Analysis of the vertices D^*D_sK, D_s^*DK, D_0D_sK, and $D_{s0}DK$ with the light-cone QCD sum rules

Z. G. Wang[1,*,†] and S. L. Wan[2]

[1]Department of Physics, North China Electric Power University, Baoding 071003, People's Republic of China
[2]Department of Modern Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China

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In this article, we analyze the vertices D^*D_sK, D_s^*DK, D_0D_sK, and $D_{s0}DK$ within the framework of the light-cone QCD sum rules approach in an unified way. The strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are important parameters in evaluating the charmonium absorption cross sections in searching for the quark-gluon plasmas. Our numerical values of the $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are compatible with the existing estimations although somewhat smaller; the $SU(4)$ symmetry breaking effects are very large, about 60%. For the charmed scalar mesons D_0 and D_{s0}, we take the point of view that they are the conventional $c\bar{u}$ and $c\bar{s}$ mesons, respectively, and calculate the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ with the vector interpolating currents. The numerical values of the scalar-D_sK and scalar-DK coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ are compatible with the existing estimations—the large values support the hadronic dressing mechanism. Furthermore, we study the dependence of the four strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ on the nonperturbative parameter a_4 of the twist-2 K meson light-cone distribution amplitude.

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I. INTRODUCTION

The suppression of the J/ψ production in relativistic heavy ion collisions may be one of the important signatures to identify the possible phase transition to the quark-gluon plasma [1]. The dissociation of the J/ψ in the quark-gluon plasma due to color screening can lead to a reduction of its production, however, the J/ψ suppression may be already present in the hadron-nucleus collisions. It is necessary to separate the absorption of the J/ψ by the nucleons and by the comover light mesons (π, K, ρ, ω, etc.) before we can make a definitive conclusion about the formation of the quark-gluon plasma. It is of great importance to understand the J/ψ production and absorption mechanisms in the hadronic matter. The values of the J/ψ absorption cross sections by the light hadrons are not known empirically; we have to resort to some theoretical approaches. Among existing approaches for evaluating the charmonium absorption cross sections by the light hadrons, the one-meson exchange model and the effective $SU(4)$ theory are typical [2,3]. The detailed knowledge about the hadronic vertices or the strong coupling constants which are basic parameters in the effective Lagrangians is of great importance.

The discovery of the two strange-charmed mesons D_{s0} and D_{s1} with spin-parity 0^+ and 1^+, respectively, has triggered hot debate on their nature, understructures, and whether it is necessary to introduce the exotic states [4]. The mass of the D_{s0} is significantly lower than the values of the 0^+ state mass from the quark models and lattice simulations [5]. The difficulties to identify the D_{s0} and D_{s1} states with the conventional $c\bar{s}$ mesons are rather similar to those appearing in the light scalar mesons below 1 GeV. Among the various explanations, the hadronic dressing mechanism is typical. The scalar mesons $a_0(980)$, $f_0(980)$, D_0, and D_{s0} may have bare $q\bar{q}$, $\bar{c}u$, and $c\bar{s}$ kernels in the P-wave states with strong coupling to the nearby threshold, respectively. The S-wave virtual intermediate hadronic states (or the virtual mesons loops) play a crucial role in the composition of those bound states (or resonances due to the masses below or above the thresholds). The hadronic dressing mechanism (or unitarized quark models) takes the point of view that the $f_0(980)$, $a_0(980)$, D_0, and D_{s0} mesons have small $q\bar{q}$, $\bar{c}u$, and $c\bar{s}$ kernels of the typical $q\bar{q}$, $c\bar{u}$, and $c\bar{s}$ mesons size, respectively. The strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional scalar $q\bar{q}$, $c\bar{u}$, and $c\bar{s}$ mesons in the constituent quark models and enrich the pure $q\bar{q}$, $c\bar{u}$, and $c\bar{s}$ states with other components [6,7]. Those mesons may spend part (or most part) of their lifetime as virtual $K\bar{K}$, D_sK, and DK states [6,7]. It is interesting to study the possibility of the hadronic dressing mechanism.

In this paper, we calculate the values of the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$, while the nonperturbative matrix elements are parameterized by the light-cone distribution amplitudes which are classified according to their twists instead of the vacuum condensates [8,9]. Furthermore, we study the dependence of the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ on the coefficient a_4 of the twist-2 K meson light-cone distribution amplitude $\phi_K(u)$ and estimate the values of the nonperturbative parameter. It is very difficult to determine the a_4 with the QCD sum rules; the values of the a_4 suffer

*Corresponding author.
†Electronic address: wangzgyiti@yahoo.com.cn

from large uncertainties, as it concerns high-dimension vacuum condensates which are known poorly [8–12]. It is of great importance to determine the values directly from the experimental data.

The paper is arranged as follows: In Sec. II, we derive the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. In Sec. III, we give the numerical results and discussions. Finally, in Sec. IV is the conclusion.

II. STRONG COUPLING CONSTANTS $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, AND $G_{D_{s0}DK}$ WITH LIGHT-CONE QCD SUM RULES

In the following, we write down the definitions for the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$,

$$\begin{aligned}\langle D^*(q+P)D_s(q)|K(P)\rangle &= G_{D^*D_sK}(P-q)\cdot\epsilon,\\ \langle D_s^*(q+P)D(q)|K(P)\rangle &= G_{D_s^*DK}(P-q)\cdot\epsilon,\\ \langle D_0(q+P)D_s(q)|K(P)\rangle &= G_{D_0D_sK},\\ \langle D_{s0}(q+P)D(q)|K(P)\rangle &= G_{D_{s0}DK}.\end{aligned} \tag{1}$$

Here the ϵ_μ are the polarization vectors of the mesons D^* and D_s^*. We study the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ with the interpolating currents $J_{D_s}(x)$, $J_D(x)$, $J_\mu^{D_s}(x)$, and $J_\mu^D(x)$ in an unified way, and we choose the two-point correlation functions $\Pi_\mu^1(P,q)$ and $\Pi_\mu^2(P,q)$,

$$\Pi_\mu^1(P,q) = i\int d^4x e^{-iq\cdot x}\langle 0|T\{J_\mu^D(0)J_{D_s}(x)\}|K(P)\rangle, \tag{2}$$

$$\Pi_\mu^2(P,q) = i\int d^4x e^{-iq\cdot x}\langle 0|T\{J_\mu^{D_s}(0)J_D(x)\}|K(P)\rangle, \tag{3}$$

$$\begin{aligned}J_\mu^D(x) &= \bar u(x)\gamma_\mu c(x), & J_\mu^{D_s}(x) &= \bar s(x)\gamma_\mu c(x),\\ J_D(x) &= \bar c(x)i\gamma_5 u(x), & J_{D_s}(x) &= \bar c(x)i\gamma_5 s(x).\end{aligned} \tag{4}$$

The correlation functions $\Pi_\mu^{1(2)}(P,q)$ can be decomposed as

$$\Pi_\mu^{1(2)}(P,q) = \Pi_P^{1(2)}(q^2,(q+P)^2)P_\mu + \Pi_q^{1(2)}(q^2,(q+P)^2)q_\mu, \tag{5}$$

due to the Lorentz covariance. In this article, we derive the sum rules with the tensor structures P_μ and q_μ, respectively, and make detailed studies.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [13], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_{D_s}(x)$ [$J_D(x)$] and $J_\mu^D(x)$ [$J_\mu^{D_s}(x)$] into the correlation function Π_μ^1 (Π_μ^2) to obtain the hadronic representation. After isolating the ground states and the first orbital excited states contributions from the pole terms of the D_s, D^*, and D_0 (D, D_s^*, and D_{s0}) mesons, the correlation function Π_μ^1 (Π_μ^2) can be expressed in terms of the strong coupling constants G and the decay constants f_M of the heavy mesons. The explicit expressions are presented in the appendix. We use the standard definitions for the decay constants f_M ($f_{D_s}, f_D, f_{D_s^*}, f_{D^*}, f_{D_{s0}}, f_{D_0}$) of the heavy mesons,

$$\begin{aligned}\langle 0|J_D(0)|D(q)\rangle &= \frac{f_D m_D^2}{m_c+m_u},\\ \langle 0|J_{D_s}(0)|D_s(q)\rangle &= \frac{f_{D_s} m_{D_s}^2}{m_c+m_s},\\ \langle 0|J_\mu^D(0)|D^*(q)\rangle &= f_{D^*}m_{D^*}\epsilon_\mu,\\ \langle 0|J_\mu^{D_s}(0)|D_s^*(q)\rangle &= f_{D_s^*}m_{D_s^*}\epsilon_\mu,\\ \langle 0|J_\mu^D(0)|D_0(q)\rangle &= f_{D_0}q_\mu,\\ \langle 0|J_\mu^{D_s}(0)|D_{s0}(q)\rangle &= f_{D_{s0}}q_\mu.\end{aligned} \tag{6}$$

The quarks c and s have finite and nonequal masses. The nonconservation of the vector currents $J_\mu^{D_s}(x)$ and $J_\mu^D(x)$ can lead to the nonvanishing couplings to the scalar mesons D_{s0} and D_0 beside the vector mesons D_s^* and D^*. We can study the properties of those mesons with the two interpolating currents $J_\mu^{D_s}(x)$ and $J_\mu^D(x)$ in an unified way. Here we have not shown the contributions from the high resonances and continuum states explicitly as they are suppressed due to the double Borel transformation. The numerical values of the fractions

$$\frac{m_{D^*}^2 - m_{D_s}^2 + m_K^2}{m_{D^*}^2}, \qquad \frac{m_{D_s^*}^2 - m_D^2 + m_K^2}{m_{D_s^*}^2}$$

are less than 30% and the corresponding spectral densities for the ground states are greatly suppressed. The tensor structures with q_μ are especially suitable for studying the first orbital excited states D_0 and D_{s0} with the vector currents. The numerical values of the fractions

$$\frac{m_{D^*}^2 + m_{D_s}^2 - m_K^2}{m_{D^*}^2}, \qquad \frac{m_{D_s^*}^2 + m_D^2 - m_K^2}{m_{D_s^*}^2}$$

are about 2. The tensor structures with P_μ are especially suitable for studying the ground states D^* and D_s^* with the vector currents.

