + \frac{(e + f + f^1)}{2} \tag{12}$$

where the values of the constants are $d_1 = 5.46$, $d_2 = -7.22$, $f = 13.35$, $e = 4.53$ [43], $d^1 = -4.04$, and $f^1 = -3.63$ [44]. $G = 1$ when $\Gamma = \gamma_0$.

The matching of the continuum static theory to continuum QCD was done using [44]

$$Z_{Q{\rm stat}} = 1 + \frac{g^2}{12\pi^2}\left(-\frac{3}{2}\log(\mu^2/m_b^2) - 2\right). \tag{13}$$

where m_b is the mass of the bottom quark. The equivalent expression for Eq. (13), for arbitrary Γ matrix, is in [44]. The matching factor $Z_S^{\rm static}$ in equation is the product of $Z(\Gamma = \gamma_0)$ with $Z_{Q{\rm stat}}$ to leading order in the square of the coupling g^2.

The improvement coefficients have not been calculated for the P-wave decay constant and we will therefore not include them in our analysis. We used a simple boosted coupling ($g^2/(u_0)^4$ and $g^2 = \frac{6}{\beta}$) to compute the perturbative matching factors.

We use the mass (5279 + 400 MeV) for the mass factor in Eq. (10). The experimental spectrum of the P-wave B mesons is not very well determined at the moment [6,9]. Lattice calculations predict that the lowest P-wave meson will be roughly 400 MeV above the S-wave states. So we add 400 MeV to the mass of the B^+ state [6]. This corresponds to a 4% effect on the decay constant. We prefer not to use the HQET scaling law of the decay constant to quote numbers for the decay constant of the 0^+ charm-light meson, as is done by Jugeau *et al.* [17], because it is known that $1/M$ corrections to the static limit of the decay constant of the pseudoscalar heavy-light meson are large.

The unrenormalized data for the amplitudes are in Table I. The final column shows the value for $f_P^{\rm static}$ in the static limit in physical units. The value of $r_0 = 0.5$ fm was used to convert the data into physical units.

As the renormalization factor for the ALPHA static action has not yet been determined, we cannot use that data to quote a physical number. What is disappointing is that the ALPHA static action does not produce any reduction in the statistical errors over the standard Eichten-Hill static action in our case. For the static-light pseudoscalar

TABLE I. Amplitudes and decay constants for the P_- static-light mesons. Further information about the lattice parameters is in [40].

Name	formalism	$\kappa_{\rm sea}$	$Z_L^{\rm static}$	$f_P^{\rm static}$ MeV
DF3	Eichten-Hill	0.1350	0.240^{+31}_{-27}	302(39)
DF3	Fuzzed ALPHA	0.1350	0.199^{+35}_{-20}	
DF4	Eichten-Hill	0.1355	0.234^{+43}_{-18}	322(59)
DF4	Fuzzed ALPHA	0.1355	0.174^{+30}_{-20}	
DF6	Eichten-Hill	0.1358	0.182^{+52}_{-36}	272(78)
DF6	Fuzzed ALPHA	0.1358	0.131^{+41}_{-35}	

meson decay constant the statistical errors were significantly smaller for the ALPHA static action than the Eichten-Hill action [40].

In [13] it was shown that the $DF3$ data set corresponded to sea quarks with mass around the strange quark mass. Hence we quote the $f_{P_s}^{\text{static}}$ decay constant as 302(39) MeV.

The value of the P-wave 0^+ decay constant at the strange quark mass is of interest to compare to model calculations or for future decays of the B_s meson measured at a hadronic experiment such as LHCb, CDF, or D0. The decay constant required for the factorization analysis of the heavy to heavy $\bar{B} \to D^{\star\star}\pi$ is the f_P^{static} decay constant in the chiral limit [17].

To study the light P-wave 0^+ decay constant the chiral extrapolation must be discussed. The importance of chiral logs in the mass extrapolation of the f_B decay constant has only recently been observed as the dominant systematic error in the determination of f_B/f_{B_s} [45,46].