Now we carry out the operator product expansion near the light-cone $x^2 \approx 0$ to obtain the representation at the level of quark-gluon degrees of freedom for the correlation functions Π_μ^1 and Π_μ^2. In the following, we briefly outline the operator product expansion for the correlation functions Π_μ^1 and Π_μ^2 in perturbative QCD theory. The calculations are performed at the large spacelike momentum regions $(q+P)^2 \ll 0$ and $q^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge first [10],

$$\langle 0|T\{q_i(x_1)\bar{q}_j(x_2)\}|0\rangle = i\int\frac{d^4k}{(2\pi)^4}e^{-ik(x_1-x_2)}\Big\{\frac{\not k+m}{k^2-m^2}\delta_{ij} - \int_0^1 dv g_s G^{\mu\nu}_{ij}(vx_1+(1-v)x_2)\times\Big[\frac{1}{2}\frac{\not k+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\Big]\Big\}. \quad (7)$$

Here $G^{\mu\nu}$ is the gluonic field strength and g_s denotes the strong coupling constant. Substituting the above c quark propagator and the corresponding K meson light-cone distribution amplitudes into the correlation functions Π^1_μ and Π^2_μ in Eqs. (2) and (3) and completing the integrals over the variables x and k, finally we obtain the representation at the level of quark-gluon degrees of freedom; the explicit expressions are presented in the appendix. In calculation, we have used the two-particle and three-particle K meson light-cone distribution amplitudes [8–12]. The explicit expressions are also presented in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules approach [8–12]. In this paper, the energy scale μ is chosen to be $\mu = 1$ GeV.

We perform the double Borel transformation with respect to the variables $Q_1^2 = -(q+P)^2$ and $Q_2^2 = -q^2$ for the correlation functions $\Pi_P^{1(2)}$ and $\Pi_q^{1(2)}$ and obtain the analytical expressions for those invariant functions (explicit expressions are presented in the appendix).

In order to match the duality regions below the thresholds s_0 and s_0' for the interpolating currents $J^D_\mu(x)(J^{D_s}_\mu(x))$ and $J_{D_s}(x)(J_D(x))$, respectively, we can express the correlation functions $\Pi_P^{1(2)}$ and $\Pi_q^{1(2)}$ at the level of quark-gluon degrees of freedom into the following form:

$$\Pi^{1(2)}_{P(q)}(q^2,(q+P)^2) = \int ds\int ds'\frac{\rho(s,s')}{\{s-(q+P)^2\}(s'-q^2)}, \quad (8)$$

then we perform the double Borel transformation with respect to the variables $Q_1^2 = -(q+P)^2$ and $Q_2 = -q^2$ directly. However, the analytical expressions for the spectral densities $\rho(s,s')$ are hard to obtain; we need to resort to some approximations. As the contributions from the higher twist terms are suppressed by more powers of $\frac{1}{-q^2}$ or $\frac{1}{-(q+P)^2}$, the continuum subtractions will not affect the results remarkably. Here we will use the expressions in Eqs. (A10) and (A11) for the three-particle (quark-antiquark-gluon) twist-3, twist-4 terms, and the two-particle twist-4 terms. In fact, their contributions are of minor importance; the dominating contributions come from the two-particle twist-2 and twist-3 terms involving the $\phi_K(u)$, $\phi_p(u)$, and $\phi_\sigma(u)$. We perform the same trick as the authors of Refs. [10,14] and expand the amplitudes $\phi_K(u)$, $\phi_p(u)$, and $\phi_\sigma(u)$ in terms of polynomials of $1-u$,

$$\phi_K(u),\, \phi_p(u),\, \phi_\sigma(u),\, \frac{d}{du}\phi_\sigma(u) \quad\Rightarrow\quad \sum_{k=0}^N b_k(1-u)^k = \sum_{k=0}^N b_k\Big(\frac{s-m_c^2}{s-q^2}\Big)^k, \quad (9)$$

then introduce the variable s' and the spectral densities are obtained. After straightforward but cumbersome calculations, we can obtain the final expressions for the double Borel transformed correlation functions $\Pi_\mu^{1(2)}$ at the level of quark-gluon degrees of freedom below the thresholds. The masses of the charmed mesons are $M_{D^*} = 2.012$ GeV, $M_{D_s^*} = 2.112$ GeV, $M_D = 1.865$ GeV, $M_{D_s} = 1.97$ GeV, $M_{D_0} = 2.40$ GeV, and $M_{D_{s0}} = 2.317$ GeV. The ratios are $\frac{M_D}{M_D+M_{D_{s0}}} \approx 0.45$, $\frac{M_D}{M_D+M_{D_s^*}} \approx 0.47$, $\frac{M_{D_s}}{M_{D_s}+M_{D^*}} \approx 0.49$, and $\frac{M_{D_s}}{M_{D_s}+M_{D_0}} \approx 0.45$ [15]. There exist overlapping working windows for the two Borel parameters M_1^2 and M_2^2. It is convenient to take the value $M_1^2 = M_2^2$, $u_0 = \frac{M_1^2}{M_1^2+M_2^2} = \frac{1}{2}$, $M^2 = \frac{M_1^2M_2^2}{M_1^2+M_2^2} = \frac{1}{2}M_1^2$; furthermore, the K meson light-cone distribution amplitudes are known quite well at the value $u_0 = \frac{1}{2}$ comparing with the values at the end points. We can introduce the threshold parameter s_0 and make the simple replacement,

$$\exp\Big\{-\frac{m_c^2+u_0(1-u_0)m_K^2}{M^2}\Big\}\rightarrow\exp\Big\{-\frac{m_c^2+u_0(1-u_0)m_K^2}{M^2}\Big\} - \exp\Big\{-\frac{s_0}{M^2}\Big\}$$

to subtract the contributions from the higher resonances and continuum states [10]. Finally we obtain the following sum rules:

$$-\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{m_c+m_s}\frac{m_{D^*}^2+m_{D_s}^2-m_K^2}{m_{D^*}^2}\times\exp\Big\{-\frac{m_{D^*}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\Big\} = AA; \quad (10)$$

$$-\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_D m_D^2}{m_c+m_u}\frac{m_{D_s^*}^2+m_D^2-m_K^2}{m_{D_s^*}^2}\times\exp\Big\{-\frac{m_{D_s^*}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\Big\} = BB; \quad (11)$$

$$\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{m_c+m_s}\frac{m_{D^*}^2-m_{D_s}^2+m_K^2}{m_{D^*}^2}\exp\left\{-\frac{m_{D^*}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}+\frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{m_c+m_s}\exp\left\{-\frac{m_{D_0}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}=CC; \quad (12)$$

$$\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{m_c+m_u}\frac{m_{D_s^*}^2-m_D^2+m_K^2}{m_{D_s^*}^2}\exp\left\{-\frac{m_{D_s^*}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}+\frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{m_c+m_u}\exp\left\{-\frac{m_{D_{s0}}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}=DD; \quad (13)$$

$$-\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{m_c+m_s}\frac{m_{D^*}^2+m_{D_s}^2-m_K^2}{m_{D^*}^2}\exp\left\{-\frac{m_{D^*}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}+\frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{m_c+m_s}\exp\left\{-\frac{m_{D_0}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}=EE; \quad (14)$$

$$-\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{m_c+m_u}\frac{m_{D_s^*}^2+m_D^2-m_K^2}{m_{D_s^*}^2}\exp\left\{-\frac{m_{D_s^*}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}+\frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{m_c+m_u}\exp\left\{-\frac{m_{D_{s0}}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}=FF. \quad (15)$$

The explicit expressions of the notations AA, BB, CC, DD, EE, and FF are lengthy and given explicitly in the appendix. A slightly different manipulation (with the techniques taken in Refs. [16,17]) for the dominating contributions come from the terms involving the two-particle twist-2 and twist-3 light-cone distribution amplitudes $\phi_K(u)$, $\phi_p(u)$, and $\phi_\sigma(u)$ leads to the sum rules with the same type as in Ref. [17]. However, those type sum rules are not stable with respect to the variations of the Borel parameter M^2. Here we will not show the expressions explicitly for simplicity. It is no surprise that the QCD sum rules as a QCD model have both advantages and shortcomings.

III. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters are taken as $m_s = (140 \pm 10)$ MeV, $m_c = (1.25 \pm 0.10)$ GeV, $\lambda_3 = 1.6 \pm 0.4$, $f_{3K} = (0.45 \pm 0.15) \times 10^{-2}$ GeV^2, $\omega_3 = -1.2 \pm 0.7$, $\omega_4 = 0.2 \pm 0.1$, $a_2 = 0.25 \pm 0.15$, $a_1 = 0.06 \pm 0.03$ [8–12], $f_K = 0.160$ GeV, $m_K = 498$ MeV, $m_{D_{s0}} = 2.317$ GeV, $m_D = 1.865$ GeV, $m_{D_s} = 1.97$ GeV. In this paper, we take the values of the a_4 to be zero and explore the dependence of the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ on this parameter.

For the threshold parameter $s_{D_0}^0$, we can use the experimental data as a guide, $m_{D_0} = 2.40$ GeV, $\Gamma_{m_{D_0}} = 283$ MeV [15], and choose the values $s_{D_0}^0 = (6.8 - 7.2)$ GeV^2 to subtract the contributions from the high resonances and continuum states. The mass and width of the D_0 from the Belle and Focus collaborations are $m_{D_0} = 2308 \pm 17 \pm 15 \pm 28$ MeV, $\Gamma_{D_0} = 276 \pm 21 \pm 18 \pm 60$ MeV [18], $m_{D_0} = 2407 \pm 21 \pm 35$ MeV, $\Gamma_{D_0} = 240 \pm 55 \pm 59$ MeV [19]. The predictions from the constituent quark models are $m_{D_0} = 2.4$ GeV [5]. The values of the mass from the two collaborations have a difference of about 100 MeV; we take the value $m_{D_0} = 2.4$ GeV as the input parameter. Our final numerical results for the large strong coupling constant $G_{D_0D_sK}$ support smaller values for the D_0 if the same mechanism takes place for both the charmed scalar mesons D_0 and D_{s0}. Furthermore, the strong coupling constant $G_{D_0D_sK}$ is not sensitive to the values of the m_{D_0}, taking the values $m_{D_0} = 2.4$ GeV or $m_{D_0} = 2.3$ GeV cannot change the conclusion qualitatively or quantitatively.

For the threshold parameters $s_{D^*}^0$, $s_{D_s^*}^0$, and $s_{D_{s0}}^0$, the experimental values of the masses are $m_{D^*} = 2.01$ GeV, $m_{D_s^*} = 2.112$ GeV, and $m_{D_{s0}} = 2.317$ GeV; the widths are very narrow [15]. We can choose the values of the threshold parameters $s_{D^*}^0 = (4.7 - 5.1)$ GeV^2, $s_{D_s^*}^0 = (4.8 - 5.2)$ GeV^2, and $s_{D_{s0}}^0 = (7.0 - 7.4)$ GeV^2 to subtract the contributions from the high resonances and continuum states. From Figs. 1–3, we can see that the numerical values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_0D_sK}$ are not sensitive to the threshold parameters s^0 in those regions; the values we chose here are reasonable.

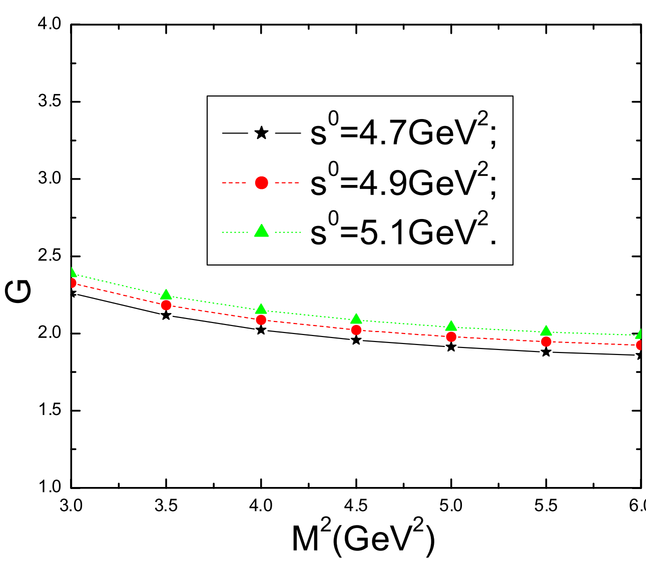

FIG. 1 (color online). The $G_{D^*D_sK}$ with the parameters M^2 and $s_{D^*}^0$ from Eq. (10).

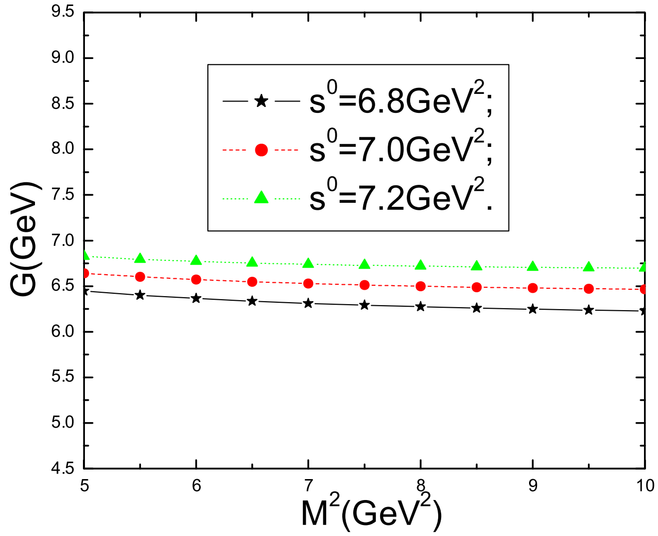

FIG. 2 (color online). The $G_{D_0D_sK}$ with the parameters M^2 and $s^0_{D_0}$ from Eq. (12).

The values of the decay constants $f_D, f_{D_s}, f_{D^*}, f_{D_s^*}, f_{D_0}$, and $f_{D_{s0}}$ vary in a large range, for example, $f_D = (0.17 \pm 0.01)$ GeV, $f_{D^*} = (0.24 \pm 0.02)$ GeV [10], $f_{D_0} = (0.217 \pm 0.025)$ GeV, $m_{D_0} = 2.272$ GeV [20], $f_{D_{s0}} = (0.225 \pm 0.025)$ GeV [21], $f_D = (0.177 \pm 0.021)$ GeV, $f_{D_s} = (0.205 \pm 0.022)$ GeV [22], $f_{D_0} = (0.17 \pm 0.02)$ GeV [23] from the QCD sum rules; $f_{D_s} = 0.268$ GeV, $f_{D_s^*} = 0.315$ GeV, $f_D = 0.234$ GeV, $f_{D^*} = 0.310$ GeV [24], $f_{D_s^*} = 0.375 \pm 0.024$ GeV, $f_{D^*} = 0.340 \pm 0.023$ GeV [25], $f_D = 0.238$ GeV, $f_{D_s} = 0.241$ GeV [26] from the potential models; $f_{D_s^*} = 326^{+21}_{-17}$ MeV, $f_{D^*} = 223^{+23}_{-19}$ MeV [27] from the quark models; and $f_D = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV from the experimental data [28]. For a review of the values of the decay constants for the mesons D and D_s from the QCD sum rules and lattice QCD, one can consult the second article of Ref. [9].

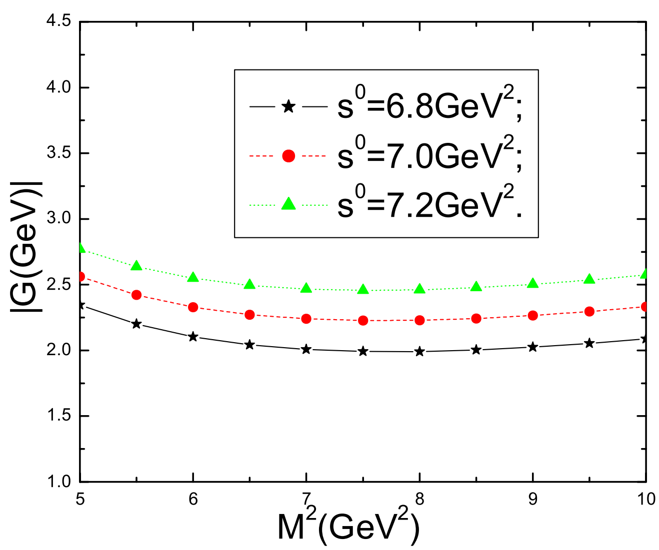

FIG. 3 (color online). The $G_{D_0D_sK}$ with the parameters M^2 and $s^0_{D_0}$ from Eq. (14).

We take the following constraints for the decay constants:

$$1.0 < \frac{f_{D_{s0}}}{f_{D_0}} \approx \frac{f_{D_s^*}}{f_{D^*}} \approx \frac{f_{D_s}}{f_D} < 1.1, \qquad (16)$$

and choose the values

$$\begin{aligned} f_{D_s} &= (0.25 \pm 0.02)\ \text{GeV}, \\ f_D &= (0.23 \pm 0.02)\ \text{GeV}, \\ f_{D_{s0}} &= (0.225 \pm 0.025)\ \text{GeV}, \\ f_{D_0} &= (0.217 \pm 0.020)\ \text{GeV}, \\ f_{D_s^*} &= (0.26 \pm 0.02)\ \text{GeV}, \\ f_{D^*} &= (0.24 \pm 0.02)\ \text{GeV}. \end{aligned} \qquad (17)$$

In numerical calculation, we observe that the values of the strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ are sensitive to the six hadronic parameters. Small variations of those parameters can lead to relatively large changes for the numerical values. Refining the six hadronic parameters is of great importance.

The Borel parameters in Eqs. (10) and (11) are taken as $M_1^2 = M_2^2 = (6 - 12)$ GeV^2 and $M^2 = (3 - 6)$ GeV^2. In those regions, the values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are rather stable from the sum rules in Eqs. (10) and (11) with the simple subtraction, which are shown, for example, in Figs. 1 and 4–7 for the strong coupling constant $G_{D^*D_sK}$. Similar figures can be obtained if the values of the strong coupling constant $G_{D_s^*DK}$ are plotted. We only show the numerical values from the sum rules in Eq. (10), (12), and (14) explicitly for simplicity.