The equivalent chiral perturbation theory calculation for the P-wave decay constant (to our knowledge) has not yet been done. There are calculations of the masses of heavy-light mesons to one loop in heavy-light chiral perturbation theory [47,48]. UKQCD has previously computed the relevant hadronic coupling [49]. This result is compared against experiment in [47].

The data for the P-wave decay constant in Table I are not really precise enough to determine the quark mass dependence. A simple linear fit against the square of the pion mass of the data in Table I gives $f_P^{\text{static}} = 294(88)$ MeV in the chiral limit of zero mass light quarks.

IV. CONCLUSION

We have computed the decay constant of 0^+ heavy-light mesons using an unquenched lattice QCD calculation at a single lattice spacing. For the static-light decay constant, we used sea quarks as low at a third of the strange quark mass. For the calculation of the 0^+ decay constant, we did a partially quenched analysis with the sea quark fixed at the strange quark mass.

We obtain the $f_{P_s}^{\text{static}}$ decay constant as 302(39) MeV. Given the qualifications mentioned in the previous section, we obtain $f_P^{\text{static}} = 294(88)$ MeV. The data for f_P^{static} from various models are summarized in [17]. Two QCD sum rule results are $f_P^{\text{static}} = 304 \pm 40$ [2] and 377 ± 53 MeV [50]. Using clover quarks for the charm quark, we obtain $f_{D_{0^+}} = 360(90)$ MeV and $f_{D_{s0^+}} = 340(110)$ MeV. The magnitude of these decay constants can be compared against the value of the f_{D_s} and f_D pseudoscalar decay constants. From unquenched lattice QCD, the Fermilab, MILC and HPQCD collaborations [51] obtained: $f_{D_s} = 249 \pm 3 \pm 16$ MeV and $f_{D^+} = 201 \pm 3 \pm 17$ MeV. The CLEO-c collaboration have recently reported [52] the experimental result $f_{D^+} = 223(17)(3)$ MeV.

Although the systematic errors are different in the two lattice calculations, we can make the qualitative statement that the P-wave and S-wave decay constants of charm-light mesons are similar in magnitude. This calculation does not support a suppression of the decay constant of the P-wave heavy-light meson relative to the decay constant of the S-wave heavy-light meson.

TABLE II. Comparison of decay constants of charm-light D_{0^+} meson.

Method	g_{0+} MeV
This work	200 ± 50
QCD sum rules [1]	170 ± 20
Lattice QCD [17]	122 ± 43
$B \to D^{\star\star}\pi$ [17]	206 ± 120

The only decay constant of a light P-wave meson that is sometimes calculated from lattice QCD is that of the a_1 meson. The decay constant of the a_1 meson, denoted as f_{a_1}, is usually computed from a definition that has dimension MeV^2. If we use the results from Wingate *et al.* [53], then the value of $\frac{f_{a_1}}{M_{a_1}}$ is 240 MeV. This is larger than the value of the pion decay constant 131 MeV, showing a similar trend to the heavy-light case.

Kurth and Sommer [54] have noted that there are potential problems with extrapolating in the heavy-quark mass unless the continuum limit has been taken. Also see Kronfeld [55] for a discussion of the problems combining static and propagating heavy-light data. Given these theoretical concerns, the data is consistent with the view that the decay constant of the static-light meson is larger than that of the 0^+ charm-light meson, if we use definition of the decay constant in Eq. (1) (our value of g_0 is in Table II), but the errors need to be reduced for a definitive statement.

Jugeau *et al.* [17]. have collected together a number of different calculations of the 0^+ decay constant of a charm-light meson. To compare our results to other calculations we use the normalization in Eq. (1). Hence we multiply our result by $m_c/M_{D_s(2317)} = 1.27/2.317$ and compare to other calculations in Table II. There is reasonable agreement between different determinations. To reduce the errors on the lattice results requires a calculation of similar effort to that done to compute the decay constants of the heavy-light pseudoscalar mesons [51].

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