The Borel parameters in Eqs. (12)–(15) are chosen as $M_1^2 = M_2^2 = (10 - 20)$ GeV^2 and $M^2 = (5 - 10)$ GeV^2. In those regions, the values of the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ are rather stable from the sum rules in Eqs. (12) and (13) with the simple subtraction, which are shown in Figs. 2, 4–6, and 8 for an illustration. However, the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ from the sum rules in Eqs. (14) and (15) have a negative sign compared with the corresponding ones from the sum rules in Eqs. (12) and (13)—and much smaller absolute values. The fractions

$$\frac{m_{D^*}^2 + m_{D_s}^2 - m_K^2}{m_{D^*}^2}, \qquad \frac{m_{D_s^*}^2 + m_D^2 - m_K^2}{m_{D_s^*}^2}$$

are about 2. In the sum rules in Eqs. (10) and (11), the ground state saturate condition can be satisfied safely below the threshold $s^0_{D^*}$ ($s^0_{D_s^*}$). The vector interpolating current $J^D_\mu(x)$ [$J^{D_s}_\mu(x)$] has both nonvanishing couplings

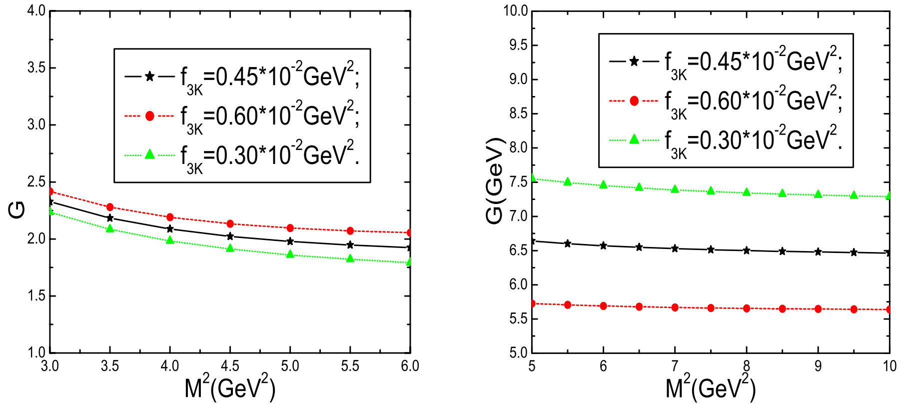

FIG. 4 (color online). The $G_{D^*D_sK}$ and $G_{D_0D_sK}$ with the parameters M^2 and f_{3K} from Eq. (10) and (12), respectively.

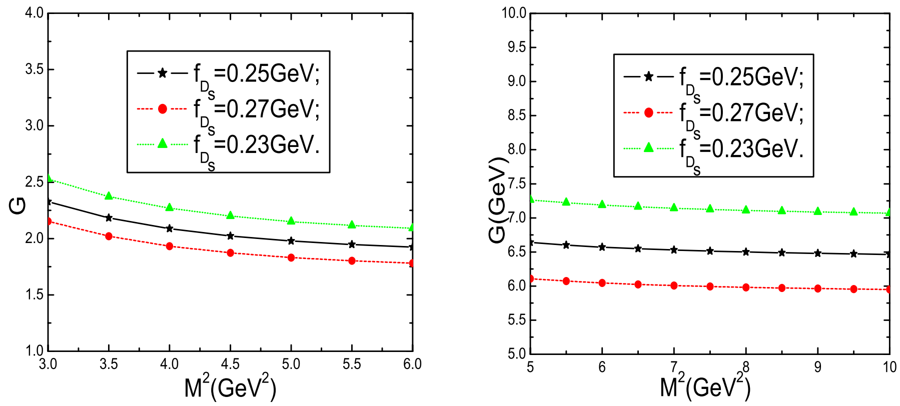

FIG. 5 (color online). The $G_{D^*D_sK}$ and $G_{D_0D_sK}$ with the parameters M^2 and f_{D_s} from Eq. (10) and (12), respectively.

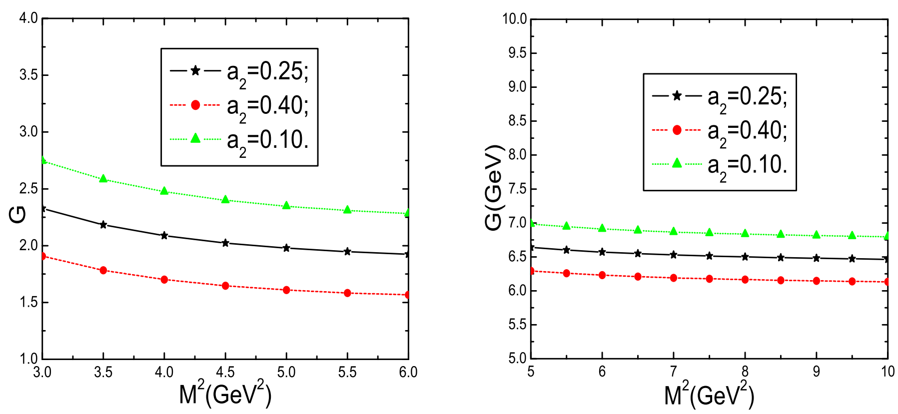

FIG. 6 (color online). The $G_{D^*D_sK}$ and $G_{D_0D_sK}$ with the parameters M^2 and a_2 from Eq. (10) and (12), respectively.

to the vector state D^* (D_s^*) and to the scalar state D_0 (D_{s0}). There are two hadronic states: the ground state D^* (D_s^*) and the first orbital excited state D_0 (D_{s0}) in the channel $\bar{c}u$ ($\bar{c}s$) below the threshold $s_{D_0}^0$ ($s_{D_{s0}}^0$). The ground states D^* and D_s^* are not suppressed due to the factor 2. The sum rules in Eqs. (14) and (15) are not suitable for studying the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$; our final numerical values support this assumption. We show this fact in Fig. 3 for an illustration.

We determine the values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ from Eqs. (10) and (11), respectively, then use those values as the input parameters and calculate the values of the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ from the Eqs. (12)–(15), respectively.

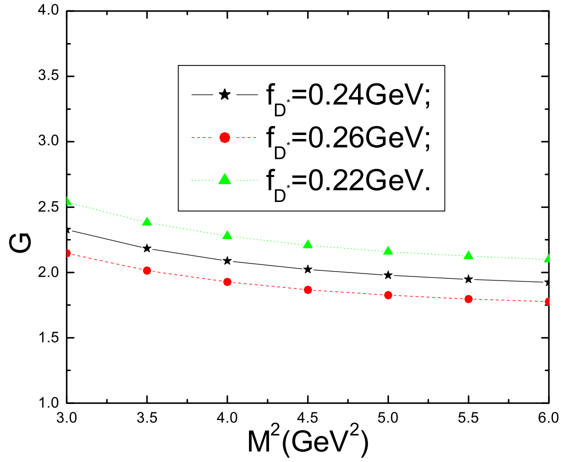

FIG. 7 (color online). The $G_{D^*D_sK}$ with the parameters M^2 and f_{D^*} from Eq. (10).

The uncertainties of the five parameters ω_4, ω_3, λ_3, m_c, and a_1 cannot lead to large uncertainties for the numerical values. The main uncertainties come from the ten parameters f_{3K}, m_s, a_2, η_4, f_D, f_{D^*}, $f_{D_s^*}$, f_{D_0}, f_{D_s}, and $f_{D_{s0}}$; small variations of those parameters can lead to relatively large changes for the numerical values, which are shown in Figs. 4–8 for illustration.

Taking into account all the uncertainties, we obtain the numerical results for the strong coupling constants:

$$\begin{aligned} G_{D^*D_sK} &= 2.02^{+0.84}_{-0.56}, \qquad G_{D_0D_sK} = 6.5^{+1.8}_{-1.5}\ \text{GeV}, \\ G_{D_s^*DK} &= 1.84^{+0.91}_{-0.63}, \qquad G_{D_{s0}DK} = 5.9^{+1.7}_{-1.6}\ \text{GeV}, \end{aligned} \tag{18}$$

which are shown in Figs. 9 and 10, respectively.

The strong coupling constants $G_{D^*D_sK}$, $G_{D_s^*DK}$, $G_{D_0D_sK}$, and $G_{D_{s0}DK}$ can be related to the parameters g and h in the heavy-light Chiral perturbation theory [29,30],

$$G_{SP\pi} = \sqrt{m_S m_P}\frac{m_S^2 - m_P^2}{m_S}\frac{|h|}{f_\pi}, \qquad G_{VP\pi} = \frac{2\sqrt{m_P m_V}}{f_\pi}g.$$

Here, the S are the heavy scalar mesons with 0^+, the P are the heavy pseudoscalar mesons with 0^-, the V are the heavy vector mesons with 1^-, and the π stand for the light pseudoscalar mesons.

The parameter g has been calculated with the light-cone QCD sum rules [31–33], the quark models [34,35] and extracted from the experimental data [36,37]. The values vary in a large range. The corresponding values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ in the $SU(3)$ limit for the light pseudoscalar mesons are listed in the Table. I. From the table, we can see that our numerical results are compatible with the existing estimations, although somewhat smaller.

The values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are sensitive to the nonperturbative parameter a_4. If we take a larger value rather than zero, larger values of the

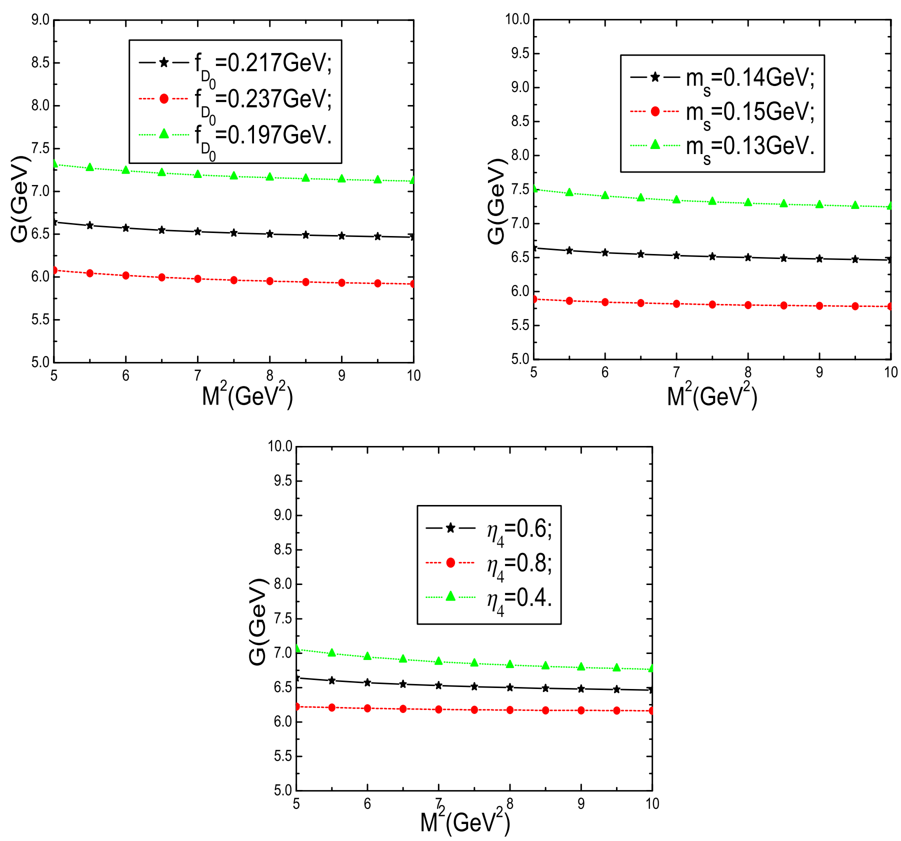

FIG. 8 (color online). The $G_{D_0D_sK}$ with the parameters M^2 and f_{D_0}, m_s, η_4 from Eq. (12).

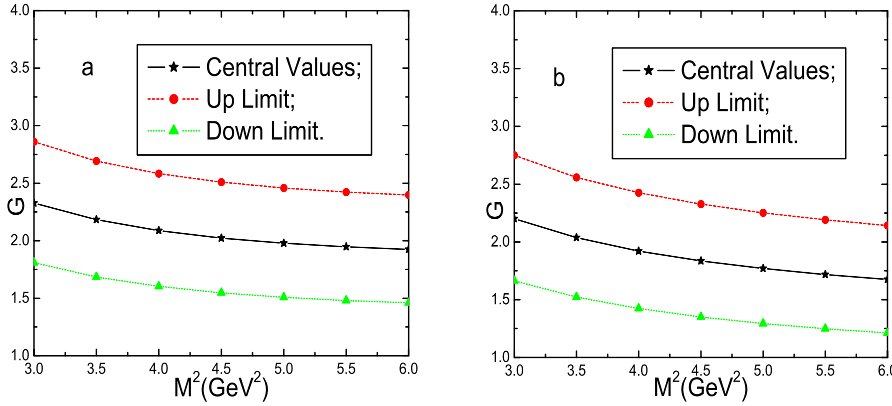

FIG. 9 (color online). The (a) $G_{D^*D_sK}$ and (b) $G_{D_s^*DK}$ with the parameter M^2 from Eq. (10) and (11), respectively.

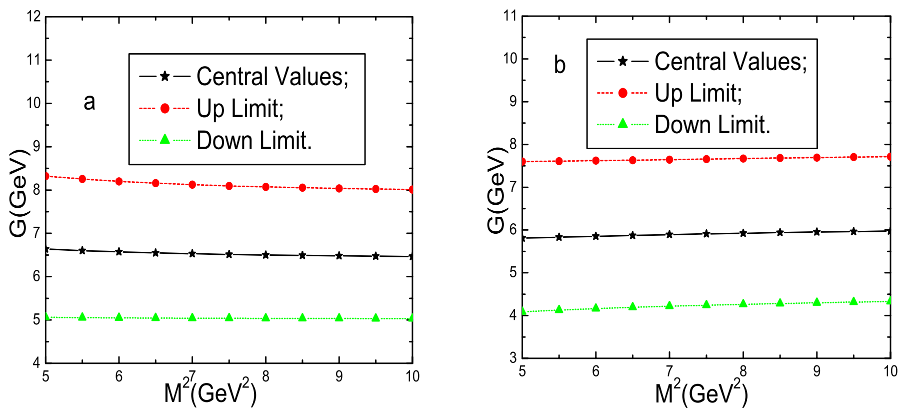

FIG. 10 (color online). The (a) $G_{D_0D_sK}$ and (b) $G_{D_{s0}DK}$ with the parameter M^2 from Eq. (12) and (13), respectively.

$G_{D^*D_sK}$ and $G_{D_s^*DK}$ are obtained. The $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are more sensitive to the a_4 comparing with the $G_{D_0D_sK}$ and $G_{D_{s0}DK}$, which are shown in Fig. 11. In fact, the largest uncertainties come from the uncertainties of the a_4; they are ideal channels to determine this parameter directly from the experimental data. Once the experimental data for the values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are available, powerful constraints can be put on the range of the parameter a_4. If we take the values from the QCD sum rules as input parameters [38], $G_{D^*D_sK} = 3.02 \pm 0.14$ and $G_{D_s^*DK} = 2.84 \pm 0.31$, very large values of the a_4 are obtained.

TABLE I. Numerical values of the parameter g, and the corresponding values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ in the $SU(3)$ limit. Here we have double the values of our numerical results and the ones from Ref. [38] due to the difference between the definitions for the strong coupling constants.

$\lvert g\rvert$	$G_{D^*D_sK}$	$G_{D_s^*DK}$	Reference
0.38 ± 0.08	9.5 ± 2.0	9.4 ± 2.0	[29]
	6.04 ± 0.28	5.68 ± 0.62	[38]
0.34 ± 0.10	8.5 ± 2.5	8.4 ± 2.5	[31]
0.28	7.0	6.9	[32]
0.35 ± 0.10	8.7 ± 2.5	8.7 ± 2.5	[33]
0.50 ± 0.02	12.4 ± 0.5	12.4 ± 0.5	[34]
0.61	15.2	15.1	[35]
0.59 ± 0.07	14.7 ± 1.7	14.6 ± 1.7	[36]
$0.27^{+0.06}_{-0.03}$	$6.7^{+1.5}_{-0.7}$	$6.7^{+1.5}_{-0.7}$	[37]
$0.16^{+0.07}_{-0.05}$	$4.04^{+1.68}_{-1.12}$	$3.68^{+1.82}_{-1.26}$	This work

The parameter h has been estimated with the light-cone QCD sum rules [30], the quark models [35], Adler-Weisberger type sum rules [39], and extracted from the experimental data [40]. The values are listed in Table II. From those values we can estimate the values of the corresponding strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ in the $SU(3)$ limit for the light pseudoscalar mesons. The value of the dimensionless effective coupling constant $\Gamma/k = 0.46(9)$ from Lattice QCD [42] is somewhat smaller than the values extracted from the experimental data $\Gamma/k = 0.73^{+28}_{-24}$. Here the Γ is the decay width and the k is the decay momentum. Our numerical values $G_{D_0D_sK} = 6.5^{+1.8}_{-1.5}$ GeV and $G_{D_{s0}DK} = 5.9^{+1.7}_{-1.6}$ GeV are compatible with the existing estimations in Refs. [30,35,39,40], although they are somewhat smaller

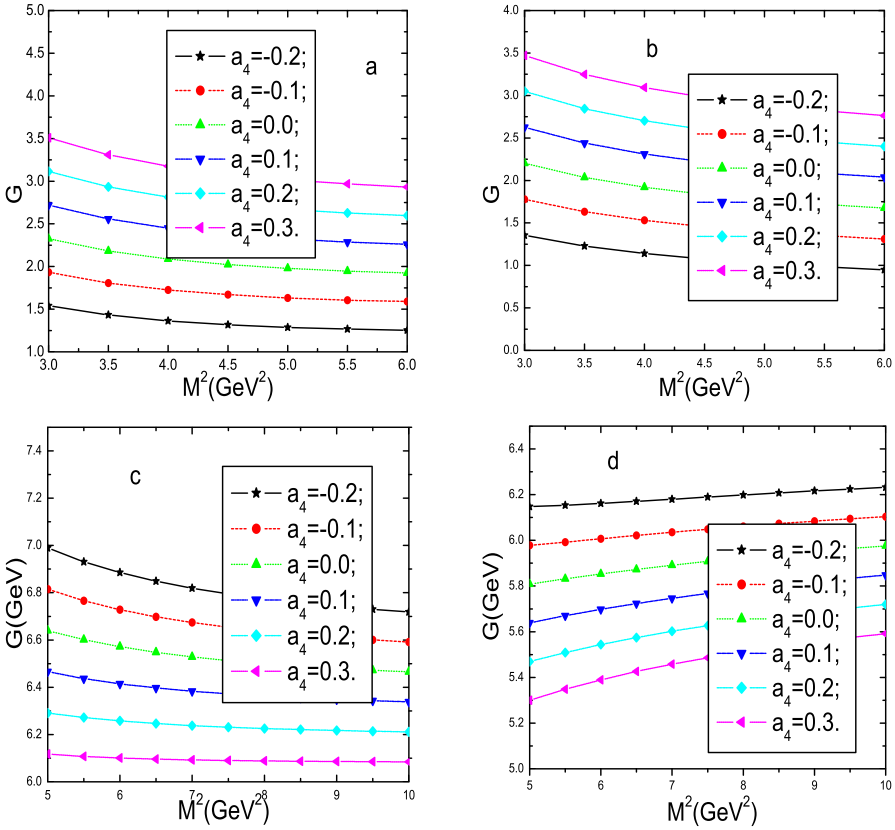

FIG. 11 (color online). The (a) $G_{D^*D_sK}$, (b) $G_{D_s^*DK}$, (c) $G_{D_0D_sK}$, (d) $G_{D_{s0}DK}$ with the parameters M^2 and a_4 from Eq. (10)–(13), respectively.

compared with the values obtained in Ref. [17] with the scalar interpolating current for the D_{s0} meson, and they are about 2–3 times as large as the energy scale $M_{D_{s0}} = 2.317$ GeV and favor the hadronic dressing mechanism. For a short discussion about the hadronic dressing mechanism, one can consult Ref. [17], or one can consult the original literatures for the details [6,7].

The large values of the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ obviously support the hadronic dressing mechanism, the D_0 and D_{s0} (just like the scalar mesons $f_0(980)$ and $a_0(980)$, see Ref. [16]) can be taken as having small scalar $c\bar{u}$ and $c\bar{s}$ kernels of typical meson size with large virtual S-wave D_sK and DK cloud, respectively. In Ref. [41], the authors analyze the unitarized two-meson scattering amplitudes from the heavy-light chiral Lagrangian and observe that the scalar meson D_{s0} appears as the bound state pole with the strong coupling constant $G_{D_{s0}DK} = 10.203$ GeV. Our numerical results $G_{D_{s0}DK} = 5.9^{+1.7}_{-1.6}$ GeV are smaller; the values of our previous work $G_{D_{s0}DK} = 9.3^{+2.7}_{-2.1}$ GeV with the scalar interpolating current are more satisfactory [17].

TABLE II. Numerical values of the parameter h and the corresponding values of the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ in the $SU(3)$ limit.

$\lvert h\rvert$	$G_{D_0D_sK}$(GeV)	$G_{D_{s0}DK}$(GeV)	Reference
$0.88^{+0.26}_{-0.20}$	$9.4^{+2.8}_{-2.1}$	$9.3^{+2.7}_{-2.1}$	[17]
		10.203	[41]
0.536	5.7	5.68	[35]
0.52 ± 0.17	5.5 ± 1.8	5.5 ± 1.8	[30]
<0.93	<9.9	<9.86	[39]
$0.57 - 0.74$	$6.1 - 7.9$	$6.0 - 7.8$	[40]
$0.61^{+0.17}_{-0.14}$ (or $0.56^{+0.16}_{-0.15}$)	$6.5^{+1.8}_{-1.5}$	$5.9^{+1.7}_{-1.6}$	This work

IV. CONCLUSIONS

We have analyzed the vertices D^*D_sK, D_s^*DK, D_0D_sK, and $D_{s0}DK$ within the framework of the light-cone QCD sum rules approach in an unified way. The strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are important parameters in evaluating the charmonium absorption cross sections in searching for the quark-gluon plasmas. Our numerical values of the $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are compatible with the existing estimations although somewhat smaller—the $SU(4)$ symmetry breaking effects are very large, about 60%, the approximation of the $SU(4)$ symmetry $G_{D^*D_sK} = G_{D_s^*DK} = 5.0$ is not suitable [3]. For the scalar mesons D_0 and D_{s0}, we take the point of view that they are the conventional $c\bar{u}$ and $c\bar{s}$ meson, respectively, and calculate the strong coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. The numerical values of the scalar-D_sK and scalar-DK coupling constants $G_{D_0D_sK}$ and $G_{D_{s0}DK}$ are compatible with the existing estimations although somewhat smaller; the large values support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$ and $a_0(980)$, the scalar mesons D_0 and D_{s0} may have small $c\bar{u}$ and $c\bar{s}$ kernels of typical $c\bar{u}$ and $c\bar{s}$ mesons size, respectively. The strong coupling to virtual intermediate hadronic states (or the virtual mesons loops) can result in smaller mass than the conventional scalar mesons $c\bar{u}$ and $c\bar{s}$ in the constituent quark models and enrich the pure states $c\bar{u}$ and $c\bar{s}$ with other components. The D_0 and D_{s0} may spend part (or most part) of their lifetime as virtual D_sK and DK states. Furthermore, we studied the dependence of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ on the nonperturbative parameter a_4 of the twist-2 K meson light-cone distribution amplitude. The values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are more sensitive to the a_4 compared with the $G_{D_0D_sK}$ and $G_{D_{s0}DK}$. The largest uncertainties come from the uncertainties of the a_4; they are the ideal channels to determine the parameter directly from the experimental data. Once the experimental data for the values of the strong coupling constants $G_{D^*D_sK}$ and $G_{D_s^*DK}$ are available, powerful constraints can be put on the range of the parameter a_4.

ACKNOWLEDGMENTS

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APPENDIX

The explicit expressions of the correlation functions Π_μ^1 and Π_μ^2 in the hadronic representation,

$$\begin{aligned}
\Pi_\mu^1 &= \frac{\langle 0 \mid J_\mu^D \mid D^*(q+P)\rangle\langle D^*D_s \mid K\rangle\langle D_s(q)|J_{D_s} \mid 0\rangle}{\{m_{D^*}^2-(q+P)^2\}(m_{D_s}^2-q^2)} + \frac{\langle 0 \mid J_\mu^D \mid D_0(q+P)\rangle\langle D_0D_s \mid K\rangle\langle D_s(q)|J_{D_s} \mid 0\rangle}{\{m_{D_0}^2-(q+P)^2\}(m_{D_s}^2-q^2)} + \cdots \\
&= \frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2(P-q)\cdot\epsilon\epsilon_\mu}{(m_c+m_s)\{m_{D^*}^2-(q+P)^2\}(m_{D_s}^2-q^2)} + \frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2(q+P)_\mu}{(m_c+m_s)\{m_{D_0}^2-(q+P)^2\}(m_{D_s}^2-q^2)} + \cdots \\
&= \left\{\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{(m_c+m_s)\{m_{D^*}^2-(q+P)^2\}(m_{D_s}^2-q^2)}\frac{m_{D^*}^2-m_{D_s}^2+m_K^2}{m_{D^*}^2} + \frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{(m_c+m_s)\{m_{D_0}^2-(q+P)^2\}(m_{D_s}^2-q^2)}\right\}q_\mu \\
&\quad + \left\{-\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{(m_c+m_s)\{m_{D^*}^2-(q+P)^2\}(m_{D_s}^2-q^2)}\frac{m_{D^*}^2+m_{D_s}^2-m_K^2}{m_{D^*}^2} + \frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{(m_c+m_s)\{m_{D_0}^2-(q+P)^2\}(m_{D_s}^2-q^2)}\right\}P_\mu \\
&\quad + \cdots,
\end{aligned} \tag{A1}$$

$$\begin{aligned}
\Pi_\mu^2 &= \frac{\langle 0 \mid J_\mu^{D_s} \mid D_s^*(q+P)\rangle\langle D_s^*D \mid K\rangle\langle D(q)|J_D \mid 0\rangle}{\{m_{D_s^*}^2-(q+P)^2\}(m_D^2-q^2)} + \frac{\langle 0 \mid J_\mu^{D_s} \mid D_{s0}(q+P)\rangle\langle D_{s0}D \mid K\rangle\langle D(q)|J_D \mid 0\rangle}{\{m_{D_{s0}}^2-(q+P)^2\}(m_D^2-q^2)} + \cdots \\
&= \frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2(P-q)\cdot\epsilon\epsilon_\mu}{(m_c+m_u)\{m_{D_s^*}^2-(q+P)^2\}(m_D^2-q^2)} + \frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2(q+P)_\mu}{(m_c+m_u)\{m_{D_{s0}}^2-(q+P)^2\}(m_D^2-q^2)} + \cdots \\
&= \left\{\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{(m_c+m_u)\{m_{D_s^*}^2-(q+P)^2\}(m_D^2-q^2)}\frac{m_{D_s^*}^2-m_D^2+m_K^2}{m_{D_s^*}^2} + \frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{(m_c+m_u)\{m_{D_{s0}}^2-(q+P)^2\}(m_D^2-q^2)}\right\}q_\mu \\
&\quad + \left\{-\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{(m_c+m_u)\{m_{D_s^*}^2-(q+P)^2\}(m_D^2-q^2)}\frac{m_{D_s^*}^2+m_D^2-m_K^2}{m_{D_s^*}^2} + \frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{(m_c+m_u)\{m_{D_{s0}}^2-(q+P)^2\}(m_D^2-q^2)}\right\}P_\mu \\
&\quad + \cdots.
\end{aligned} \tag{A2}$$

The explicit expressions of the correlation functions Π_μ^1 and Π_μ^2 at the level of quark-gluon degrees of freedom,

$$\Pi^1_\mu = q_\mu\bigg\{-\frac{f_K m_K^2}{m_s}\int_0^1 du\frac{1}{(q+uP)^2-m_c^2}\bigg[\phi_p(u)-\frac{d}{6du}\phi_\sigma(u)\bigg]+m_c f_K m_K^2\int_0^1 du\int_0^u dt\frac{B(t)}{\{(q+uP)^2-m_c^2\}^2}$$
$$+f_{3K}m_K^2\int_0^1 dv\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s\frac{(2v-3)T(\alpha_u,\alpha_g,\alpha_s)}{\{[q+((1-v)\alpha_g+\alpha_s)P]^2-m_c^2\}^2}$$
$$-4m_c f_K m_K^4\int_0^1 dv v\int_0^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{\Phi(1-\alpha-\beta,\beta,\alpha)}{\{[q+(1-v\alpha_g)P]^2-m_c^2\}^3}$$
$$+4m_c f_K m_K^4\int_0^1 dv\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\{[q+((1-v)\alpha_g+\alpha_s)P]^2-m_c^2\}^3}\bigg\} \tag{A3}$$

$$+P_\mu\bigg\{-\frac{f_K m_K^2}{m_s}\int_0^1 du\frac{u\phi_p(u)}{(q+uP)^2-m_c^2}+m_c f_K m_K^2\int_0^1 du\int_0^u dt\frac{uB(t)}{\{(q+uP)^2-m_c^2\}^2}+\frac{f_K m_K^2}{6m_s}\int_0^1\phi_\sigma(u)\bigg\{\bigg[1-u\frac{d}{du}\bigg]$$
$$\times\frac{1}{(q+uP)^2-m_c^2}-\frac{2m_c^2}{[(q+uP)^2-m_c^2]^2}\bigg\}+m_c f_K\int_0^1 du\bigg\{\frac{\phi_K}{(q+uP)^2-m_c^2}-\frac{m_K^2m_c^2}{2}\frac{A(u)}{[(q+uP)^2-m_c^2]^3}\bigg\}$$
$$+f_{3K}m_K^2\int_0^1 dv\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s\frac{[(1-v)\alpha_g+\alpha_s](2v-3)T(\alpha_u,\alpha_g,\alpha_s)}{\{[q+((1-v)\alpha_g+\alpha_s)P]^2-m_c^2\}^2}$$
$$-2f_{3K}\int_0^1 dv v\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s T(\alpha_u,\alpha_g,\alpha_s)\frac{d}{du}\frac{1}{(q+uP)^2-m_c^2}\bigg|_{u=(1-v)\alpha_g+\alpha_s}$$
$$-4m_c f_K m_K^4\int_0^1 dv v\int_0^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-v\alpha_g)\Phi(1-\alpha-\beta,\beta,\alpha)}{\{[q+(1-v\alpha_g)P]^2-m_c^2\}^3}$$
$$+4m_c f_K m_K^4\int_0^1 dv\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\frac{((1-v)\alpha_g+\alpha_s)\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\{[q+((1-v)\alpha_g+\alpha_s)P]^2-m_c^2\}^3}$$
$$-m_c f_K m_K^2\int_0^1 dv\int_0^1 d\alpha_g\int_0^{1-\alpha_g}d\alpha_s\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\{[q+((1-v)\alpha_g+\alpha_s)P]^2-m_c^2\}^2}\bigg\},$$

$$\Pi^2_\mu = \Pi^1_\mu(u\leftrightarrow 1-u;\alpha_u\leftrightarrow\alpha_s). \tag{A4}$$

The light-cone distribution amplitudes of the K meson,

$$\langle 0|\bar u(0)\gamma_\mu\gamma_5 s(x)|K(P)\rangle = if_K P_\mu\int_0^1 due^{-iuP\cdot x}\bigg\{\varphi_K(u)+\frac{m_K^2x^2}{16}A(u)\bigg\}+f_K m_K^2\frac{ix_\mu}{2P\cdot x}\int_0^1 due^{-iuP\cdot x}B(u),$$
$$\langle 0|\bar u(0)i\gamma_5 s(x)|K(P)\rangle = \frac{f_K M_K^2}{m_s}\int_0^1 due^{-iuP\cdot x}\varphi_p(u),$$
$$\langle 0|\bar u(0)\sigma_{\mu\nu}\gamma_5 s(x)|K(P)\rangle = i(P_\mu x_\nu-P_\nu x_\mu)\frac{f_K M_K^2}{6m_s}\int_0^1 due^{-iuP\cdot x}\varphi_\sigma(u),$$
$$\langle 0|\bar u(0)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(x)|K(P)\rangle = f_{3K}\{(P_\mu P_\alpha g^\perp_{\nu\beta}-P_\nu P_\alpha g^\perp_{\mu\beta})-(P_\mu P_\beta g^\perp_{\nu\alpha}-P_\nu P_\beta g^\perp_{\mu\alpha})\}\times\int\mathcal{D}\alpha_i\varphi_{3K}(\alpha_i)e^{-iP\cdot x(\alpha_s+v\alpha_g)},$$
$$\langle 0|\bar u(0)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(x)|K(P)\rangle = P_\mu\frac{P_\alpha x_\beta-P_\beta x_\alpha}{P\cdot x}f_K m_K^2\int\mathcal{D}\alpha_i A_\parallel(\alpha_i)e^{-iP\cdot x(\alpha_s+v\alpha_g)}+f_K m_K^2(P_\beta g_{\alpha\mu}-P_\alpha g_{\beta\mu})\times\int\mathcal{D}\alpha_i A_\perp(\alpha_i)e^{-iP\cdot x(\alpha_s+v\alpha_g)},$$
$$\langle 0|\bar u(0)\gamma_\mu g_s\tilde G_{\alpha\beta}(vx)s(x)|K(P)\rangle = P_\mu\frac{P_\alpha x_\beta-P_\beta x_\alpha}{P\cdot x}f_K m_K^2\int\mathcal{D}\alpha_i V_\parallel(\alpha_i)e^{-iP\cdot x(\alpha_s+v\alpha_g)}+f_K m_K^2(P_\beta g_{\alpha\mu}-P_\alpha g_{\beta\mu})\times\int\mathcal{D}\alpha_i V_\perp(\alpha_i)e^{-iP\cdot x(\alpha_s+v\alpha_g)}, \tag{A5}$$

where the operator $\tilde G_{\alpha\beta}$ is the dual of the $G_{\alpha\beta}$, $\tilde G_{\alpha\beta}=\frac{1}{2}\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$, $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$, $\Phi(\alpha_1,\alpha_2,\alpha_3)=A_\perp+V_\perp+A_\parallel+V_\parallel$, and $\Psi(\alpha_1,\alpha_2,\alpha_3)=A_\parallel+V_\parallel-2A_\perp-2V_\perp$. The light-cone distribution amplitudes are parameterized as

$$\begin{aligned}
\phi_K(u,\mu) &= 6u(1-u)\{1+a_1C_1^{3/2}(2u-1)+a_2C_2^{3/2}(2u-1)+a_4C_4^{3/2}(2u-1)\},\\
\varphi_p(u,\mu) &= 1+\left\{30\eta_3-\frac{5}{2}\rho^2\right\}C_2^{1/2}(2u-1)+\left\{-3\eta_3\omega_3-\frac{27}{20}\rho^2-\frac{81}{10}\rho^2a_2\right\}C_4^{1/2}(2u-1),\\
\varphi_\sigma(u,\mu) &= 6u(1-u)\left\{1+\left[5\eta_3-\frac{1}{2}\eta_3\omega_3-\frac{7}{20}\rho^2-\frac{3}{5}\rho^2a_2\right]C_2^{3/2}(2u-1)\right\},\\
T(\alpha_i,\mu) &= 360\alpha_u\alpha_s\alpha_g^2\left\{1+\lambda_3(\alpha_u-\alpha_s)+\omega_3\frac{1}{2}(7\alpha_g-3)\right\},\\
V_\parallel(\alpha_i,\mu) &= 120\alpha_u\alpha_s\alpha_g(v_{00}+v_{10}(3\alpha_g-1)),\\
A_\parallel(\alpha_i,\mu) &= 120\alpha_u\alpha_s\alpha_g a_{10}(\alpha_s-\alpha_u),\\
V_\perp(\alpha_i,\mu) &= -30\alpha_g^2\left\{h_{00}(1-\alpha_g)+h_{01}[\alpha_g(1-\alpha_g)-6\alpha_u\alpha_s]+h_{10}\left[\alpha_g(1-\alpha_g)-\frac{3}{2}(\alpha_u^2+\alpha_s^2)\right]\right\},\\
A_\perp(\alpha_i,\mu) &= 30\alpha_g^2(\alpha_u-\alpha_s)\left\{h_{00}+h_{01}\alpha_g+\frac{1}{2}h_{10}(5\alpha_g-3)\right\},\\
A(u,\mu) &= 6u(1-u)\left\{\frac{16}{15}+\frac{24}{35}a_2+20\eta_3+\frac{20}{9}\eta_4+\left[-\frac{1}{15}+\frac{1}{16}-\frac{7}{27}\eta_3\omega_3-\frac{10}{27}\eta_4\right]C_2^{3/2}(2u-1)\right.\\
&\quad\left.+\left[-\frac{11}{210}a_2-\frac{4}{135}\eta_3\omega_3\right]C_4^{3/2}(2u-1)\right\}+\left\{-\frac{18}{5}a_2+21\eta_4\omega_4\right\}\{2u^3(10-15u+6u^2)\log u\\
&\quad+2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\log\bar{u}+u\bar{u}(2+13u\bar{u})\},\\
g_K(u,\mu) &= 1+g_2C_2^{1/2}(2u-1)+g_4C_4^{1/2}(2u-1),\\
B(u,\mu) &= g_K(u,\mu)-\phi_K(u,\mu),
\end{aligned} \tag{A6}$$

where

$$\begin{aligned}
&h_{00}=v_{00}=-\frac{\eta_4}{3},\qquad a_{10}=\frac{21}{8}\eta_4\omega_4-\frac{9}{20}a_2,\qquad v_{10}=\frac{21}{8}\eta_4\omega_4,\qquad h_{01}=\frac{7}{4}\eta_4\omega_4-\frac{3}{20}a_2,\\
&h_{10}=\frac{7}{2}\eta_4\omega_4+\frac{3}{20}a_2,\qquad g_2=1+\frac{18}{7}a_2+60\eta_3+\frac{20}{3}\eta_4,\qquad g_4=-\frac{9}{28}a_2-6\eta_3\omega_3,
\end{aligned} \tag{A7}$$

where $C_2^{1/2}$, $C_4^{1/2}$, and $C_2^{3/2}$ are Gegenbauer polynomials, $\eta_3=\frac{f_{3K}}{f_K}\frac{m_q+m_s}{M_K^2}$ and $\rho^2=\frac{m_s^2}{M_K^2}$ [8–12].

The explicit expressions of the Borel transformed correlation functions $B_M\Pi_\mu^1$ and $B_M\Pi_\mu^2$ in the hadronic representation,

$$\begin{aligned}
B_M\Pi_\mu^1 &= \left\{\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{m_c+m_s}\frac{m_{D^*}^2-m_{D_s}^2+m_K^2}{m_{D^*}^2}\exp\left\{-\frac{m_{D^*}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}+\frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{m_c+m_s}\exp\left\{-\frac{m_{D_0}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}\right\}q_\mu\\
&\quad+\left\{-\frac{G_{D^*D_sK}m_{D^*}f_{D^*}f_{D_s}m_{D_s}^2}{m_c+m_s}\frac{m_{D^*}^2+m_{D_s}^2-m_K^2}{m_{D^*}^2}\exp\left\{-\frac{m_{D^*}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}\right.\\
&\quad\left.+\frac{G_{D_0D_sK}f_{D_0}f_{D_s}m_{D_s}^2}{m_c+m_s}\exp\left\{-\frac{m_{D_0}^2}{M_1^2}-\frac{m_{D_s}^2}{M_2^2}\right\}\right\}P_\mu+\cdots,
\end{aligned} \tag{A8}$$

$$\begin{aligned}
B_M\Pi_\mu^2 &= \left\{\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{m_c+m_u}\frac{m_{D_s^*}^2-m_D^2+m_K^2}{m_{D_s^*}^2}\exp\left\{-\frac{m_{D_s^*}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}+\frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{m_c+m_u}\exp\left\{-\frac{m_{D_{s0}}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}\right\}q_\mu\\
&\quad+\left\{-\frac{G_{D_s^*DK}m_{D_s^*}f_{D_s^*}f_Dm_D^2}{m_c+m_u}\frac{m_{D_s^*}^2+m_D^2-m_K^2}{m_{D_s^*}^2}\exp\left\{-\frac{m_{D_s^*}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}\right.\\
&\quad\left.+\frac{G_{D_{s0}DK}f_{D_{s0}}f_Dm_D^2}{m_c+m_u}\exp\left\{-\frac{m_{D_{s0}}^2}{M_1^2}-\frac{m_D^2}{M_2^2}\right\}\right\}P_\mu+\cdots.
\end{aligned} \tag{A9}$$

Here we have not shown the contributions from the high resonances and continuum states explicitly for simplicity.

The explicit expressions of the Borel transformed correlation functions $B_M\Pi_\mu^1$ and $B_M\Pi_\mu^2$ at the level of quark-gluon degrees of freedom,

$$\begin{aligned}
B_M\Pi_\mu^1 = {}& q_\mu \exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}\left[\phi_p(u_0)-\frac{d}{6du_0}\phi_\sigma(u_0)\right]+m_c f_K m_K^2\int_0^{u_0}dtB(t)\right.\\
&-f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{[2(\alpha_s-u_0)-\alpha_g]T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g^2}\\
&-\frac{2m_c f_K m_K^4}{M^2}\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2}\\
&\left.+\frac{2m_c f_K m_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha+\int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\right\}\\
&+P_\mu\exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}u_0\phi_p(u_0)+m_c f_K m_K^2 u_0\int_0^{u_0}dtB(t)\right.\\
&-\frac{f_K m_K^2}{6m_s}\left[\phi_\sigma(u_0)M^2+\frac{d}{du_0}u_0\phi_\sigma(u_0)M^2+2m_c^2\phi_\sigma(u_0)\right]+m_c f_K\left[-\phi_K(u_0)M^2+\frac{m_K^2 m_c^2 A(u_0)}{4M^2}\right]\\
&+3u_0 f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\\
&+2f_{3K}\left[M^2\frac{d}{du_0}-u_0m_K^2\right]\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\alpha_g+\alpha_s-u_0}{\alpha_g^2}T(\alpha_u,\alpha_g,\alpha_s)\\
&-\frac{2u_0 m_c f_K m_K^4}{M^2}\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2}\\
&+\frac{2u_0 m_c f_K m_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha+\int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\\
&\left.+m_c f_K m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\right\},
\end{aligned}\tag{A10}$$

$$B_M\Pi_\mu^2 = B_M\Pi_\mu^1(u\leftrightarrow 1-u;\alpha_s\leftrightarrow\alpha_u).\tag{A11}$$

Here $u_0=\frac{M_1^2}{M_1^2+M_2^2}$, $M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2}$.

The explicit expressions of the notations AA, BB, CC, DD, EE, and FF,

$$\begin{aligned}
AA = {}&\left\{\exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\}-\exp\left\{-\frac{s_{D^*}^0}{M^2}\right\}\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}u_0\phi_p(u_0)-m_c f_K\phi_K(u_0)M^2\right.\\
&\left.-\frac{f_K m_K^2}{6m_s}\left[\phi_\sigma(u_0)M^2+\frac{d}{du_0}u_0\phi_\sigma(u_0)M^2+2m_c^2\phi_\sigma(u_0)\right]\right\}+\exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\}\left\{m_c f_K m_K^2 u_0\int_0^{u_0}dtB(t)\right.\\
&+\frac{f_K m_K^2 m_c^3 A(u_0)}{4M^2}+3u_0 f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\\
&+2f_{3K}\left[M^2\frac{d}{du_0}-u_0m_K^2\right]\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\alpha_g+\alpha_s-u_0}{\alpha_g^2}T(\alpha_u,\alpha_g,\alpha_s)-\frac{2u_0 m_c f_K m_K^4}{M^2}\\
&\times\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2}+\frac{2u_0 m_c f_K m_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha\right.\\
&\left.\left.+\int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}+m_c f_K m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\right\};
\end{aligned}\tag{A12}$$

$$BB = \left\{\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]-\exp\left[-\frac{s_{D_s^*}^0}{M^2}\right]\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}u_0\phi_p(u_0)-m_c f_K\phi_K(u_0)M^2-\frac{f_K m_K^2}{6m_s}\left[\phi_\sigma(u_0)M^2 + \frac{d}{du_0}u_0\phi_\sigma(u_0)M^2+2m_c^2\phi_\sigma(u_0)\right]\right\}+\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]\left\{m_c f_K m_K^2 u_0\int_0^{u_0}dtB(t)+\frac{f_K m_K^2 m_c^3 A(u_0)}{4M^2} + 3u_0 f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}+2f_{3K}\left[M^2\frac{d}{du_0}-u_0m_K^2\right] \times\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\alpha_g+\alpha_s-u_0}{\alpha_g^2}T(\alpha_u,\alpha_g,\alpha_s)-\frac{2u_0m_cf_Km_K^4}{M^2} \times\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2}+\frac{2u_0m_cf_Km_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha + \int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}+m_cf_Km_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\right\}\Bigg|_{\alpha_s\leftrightarrow\alpha_u}; \tag{A13}$$

$$CC = \left\{\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]-\exp\left[-\frac{s_{D_0}^0}{M^2}\right]\right\}\frac{f_K m_K^2 M^2}{m_s}\left\{\phi_p(u_0)-\frac{d}{6du_0}\phi_\sigma(u_0)\right\} + \exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]\left\{m_cf_Km_K^2\int_0^{u_0}dtB(t)-f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{[2(\alpha_s-u_0)-\alpha_g]T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g^2} -\frac{2m_cf_Km_K^4}{M^2}\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2} +\frac{2m_cf_Km_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha+\int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\right\}; \tag{A14}$$

$$DD = \left\{\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]-\exp\left[-\frac{s_{D_{s0}}^0}{M^2}\right]\right\}\frac{f_K m_K^2 M^2}{m_s}\left\{\phi_p(u_0)-\frac{d}{6du_0}\phi_\sigma(u_0)\right\} + \exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]\left\{m_cf_Km_K^2\int_0^{u_0}dtB(t)-f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{[2(\alpha_s-u_0)-\alpha_g]T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g^2} -\frac{2m_cf_Km_K^4}{M^2}\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2} +\frac{2m_cf_Km_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha+\int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\right\}\Bigg|_{\alpha_s\leftrightarrow\alpha_u}; \tag{A15}$$

$$EE = \left\{\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]-\exp\left[-\frac{s_{D_0}^0}{M^2}\right]\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}u_0\phi_p(u_0)-m_c f_K\phi_K(u_0)M^2 -\frac{f_K m_K^2}{6m_s}\left[\phi_\sigma(u_0)M^2 + \frac{d}{du_0}u_0\phi_\sigma(u_0)M^2+2m_c^2\phi_\sigma(u_0)\right]\right\}+\exp\left[-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right]\left\{m_c f_K m_K^2 u_0\int_0^{u_0}dtB(t) +\frac{f_K m_K^2 m_c^3 A(u_0)}{4M^2} + 3u_0 f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g} +2f_{3K}\left[M^2\frac{d}{du_0}-u_0m_K^2\right]\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\alpha_g+\alpha_s-u_0}{\alpha_g^2}T(\alpha_u,\alpha_g,\alpha_s)-\frac{2u_0m_cf_Km_K^4}{M^2} \times\int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2}+\frac{2u_0m_cf_Km_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha + \int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}+m_cf_Km_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\right\}; \tag{A16}$$

$$FF = \left\{\exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\} - \exp\left\{-\frac{s^0_{D_{s0}}}{M^2}\right\}\right\}\left\{\frac{f_K m_K^2 M^2}{m_s}u_0\phi_p(u_0) - m_c f_K \phi_K(u_0)M^2 - \frac{f_K m_K^2}{6m_s}\left[\phi_\sigma(u_0)M^2 + \frac{d}{du_0}u_0\phi_\sigma(u_0)M^2 + 2m_c^2\phi_\sigma(u_0)\right]\right\} + \exp\left\{-\frac{u_0(1-u_0)m_K^2+m_c^2}{M^2}\right\}\left\{m_c f_K m_K^2 u_0\int_0^{u_0}dtB(t) + \frac{f_K m_K^2 m_c^3 A(u_0)}{4M^2} + 3u_0 f_{3K}m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{T(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g} + 2f_{3K}\left[M^2\frac{d}{du_0} - u_0 m_K^2\right] \times \int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\alpha_g+\alpha_s-u_0}{\alpha_g^2}T(\alpha_u,\alpha_g,\alpha_s) - \frac{2u_0 m_c f_K m_K^4}{M^2} \times \int_{1-u_0}^1 d\alpha_g\int_0^{\alpha_g}d\beta\int_0^{1-\beta}d\alpha\frac{(1-u_0)\Phi(1-\alpha-\beta,\beta,\alpha)}{\alpha_g^2} + \frac{2u_0 m_c f_K m_K^4}{M^2}\left[\int_0^{1-u_0}d\alpha_g\int_{u_0-\alpha_g}^{u_0}d\alpha_s\int_0^{\alpha_s}d\alpha + \int_{1-u_0}^1 d\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}d\alpha_s\int_0^{\alpha_s}d\alpha\right]\frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g} + m_c f_K m_K^2\int_0^{u_0}d\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\frac{\Psi(\alpha_u,\alpha_g,\alpha_s)}{\alpha_g}\right\}\Bigg|_{\alpha_s\leftrightarrow\alpha_u}. \tag{A17}$$

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